Exercise (8.4)

2 mark questions:

1. If $P(A) = \frac{2}{3}$, $P(B) = \frac{2}{5}$, $P(A \cup B) = \frac{1}{3}$ then find $P(A \cap B)$.

Solution:

Given: $P(A) = \frac{2}{3}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{1}{3}$

To find: $P(A \cap B)$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{2}{3} + \frac{2}{5} - \frac{1}{3}$$

$$\begin{array}{r|l} 3 & 3, 5, 3 \\ 5 & 1, 5, 1 \\ & 1, 1, 1 \end{array}$$

LCM = 15

$$P(A \cap B) = \frac{(2 \times 5) + (2 \times 3) - (1 \times 5)}{15}$$

$$= \frac{10 + 6 - 5}{15}$$

$$= \frac{16 - 5}{15}$$

$$\boxed{P(A \cap B) = \frac{11}{15}}$$

2. A and B are two events such that $P(A) = 0.42$, $P(B) = 0.48$, $P(A \cap B) = 0.16$. Find i) $P(\text{not } A)$ ii) $P(\text{not } B)$ iii) $P(A \text{ or } B)$.

Solution:

Given: $P(A) = 0.42$, $P(B) = 0.48$, $P(A \cap B) = 0.16$

To find: i) $P(\text{not } A) = P(\bar{A})$

$P(A) + P(\bar{A}) = 1$
$\therefore P(\bar{A}) = 1 - P(A)$

$$= 1 - P(A)$$

$$= 1 - 0.42$$

$$\boxed{P(\text{not } A) = 0.58}$$

ii) $P(\text{not } B) = P(\bar{B})$

$P(B) + P(\bar{B}) = 1$
$* \; P(\bar{B}) = 1 - P(B)$

$$= 1 - P(B)$$

$$= 1 - 0.48$$

$$\boxed{P(\text{not } B) = 0.52}$$

iii) $P(A \text{ or } B) = P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B)$$

$$= 0.42 + 0.48 - 0.16$$

$$= 0.90 - 0.16$$

$$\begin{array}{r} 0.90 \\ -\,0.16 \\ \hline +\,0.74 \end{array}$$

$$\boxed{P(A \text{ or } B) = 0.74}$$

③. If A and B are two mutually exclusive events of a random experiment and $P(\text{not } A) = 0.45$, $P(A\cup B) = 0.65$ then find $P(B)$.

Solution:

Given: $P(\text{not } A) = P(\bar{A}) = 0.45$, $P(A\cup B) = 0.65$

To find: $P(B)$.

$P(\bar{A}) = 0.45$ $\quad [P(\bar{A}) = 1 - P(A)]$

$1 - P(A) = 0.45$

$1 - 0.45 = P(A)$.

$\boxed{P(A) = 0.55}$

$$\begin{array}{r} 1.00 \\ -0.45 \\ \hline +0.55 \\ \hline \end{array}$$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$.

[Given that A and B are mutually exclusive events. $\therefore P(A\cap B) = 0$.]

$\therefore P(A\cup B) = P(A) + P(B) - 0$

$0.65 = 0.55 + P(B)$

$P(B) = 0.65 - 0.55$

$\boxed{P(B) = 0.10}$

$$\begin{array}{r} 0.65 \\ -0.55 \\ \hline +0.10 \\ \hline \end{array}$$

④. The probability that atleast one of A and B occur is 0.6. If A and B occur simultaneously with probability 0.2, then find $P(\bar{A}) + P(\bar{B})$.

Solution:

Given: $P(A\cup B) = 0.6$, $P(A\cap B) = 0.2$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$0.6 = P(A) + P(B) - 0.2$

$0.6 + 0.2 = P(A) + P(B)$.

$\therefore \boxed{P(A) + P(B) = 0.8}$

To find: $P(\bar{A}) + P(\bar{B})$.

$P(\bar{A}) + P(\bar{B}) = [1 - P(A)] + [1 - P(B)]$

$= 1 - P(A) + 1 - P(B)$

$= 2 - P(A) - P(B)$

[' $-$ ' is common. Taking out ' $-$ ' we get]

$= 2 - (P(A) + P(B))$

$= 2 - 0.8$

$\boxed{P(\bar{A}) + P(\bar{B}) = 1.2}$

$$\begin{array}{r} 2.0 \\ -0.8 \\ \hline +1.2 \\ \hline \end{array}$$

5. The probability of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events then find the probability that neither A nor B happen.

Solution: $P(A) = 0.5$, $P(B) = 0.3$.

Given that A and B are mutually exclusive events $\therefore P(A \cap B) = 0$.

$$P(\text{neither } A \text{ nor } B) = P(\bar{A} \cap \bar{B})$$
$$= P(\overline{A \cup B})$$
$$= 1 - P(A \cup B).$$
$$= 1 - [P(A) + P(B) - P(A \cap B)]$$
$$= 1 - [P(A) + P(B) - 0]$$
$$= 1 - [P(A) + P(B)]$$
$$= 1 - [0.5 + 0.3]$$
$$= 1 - 0.8$$

$$\boxed{P(\text{neither } A \text{ nor } B) = 0.2}$$

6. Two dice are rolled once. Find the probability of getting an even number on the first die or a total of face sum 8.

Solution: Given: Two dice.

$$S = \left\{ \begin{array}{l} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{array} \right\}$$

$n(S) = 36$.

Let A be the event of getting even number of first die.

$$A = \left\{ \begin{array}{l} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),3, \quad (4,4),(4,5)(4,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{array} \right\}$$

$n(A) = 18$

$$P(A) = \frac{n(A)}{n(S)} = \frac{18}{36}$$

$$\boxed{P(A) = \frac{18}{36}}$$

Let B = Total of face sum as 8.

$B = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$

$n(B) = 5$. $P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$.

$\therefore \boxed{P(B) = \frac{5}{36}}$

$A \cap B = \{(2,6), (4,4), (6,2)\}$.

$n(A \cap B) = 3$.

$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{36}$.

$\therefore \boxed{P(A \cap B) = \frac{3}{36}}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36}$

$= \frac{18+5}{36} - \frac{3}{36}$

$= \frac{23}{36} - \frac{3}{36} = \frac{23-3}{36}$

$= \frac{\cancel{20}^{5}}{\cancel{36}_{9}}$

$\boxed{P(A \cup B) = \frac{5}{9}}$

7. From a well-shuffled pack of 52 cards a card is drawn at random. Find the probability of it being either a red king or a black queen.

Solution: Given : $n(S) = 52$.

No. of Red cards = 26	No. of Black Cards = 26
Red king cards = 2	Black queen cards = 2
No. of Red king cards $n(K) = 2$	No. of black queen cards $n(Q) = 2$

$P(K) = \frac{n(K)}{n(S)} = \frac{2}{52}$ $P(Q) = \frac{n(Q)}{n(S)} = \frac{2}{52}$

$\therefore \boxed{P(K) = \frac{2}{52}}$ $\boxed{P(Q) = \frac{2}{52}}$

$P(K \cup Q) = P(K) + P(Q) - P(K \cap Q)$.

(Since K and Q are mutually exclusive) $[\therefore P(K \cap Q) = 0]$

$P(K \cup Q) = P(K) + PQ - 0$

$= \frac{2}{52} + \frac{2}{52} = \frac{\cancel{4}^{1}}{\cancel{52}_{13}}$

$\boxed{P(K \cup Q) = \frac{1}{13}}$

8) A box contains cards numbered 3, 5, 7, 9, -----, 35, 37. A card is drawn at random from the box. Find the probability that the drawn card have either multiples of 7 or a prime number.

Solution: Sample space.

$S = \{3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37\}$

$\boxed{n(S) = 18}$

Let $A \Rightarrow$ multiple of 7.

$A = \{7, 21, 35\}$ $n(A) = 3$

$\therefore \boxed{P(A) = \frac{3}{18}}$

Let $B \Rightarrow$ prime number cards.

$B = \{3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37\}$

$n(B) = 11$

$\boxed{P(B) = \frac{11}{18}}$

$\therefore A \cap B = \{7\}$. $n(A \cap B) = 1$.

$\therefore P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{18}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{3}{18} + \frac{11}{18} - \frac{1}{18}$

$= \frac{3+11-1}{18} = \frac{14-1}{18}$

$\boxed{P(A \cup B) = \frac{13}{18}}$

9). Three unbiased coins are tossed once. Find the probability of getting atmost 2 tails or atleast 2 heads.

Solution: Sample Space.

$S = \{HHH, HHT, HTH, THH, TTH, HTT, THT, TTT\}$

$n(S) = 8$

Let $A \Rightarrow$ atmost 2 tails.

$A = \{HHH, HHT, HTH, THH, TTH, HTT, THT\}$.

$n(A) = 7$ $P(A) = \frac{n(A)}{n(S)} = \frac{7}{8}$

$\therefore \boxed{P(A) = \frac{7}{8}}$

Let $B \Rightarrow$ atleast 2 heads.

$B = \{HHH, THH, HHT, HTH\}$.

$n(B) = 4$. $\boxed{P(B) = \frac{4}{8}}$

$\therefore A \cap B = \{HHH, HHT, HTH, THH\}$.

$n(A \cap B) = 4$. $\boxed{P(A \cap B) = \frac{4}{8}}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{7}{8} + \cancel{\frac{4}{8}} - \cancel{\frac{4}{8}}$

$\boxed{P(A \cup B) = \frac{7}{8}}$

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(10). The probability that a person will get an electrification contract is $\frac{3}{5}$ and the probability that he will not get plumbing contract is $\frac{5}{8}$. The probability of getting atleast one contract is $\frac{5}{7}$. what is the probability that he will get both.

Solution: Given: $P(A) = \frac{3}{5}$, $P(\bar{B}) = \frac{5}{8}$, $P(A \cup B) = \frac{5}{7}$

Tofind: The probability of getting both the contracts $P(A \cap B) = $ —

Given that $P(\bar{B}) = \frac{5}{8}$.

$P(B) + P(\bar{B}) = 1$

$P(B) = 1 - P(\bar{B})$

$P(B) = 1 - \frac{5}{8}$

$\boxed{P(B) = \frac{3}{8}}$

Tofind: $P(A \cap B)$.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$P(A \cap B) = \frac{3}{5} + \frac{3}{8} - \frac{5}{7}$

5, 8, 7
LCM = 5 × 8 × 7
LCM = 280

$= \frac{(3 \times 8 \times 7) + (3 \times 5 \times 7) - (5 \times 5 \times 8)}{280}$

$P(A \cap B) = \frac{168 + 105 - 200}{280}$

$P(A \cap B) = \frac{273 - 200}{280}$

$\boxed{P(A \cap B) = \frac{73}{280}}$

(11) In a town of 8000 people, 1300 are over 50 years and 3000 are females. Its known that 30% of the females are over 50 years. what is the probability that a chosen individual from the town is either a female or over 50 years?

Solution:

Let $A \Rightarrow$ Female, $B \Rightarrow$ over 50 years.

Given: $n(S) = 8000$, $n(A) = 3000$, $n(B) = 1300$

$n(A\cap B) = 30\%$ of 3000 are over 50 years.

$$= \frac{3\cancel{0}}{10\cancel{0}} \times 300\cancel{0}$$

$$= 3 \times 300$$

$$\boxed{n(A\cap B) = 900}$$

$P(A) = \frac{3000}{8000}$ | $P(B) = \frac{1300}{8000}$ | $P(A\cap B) = \frac{900}{8000}$

$$\therefore P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$= \frac{3000}{8000} + \frac{1300}{8000} - \frac{900}{8000}$$

3000 + 1300 = 4300; 4300 − 900 = 3400

$$= \frac{3000+1300-900}{8000} = \frac{340\cancel{0}}{800\cancel{0}}$$

$$P(A\cup B) = \frac{\overset{17}{\cancel{34}}}{\underset{40}{\cancel{80}}}.$$

$$\therefore \boxed{P(A\cup B) = \frac{17}{40}}$$

(12). A coin is tossed thrice. Find the probability of getting exactly two heads or atleast one tail or two consective heads.

Solution:

$S = \{HHH, HHT, HTH, THH, THT, TTH, HTT, TTT\}$

$$\boxed{n(S) = 8}$$

A = Exactly two heads	B = Atleast one tail	C = consecutive Two heads.
$A = \{HHT, HTH, THH\}$ $n(A) = 3$ $\boxed{P(A) = \frac{3}{8}}$	$B = \{HHT, THH, HTH, THT, TTH, HTT, TTT\}$ $n(B) = 7$ $\boxed{P(B) = \frac{7}{8}}$	$C = \{HHH, HHT, THH\}$ $n(C) = 3$ $\boxed{P(C) = \frac{3}{8}}$
$A\cap B = \{HHT, HTH, THH\}$ $n(A\cap B) = 3$ $P(A\cap B) = \frac{3}{8}$	$B\cap C = \{HHT, THH\}$ $n(B\cap C) = 2$ $P(B\cap C) = \frac{2}{8}$	$C\cap A = \{HHT, THH\}$ $n(C\cap A) = 2$ $P(C\cap A) = \frac{2}{8}$

$A\cap B\cap C = \{HHT, THH\}$ $\therefore n(A\cap B\cap C) = 2$

$P(A\cap B\cap C) = \frac{2}{8}$.

$$\therefore P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$$

$$= \frac{3}{8} + \frac{7}{8} + \cancel{\frac{3}{8}} - \cancel{\frac{3}{8}} - \frac{2}{8} - \cancel{\frac{2}{8}} + \cancel{\frac{2}{8}}$$

$$= \frac{3+7-2}{8} = \frac{\overset{1}{\cancel{8}}}{\cancel{8}} = 1$$

$$\boxed{P(A\cup B\cup C) = 1}$$

(13). Given: $P(B) = 2P(A)$, $P(C) = 3P(A)$.

$P(A\cap B) = \frac{1}{6}$, $P(B\cap C) = \frac{1}{4}$, $P(A\cap C) = \frac{1}{8}$.

$P(A\cap B\cap C) = \frac{1}{15}$, $P(A\cup B\cup C) = \frac{9}{10}$.

To find: $P(A)$, $P(B)$ and $P(C)$ ...

$$P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C) + P(A\cap B\cap C)$$

$$\frac{9}{10} = P(A)+2P(A)+3P(A)-\frac{1}{6}-\frac{1}{4}-\frac{1}{8}+\frac{1}{15}$$

$$\frac{9}{10} = 6P(A)-\frac{1}{6}-\frac{1}{4}-\frac{1}{8}+\frac{1}{15}$$

$$\frac{9}{10} = 6P(A)-\frac{1}{6}-\left(\frac{4+1}{8}\right)+\frac{1}{15}$$

$$\frac{9}{10} = 6P(A)-\frac{1}{6}-\frac{3}{8}+\frac{1}{15}$$

$$6P(A) = \frac{9}{10}-\frac{1}{15}+\frac{1}{6}+\frac{3}{8}$$

$$= \left(\frac{27-2}{30}\right)+\frac{1}{6}+\frac{3}{8}$$

$$6P(A) = \frac{\cancel{25}^{5}}{\cancel{30}_{6}}+\frac{1}{6}+\frac{3}{8}$$

$$6P(A) = \left(\frac{5}{6}+\frac{1}{6}\right)+\frac{3}{8}$$

$$6P(A) = \frac{\cancel{6}}{\cancel{6}}+\frac{3}{8}$$

$$6P(A) = 1+\frac{3}{8}$$

$$6P(A) = \frac{8+3}{8}$$

* $P(A) = \frac{11}{6\times 8} \Rightarrow$ * $\boxed{P(A) = \frac{11}{48}}$

* $P(B) = 2P(A) = 2\times\frac{11}{48}$

$P(B) = \frac{\cancel{2}\times 11}{\cancel{48}_{24}}$ * $\boxed{P(B) = \frac{11}{24}}$

* $P(C) = 3\times P(A) = \cancel{3}\times\frac{11}{\cancel{48}_{16}}$

* $\boxed{P(C) = \frac{11}{16}}$

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