28-10-18

Unit - 11

Integral Calculus:-

Type 1:-

(i) $\int \frac{dx}{a^2-x^2} = \int \frac{dx}{(a+x)(a-x)}$

$$\frac{1}{a^2-x^2} = \frac{A}{a+x} + \frac{B}{a-x}$$

$\boxed{x=-a}$

$$1 = A(a-x) + B(a+x)$$
$$1 = A(a+a) + B(a-a)$$
$$1 = 2Aa$$
$$\boxed{\frac{1}{2a} = A}$$

$\boxed{x=a}$

$$1 = A(a-a) + B(a+a)$$
$$1 = 2aB$$
$$\boxed{1/2a = B}$$

$$= 1/2a\int \frac{1}{a+x}dx + 1/2a\int \frac{1}{a-x}dx$$

$$= 1/2a\left[\log(a+x) - \cancel{+}\log(a-x)\right]$$

$$= 1/2a\left[\frac{\log(a+x)}{\log(a-x)}\right]$$

$$= 1/2a \log\left(\frac{a+x}{a-x}\right) + c$$

Try (iii) $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$

Sol Let $I = \int \frac{dx}{a^2+x^2}$

Put $x = a\tan\theta$ $\quad dx = a\sec^2\theta\, d\theta$

$\tan\theta = \frac{x}{a}$

$\theta = \tan^{-1}\left(\frac{x}{a}\right)$

$$I = \int \frac{dx}{a^2+a^2\tan^2\theta} = \int \frac{a\sec^2\theta}{a^2(1+\tan^2\theta)}\, d\theta$$

$$= \int \frac{\sec^2\theta}{a\sec^2\theta}\, d\theta$$

$$= \frac{1}{a}\int d\theta$$

$$= \frac{1}{a}\theta + c$$

$$= \frac{1}{a}\tan^{-1}(x/a) + c$$

(ii) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)+c$

Sol $I = \int \frac{dx}{x^2+a^2}$

$$\frac{1}{x^2+a^2} = \frac{A}{x+a} + \frac{B}{x-a}$$

$$1 = A(x-a) + B(x+a)$$

$\boxed{x=-a}$ $\quad 1 = A(-2a) + B(-a+a)$

$$\boxed{-\frac{1}{2a} = A}$$

$\boxed{x=a}$ $\quad 1 = A(a-a) + B(a+a)$

$1 = 2Ba$

$\boxed{\frac{1}{2a} = B}$

$$= \frac{1}{2a}\int \frac{-1}{x+a}\,dx + \frac{1}{2a}\int \frac{1}{x-a}\,dx$$

$$= \frac{1}{2a}\left[-\log(x+a) + \log(x-a)\right]$$

$$= \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right) + c$$

(iv) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}(x/a) + c$

Sol Put $x = a\sin\theta \qquad dx = a\cos\theta\, d\theta$

$\frac{x}{a} = \sin\theta$

$\sin^{-1}(x/a) = \theta$

$$I = \int \frac{a\cos\theta\, d\theta}{\sqrt{a^2 - a^2\sin^2\theta}}$$

$$= \int \frac{a\cos\theta\, d\theta}{a\sqrt{\cos^2\theta}}$$

$$= \int d\theta$$

$$= \theta + c$$

$$= \sin^{-1}(x/a) + c$$

v) $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right| + c$

Sol

$\int \frac{dx}{\sqrt{x^2-a^2}}$

Put $x = a\sec\theta$ $dx = a\sec\theta\tan\theta\, d\theta$

$\frac{x}{a} = \sec\theta$

$\sec^{-1}\left(\frac{x}{a}\right) = \theta$

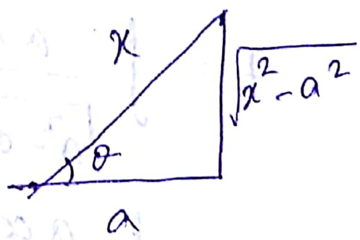
$= \int \frac{a\sec\theta\tan\theta\, d\theta}{\sqrt{a^2\sec^2\theta - a^2}}$

$= \int \frac{a\sec\theta\tan\theta}{a\sqrt{(\sec^2\theta-1)}}\, d\theta$

$= \int \frac{\sec\theta\tan\theta}{\tan\theta}\, d\theta$

$= \int \sec\theta\, d\theta$

$= \log \sec\theta + \tan\theta + c$



$\sec\theta = x/a$

$\tan\theta = \frac{\sqrt{x^2-a^2}}{a}$

$= \log\left|\frac{x}{a} + \frac{\sqrt{x^2-a^2}}{a}\right| + c$

$= \log\left|x+\sqrt{x^2-a^2}\right| - \log a + c$

$= \log\left|x+\sqrt{x^2-a^2}\right| + c_1$ $\quad [\because c_1 = c - \log a$

vi) $\int \frac{dx}{\sqrt{a^2+x^2}} = \log\left|x+\sqrt{x^2+a^2}\right| + c,$

Sol let $I = \int \frac{dx}{\sqrt{a^2+x^2}}$

Put $x = a\tan\theta \quad dx = a\sec^2\theta\, d\theta$

$\theta = \tan^{-1}\left(\frac{x}{a}\right)$

$\tan\theta = x/a$

$\sec\theta = \frac{\sqrt{a^2+x^2}}{a}$

$I = \int \frac{a\sec^2\theta\, d\theta}{\sqrt{a^2\tan^2\theta + a^2}}$

$= \int \frac{a\sec^2\theta}{a\sqrt{\tan^2\theta+1}}\, d\theta$

$= \int \sec\theta\, d\theta$

$= \log\left|\sec\theta + \tan\theta\right| + c$

$= \log\left|\frac{x}{a} + \sqrt{\frac{x^2}{a^2}+1}\right| + c$

$= \log\left|x + \sqrt{x^2+a^2}\right| - \log a + c$

$= \log\left|x+\sqrt{x^2+a^2}\right| + c_1 \quad (\because c_1 = c - \log a)$

Remark

$a^2 - x^2 \Rightarrow x = a\sin\theta$

$a^2 + x^2 \Rightarrow x = a\tan\theta$

$x^2 - a^2 \Rightarrow x = a\sec\theta$

Example 11.38

Evaluate:-

(i) $\int \frac{1}{(x-2)^2+1} dx$

Let $I = \int \frac{1}{(x+2)^2+1} dx$

Put $t = x+2$ $\qquad I = \int \frac{1}{1+t^2} dt$

$\boxed{dt = dx}$

$= \tan^{-1}(t) + c$

$= \tan^{-1}(x+2) + c$

(ii) Let $I = \int \frac{x^2}{x^2+5} dx$

$I = \int \frac{x^2+5-5}{x^2+5} dx$

$= \int \frac{x^2+5}{x^2+5} - \frac{5}{x^2+5} dx$

$= \int 1\, dx - 5\int \frac{1}{x^2-5} dx$

$= x - 5\int \frac{1}{x^2-(\sqrt{5})^2} dx$

$= x - 5\frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + c$

$= x - \sqrt{5} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + c$

(iii) $\int \frac{1}{\sqrt{1+4x^2}}\,dx$

$$I = \int \frac{1}{\sqrt{1+(2x)^2}}\,dx$$

Put $t = 2x$

$dt = 2\,dx$

$\boxed{\frac{dt}{2} = dx}$

$$I = \frac{1}{2}\int \frac{1}{\sqrt{1+t^2}}\,dt$$

$\left(\because \int \frac{dx}{\sqrt{a^2+x^2}} = \log\left|x+\sqrt{x^2+a^2}\right| + c\right)$

$$= \frac{1}{2}\log\left|t+\sqrt{t^2+1}\right| + c$$

$$= \frac{1}{2}\log\left|2x+\sqrt{(2x)^2+1}\right| + c$$

$$I = \frac{1}{2}\log\left|2x+\sqrt{4x^2+1}\right| + c$$

(iv) $I = \int \frac{1}{\sqrt{4x^2-25}}\,dx$

$$= \int \frac{1}{\sqrt{(2x)^2-5^2}}\,dx$$

Put $t = 2x$

$dt = 2\,dx$

$\frac{dt}{2} = dx$

$$= \frac{1}{2}\int \frac{1}{\sqrt{t^2-5^2}}\,dx$$

$\left(\because \int \frac{1}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right| + c\right)$

$$= \frac{1}{2}\log\left|t+\sqrt{t^2-5^2}\right| + c$$

$$= \frac{1}{2}\log\left|2x+\sqrt{4x^2-25}\right| + c$$

Type II :

Integrals form $\int \frac{dx}{ax^2+bx+c}$ & $\int \frac{dx}{\sqrt{ax^2+bx+c}}$

Example 11 : 39

(i) $\int \frac{1}{x^2-2x+5}\,dx$

$$\int \frac{1}{x^2-2x+5}\,dx = \int \frac{1}{x^2-2(1)x+1^2+2^2}\,dx$$

$$= \int \frac{1}{(x-1)^2+2^2}\,dx \quad \left(\because \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x-1}{2}\right)+c$$

(ii) $\int \frac{1}{\sqrt{x^2+12x+11}}$

$$= \int \frac{1}{\sqrt{x^2+2(6)x+11}}\,dx$$

$$= \int \frac{1}{\sqrt{x^2+2(6x)+36-36+11}}\,dx$$

$$= \int \frac{1}{\sqrt{(x+6)^2-25}}\,dx \quad \left(\because \int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right|+c\right)$$

$$= \log\left|x+6+\sqrt{(x+6)^2-5^2}\right|+c$$

$$= \log\left|x+6+\sqrt{x^2+12x+11}\right|+c$$

(iii) $\int \frac{1}{\sqrt{12+4x-x^2}} dx = \int \frac{1}{\sqrt{12-(x^2-4x)}} dx$

$= \int \frac{1}{\sqrt{12-\{(x-2)^2-4\}}} dx$

$= \int \frac{1}{\sqrt{16-(x-2)^2}} dx$

$= \int \frac{1}{\sqrt{4^2-(x-2)^2}} dx$

$= \sin^{-1}\left(\frac{(x-2)}{4}\right) + c$

Exercise 11.10

1) (i) $\frac{1}{4-x^2}$

$\int \frac{dx}{4-x^2} = \int \frac{1}{2^2-x^2} dx$ $\qquad \therefore \int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log\left(\frac{a+x}{a-x}\right) + c$

$= \frac{1}{2(2)} \log\left(\frac{2+x}{2-x}\right) + c$

$= \frac{1}{4} \log\left(\frac{2+x}{2-x}\right) + c$

(ii) $\frac{1}{25-4x^2} = \int \frac{1}{5^2-(2x)^2} dx$

$= \frac{1}{2(5)(2)} \log\left(\frac{5+2x}{5-2x}\right) + c$

$= \frac{1}{20} \log\left(\frac{5+2x}{5-2x}\right) + c$

(iii) $\int \frac{1}{9x^2-4}\,dx = \int \frac{1}{(3x)^2-2^2}\,dx$

$= \frac{1}{2(2)(3)} \log\left|\frac{3x-2}{3x+2}\right| + c$

$= \frac{1}{12} \log\left|\frac{3x-2}{3x+2}\right| + c$

(2) (i) $\int \frac{1}{6x-7-x^2}\,dx = \int \frac{1}{-7-\{(x^2-6x)\}}\,dx$

$= \int \frac{1}{-7-\{(x-3)^2-9\}}\,dx$

$= \int \frac{1}{2-(x-3)^2}\,dx$

$= \int \frac{1}{(\sqrt{2})^2-(x-3)^2}\,dx$ $\quad \int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)+c$

$= \frac{1}{2(\sqrt{2})} \log\left|\frac{\sqrt{2}+(x-3)}{\sqrt{2}-(x-3)}\right| + c$

(ii) $\int \frac{1}{(x+1)^2-25}\,dx = \int \frac{1}{(x+1)^2-5^2}\,dx$

$= \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right) + c$

$= \frac{1}{2(5)}\log\left(\frac{x+1-5}{x+1+5}\right) + c$

$= \frac{1}{10}\log\left(\frac{x-4}{x+6}\right) + c$

(iii) $\int \frac{1}{\sqrt{x^2+4x+2}}\,dx = \int \frac{1}{\sqrt{(x+2)^2-4+2}}\,dx$

$= \int \frac{1}{\sqrt{(x+2)^2-2}}\,dx$

$= \int \frac{1}{\sqrt{(x+2)^2-(\sqrt{2})^2}}\,dx$

$= \log\left|x+2+\sqrt{(x+2)^2-(\sqrt{2})^2}\right|+c$

$= \log\left|x+2+\sqrt{x^2+4+2}\right|+c$

3. (i) $\int \frac{1}{\sqrt{(2+x)^2-1}}\,dx$

$\cancel{t=2+x}$ $\quad I = \cancel{\int \frac{1}{t^2-1}\,dx}$

$\boxed{\cancel{dt=dx}}$ $\quad = \cancel{\sec^{-1}(t)+C}$

$= \cancel{\sec^{-1}(2+x)}+C$

$\int \frac{1}{\sqrt{(2+x)^2-1}}\,dx = \log\left|(2+x)+\sqrt{(2+x)^2-1}\right|+C$

$= \log\left|(2+x)+\sqrt{x^2+2x+3}\right|+C$

$\left(\because \int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x+(x^2-a^2)\right|+C\right)$

(ii) $\int \frac{1}{\sqrt{x^2-4x+5}}\,dx = \int \frac{dx}{\sqrt{(x-2)^2-4+5}}$

$= \int \frac{dx}{\sqrt{(x-2)^2+1}}$

$\left[\because \int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x+\sqrt{x^2+a^2}\right|+c\right]$

$= \log\left|(x-2)+\sqrt{(x-2)^2+1}\right|+c$

$= \log\left|x-2+\sqrt{x^2+4-4x+1}\right|+c$

$= \log\left|x-2+\sqrt{x^2-4x+5}\right|+c$

(iii) $\int \frac{1}{\sqrt{9+8x-x^2}}\,dx = \int \frac{dx}{\sqrt{9-(x^2-8x)}}$

$= \int \frac{dx}{\sqrt{9-\{(x-4)^2-16\}}}$

$= \int \frac{dx}{\sqrt{25-(x-4)^2}}$

$= \int \frac{dx}{\sqrt{(5)^2-(x-4)^2}}$

$= \sin^{-1}\left(\frac{x-4}{5}\right)+c$

Type III.

Integrals $\int \frac{px+q}{ax^2+bx+c}\,dx$ & $\int \frac{px+q}{\sqrt{ax^2+bx+c}}\,dx$

$px+q = A\frac{d}{dx}(ax^2+bx+c)+B$

$= A(2ax+b)+B \Rightarrow A\log|ax^2+bx+c| + B\int\frac{1}{ax^2+bx+c}dx$

$A\left(2\sqrt{ax^2+bx+c}\right) + B\int\frac{1}{\sqrt{ax^2+bx+c}}dx$

Example 11.40

Evaluate (i) $\int \frac{3x+5}{x^2+4x+7}\,dx$

Let $I = \int \frac{3x+5}{x^2+4x+7}\,dx$

$3x+5 = A\frac{d}{dx}(x^2+4x+7)+B$

$3x+5 = A(2x+4)+B$

Comparing the coefficient of like terms,

$2A = 3 \Rightarrow \boxed{A = 3/2}$

$5 = 4A+B$

$5 = 4(3/2)+B$

$5 = 6+B$

$5-6 = B$

$\boxed{-1 = B}$

$I = \int \frac{\frac{3}{2}(2x+4)-1}{x^2+4x+7}\,dx$

$= \frac{3}{2}\int\frac{2x+4}{x^2+4x+7}dx - \int\frac{1}{x^2+4x+7}dx$

$= \frac{3}{2}\log|(x^2+4x+7)| - \int\frac{1}{(x+2)^2-4+7}dx$

$$= \frac{3}{2} \log |x^2+4x+7| - \int \frac{1}{(x+2)^2+(\sqrt{3})^2} dx$$

$$= \frac{3}{2} \log |x^2+4x+7| - \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x+2}{\sqrt{3}}\right) + C$$

(ii) $\int \frac{x+1}{x^2-3x+1} dx$

Let $I = \int \frac{x+1}{x^2-3x+1} dx$

$x+1 = A\,(x^2-3x+1)' + B$

$x+1 = A(2x-3)+B$

Comparing coefficient of like term,

$2A = 1$ $\quad$ $1 = -3A+B$

$\boxed{A = \frac{1}{2}}$ $\quad$ $1 = -3\left(\frac{1}{2}\right)+B$

$1 = -\frac{3}{2}+B$

$1+\frac{3}{2} = B$

$\boxed{\frac{5}{2} = B}$

$$I = \int \frac{\frac{1}{2}(2x-3)+\frac{5}{2}}{x^2-3x+1} dx$$

$$= \frac{1}{2}\int \frac{2x-3}{x^2-3x+1} dx + \frac{5}{2}\int \frac{1}{x^2-3x+1} dx$$

$$= \frac{1}{2}\log|x^2-3x+1| + \frac{5}{2}\int \frac{1}{\left(x-\frac{3}{2}\right)^2-\frac{9}{4}+1} dx$$

$$= \frac{1}{2}\log|x^2-3x+1| + \frac{5}{2}\int \frac{1}{\left(x-\frac{3}{2}\right)^2-\frac{5}{4}} dx$$

$$= \frac{1}{2}\log|x^2-3x+1| + \frac{5}{2}\int \frac{1}{\left(x-\frac{3}{2}\right)^2-\left(\frac{\sqrt5}{2}\right)^2}\,dx$$

$$= \frac{1}{2}\log|x^2-3x+1| + \frac{5}{2}\,\frac{1}{2\left(\frac{\sqrt5}{2}\right)}\log\left|\frac{x-\frac{3}{2}-\frac{\sqrt5}{2}}{x-\frac{3}{2}+\frac{\sqrt5}{2}}\right|+c$$

$$= \frac{1}{2}\log|x^2-3x+1| + \frac{5}{2}\left(\frac{1}{\sqrt5}\right)\log\left|\frac{2x-3-\sqrt5}{2x-3+\sqrt5}\right|+c$$

$$= \frac{1}{2}\log|x^2-3x+1| + \frac{\sqrt5}{2}\log\left|\frac{2x-3-\sqrt5}{2x-3+\sqrt5}\right|+c$$

(iii) $\int \frac{2x+3}{\sqrt{x^2+x+1}}\,dx$

$$2x+3 = A\frac{d}{dx}(x^2+x+1)+B$$

$$2x+3 = A(2x+1)+B$$

Comparing the coefficient of like terms,

$2 = 2A$ $\quad$ $3 = A+B$

$\boxed{1=A}$ $\quad$ $3 = 1+B$

$\boxed{2=B}$

$$I = \int \frac{(2x+1)+2}{\sqrt{x^2+x+1}}$$

$$= \int \frac{2x+1}{\sqrt{x^2+x+1}}\,dx + 2\int \frac{1}{\sqrt{x^2+x+1}}\,dx$$

$(x^2+x+1)^{1/2}$

$$= 2\sqrt{x^2+x+1} + 2\int \frac{1}{\sqrt{\left(x+\frac{1}{2}\right)^2-\frac{1}{4}+1}}\,dx$$

$$= 2\sqrt{x^2+x+1} + 2\int \frac{1}{\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{3}{4}\right)}}\,dx$$

$$= 2\sqrt{x^2+x+1} + 2\int \frac{1}{\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt3}{2}\right)^2}}\,dx$$

$$= 2\sqrt{x^2+x+1} + 2\log\left|x+\tfrac{1}{2}+\sqrt{\left(x+\tfrac{1}{2}\right)^2+\left(\tfrac{\sqrt{3}}{2}\right)^2}\right|$$

$$= 2\sqrt{x^2+x+1} + 2\log\left|x+\tfrac{1}{2}+\sqrt{x^2+x+1}\right| + c$$

iv) $\displaystyle\int \frac{5x-7}{\sqrt{3x-x^2+2}}\,dx$

$$5x-7 = A\frac{d}{dx}(3x-x^2+2)+B$$

$$5x-7 = A(3-2x)+B$$

Comparing the Coefficient of like terms,

$5 = -2A$ $\quad$ $-7 = +3A+B$

$\boxed{\frac{5}{-2} = A}$ $\quad$ $-7 = 3\left(-\frac{5}{2}\right)+B$

$-7 = -\frac{15}{2}+B$

$-7+\frac{15}{2} = B$

$\frac{-14+15}{2} = B \Rightarrow \boxed{\frac{1}{2} = B}$

$$I = \int \frac{\frac{-5}{2}(3-2x)+\frac{1}{2}}{\sqrt{3x-x^2+2}}\,dx$$

$$= \frac{-5}{2}\int\frac{3-2x}{\sqrt{3x-x^2+2}}\,dx + \frac{1}{2}\int\frac{1}{\sqrt{3x-x^2+2}}\,dx$$

$$= \frac{-5}{2}\cdot 2\sqrt{3x-x^2+2} + \frac{1}{2}\int\frac{1}{\sqrt{2+\frac{9}{4}-\left(x-\frac{3}{2}\right)^2}}\,dx$$

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$$= -\frac{5}{2}\left(2\sqrt{3x-x^2+2}\right)+\frac{1}{2}\int \frac{1}{\sqrt{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}}\,dx$$

$$= \frac{-5}{2}\left(2\sqrt{3x-x^2+2}\right)+\frac{1}{2}\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{17}}{2}}\right)+c$$

$$= -5\sqrt{3x-x^2+2}+\frac{1}{2}\sin^{-1}\left(\frac{2x-3}{\sqrt{17}}\right)+c$$

Ex 11.11

1) Integrate.

(i) $\dfrac{2x-3}{x^2+4x-12}$

Let $I = \int \dfrac{2x-3}{x^2+4x-12}\,dx$

$2x-3 = A\dfrac{d}{dx}(x^2+4x-12)+B$

$2x-3 = A(2x+4)+B$

Comparing the coefficient of like terms

$2 = 2A$ $\quad$ $-3 = 4A+B$

$\boxed{1=A}$ $\quad$ $-3-4 = B$

$\boxed{-7=B}$

$$I = \int \frac{(2x+4)-7}{(x^2+4x-12)}\,dx$$

$$= \int \frac{2x+4}{x^2+4x-12}\,dx - 7\int \frac{1}{x^2+4x-12}\,dx$$

$$= \log\left|x^2+4x-12\right| - 7\int \frac{1}{(x+2)^2-4-12}\,dx$$

$$= \log\sqrt{x^2+4x-12} - 7\int \frac{1}{(x+2)^2-16}\,dx$$

$$= \log\sqrt{x^2+4x-12} - 7\int \frac{1}{(x+2)^2-(4)^2}\,dx$$

$$= \log\sqrt{x^2+4x-12} - 7\left(\frac{1}{2(4)}\log\left(\frac{x+2-4}{x+2+4}\right)\right) + C$$

$$= \log\sqrt{x^2+4x-12} - \frac{7}{8}\log\left(\frac{x-2}{x+6}\right) + C$$

(ii) $\int \frac{5x-2}{2+2x+x^2}\,dx$

$$5x-2 = A\frac{d}{dx}(2+2x+x^2) + B$$

$$5x-2 = A(2x+2) + B$$

Comparing the coefficient of like terms,

$5 = 2A$ $\quad\quad -2 = 2A+B$

$\boxed{\frac{5}{2} = A}$ $\quad\quad -2 = 2\left(\frac{5}{2}\right)+B$

$-2 = 5+B$

$\boxed{-7 = B}$

$$= \frac{5}{2}\int \frac{(2x+2)}{2+2x+x^2}\,dx - 7\int \frac{1}{2+2x+x^2}\,dx$$

$$= \frac{5}{2}\log(x^2+2x+2) - 7\int \frac{1}{x^2+2x+2}\,dx$$

$$= \frac{5}{2}\log(x^2+2x+2) - 7\int \frac{1}{(x+1)^2-1+2}\,dx$$

$= \frac{5}{2}\log(x^2+2x+2) - 7\int \frac{1}{(x+1)^2+1}\,dx$

$= \frac{5}{2}\log(x^2+2x+2) - 7\int \frac{1}{(x+1)^2+(1)^2}\,dx$

$\cancel{= \frac{5}{2}\log(x^2+2x+2) - 7\frac{1}{2\sqrt{2}}\log\left(\frac{x+1-\sqrt{2}}{x+1+\sqrt{2}}\right)+c}$

$\because \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$

$= \frac{5}{2}\log(x^2+2x+2) - 7\tan^{-1}(x+1)+c$

(iii) $\frac{3x+1}{2x^2-2x+3}$

$\int \frac{3x+1}{2x^2-2x+3}\,dx \Rightarrow$

$3x+1 = A\frac{d}{dx}(2x^2-2x+3)+B$

$3x+1 = A(4x-2)+B$

comparing the coefficient of like terms,

$3 = 4A$ $\quad 1 = -2A+B$

$\boxed{\frac{3}{4} = A}$ $\quad 1 = -2\left(\frac{3}{4}\right)+B$

$1 = -\frac{3}{2}+B$

$1+\frac{3}{2} = B$

$\boxed{\frac{5}{2} = B}$

$= \frac{3}{4}\int \frac{4x-2}{2x^2-2x+3}\,dx + \frac{5}{2}\int \frac{1}{2x^2-2x+3}\,dx$

$= \frac{3}{4}\log(2x^2-2x+3) + \frac{5}{2}\int \frac{1}{2(x^2-x)^2+3}\,dx$

$= \frac{3}{4}\log(2x^2+3-2x) + \frac{5}{2}\int \frac{1}{2\{(x-\frac{1}{2})^2-\frac{1}{4}\}+3}$

$= \frac{3}{4}\log(2x^2+3-2x) + \frac{5}{2}\int \frac{1}{2(x-\frac{1}{2})^2 - \frac{1}{2} + 3} dx$

$= \frac{3}{4}\log(2x^2+3-2x) + \frac{5}{2}\int \frac{1}{2(x-\frac{1}{2})^2 + \frac{5}{2}} dx$

$= \frac{3}{4}\log(2x^2+3-2x) + \frac{5}{2}\int \frac{1}{[\sqrt{2}(x-\frac{1}{2})]^2 + \left(\frac{\sqrt{5}}{\sqrt{2}}\right)^2} dx$

$= \frac{3}{4}\log(2x^2+3-2x) + \frac{5}{2} \cdot \frac{1}{\frac{\sqrt{5}}{\sqrt{2}}} \tan^{-1}\left(\frac{\sqrt{2}(x-\frac{1}{2})}{\sqrt{5}/\sqrt{2}}\right) + C$

$= \frac{3}{4}\log(2x^2+3-2x) + \frac{\sqrt{5}}{\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{5}}\right) + C$

$\therefore \frac{5}{2} \times \frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{5}}{\sqrt{2}}$

2) (i) $\int \frac{2x+1}{\sqrt{9+4x-x^2}} dx$

$2x+1 = A\frac{d}{dx}(9+4x-x^2) + B$

$2x+1 = (4-2x)A + B$

$2 = -2A$ $\qquad$ $1 = 4A + B$

$\boxed{-1 = A}$ $\qquad$ $1 = 4(-1) + B$

$1 + 4 = B$

$\boxed{5 = B}$

$= -\int \frac{dx\,(4-2x)}{\sqrt{9+4x-x^2}} + 5\int \frac{dx}{\sqrt{9+4x-x^2}}$

$= -2\sqrt{9+4x-x^2} + 5\int \frac{dx}{\sqrt{9-(x^2-4x)}}$

$$= -2\sqrt{9+4x-x^2} + 5\int \frac{dx}{\sqrt{9-\{(x-2)^2-4\}}}$$

$$= -2\sqrt{9+4x-x^2} + 5\int \frac{dx}{\sqrt{13-(x-2)^2}}$$

$$= -2\sqrt{9+4x-x^2} + 5\int \frac{dx}{\sqrt{(\sqrt{13})^2-(x-2)^2}}$$

$$= -2\sqrt{9+4x-x^2} + 5\sin^{-1}\left(\frac{x-2}{\sqrt{13}}\right) + c$$

$$= 5\sin^{-1}\left(\frac{x-2}{13}\right) - 2\sqrt{9+4x-x^2} + c$$

(ii) $\int \frac{x+2}{\sqrt{x^2-1}}\,dx$

$$x+2 = A\frac{d}{dx}(x^2-1)+B$$

$$x+2 = A(2x)+B$$

$1 = 2A$ $\quad\boxed{2 = B}$

$\boxed{1/2 = A}$

$$= \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}\,dx + 2\int \frac{1}{\sqrt{x^2-1}}\,dx$$

$$= \frac{1}{\cancel{2}}\cdot\cancel{2}\,\cancel{\log}\left(\sqrt{x^2-1}\right) + 2\log\left(x+\sqrt{x^2-1}\right) + c$$

$$= \cancel{\log}\sqrt{x^2-1} + 2\log\left(x+\sqrt{x^2-1}\right) + c$$

(iii) $\int \frac{2x+3}{\sqrt{x^2+4x+1}} dx$

$2x+3 = A\frac{d}{dx}(x^2+4x+1) + B$

$2x+3 = A(2x+4) + B$

$2 = 2A$ | $3 = 4A + B$

$\boxed{A=1}$ | $3 = 4 + B$

$\boxed{-1 = B}$

$\int \frac{dx\,(2x+4)}{\sqrt{x^2+4x+1}} + \int \frac{-1}{\sqrt{x^2+4x+1}} dx$

$2\left(\sqrt{x^2+4x+1}\right) - \int \frac{dx}{\sqrt{(x+2)^2-4+1}}$

$2\left(\sqrt{x^2+4x+1}\right) - \int \frac{dx}{\sqrt{(x+2)^2-3}}$

$2\sqrt{x^2+4x+1} - \int \frac{dx}{\sqrt{(x+2)^2-(\sqrt{3})^2}}$

$2\sqrt{x^2+4x+1} - \log\left|x+2+\sqrt{(x+2)^2-(\sqrt{3})^2}\right| + C$

$2\sqrt{x^2+4x+1} - \log\left|x+2+\sqrt{x^2+4x+4-3}\right| + C$

$2\sqrt{x^2+4x+1} - \log\left|x+2+\sqrt{x^2+4x+1}\right| + C$

Type IV

Integrals form $\int\sqrt{a^2 \pm x^2}\,dx$, $\int\sqrt{x^2-a^2}\,dx$

(i) $\int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+c$

Proof Let $I = \int\sqrt{a^2-x^2}\,dx$

$u = \sqrt{a^2-x^2}$ $\boxed{dv = dx}$

$u = (a^2-x^2)^{1/2}$ $\boxed{v = x}$

$du = -2x \frac{1}{2}(a^2-x^2)^{-1/2}$

$$\boxed{du = \frac{-2x}{2\sqrt{a^2-x^2}}\,dx}$$

$\int u\,dv = -\int v\,du + uv$

$$I = \sqrt{a^2-x^2}\cdot x - \int x \cdot \frac{(-2x)}{2\sqrt{a^2-x^2}}\,dx$$

$$= x\sqrt{a^2-x^2} - \int \frac{-2x^2}{2\sqrt{a^2-x^2}}\,dx$$

$$= x\sqrt{a^2-x^2} - \int \frac{a^2-x^2-a^2}{\sqrt{a^2-x^2}}\,dx$$

$$= x\sqrt{a^2-x^2} - \int \frac{a^2-x^2}{\sqrt{a^2-x^2}}\,dx + \int \frac{a^2}{\sqrt{a^2-x^2}}\,dx$$

$$= x\sqrt{a^2-x^2} - \int\sqrt{a^2-x^2}\,dx + a^2\int\frac{1}{\sqrt{a^2-x^2}}\,dx$$

$$I = x\sqrt{a^2-x^2} - I + a^2\sin^{-1}\left(\frac{x}{a}\right)$$

$$2I = x\sqrt{a^2-x^2} + a^2\sin^{-1}(x/a)$$

$$I = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}(x/a) + c$$

(ii) $\int \sqrt{x^2-a^2}\, dx$

Let $I = \int \sqrt{x^2-a^2}\, dx$

$dv = dx$

$\boxed{v = x}$

$u = \sqrt{x^2-a^2}$

$du = (x^2-a^2)^{1/2}\, d$

$= \frac{1}{2}(x^2-a^2)^{-1/2} \cdot 2x\, dx$

$$\boxed{du = \frac{2x}{2\sqrt{x^2-a^2}}\, dx}$$

$\int u\,dv = uv - \int v\, du$

$$= x\sqrt{x^2-a^2} - \int \frac{x\, 2x}{2\sqrt{x^2-a^2}}\, dx$$

$$= x\sqrt{x^2-a^2} - \int \frac{x^2}{\sqrt{x^2-a^2}}\, dx$$

$$= x\sqrt{x^2-a^2} - \int \frac{x^2-a^2+a^2}{\sqrt{x^2-a^2}}\, dx$$

$$= x\sqrt{x^2-a^2} - \int \frac{x^2-a^2}{\sqrt{x^2-a^2}}\, dx \mp \int \frac{a^2}{\sqrt{x^2-a^2}}\, dx$$

$$= x\sqrt{x^2-a^2} - \int \sqrt{x^2-a^2}\, dx \mp a^2 \int \frac{1}{\sqrt{x^2-a^2}}\, dx$$

$$2I = x\sqrt{x^2-a^2} \mp a^2 \log\left|x + \sqrt{x^2-a^2}\right| + c_1$$

$$I = \frac{x}{2}\sqrt{x^2-a^2} \mp \frac{a^2}{2} \log\left|x + \sqrt{x^2-a^2}\right| + c_1$$

(iii) $\int \sqrt{x^2+a^2}\,dx$

Let $I = \int \sqrt{x^2+a^2}\,dx$

$u = \sqrt{x^2+a^2}$ $\qquad dv = dx$

$\boxed{v = x}$

$du = \frac{1}{2}(x^2+a^2)^{-1/2}\, 2x\, dx$

$$\boxed{du = \frac{x}{\sqrt{x^2+a^2}}\,dx}$$

$\int u\,dv = uv - \int v\,du$

$$I = x\sqrt{x^2+a^2} - \int \frac{x^2}{\sqrt{x^2+a^2}}\,dx$$

$$= x\sqrt{x^2+a^2} - \int \frac{x^2+a^2-a^2}{\sqrt{x^2+a^2}}\,dx$$

$$= x\sqrt{x^2+a^2} - \int \frac{x^2+a^2}{\sqrt{x^2+a^2}}\,dx + a^2\int \frac{1}{\sqrt{x^2+a^2}}\,dx$$

$$I = x\sqrt{x^2+a^2} - \int \sqrt{x^2+a^2}\,dx + a^2 \log\left|x+\sqrt{x^2+a^2}\right| + c_1$$

$$2I = x\sqrt{x^2+a^2} + a^2 \log\left|x+\sqrt{x^2+a^2}\right| + c$$

$$I = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + c$$

Ex 11.12

2) (i) $\sqrt{9-(2x+5)^2}$ $= \int \sqrt{3^2-(2x+5)^2}\,dx$

$$= \frac{1}{2}\,\frac{2x+5}{2}\sqrt{3^2-(2x+5)^2} + \frac{9}{2}\sin^{-1}\left(\frac{2x+5}{3}\right) + c$$

$$= \frac{1}{2}\,\frac{2x+5}{2}\sqrt{9-(2x+5)^2} + \frac{9}{2}\sin^{-1}\left(\frac{2x+5}{3}\right) + c$$

$$= \frac{2x+5}{4}\sqrt{9-(2x+5)^2} + \frac{9}{4}\sin^{-1}\left(\frac{2x+5}{3}\right) + c$$

(ii) $\sqrt{81+(2x+1)^2}$

$$\int \sqrt{9^2+(2x+1)^2}\,dx = \frac{1}{2}\left[\frac{2x+1}{2}\sqrt{(2x+1)^2+9^2}+\frac{9^2}{2}\log\left|2x+1+\sqrt{(2x+1)^2+9^2}\right|\right]+c$$

$$= \frac{1}{4}\left[(2x+1)\sqrt{81+(2x+1)^2}+\frac{81}{2}\log\left|2x+1+\sqrt{81+(2x+1)^2}\right|\right]$$

(iii) $\sqrt{(x+1)^2-4}$

$$\int\sqrt{(x+1)^2-2^2}\,dx = \frac{x+2}{2}\sqrt{(x+1)^2-4}-\frac{4}{2}\log\left|x+1+\sqrt{(x+1)^2-4}\right|+c$$

$$= \frac{x+2}{2}\sqrt{(x+1)^2-4}-2\log\left|x+1+\sqrt{(x+1)^2-4}\right|+c$$

① (i) $\int\sqrt{x^2+2x+10}\,dx$

$$I = \int\sqrt{(x+1)^2-1+10}\,dx = \int\sqrt{(x+1)^2+3^2}\,dx$$

$$= \frac{x+1}{2}\sqrt{(x+1)^2+9}+\frac{9}{2}\log\left|x+1+\sqrt{(x+1)^2+9}\right|+c$$

(ii) $\int\sqrt{x^2-2x-3}\,dx = \int\sqrt{(x-1)^2-1-3}\,dx$

$$= \int\sqrt{(x-1)^2-4}\,dx$$

$$= \int\sqrt{(x-1)^2-2^2}\,dx$$

$$= \frac{x-1}{2}\sqrt{(x-1)^2-4}-2\log\left|x-1+\sqrt{(x-1)^2-4}\right|+c$$

$$= \frac{x-1}{2}\sqrt{x^2-2x-3}-2\log\left|x-1+\sqrt{x^2-2x-3}\right|+c$$

(iii) $\int\sqrt{(6-x)(x-4)}$

$$\int\sqrt{(6-x)(x-4)}\,dx = \int\sqrt{6x-24-x^2+4x}\,dx$$

$$= \int\sqrt{-x^2+10x-24}\,dx$$

$$= \int\sqrt{-(x+5)^2+25-24}\,dx$$

$$= \int\sqrt{-(x-5)^2+1}\,dx$$

$$= \int\sqrt{-(x-5)^2+1^2}\,dx$$

$$= \int\sqrt{1-(x-5)^2}\,dx$$

$$= \frac{x-5}{2}\sqrt{1-(x-5)^2} + \frac{a1}{2}\sin^{-1}\left(\frac{x-5}{1}\right)+c$$

$$= \frac{x-5}{2}\sqrt{-x^2+24+10x} + \frac{1}{2}\sin^{-1}(x-5)+c \quad //$$

Ex 11.9

1) $e^x(\tan x + \log \sec x)$

$$I = \int e^x(\tan x + \log \sec x)\,dx$$

$f(x) = \log \sec$

$f'(x) = \frac{1}{\sec x}\sec x \tan x$

$$= e^x f(x) + c$$

$$= e^x \log \sec x + c$$

(ii) $\int e^x\left(\frac{x-1}{2x^2}\right)dx$

$$I = \frac{1}{2}\int e^x\left(\frac{x-1}{x^2}\right)dx$$

$$= \frac{1}{2}\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$$

$f(x) = 1/x$

$f'(x) = -1/x^2$

$$= \frac{1}{2}\int e^x f(x) + c$$

$$= \frac{1}{2}e^x\left(\frac{1}{x}\right)+c$$

(iii) $\int e^x \sec x(1+\tan x)\,dx$

$$I = \int e^x \sec x\,(1+\tan x)\,dx$$

$$= \int e^x(\sec x + \sec x \tan x)\,dx$$

$$= e^x f(x) + c$$

$f(x) = \sec x$

$f'(x) = \sec x \tan x$

$$= e^x \sec x + C$$

(iv) $e^x \left(\frac{2+\sin 2x}{1+\cos 2x}\right)$

$$I = \int e^x\left(\frac{2+\sin 2x}{1+\cos 2x}\right)dx$$

$\sin 2x = 2\sin x\cos x$

$1+\cos 2x = 2\cos^2 x$

$$= \int e^x\left[\frac{2+2\sin x\cos x}{2\cos^2 x}\right]dx$$

$$= \int e^x\left[\frac{1+\sin x\cos x}{\cos^2 x}\right]dx$$

$$= \int e^x(\sec^2 x + \tan x)\,dx$$

$$= \int e^x(\tan x + \sec^2 x)\,dx$$

$$= e^x f(x) + c$$

$$= e^x \tan x + C$$

Ex 11.7

3) (i) $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$

$I = \int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$ $\quad t = \sin^{-1} x$

$dt = \frac{1}{\sqrt{1-x^2}} dx$

$= \int t \sin t \, dt$

$= t(-\cos t) - 1(-\sin t)$

$= -t \cos t + \sin t + c$

$= -\sin^{-1} x \cos t + \sin(\sin^{-1} x) + c$

$= -\sin^{-1} x \sqrt{1-\sin^2 t} + x + c$

$= -\sin^{-1} x \sqrt{1-x^2} + x + c$

(ii) $\int x^5 e^{x^2} dx$

$I = \int x^5 e^{x^2} dx$ $\quad x^2 = t$

$2x\,dx = dt$

$= \int t^2 e^t \frac{dt}{2}$ $\quad x\,dx = \frac{dt}{2}$

$= \frac{1}{2} \int t^2 e^t dt$

$= \frac{1}{2}\left[t^2 e^t - 2te^t + 2e^t\right] + c$

$= \frac{1}{2}\left((x^2)^2 e^{x^2} - 2x^2 e^{x^2} + 2e^{x^2}\right) + c$

$= \frac{1}{2} e^{x^2}(x^4 - 2x^2 + 2) + c$

Ex 11.6

1 (i) $\int \frac{x}{\sqrt{1+x^2}}\,dx$

$u = 1+x^2$

$du = 2x\,dx$

$x\,dx = \frac{du}{2}$

$$I = \int \frac{x}{\sqrt{1+x^2}}\,dx$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$$

$$= \frac{1}{2}\int u^{-1/2}\,du$$

$$= \frac{1}{2}\,\frac{u^{1/2}}{1/2} + c$$

$$= u^{1/2} + c$$

$$= \sqrt{1+x} + c$$

(ii) $I = \int \frac{x^2}{1+x^6}\,dx$

$x^3 = t$

$3x^2\,dx = dt$

$x^2\,dx = \frac{dt}{3}$

$$= \frac{1}{3}\int \frac{dt}{1+t^2}$$

$$= \frac{1}{3}\tan^{-1}(t) + c$$

$$= \frac{1}{3}\tan^{-1}(x^3) + c$$

(8) $\int \frac{\sin 2x}{a^2+b^2\sin^2 x}\,dx$

$f(x) = a^2+b^2\sin^2 x$

$f'(x) = 0 + b^2(2\sin x\cos x)$

$= b^2\sin 2x$

$$= \frac{1}{b^2}\int \frac{b^2\sin 2x}{a^2+b^2\sin^2 x}\,dx$$

$$= \frac{1}{b}\log\left|a^2+b^2\sin^2 x\right| + c$$

(3) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

$f(x) = e^x + e^{-x}$

$f'(x) = e^x - e^{-x}$

$I = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

$= \log(e^x + e^{-x}) + c$

(4) $\frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}}$

$I = \int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}} dx$

$f(x) = x^{10} + 10^x$

$f'(x) = 10x^9 + 10^x \log_e 10$

$= \log |10^x + x^{10}| + c$

(5) $I = \int \frac{\sin\sqrt{x}}{\sqrt{x}} dx$

$t = \sqrt{x}$

$dt = \frac{1}{2\sqrt{x}} dx$

$2dt = \frac{1}{\sqrt{x}} dx$

$= 2\int \sin t \, dt$

$= 2(-\cos t) + c$

$= -2\cos\sqrt{x} + c$

(6) $\int \frac{\cot x}{\log(\sin x)} dx$

$f(x) = \log \sin x$

$f'(x) = \frac{1}{\sin x} \cos x$

$= \cot x$

$I = \log |\log \sin x| + c$

9) $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx = I$

$t = \sin^{-1} x$

$dt = \frac{1}{\sqrt{1-x^2}} dx$

$= \int t \, dt$

$= \frac{t^2}{2} + c$

$= \frac{(\sin^{-1} x)^2}{2} + c$

(10) $\int \frac{\sqrt{x}}{1+\sqrt{x}} dx = I$

$1+\sqrt{x} = t$

$\sqrt{x} = t-1$

$x = (t-1)^2$

$dx = 2t-2$

$I = \int \frac{t-1}{1+t-1} (2t-2) dt$

$= 2\int \frac{t^2-1-2t}{t} dt$

$= 2\int \left(t - \frac{1}{t} - 2\right) dt$

$= 2\left(\frac{t^2}{2} - \log t - 2t\right) + c$

$= (1+\sqrt{x})^2 - 2\log(1+\sqrt{x}) - 4(1+\sqrt{x}) + c$

(11) $\int \frac{1}{x \log x \log(\log x)} dx$

$f(x) = \log \log x$

$f'(x) = \frac{1}{\log x} \cdot \frac{1}{x}$

$I = \int \frac{\frac{1}{\log x \cdot x}}{\log \log x} dx$

$= \log |\log \log x| + c$

12) $\int \alpha\beta x^{\alpha-1} e^{-\beta x^{\alpha}} dx$

$t = x^{\alpha}$

$dt = \alpha x^{\alpha-1} dx$

$I = \int \alpha\beta x^{\alpha-1} e^{-\beta x^{\alpha}} dx$

$= \int \beta e^{-\beta t} dt$

$= \beta \dfrac{e^{-\beta t}}{\beta} + c$

$= -e^{-\beta t} + c$

$= -e^{-\beta x^{\alpha}} + c$

13) $\int \tan x \sqrt{\sec x}\, dx$

$t = \sec x$

$dt = \sec x \tan x\, dx$

$\dfrac{dt}{\sec x} = \tan x\, dx$

$\dfrac{dt}{t} = \tan x\, dx$

$I = \int \tan \sqrt{\sec x}\, dx$

$= \int \sqrt{t} \dfrac{dt}{t}$

$= \int \dfrac{1}{\sqrt{t}} dt$

$= 2\sqrt{t} + c$

$= 2\sqrt{\sec x} + c$

14) $\int x(1-x)^{17} dx$

$1-x = t \Rightarrow 1-t = x$

$-dx = dt$

$I = \int (1-t)(t)^{17} dx$

$= \int t^{17} - t^{18} (-dt)$

$= -\dfrac{t^{18}}{18} + \dfrac{t^{19}}{19} + c$

$= -\dfrac{(1-x)^{18}}{18} + \dfrac{(1-x)^{19}}{19} + c$

15) $I = \int \sin^5 x \cos^3 x \, dx$

$= \int \sin^5 x (1-\sin^2 x)\cos x \, dx \quad \because 1-\sin^2 x = \cos^2 x$

$\sin x = t$

$\cos x \, dx = dt$

$= \int (\sin^5 x - \sin^7 x)\cos x \, dx$

$= \int (t^6 - t^7) dt$

$= \frac{t^7}{7} - \frac{t^8}{8} + c$

$= \frac{\sin^7 x}{7} - \frac{\sin^8 x}{8} + c$

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16) $\int \frac{\cos x}{\cos(x-a)} dx$

$\int \frac{\cos(x-a+a)}{\cos(x-a)} dx$

$\int \frac{\cos(x-a)\cos a + \sin(x-a)\sin a}{\cos(x-a)} dx$

$\int (\cos a + \tan(x-a)\sin a) \, dx$

$x \cos a + \sin a \int \tan(x-a) dx$

$x \cos a + \sin a \log|\sec(x-a)| + c$

17) $\int \frac{\operatorname{cosec} x}{\log(\tan x/2)} dx$ ~~$\tan\frac{x}{2} = t$~~

W.K.T $\int \frac{f'(x)}{f(x)} dx = \log f(x) + c$ ~~log~~

$= \log\left(\log\left(\tan x/2\right)\right) + c$