Padasalai's Telegram Groups!

(தலைப்பிற்கு கீழே உள்ள லிங்கை கிளிக் செய்து குழுவில் இணையவும்!)

- Padasalai's NEWS - Group
https://t.me/joinchat/NIfCqVRBNj9hhV4wu6_NqA

- Padasalai's Channel - Group
https://t.me/padasalaichannel

- Lesson Plan - Group
https://t.me/joinchat/NIfCqVWwo5iL-21gpzrXLw

- 12th Standard - Group
https://t.me/Padasalai_12th

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- 9th Standard - Group
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Unit - 6 Gravitation

Kepler's Law :

First law:

All the planets in the solar system orbit the sun in elliptical orbits with the sun at one of the foci.

Second law:

The radial vector line joining the sun to a planet sweeps equal areas in equal intervals of time.

Third law;

The ratio of the square of the time period of planet to the cubic power of semi major axis is constant for all the planets in the solar system.

Measurement of Radius of Earth :

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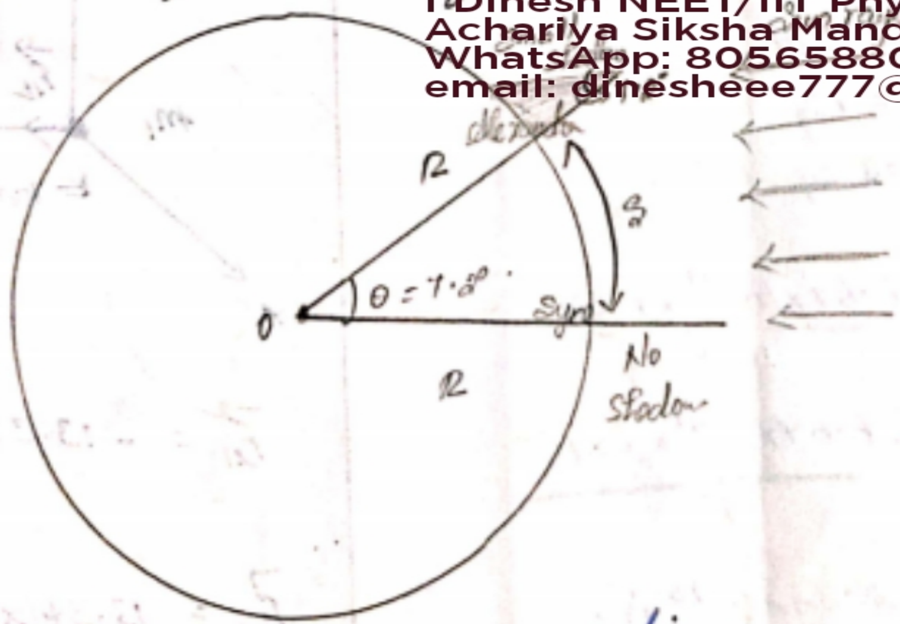

$\theta = 7.2° = \frac{1}{8}$ radian

$\theta = 1/8$ radian

$S = R\theta$.

Here,

S - length of arc b/w two cities.

R - radius of earth.

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$S = R\theta$

$S = 500$ miles

$R = \frac{500}{\theta}$

$= \frac{500 \text{ miles}}{1/8}$

$= 500 \times 8$ miles

$R = 4000$ miles.

1 mile is equal to 1.609 km.

$R = 4000 \times 1.609$

$= 4 \times 1609$

$R \simeq 6436$ km.

$\therefore$ The exact value of $R = 6378$ km.

Universal law of Gravitation:

Gravitational constant, $G = 6.626 \times 10^{-11} Nm^2 kg^{-2}$

$$\vec{F} = -\frac{G M_1 M_2}{r^2} \hat{r}.$$

Example 6.1

Force of attraction, $\vec{F} = -\frac{G m_1 m_2}{r^2} \hat{r}$.

given:

$r = 10m$

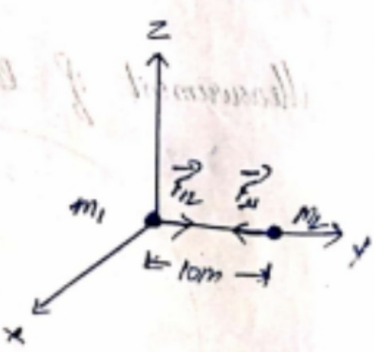
$m_1 = 1 kg$

$m_2 = 2 kg$.

$$F = \frac{6.67 \times 10^{-11} \times 1 \times 2}{100}$$

$$= \frac{6.67 \times 10^{-11} \times 2 \times 10^{-2}}{1}$$

$F = 13.34 \times 10^{-13} N$.



$\vec{F}_{21} = -13.34 \times 10^{-13} \hat{j}$

$\vec{F}_{12} = 13.34 \times 10^{-13} \hat{j}$

Since $\hat{r} = -\hat{j}$

By Newton's third law.

$\vec{F}_{12} = -\vec{F}_{21}$

(3)

Newton's inverse square law:

Centripetal acceleration,

$$a = \frac{-v^2}{r} \rightarrow (1).$$

Velocity, $v = \frac{2\pi r}{T} \rightarrow (2)$.

Where,

r - distance of the planet from centre of the orbit.

T - Time period.

sub. equ (2) in (1).

$$a = \frac{\left(-\frac{2\pi r}{T}\right)^2}{r}$$

$$= \frac{-\frac{4\pi^2 r^2}{T^2}}{r}$$

$$= -\frac{4\pi^2 r^{\cancel{2}}}{T^2 \cancel{r}}$$

$$a = -\frac{4\pi^2 r}{T^2} \rightarrow (3).$$

From Newton's second law,

$$F = ma \rightarrow (4).$$

sub. equ (3) in (4).

$$F = m\left(\frac{-4\pi^2 r}{T^2}\right)$$

$$F = -\frac{4\pi^2 m r}{T^2} \rightarrow (5)$$

From Kepler's third law,

$$\frac{r^3}{T^2} = K \text{ (constant)}.$$

$$\frac{r \times r^2}{T^2} = K$$

$$\frac{r}{T^2} = \frac{K}{r^2}. \rightarrow (6).$$

sub. equ (6) in (5).

$$F = -\frac{4\pi^2 m K}{r^2} \rightarrow (7).$$

((-) sign implies that the force is attractive and it act towards the centre)

Here,

$$4\pi^2 K = GM.$$

$$F = -\frac{GMm}{r^2}.$$

(∴ (-) sign implies gravitational force is attractive)

Example 6.2

The gravitational force by the apple due to Earth.

$$F = -\frac{G M_E M_A}{R^2} \rightarrow ①$$

Here,

M_A - Mass of the apple.

M_E - Mass of Earth

R - Radius of Earth.

From, Newton's second law,

$$F = ma \rightarrow ②$$

sub. equ ① in ②.

$$M_A a_A = -\frac{G M_E M_A}{R^2}$$

$$a_A = -\frac{G M_E}{R^2} \rightarrow ③$$

Here,

a_A - acceleration of apple that is equal to g.

$$F = -\frac{G M_E M_m}{R_m^2} \rightarrow ④.$$

Here,

R_m - distance of the moon from Earth.

M_m - Mass of the Moon.

From, Newton's second law,

$$F = ma. \rightarrow ⑤.$$

sub. equ ⑤ in ④.

$$M_m a_m = -\frac{G M_E M_m}{R_m^2}$$

$$a_m = -\frac{G M_E}{R_m^2} \rightarrow ⑥.$$

Divide the equ. ③ & ⑥.

$$\frac{a_A}{a_m} = \frac{\frac{-G M_E}{R^2}}{\frac{-G M_E}{R_m^2}}$$

$$= \frac{-G M_E}{R^2} \times \frac{R_m^2}{-G M_E}$$

$$\frac{a_A}{a_m} = \frac{R_m^2}{R^2}$$

From, Hipparchus measurement, Moon is 60 times that of Earth radius.

$$R_m = 60R.$$

$$\frac{a_A}{a_m} = \frac{(60R)^2}{R^2}$$

$$= \frac{3600 R^2}{R^2}$$

$$= 3600$$

∴ The apple acceleration is 3600 times than the acc. of Moon.

We know that,

Apple's acceleration $g = 9.8$ m/s²

$$\frac{a_A}{a_m} = \frac{9.8}{a_m}$$

We know that,

$a_m = 0.00272$

$$\frac{a_n}{a_m} = \frac{9.8}{0.00272}$$

$= 3600.$

$\therefore$ Law of gravitation is Proved.

Gravitational Constant:

$$F = -\frac{GM_E m}{R_E^2} \rightarrow ①.$$

Here,

M_E - mass of the earth

m - mass of the object

G - Gravitational constant

R_E - Radius of the earth.

From, Newton's second law,

$F = ma.$

Here, $a = g$

$F = mg. \rightarrow ②.$

sub. equ ② in ①.

$$\cancel{m}g = -\frac{GM_E \cancel{m}}{R_E^2}$$

$$g = -\frac{GM_E}{R_E^2}$$

$$-G = \frac{gR_E^2}{M_E}$$

$$G = \frac{gR_E^2}{M_E} \rightarrow ③.$$

From, Center of the Earth,

$$F = -\frac{GM_E M}{r^2} \rightarrow ④.$$

here,

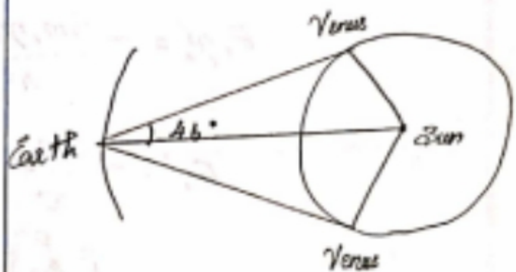
M = Mass of object distance from ...

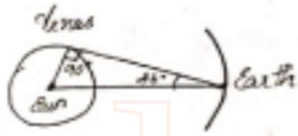
sub equ. ③ in ④.

$$F = -\frac{gR_E^2}{\cancel{M_E}} \cdot \frac{\cancel{M_E} M}{r^2}$$

$$F = -gM\frac{R_E^2}{r^2}$$

Kepler's third law and the Astronomical distance.





$$\sin\theta = \frac{r}{R}$$

$r = R\sin\theta$

here, $R = 1AU$

$r = (1AU)(\sin\theta)$

$\theta = 46°$

$r = (1AU)(\sin 46).$

$\sin 46° = 0.77$

$r = (0.77)(1AU)$

$r = 0.77AU$

$\therefore$ Venus at a distance of 0.77AU from sun.

$\therefore$ Mercury at a distance of 22.5° and sun is calculated as 0.38AU.

Gravitational field:

$$F_{21} = -\frac{Gm_1m_2}{r^2}\hat{r} \rightarrow (1).$$

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Here,

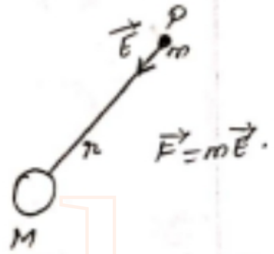
$$E_1 = \frac{F_{21}}{m_2}$$

$$F_{21} = E_1 m_2 \rightarrow (2).$$

sub. (2) in (1).

$$E_1 \cancel{m_2} = -\frac{Gm_1\cancel{m_2}}{r^2}\hat{r}.$$

$$E_1 = -\frac{Gm_1}{r^2}\hat{r} \rightarrow (3).$$



The gravitational force experienced by m due to it is

$$F_m = m\vec{E} \rightarrow (4).$$

we know that,

By newton's second law,

$$F = m\vec{a} \rightarrow (5).$$

from equ (4) & (5).

$$\cancel{m}\vec{a} = \cancel{m}\vec{E}$$

$$\boxed{\vec{a} = \vec{E}.}$$

Points to notation:

(I) The unit of gravitational field is Newton per kilogram (N/Kg) or ms^{-2}.

Superposition principle for gravitational field:

The total gravitational field at a point P due to all the mass is given by the vector sum of the gravitational field due to the individual ... principle is known as superposition of gravitational fields.

$m_1, m_2, \ldots m_n$ - 'n' particles of masses.

$r_1, r_2, r_3, r_4 \ldots$ - position of masses.

$$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2 + \ldots \vec{E}_n$$

$$= -\frac{Gm_1}{r_1^2}\hat{r}_1 - \frac{Gm_2}{r_2^2}\hat{r}_2 - \ldots - \frac{Gm_n}{r_n^2}\hat{r}_n$$

$$= -\sum_{i=1}^{n} \frac{Gm_i}{r_i^2}\hat{r}_i$$

Example 6.3

Gravitational field due to m_1 at a point P is given by.

$$\vec{E}_1 = -\frac{Gm_1}{a^2}\hat{j}.$$

Gravitational field due to m_2 at a point P is given by.

$$\vec{E}_2 = -\frac{Gm_2}{a^2}\hat{i}$$

$$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2$$

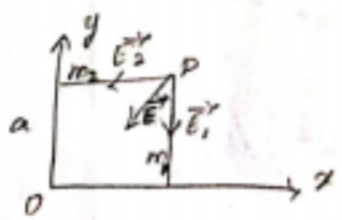
$$= -\frac{Gm_1}{a^2}\hat{j} - \frac{Gm_2}{a^2}\hat{i}.$$

$$= -\frac{G}{a^2}(m_1\hat{j} + m_2\hat{i}).$$

When, $m_1 = m_2 = m$.

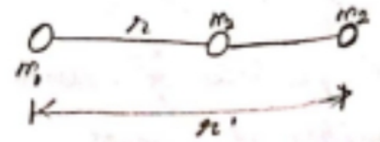
$$\vec{E}_{total} = -\frac{Gm}{a^2}(\hat{i}+\hat{j}).$$

$(\hat{i}+\hat{j} = \hat{j}+\hat{i}.)$ as vector obeys commutation laws.

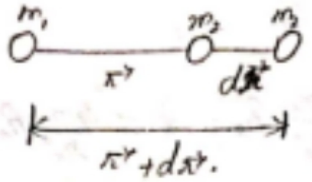


$\therefore$ The magnitude of $\vec{E}_{total} = \frac{Gm}{a^2}$.

Gravitational potential energy:



(a)



Work done,

$$dW = \vec{F}_{ext} \cdot d\vec{r} \rightarrow (1).$$

Work done against gravitational force.

$$|\vec{F}_{ext}| = |\vec{F}_G| = \frac{G m_1 m_2}{r^2}\hat{r} \rightarrow (2).$$

sub. equ (2) in (1).

$$dW = \frac{Gm_1m_2}{r^2}\hat{r}\cdot d\vec{r} \rightarrow (3).$$

We know that,

$$d\vec{r} = dr\,\hat{r} \rightarrow (4)$$

$$dW = \frac{Gm_1m_2}{r^2}\hat{r}\cdot dr\,\hat{r}. \rightarrow (5).$$

$\hat{r}\cdot\hat{r} = 1$ (since both are unit vectors).

$$dW = \frac{Gm_1m_2}{r^2}dr \rightarrow (6).$$

Integrating with r to r'.

$$W = \int_{r'}^{r} \frac{Gm_1m_2\,dr}{r^2} \rightarrow (7).$$

$$W = Gm_1m_2\int_{r'}^{r} \frac{1}{r^2}\,dr$$

$$= Gm_1m_2\int_{r'}^{r} r^{-2}\,dr.$$

$$= Gm_1m_2\left[\frac{r^{-2+1}}{-2+1}\right]_{r'}^{r}$$

$$W = Gm_1m_2\left[\frac{r^{-1}}{-1}\right]_{r'}^{r}$$

$$= -Gm_1m_2\left[+r^{-1}\right]_{r'}^{r}$$

$$= -Gm_1m_2\left[\frac{1}{r}\right]_{r'}^{r}$$

$$= -Gm_1m_2\left[\frac{1}{r}-\frac{1}{r'}\right].$$

$$W = -\frac{Gm_1m_2}{r} + \frac{Gm_1m_2}{r'}. \rightarrow (8)$$

$$W = U(r) - U(r').$$

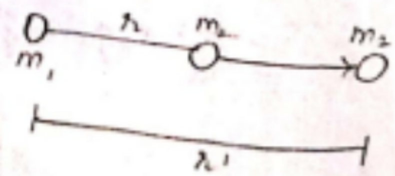
Where,

$$U(r) = -\frac{Gm_1m_2}{r}$$

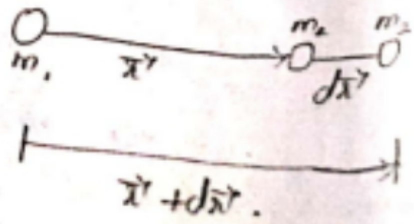
$$U(r') = \frac{Gm_1m_2}{r'}.$$

Case 1: If $r < r'$.

(i)



(b)



gravitational force is attractive, m_2 is attached by m_1. m_2 can remove from r to r'. Therefore, the work done is said to be negative.

Case 2: If $r > r'$.

$r' = \infty$. equ ③ becomes zero.

$$W = -\frac{Gm_1m_2}{r} + 0.$$

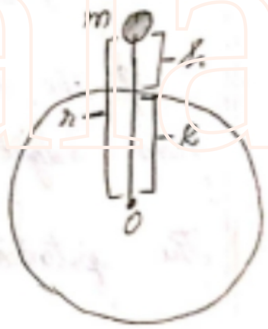
$$U(r) = -\frac{Gm_1m_2}{r}.$$

∴ The gravitational potential energy is always negative because when two masses come together and from infinity. work is done by the system.

Gravitational potential energy near the surface of the earth.

$h << R.$

$$U = -\frac{GM_em}{r} \rightarrow ①.$$



here,

$$r = R_e + h \rightarrow ②.$$

sub. equ ② in ①.

$$U = -\frac{GM_em}{(R_e+h)} \rightarrow ③.$$

here, $h << R$.

$$U = -\frac{GM_em}{R_e\left[1+\frac{h}{R_e}\right]}.$$

$$U = -\frac{GM_em}{R_e}\left(1+\frac{h}{R_e}\right)^{-1} \rightarrow ④.$$

By Binomial expansion

$$U = -\frac{GM_em}{R_e}\left[1-\frac{h}{R_e}\right] \rightarrow ⑤.$$

We know that,

$$mg = +\frac{GM_E m}{R_E^2}$$

$$mg = +\frac{GM_E m}{R_E R_E}.$$

$$R_E mg = +\frac{GM_E m}{R_E} \rightarrow (6).$$

sub. equ (6) in (5).

$$U = -mgR_E\left[1-\frac{h}{R_E}\right]$$

$$U = -mg\cancel{R_E}\left[\frac{R_E-h}{\cancel{R_E}}\right]$$

$$U = -mg(R_E-h).$$

$$U = -mgR_E + mgh \rightarrow (7).$$

If the object is taken from height h_1 to h_2.

The potential energy at h_1 is

$$U(h_1) = -mgR_E + mgh_1 \rightarrow (8).$$

The potential energy at h_2 is

$$U(h_2) = -mgR_E + mgh_2 \rightarrow (9).$$

Difference b/w the equ (9) & (8).

$$U(h_2) - U(h_1) = (-mgR_E + mgh_2) - (-mgR_E + mgh_1)$$

$$= -\cancel{mgR_E} + mgh_2 + \cancel{mgR_E} - mgh_1$$

$$= mgh_2 - mgh_1$$

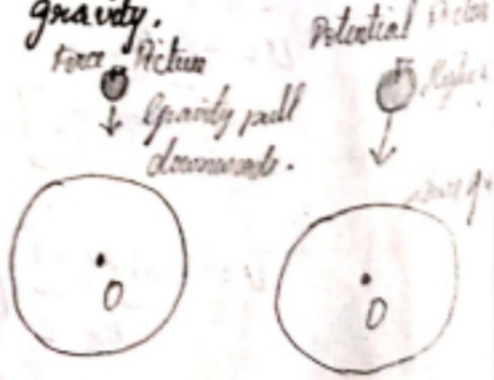
$$U(h_2) - U(h_1) = mg(h_2 - h_1).$$

Gravitational Potential $V(r)$:

The gravitational potential at a distance r due to a mass is defined as the amount of work required to bring unit mass from infinity to the distance r and is denoted as V. It's unit is Jkg^{-1}.

$$V(r) = -\frac{Gm_1}{r} \rightarrow (1).$$

Apple falling freeling under gravity.



Gravitational P.E $V(r)$ at point of height (h),
here, $r = (R+h)$.

$$V(R+h) = -\frac{GM_e}{(R+h)} \rightarrow ②.$$

Gravitational potential $V(r)$ on surface of Earth, $r = R$.

$$V(R) = -\frac{GM_e}{R} \rightarrow ③.$$

Compare equ ② & ③.

$$V(R) < V(R+h) \rightarrow ④.$$

The gravitational P.E at this point,

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$$V(h) = \frac{U(h)}{m}$$

here, $U(h) = mgh$

$$= \frac{mgh}{m}$$

$$V(h) = gh$$

Example: 6.6

$U(r)$ - Gravitational potential energy

$$U(r) = -\frac{Gm_1m_2}{r}$$

P.E for each pair of particles.

$$U = -\frac{Gm_1m_2}{r_{12}} - \frac{Gm_1m_3}{r_{13}} - \frac{Gm_1m_4}{r_{14}} - \frac{Gm_2m_3}{r_{23}} - \frac{Gm_2m_4}{r_{24}} - \frac{Gm_3m_4}{r_{34}}.$$

here, r_{12}, r_{13} ... are distance between pair of particles.

$$r_{14}^2 = R^2+R^2$$

$$r_{14}^2 = 2R^2$$

square on both sides

$$r_{14} = \sqrt{2R^2}$$

$$= \sqrt{2}R$$

$$r_{12} = r_{23} = r_{34}$$

$$r_{13} = r_{24} = 2R.$$

$$U = -\frac{Gm_1m_2}{\sqrt{2}R} - \frac{Gm_1m_3}{2R} - \frac{Gm_1m_4}{\sqrt{2}R} - \frac{Gm_2m_3}{\sqrt{2}R} - \frac{Gm_2m_4}{2R} - \frac{Gm_3m_4}{\sqrt{2}R}$$

$$U = -\frac{G}{R}\left[\frac{m_1m_2}{\sqrt{2}} + \frac{m_1m_3}{2} + \frac{m_1m_4}{\sqrt{2}} + \frac{m_2m_3}{\sqrt{2}} + \frac{m_2m_4}{2} + \frac{m_3m_4}{\sqrt{2}}\right]$$

here,

$$m_1 = m_2 = m_3 = m_4 = M.$$

$$U = -\frac{G}{R}\left[\frac{M^2}{\sqrt{2}} + \frac{M^2}{2} + \frac{M^2}{\sqrt{2}} + \frac{M^2}{\sqrt{2}} + \frac{M^2}{2} + \frac{M^2}{\sqrt{2}}\right].$$

$$= -\frac{GM^2}{R}\left[\frac{1}{\sqrt{2}} + \frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{2} + \frac{1}{\sqrt{2}}\right].$$

$$= -\frac{GM^2}{R}\left[\frac{4}{\sqrt{2}} + \frac{\cancel{2}}{\cancel{2}}\right]$$

$$= -\frac{GM^2}{R}\left[1 + \frac{4}{\sqrt{2}}\right].$$

$$U = -\frac{GM^2}{R}\left(1 + \frac{2\times 2}{\sqrt{2}}\right)$$

$$= -\frac{GM^2}{R}\left[1 + \frac{2\times\sqrt{2}\times\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}\right]$$

$$= -\frac{GM^2}{R}\left[1 + 2\sqrt{2}\right].$$

The gravitational P.E (V) at a point O.

$$V_0(r) = -\frac{Gm_1}{R} - \frac{Gm_2}{R} - \frac{Gm_3}{R} - \frac{Gm_4}{R}$$

If $m_1 = m_2 = m_3 = m_4 = M.$

$$V_0(r) = -\frac{GM}{R} - \frac{GM}{R} - \frac{GM}{R} - \frac{GM}{R}$$

$$V_0(r) = -\frac{4GM}{R}$$

Acceleration due to gravity of the earth.

$$\vec{F} = -\frac{GmM_e}{R_e^2}\hat{r} \rightarrow (1).$$

We know that Newton second law,

$$\vec{F} = m\vec{a} \rightarrow (2).$$

sub eq ② in ①.

$$m\vec{a} = -\frac{GmM_e}{R_e^2}\hat{r}.$$

$$\vec{a} = -\frac{GM_e}{R_e^2}\hat{r} \rightarrow ③.$$

here,

$\vec{a} = g$.

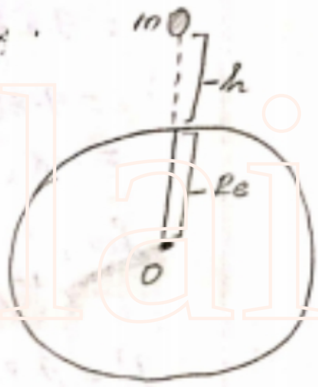
$$|g| = \frac{GM_e}{R_e^2} \rightarrow ④.$$

Define:

Acceleration due to gravity:

The acceleration experienced by the object near the surface of the earth due to its gravity is called acceleration due to gravity.

Variation of g with altitude.



$$g' = \frac{GM}{(R_e+h)^2} \rightarrow ⑤. \qquad g' = \frac{GM}{R_e^2+h^2}$$

$$g' = \frac{GM}{R_e^2\left(1+\frac{h}{R_e}\right)^2}$$

$$g' = \frac{GM}{R_e^2}\left[1+\frac{h}{R_e}\right]^{-2}$$

If $h \ll R_e$

Use Binomial expansion.

$$g' = \frac{GM}{R_e^2}\left(1-2\frac{h}{R_e}\right) \rightarrow ⑦.$$

here,

$$g = \frac{GM}{R_e^2} \rightarrow ⑥.$$

sub. ⑥ in ⑦.

$$g' = g\left(1 - \frac{2h}{R_e}\right)$$

Since $g' < g$. height increases g decreases

Example 6.7

1.(a) Given:

$h = 15\ m$

$g = ?$

$$g' = g\left(1 - 2\frac{h}{R_e}\right)$$

$$= 9.8\left(1 - \frac{2 \times 15}{6400 \times 10^3}\right).$$

$$= 9.8\left(1 - \frac{\cancel{2} \times 15}{64 \times 10^5}\right)$$

$$= 9.8\left(1 - \frac{15 \times 10^{-5}}{32}\right)$$

$$= 9.8(1 - 0.469 \times 10^{-5}).$$

$$= 9.8(1 - 0.00000469)$$

Since,

$$1 - 0.00000469 \cong 1$$

$$g' = 9.8(1)$$

$$g' = g.$$

(b). Given:

$h = 1600\ km$

$= 1600 \times 10^3.$

$$g' = g\left(1 - 2\frac{h}{R_e}\right)$$

$$= g\left(1 - \frac{2 \times 1600 \times 10^3}{6400 \times 10^3}\right)$$

$$= g\left(1 - \frac{1}{2}\right).$$

$$= g\left(1-\frac{1}{2}\right)$$

$$= g\left(\frac{2-1}{2}\right)$$

$$= g\left(\frac{1}{2}\right)$$

$$g' = \frac{g}{2}$$

Vibration of g with depth:

$$g' = \frac{GM'}{(R_e-d)^2} \rightarrow (1)$$

$g' = \frac{GM}{(R_e+h)^2}$

$g = \frac{GM'}{(R_e-d)^2}$

here, M' is the mass of the earth of radius (R_e-d).

density of the earth, $\rho = \frac{M}{V} \rightarrow (2)$.

Then, density of inner earth, $\rho = \frac{M'}{V'} \rightarrow (3)$.

from (2) and (3).

$$\frac{M'}{V'} = \frac{M}{V}$$

$V = \frac{4}{3}\pi R_e^3 \quad V' = \frac{4}{3}\pi (R_e-d)^3$

$$M' = \frac{M}{V} \cdot V'$$

$$M' = \left[\frac{M}{\frac{4}{3}\pi R_e^3}\right]\left[\frac{4}{3}\pi (R_e-d)^3\right]$$

$$M' = \frac{M}{R_e^3}(R_e-d)^3 \rightarrow (4).$$

sub. equ (4) in (1).

$$g' = \frac{GM}{R_e^3}\frac{(R_e-d)^{\cancel{3}}}{\cancel{(R_e-d)^2}}$$

$$= \frac{GM}{R_e^3}(R_e-d),$$

$$= \frac{GM}{R_e^{\cancel{3}}}\cancel{R_e}\left[1-\frac{d}{R_e}\right]$$

$$= \frac{GM}{R_e^2}\left(1-\frac{d}{R_e}\right).$$

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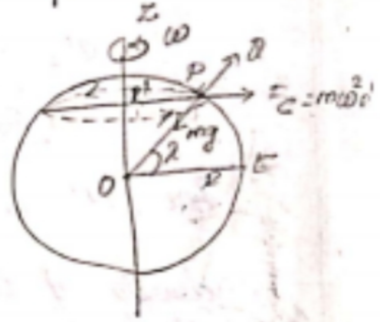
$$g' = \frac{GM\left(1-\frac{d}{R_e}\right)}{R_e^2}$$

$$= \frac{GM}{R_e^2}\left(1-\frac{d}{R_e}\right).$$

$$g' = g\left(1-\frac{d}{R_e}\right).$$ $g' < g$. As depth increases, g' decreases.

Variation of g with latitude:



The centrifugal force, $= m\omega^2 R'$

here,

$R' = R\cos\lambda \rightarrow (1)$.

where

λ - latitude.

$a = \omega^2 R$

The component of centrifugal acceleration } $a_{PQ} = \omega^2 R'\cos\lambda \rightarrow (2)$.

sub equ (1) in (2).

$a_{PQ} = \omega^2 R\cos\lambda\cos\lambda$

$a_{PQ} = \omega^2 R\cos^2\lambda$

Therefore,

$g' = g - \omega^2 R\cos^2\lambda \rightarrow (3)$.

from (3).

At equator $\lambda = 0$.

$g' = g - \omega^2 R\cos^2(0)$

$g' = g - \omega^2 R$

[acceleration due to gravity is minimum].

At poles: $\lambda = 90°$

then,

$g' = g - \omega^2 R(\cos^2 90°)$

$g' = g$. it is maximum

[At the equator. g' is ~~minimum~~].

$$g' = g - \omega^2 R\cos^2\lambda$$

Here,

$$\omega^2 R = \left(\frac{2\times 3.14}{86400}\right)^2 \times (6400\times 10^3)$$

$$= 3.4\times 10^{-2}\ ms^{-2}.$$

$$g' = 9.8 - (3.4\times 10^{-2})(\cos\lambda).$$

here,

$13^\circ = 0.2268$ rad.

$$g' = 9.8 - (3.4\times 10^{-2})(\cos 13^\circ)$$

$$g' = 9.8 - (3.4\times 10^{-2})\cos(0.9768)^2$$

$$g' = 9.7677\ ms^{-2}$$

Escape speed and orbital speed:

The initial total energy of the object, $E_i = \frac{1}{2}Mv_i^2 - \frac{GMM_E}{R_E} \rightarrow (1)$.

At infinity, the gravitational potential energy $U(\infty) = 0$

Then, $E_f = 0 \rightarrow (2)$.

from law of conservation of energy.

$$E_i = E_f \rightarrow (3)$$

sub. eqn (1) and (2) in (3).

$$\frac{1}{2}Mv_i^2 - \frac{GMM_E}{R_E} = 0.$$

$$\frac{1}{2}Mv_i^2 = \frac{GMM_E}{R_E} \rightarrow (4).$$

here, $v_i = v_e$

$$\frac{1}{2}Mv_e^2 = \frac{GMM_E}{R_E}$$

$$v_e^2 = \frac{GMM_E}{R_E}\cdot\frac{2}{M}.$$

$$v_e^2 = \frac{2GM_E}{R_E} \rightarrow (5).$$

We know that

$$g = \frac{GM_E}{R_E^2}$$

$$g = \frac{GM_E}{R_E \cdot R_E}$$

$$gR_E = \frac{GM_E}{R_E} \rightarrow (6).$$

sub. eqn (6) in (5)

$$v_e^2 = 2gR_E$$

$$v_e = \sqrt{2gR_E} \rightarrow (8).$$

sub. the value of $g = 9.8\ ms^{-2}$

$$R_e = 6400km$$

$$v_e = \sqrt{2 \times 9.8 \times 6400}$$

$$\frac{9.8 \times 2}{19.6}$$

$$v_e = \sqrt{19.6 \times 64 \times 10^2}$$

$$v_e = 11.2\ km s^{-1}$$

Satellites, orbital speed and time period.

Centripetal force = Earth's gravitational force

$$\frac{Mv^2}{(R_E+h)} = \frac{GMM_E}{(R_e+h)^2} \rightarrow (9).$$

$$v^2 = \frac{GMM_E}{(R_e+h)^2} \times \frac{(R_E+h)}{M}$$

$$v^2 = \frac{GM_E}{(R_e+h)}$$

$$v = \sqrt{\frac{GM_E}{(R_E+h)}} \rightarrow (10)$$

As h increases, the speed of the satellite decreases.

Time period of the satellite:

Circumference of satellite $= 2\pi(R_E+h)$.

speed, $v = \frac{\text{Distance travelled}}{\text{Time Taken}}$.

$$v = \frac{2\pi(R_E+h)}{T} \rightarrow (3).$$

sub. equ (3) in (2).

$$\sqrt{\frac{GM_E}{(R_E+h)}} = \frac{2\pi(R_E+h)}{T} \rightarrow (4).$$

$$T = \frac{2\pi(R_E+h)}{\sqrt{\frac{GM_E}{(R_E+h)}}}.$$

$$T = \frac{2\pi(R_E+h)}{\frac{\sqrt{GM_E}}{\sqrt{R_E+h}}}$$

$$T = \frac{2\pi(R_E+h)}{\frac{\sqrt{GM_E}}{(R_E+h)^{1/2}}}$$

$$T = \frac{2\pi(R_E+h)\cdot(R_E+h)^{1/2}}{\sqrt{GM_E}}$$

$$T = \frac{2\pi(R_E+h)^{1+1/2}}{\sqrt{GM_E}}$$

$$T = \frac{2\pi(R_E+h)^{3/2}}{\sqrt{GM_E}} \rightarrow (5).$$

squaring on both sides.

$$T^2 = \left(\frac{2\pi(R_E+h)^{3/2}}{\sqrt{GM_E}}\right)^2.$$

$$T^2 = \frac{4\pi^2 (R_E+h)^{3}}{GM_E}$$

$$T^2 = \frac{4\pi^2 (R_E+h)^3}{GM_E}$$

$$T = \sqrt{\frac{4\pi^2 (R_E+h)^3}{GM_E}}$$

$$\boxed{T = 2\pi \sqrt{\frac{(R_E+h)^3}{GM_E}}}$$

here,

$$\frac{4\pi^2}{GM_E} = \text{constant } (c).$$

$$T^2 = c\,(R_E+h)^3 \longrightarrow (5)$$

$$h \ll R_E.$$

$$T^2 = c \cdot R_E^3$$

here,

$$c = \frac{4\pi^2}{GM_E}$$

$$T^2 = \frac{4\pi^2}{GM_E} \cdot R_E^3$$

$$= \frac{4\pi^2}{GM_E} \cdot R_E^2 \cdot R_E$$

$$T^2 = \frac{4\pi^2}{\frac{GM_E}{R_E^2}} \cdot R_E$$

here, $\frac{GM_E}{R_E^2} = g$

$$T^2 = \frac{4\pi^2}{g} \cdot R_E$$

$$T = \sqrt{\frac{4\pi^2}{g} R_E}.$$

$$T = 2\pi \sqrt{\frac{R_E}{g}}$$

eq. sub. the value of, $R_E = 6.4 \times 10^6$ m.

$g = 9.8\ ms^{-2}$.

$$T = 2\pi \sqrt{\frac{6.4 \times 10^6}{9.8}}$$

$$T = 2\pi \sqrt{\frac{64 \times 10^5}{10}}$$

$$T = 2\pi \sqrt{8 \times 8 \times 10^5 \times 10^{-1}}$$

$$T = 2\pi \times 8 \sqrt{10^4}$$

$$= 2\pi \times 8 \times \sqrt{10^2 \times 10^2}$$

$$= 2\pi \times 8 \times 10^2$$

$$= 2 \times 3.14 \times 8 \times 10^2$$

$$= 50.24 \times 10^2$$

$$T = \frac{5024}{60}$$

$$\boxed{T = 83.7 \text{ minutes}}$$

Example 6.9

We know that,

Kepler's third law,

$$T^2 = c(R_E + h)^3$$

Taking cube root on both sides.

$$T^{2\times 1/3} = c^{1/3} \cdot (R_E + h)^{3 \times 1/3}$$

$$T^{2/3} = c^{1/3} (R_E + h)$$

$$\frac{T^{2/3}}{c^{1/3}} = R_E + h$$

$$\left(\frac{T^2}{c}\right)^{1/3} = R_E + h$$

We know that, $c = \frac{4\pi^2}{GM_E}$

$$\left(\frac{T^2}{\frac{4\pi^2}{GM_E}}\right)^{1/3} = R_E + h$$

$$\left(\frac{T^2 \cdot GM_E}{4\pi^2}\right)^{1/3} = R_E + h$$

$$h = \left(\frac{T^2 \cdot GM_E}{4\pi^2}\right)^{1/3} - R_E.$$

$h = \frac{(28320)^2 \cdot 6.67\times10^{-11}}{4\times 3.14 \times 3.4} - 6.4$

$= 28320 \times 6.67\times10^{-11} \times 10$

here,

h - distance of Moon from Earth.

Radius of earth, $R_E = 6.4 \times 10^6 m$

Mass of earth, $M_E = 6.02 \times 10^{24} Kg$.

Universal constant, $G = 6.67 \times 10^{-11} \frac{Nm^2}{Kg^2}$.

$T = 27 \times 24 \times 3600$

$= 28320$ min.

Energy of an orbiting satellite:

The potential energy of satellite,

$$U = -\frac{GM_s M_E}{(R_E + h)} \rightarrow (1).$$

here,

M_s - mass of satellite

M_E - mass of earth

R_E - radius of the earth.

The Kinetic energy of satellite,

$$K.E = \frac{1}{2} M_s V^2 \rightarrow (2).$$

here,

$$V = \sqrt{\frac{GM_E}{(R_E + h)}} \rightarrow (3).$$

sub. equ (3) in (2).

$$K.E = \frac{1}{2} \cdot M_s \left(\sqrt{\frac{GM_E}{R_E + h}}\right)^2$$

$$K.E = \frac{1}{2} \frac{GM_s M_E}{R_E + h}$$

$$K.E = \frac{1 (GM_s M_E)}{2(R_E + h)} \rightarrow (4).$$

Add the equ (4) & (1).

$$\text{Total energy, } E = \frac{1}{2} \frac{GM_E M_s}{(R_E + h)} + \left(-\frac{GM_s M_E}{R_E + h}\right).$$

$$= \frac{GM_E M_s}{2(R_E + h)} - \left(\frac{GM_s M_E}{R_E + h}\right).$$

$$= \frac{GM_E M_s}{R_E + h} \left[\frac{1}{2} - 1\right].$$

$$E = -\frac{1}{2} \frac{GM_E M_s}{R_E + h} \rightarrow (5).$$

Example 6.10

$$E_m = -\frac{G M_E M_m}{2(R_E+h)}.$$

Here, $(R_E+h) = R_m$

$$E_m = -\frac{G M_E M_m}{2 R_m}.$$

Where,

Mass of earth, $M_E = 6.02 \times 10^{24}$ kg.

Mass of moon, $M_m = 7.35 \times 10^{22}$ kg.

Distance b/w moon from earth } $R_m = 3.84 \times 10^5$ km. $= 3.84 \times 10^5 \times 10^3$ m.

$G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}.

$$E_m = -\frac{6.67\times10^{-11} \times 6.02\times10^{24} \times 7.35\times10^{22}}{2\times3.84\times10^5 \times10^3}$$

$$= \frac{-6.67\times6.02\times7.35\times10^{-11}\times10^{46}}{2\times3.84\times10^8}$$

$$= \frac{-6.67\times6.02\times7.35\times10^{-11}\times10^{46}\times10^{-8}}{7.68}$$

$$= \frac{-6.67\times6.02\times7.35\times10^{-19}\times10^{46}}{7.68}$$

$$= -38.42\times10^{-19}\times10^{46}.$$

$$= -38.42\times10^{46} \text{ joule}.$$

Geo - stationary and polar satellite :

From Kepler's third law,

$$T^2 = \frac{4\pi^2}{G M_E}(R_E+h)^3$$

$$(R_E+h)^3 = \frac{G M_E T^2}{4\pi^2}$$

Take cube root on both sides

$$(R_E+h) = \left(\frac{G M_E T^2}{4\pi^2}\right)^{1/3}$$

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$$R_E + h = \left(\frac{GM_E T^2}{4\pi^2}\right)^{1/3}.$$

$$h = \left(\frac{GM_E T^2}{4\pi^2}\right)^{1/3} - R_E$$

sub. time period, $T = 24$ hours $= 86400$ sec.

$h. = 36,000$ km

Apparent weight in elevator:

When a man standing in the elevator, there are two forces acting on him.

(i) gravitational force, $F = -mg\hat{j} \rightarrow (1)$.

(ii) The normal force exerted by the floor, $\vec{N} = N\hat{j} \rightarrow (2)$.

Case (i) when the elevator at rest:

here, the acceleration of the man is zero.

By newton's second law,

$$\vec{F_G} + \vec{N} = 0.$$

sub. equ (1) & (2).

$$-mg\hat{j} + N\hat{j} = 0.$$

By comparing the components,

$$N - mg = 0.$$

$$N = mg \rightarrow (3).$$

Since,

Weight, $W = N$.

the apparent weight of the man is equal to his actual weight.

Case(ii) When the elevator is moving uniformly in the upward (or) downward direction.

In uniform motion, the net force acting on the man is still zero.

Hence,

the apparent weight of man equal to his actual weight.

Case(iii) When the elevator is accelerating upwards.

Acceleration, $\vec{a} = +a\hat{j}$.

By newton's second law,

$$\vec{F}_G + \vec{N} = m\vec{a}.$$

$$-mg\hat{j} + N\hat{j} = -ma\hat{j}.$$

By comparing the components.

$$-mg + N = ma$$
$$N = mg + ma$$
$$N = m(g+a)$$

$\therefore$ Apparent weight of man is greater than his actual weight.

Case(iv) When the elevator is accelerating downwards.

Acceleration, $\vec{a} = -a\hat{j}$.

By newton's second law.

$$\vec{F}_G + \vec{N} = m\vec{a}.$$

$$-mg\hat{j} + N\hat{j} = -ma\hat{j}.$$

By comparing the components.

$$-mg + N = -ma$$
$$N = mg - ma$$
$$N = m(g-a).$$

$\therefore$ Apparent weight, $W = N = m(g-a)$. of man is lesser than his actual weight.

Weightlessness of freely falling bodies:

The normal force acting on the object is zero.

here,

$a = g. \rightarrow (1)$

We know that,

$N = m(g-a) \rightarrow (2)$.

sub. equ (1) in (2).

$N = m(g-g)$.

$N = m(0)$

$N = 0$.

Retrograde motion:

The planets moves eastwards and reverse the motion for a while and return to eastward motion again. This is called retrograde motion of planets.

Interesting Astronomical Facts:

The apparent radius of earth's umbra shadow,

$R_s = 13.2 cm$.

Apparent radius of moon, $R_m = 5.15 cm$.

The ratio, $\frac{R_s}{R_m} = \frac{13.2 \times 100}{5.15 \times 100}$

$= \frac{1320}{515}$

$\simeq 2.56$

The radius of the earth's umbra shadow,

$R_s = 2.56 \times R_m$.

The radius of moon, $R_m = 1737 \ Km$.

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The radius of the earth's umbra shadow.

$$R_S = 2.56 \times 1737$$

$$= 4446.72$$

$$\cong 4446 \text{ km}.$$

The correct radius is 4610 km.

$$\text{Percentage error} = \frac{4610 - 4446}{4610} \times 100$$

$$= \frac{164}{4610} \times 100$$

$$= 0.35 \times 10$$

$$= 3.5\%.$$

1737 × 256

10422
8685
3474
444672

4610
4446
164

$\frac{164}{4610} \times 100$

461) 1640 (.35
1383
2570