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Chapter 5
Two Dimensional Analytical Geometry-II

Equations of tangent and normal at a point P on a given conic

Tangent of a conic is a line which touches the conic at only one point and normal is a line perpendicular to the tangent and passing through the point of contact.

NOTE:

To find the equation of the tangent at the point (x_1 , y_1), we just replace in the equation of the given conic as follows:

x^2 is replaced by xx_1

y^2 is replaced by yy_1

x is replaced by $\frac{1}{2}(x + x_1)$

y is replaced by $\frac{1}{2}(y + y_1)$

xy is replaced by $\frac{1}{2}(xy_1 + yx_1)$.

So we have

(1) If the circle is $x^2 + y^2 + 2gx + 2fy + c=0$, then the equation of the tangent at (x_1,y_1) is

$$xx_1 + yy_1 + 2g \times \frac{1}{2}(x + x_1) + 2f \times \frac{1}{2}(y + y_1) + c = 0$$
$$\boldsymbol{xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0}$$

(2) If the circle is $x^2 + y^2 = r^2$, then the equation of the tangent at (x_1,y_1) is

$$\boldsymbol{xx_1 + yy_1 = r^2}$$

(3) If the parabola is $y^2 = 4ax$, then the equation of the tangent at (x_1,y_1) is

$$yy_1 = 4a \times \frac{1}{2}(x + x_1)$$
$$\boldsymbol{yy_1 = 2a(x + x_1)}$$

(4) If the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then the equation of the tangent at (x_1,y_1) is

$$\boldsymbol{\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1}$$

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(4) If the hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then the equation of the tangent at (x_1,y_1) is

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$$

(5) If the rectangular hyperbola is $xy = c^2$, then the equation of the tangent at (x_1,y_1) is

$$\frac{1}{2}(xy_1 + yx_1) = c^2$$

$$xy_1 + yx_1 = 2c^2$$

Example 5.11

Find the equations of the tangent and normal to the circle $x^2 + y^2 = 25$ at $P(-3, 4)$.

Solution:

Given equation of the circle is $x^2 + y^2 = 25$

The equation of the tangent at (x_1,y_1) is $xx_1 + yy_1=25$

[Since x^2 is replaced by xx_1 and y^2 is replaced by yy_1]

∴ The equation of the tangent at $P(x_1,y_1)= (-3, 4)$ is

$$x(-3) + y(4)=25$$

$$-3x + 4y=25$$

$$3x - 4y - 25=0$$

The equation of the normal is of the form

$$3x + 4y=k \text{ ------(1)}$$

It passes through the point $P(-3, 4)$

$$\therefore (1) \Rightarrow 3(-3) + 4(4) = k$$

$$k= -9 + 16$$

$$k=7$$

∴ The equation of the normal is

$$(1) \Rightarrow 3x + 4y=7$$

(i.e) $3x + 4y - 7 = 0$

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EXERCISE 5.1

9. Find the equation of the tangent and normal to the circle $x^2 + y^2 - 6x + 6y - 8=0$ at (2,2).

Solution:

Given equation of the circle is $x^2 + y^2 - 6x + 6y - 8=0$

The equation of the tangent at (x_1,y_1) is

$$xx_1 + yy_1 - 6\times\frac{1}{2}(x+x_1) + 6\times\frac{1}{2}(y+y_1) - 8 = 0$$

[Since x^2 is replaced by xx_1

y^2 is replaced by yy_1

x is replaced by $\frac{1}{2}(x+x_1)$

y is replaced by $\frac{1}{2}(y+y_1)$]

(ie) $xx_1 + yy_1 - 3(x+x_1) + 3(y+y_1) - 8 = 0$

∴ The equation of the tangent at (x_1,y_1)= (2, 2) is

$$x(2) + y(2) - 3(x+2) + 3(y+2) - 8 = 0$$
$$2x + 2y - 3x - 6 + 3y + 6 - 8 = 0$$
$$-x + 5y - 8 = 0$$
$$\mathbf{x - 5y + 8 = 0}$$

The equation of the normal is of the form

5x + y=k ------(1)

It passes through the point (2, 2)

∴**(1)**⇒ 5(2) + 2 = k

k= 10+2

k=12

∴ The equation of the normal is

(1)⇒ $5x + y$=12

(i.e) $5x + y - 12 = 0$

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Example 5.27

Find the equations of tangent and normal to the parabola $x^2 + 6x + 4y + 5 = 0$ at (1, –3).

Solution:

Given equation of the parabola is $x^2 + 6x + 4y + 5 = 0$

The equation of the tangent at (x_1, y_1) is

$$xx_1 + 6 \times \frac{1}{2}(x + x_1) + 4 \times \frac{1}{2}(y + y_1) + 5 = 0$$

[Since x^2 is replaced by xx_1

x is replaced by $\frac{1}{2}(x + x_1)$

y is replaced by $\frac{1}{2}(y + y_1)$]

(ie) $\mathbf{xx_1 + 3(x + x_1) + 2(y + y_1) + 5 = 0}$

∴ The equation of the tangent at $(x_1, y_1) = (1, -3)$ is

$$x(1) + 3(x + 1) + 2(y - 3) + 5 = 0$$

$$x + 3x + 3 + 2y - 6 + 5 = 0$$

$$4x + 2y + 2 = 0$$

$$\mathbf{2x + y + 1 = 0}$$

The equation of the normal is of the form

$$\mathbf{x - 2y = k}\text{------(1)}$$

It passes through the point $(1, -3)$

$$\therefore (1) \Rightarrow 1 - 2(-3) = k$$

$$k = 1 + 6$$

$$\mathbf{k = 7}$$

∴ The equation of the normal is

$$(1) \Rightarrow x - 2y = 7$$

(i.e) $\mathbf{x - 2y - 7 = 0}$

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NOTE:

5.5 The parametric forms of the conic

(1) For the circle $x^2 + y^2 = r^2$, the parametric point is $(x_1,y_1)=(r\cos\theta , r\sin\theta)$, where θ is the parameter.

(2) For the parabola $y^2=4ax$, the parametric point is $(x_1,y_1)=(at^2 , 2at)$, where t is the parameter.

(3) For the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, the parametric point is $(x_1,y_1)=(a\cos\theta , b\sin\theta)$, where θ is the parameter.

(4) For the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, the parametric point is $(x_1,y_1)=(a\sec\theta , b\tan\theta)$, where θ is the parameter.

EXERCISE 5.4

5. Find the equation of the tangent at $t = 2$ to the parabola $y^2=8x$.

Solution:

Given equation of the parabola is $y^2 =8x$ --------(1)

Comparing this with $y^2=4ax$

we have $4a=8 \Rightarrow a=2$

At $t=2$, the parametric point is $(x_1,y_1)= (at^2, 2at)$

$=(2\times4, 2\times 2 \times 2)$

$(x_1,y_1)=(8,8)$

The equation of the tangent at (x_1,y_1) is

$$(1)\Rightarrow yy_1 = 8 \times \frac{1}{2}(x+x_1)$$

[Since y^2 is replaced by yy_1 and x is replaced by $\frac{1}{2}(x+x_1)$]

$$\text{(ie) } yy_1 = 4(x+x_1)$$

∴ The equation of the tangent at $(x_1,y_1)=(8,8)$ is

$$y(8) = 4(x+8)$$

$$2y = x + 8$$

$$x - 2y + 8 = 0$$

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Example 5.28

Find the equations of tangent and normal to the ellipse $x^2 + 4y^2 = 32$ when $\theta = \frac{\pi}{4}$.

Solution:

Given equation of the ellipse $x^2 + 4y^2 = 32$ --------(1)

$$\Rightarrow \frac{x^2}{32} + \frac{y^2}{8} = 1$$

Comparing this with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

We have $a^2 = 32$ and $b^2 = 8 \Rightarrow a = 4\sqrt{2}$ and $b = 2\sqrt{2}$

At $\theta = \frac{\pi}{4}$, the parametric point is $(x_1, y_1) = (a\cos\theta, b\sin\theta)$

$$= (4\sqrt{2}\cos\frac{\pi}{4}, 2\sqrt{2}\sin\frac{\pi}{4})$$

$$= (4\sqrt{2} \times \frac{1}{\sqrt{2}}, 2\sqrt{2} \times \frac{1}{\sqrt{2}})$$

$$(x_1, y_1) = (4, 2)$$

The equation of the tangent at (x_1, y_1) is

$$(1) \Rightarrow xx_1 + 4yy_1 = 32$$

[Since x^2 is replaced by xx_1 and y^2 is replaced by yy_1]

∴ The equation of the tangent at $(x_1, y_1) = (4, 2)$ is

$$x(4) + 4y(2) = 32$$

$$4x + 8y - 32 = 0$$

$$x + 2y - 8 = 0$$

The equation of the normal is of the form

$$2x - y = k \text{------}(2)$$

It passes through the point (4, 2)

∴(2)⇒ $2(4) - 2 = k$

$k = 8 - 2$

$k = 6$

∴ The equation of the normal is

$$(2) \Rightarrow 2x - y = 6$$

(i.e) $2x - y - 6 = 0$

EXERCISE 5.4

6. Find the equations of the tangent and normal to hyperbola $12x^2 - 9y^2 = 108$ at $\theta = \frac{\pi}{3}$.

Solution:

Given equation of the ellipse $12x^2 - 9y^2 = 108$ --------(1)

$$\Rightarrow \frac{x^2}{9} - \frac{y^2}{12} = 1$$

Comparing this with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

We have $a^2 = 9$ and $b^2 = 12 \Rightarrow$ **$a=3$ and $b=2\sqrt{3}$**

At $\theta = \frac{\pi}{3}$, the parametric point is **$(x_1,y_1) = (a\sec\theta, b\tan\theta)$**

$$= (3\sec\frac{\pi}{3}, 2\sqrt{3}\tan\frac{\pi}{3})$$

$$= (3\times 2, 2\sqrt{3}\times\frac{1}{\sqrt{3}})$$

$$(x_1,y_1) = (6, 2)$$

The equation of the tangent at (x_1,y_1) is

$$(1) \Rightarrow 12xx_1 - 9yy_1 = 108$$

[Since x^2 is replaced by xx_1 and y^2 is replaced by yy_1]

∴ The equation of the tangent at $(x_1,y_1) = (6, 2)$ is

$$12x(6) - 9y(2) = 108$$
$$72x - 18y - 108 = 0$$

$$4x - y - 6 = 0$$

The equation of the normal is of the form

$$x + 4y = k \text{------}(2)$$

It passes through the point (6, 2)

$$\therefore (2) \Rightarrow 6 + 4(2) = k$$

$$k = 6 + 8$$

$$k = 14$$

∴ The equation of the normal is

$$(2) \Rightarrow x + 4y = 14$$

(i.e) $x + 4y - 14 = 0$

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