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10th Std Maths

1. Points to be familiarized by the10th Students

2. Unit Exercise Solutions for 1 - 3

வைத்ததொரு கல்வி மனப்பழக்கம் – ஔவையார்

Wish you all the Best

Each and Every 10th Student Must familiar with the following Basic and Essential Concepts which have been already studied in the previous classes.

1. Clear understanding of the various numbers such as Natural ($\mathbb{N}$), Whole ($\mathbb{W}$), Integer ($\mathbb{Z}$), Rational ($\mathbb{Q}$), Irrational ($\mathbb{Q}'$), Real numbers($\mathbb{R}$) and the differences between them.

2. Also Odd number, Even number, Prime numbers and Composite numbers upto 100, Prime factors, Perfect square numbers (1,4,9,16,25, .. etc), Perfect cube numbers. (1, 8,27,64,125,..etc)

3. Shortcuts and BODMAS in $+, -, \times, \div$ for Quickness. i.e. 190 x 30 = 5700 etc

4. Knowing all the fractions (Proper, Improper, Mixed, Like, Unlike), shortcut to find LCM for it's operations. (For example LCM of 5 and 25 is 25 because 25 is divisible by 5. LCM of 11 and 12 is (11x12) = 132 because of consecutive numbers & also for consecutive odd numbers but this not applicable consecutive odd numbers and etc like this.)

5. Proportions, Ratios and Conversion of Ratios → Fraction → Percentage → Decimal etc.

6. Decimal numbers calculations and placing correct decimal point during multiplication.

7. Sharpness of placing (+, –) signs during fundamental operations. i.e. $(-2)^2 = 4$; $(-2)^3 = -8$ etc.

8. Divisibility checks for easy cut shorting the fractions. (For 2, 3, 4, 5, 6, 8, 9, 10, 11, etc)

9. Squares of numbers up to 20. Shortcut methods to find the squaring.

1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2	11^2	12^2	13^2	14^2	15^2	16^2	17^2	18^2	19^2	20^2
1	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400

35 x 35 =(3x4)(5x5) = 1225; 65 x 65 =(6x7)(5x5) = 4225; 105 x 105 =(10x11)(5x5) = 11025
$13^2 = 169$; $\therefore 130^2 = 16900$; $1300^2 = 1690000$; $600^2 = 360000$; $2500^2 = 6250000$
$20^2 = 400$; $\therefore 21^2 = 400 + (20+21) = 441$; $19^2 = 400 - (20+19) = 361$; $29^2 = 900 - (30+29) = 841$
$99^2 = 10000 - (100+99) = 9801$; $201^2 = 40000 + (200+201) = 40401$; Practice likewise.

10. Actual method of Square rooting the numbers of perfect squares and other numbers and decimals. As per (8) we can easily find out certain square roots. If the unit places are 1, 4, 5, 6, 9 and with ending 00, 0000 etc then it may be a perfect square (not sure). But If the unit places are 2, 3, 7, 8 and ending with 0, 000, 00000, then it will never be a perfect square. (Note : A shortcut to find out square root is attached. It is much useful for the 8th chapter.)

11. Knowing of $\sqrt{2} = 1.414$; $\sqrt{3} = 1.732$; $\sqrt{5} = 2.236$; $\sqrt{6} = 2.45$; $\sqrt{10} = 3.16$ etc will be better.

12. Similarly remember the cubes of numbers up to 10 and cube roots of it.

13. Surds rules like $\sqrt{6} = \sqrt{3 \times 2} = \sqrt{3} \times \sqrt{2}$; $8\sqrt{5} + 3\sqrt{5} = 11\sqrt{5}$; $5\sqrt{7} - 4\sqrt{7} = \sqrt{7}$ etc

14. Exponents rules such as : $a^m \, x \, a^n = a^{m+n}$; $\frac{a^m}{a^n} = a^{m-n}$; $(ab)^m = a^m \times b^m$; $a^0 = 1$
$a^m = \frac{1}{a^{-m}}$; $a^{-m} = \frac{1}{a^m}$; $a^{m^n} = a^{mn}$; $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ etc

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15. The Algebraic Identities (1). $(x+y)^2 = x^2 + 2xy + y^2$; (2). $(x-y)^2 = x^2 - 2xy + y^2$
(3). $x^2 - y^2 = (x+y)(x-y)$; (4). $(x+y)^3 = x^3 + 3xy(x+y) + y^3$ $(or) = x^3 + 3x^2y + 3xy^2 + y^3$
(5). $(x+y)^3 = x^3 - 3xy(x-y) + y^3$ $(or) = x^3 - 3x^2y + 3xy^2 - y^3$
(6). $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$; (7). $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$ are very important and practice it with left to right and right to left since both will be involved in the sums.

16. Well practice in the Factorisation of quadratic equations is also very important because it is invariably used almost in all the chapters.

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17. Daily before going to sleep, remember all the formulae involved in all the chapters for 10 mts.

18. For best result obey the 1st Teachers & 2nd Parents, because they will bless in mind and not by word. If anything left here and anything you forget in the above, clear it with the near & dear.

10 கணிதம் - புள்ளியியல்

வர்க்கமூலம் காண உதவும் சூத்திரம்..

- Very useful
- Simple
- Accurate.

$$\sqrt{x \pm y} = \sqrt{x} \pm \frac{y}{2\sqrt{x}}$$

THANKS TO...
MR. SREEDHARAN,
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① எடுத்துக்காட்டு - 8.4:

$\sqrt{8} = \sqrt{9-1}$

$\sqrt{x-y} = \sqrt{x} - \frac{y}{2\sqrt{x}}$

$x=9,\ y=1 \Rightarrow \sqrt{9-1} = \sqrt{9} - \frac{1}{2\sqrt{9}}$

$= 3 - \frac{1}{2\times3} = 3 - \frac{1}{6}$

$= 3 - 0.166 = 2.834$

Book Answer: $\sigma \simeq 2.83$

② எ.கா. 8.5:

$\sqrt{8.53} = \sqrt{9-0.47} = \sqrt{9} - \frac{0.47}{2\sqrt{9}}$ (x = 9) (y = 0.47)

$= 3 - \frac{0.47}{6}$

$= 3 - 0.078 = 2.92$

Book $\simeq$ 2.9

③ எ.கா: 8.6.

$\sqrt{44.49} = \sqrt{49 - 4.51} = 7 - \frac{4.51}{2\times7}$ (x = 49, y = 4.51)

$= 7 - 0.32 = 6.68$

Book $\simeq$ 6.67

④ எ.கா: 8.7

$\sqrt{5.5 - 0.25} = \sqrt{5.25} = \sqrt{4 + 1.25}$ (x = 4, y = 1.25)

$= 2 + \frac{1.25}{2\times2} = 2 + 0.31$

$\simeq 2.31$ Book. $\simeq$ 2.29

⑤ எ.கா.: 8.8.

$\sqrt{6} = \sqrt{4+2}$ (x = 4, y = 2)

$= 2 + \frac{2}{2\times2} = 2.5$

Book $\simeq$ 2.45

⑥ எ.கா. 8.9:

$\sqrt{5.2} = \sqrt{4+1.2} = 2 + \frac{1.2}{2\times2}$ (x = 4, y = 1.2)

$= 2 + 0.3 = 2.3$

Book $\simeq$ 2.28

⑦ எ.கா. 8.11:

$\sqrt{2.58} = \sqrt{4 - 1.42}$ (x = 4, y = 1.42)

$= 2 - \frac{1.42}{2\times2} = 2 - 0.355$

$= 1.645$ Book $\simeq$ 1.6

⑧ எ.கா: 8.13:

$\sqrt{2.779} = \sqrt{4 - 1.221} = 2 - \frac{1.221}{2\times2}$ (x = 4, y = 1.221)

$= 2 - 0.305 = 1.695$

Book: 1.667

⑨ எ.கா: 8.14:

$\sqrt{35} = \sqrt{36-1} = 6 - \frac{1}{2\times6}$ (x = 36, y = 1)

$= 6 - 0.083 = 5.917$

Book → 5.9

⑩ எ.கா: 8.17

$\sqrt{19.43 - 18.40} = \sqrt{1.03} = \sqrt{1 + 0.03}$ (x = 1, y = 0.03)

$= 1 + \frac{0.03}{2} = 1.015$ 1.01 Book

$\sqrt{26.29 - 18.40} = \sqrt{7.89} = \sqrt{9 - 1.11}$ (x = 9, y = 1.11)

$= 3 - \frac{1.11}{6} = 3 - 0.183$

$= 2.817$ Book: 2.81

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10th Maths Unit Exercise Chapter – 1

1. Given : $(x^2 - 3x, \ y^2 + 4y)$ and (-2, 5) are equal

$(x^2 - 3x, y^2 + 4y) = (-2, 5)$

i.e. $x^2 - 3x = -2$	$y^2 + 4y = 5$
$x^2 - 3x + 2 = 0$	$y^2 + 4y - 5 = 0$
$(x - 2)(x - 1) = 0$	$(y + 5)(x - 1) = 0$
$x = 1, 2$	$y = -5, 1$

2 . Given : n(A x A) = 9 Also two ordered pairs = (–1, 0) and (0,1)

n(A) x n(A) = 9 ∴n(A) = 3

From the given two ordered pairs = (–1, 0) and (0,1)

A = {–1, 0, 1} ; ∴ A x A = {–1, 0, 1} x {–1, 0, 1}

= {(–1, –1), (–1, 0), (–1, 1), (0, –1), (0, 0), (0, 1), (1, –1), (1, 0), (1, 1)}

3. Given : $f(x) = \begin{cases} \sqrt{x-1}, & x \geq 1 \\ 4, & x < 1 \end{cases}$

(i) f (0) = 4 〔$\because 0 < 1$, It is satisfy 2[nd] condition〕

(ii) f (3) = $\sqrt{3-1} = \sqrt{2}$ 〔$\because 3 \geq 1$ It is satisfy 1[st] condition〕

(iii) f (a +1) = $\sqrt{(a+1)-1} = \sqrt{a}$ 〔$\because 0+1 \geq 1$ It is satisfy 1[st] condition〕

4. Given : A = {9, 10, 11, 12, 13, 14, 15, 16, 17}

f (n) = the highest prime factor (Note : 1 is neither a prime nor a composite)

f (9) = (9 = 3 x 3), ∴the highest prime = 3

f (10) = (10 = 2 x 5), ∴the highest prime = 5

f (11) = It's prime number, (11 = 1x11) ∴the highest prime = 11

f (12) = (12 = 2 x 2 x 3), ∴the highest prime = 3

f (13) = It's prime number,(13 = 1x13) ∴the highest prime = 13

f (14) = (14 = 2 x 7), ∴the highest prime = 7

f (15) = (15 = 3 x 5), ∴the highest prime = 5

f (16) = (16 = 2 x 2 x 2 x 2), ∴the highest prime = 2

f (17) = It's prime number, (17 = 1x17) ∴the highest prime = 17

f = {(9, 3), (10, 5), (11, 11), (12, 3), (13, 13), (14, 7), (15, 5), (16, 2), (17, 17)}

Range f = {2, 3, 5, 7, 11, 13, 17}

5. Given : $f(x) = \sqrt{1 + \sqrt{1 - \sqrt{1 - x^2}}}$

When x = 0 ; $f(0) = \sqrt{1 + \sqrt{1 - \sqrt{1 - 0}}} = 1$

When x = 1 ; $f(1) = \sqrt{1 + \sqrt{1 - \sqrt{1 - 1}}} = \sqrt{2}$

When x = –1 ; $f(-1) = \sqrt{1 + \sqrt{1 - \sqrt{1 - 1}}} = 1$

When x = 2 ; $f(2) = \sqrt{1 + \sqrt{1 - \sqrt{1 - 4}}} = \sqrt{1 + \sqrt{1 - \sqrt{-3}}}$ is an imaginary

When x = –2 ; $f(-2) = \sqrt{1 + \sqrt{1 - \sqrt{1 - 4}}} = \sqrt{1 + \sqrt{1 - \sqrt{-3}}}$ is an imaginary

From the above, except (–1, 0, 1) the result for the other value of x become an imaginary one.

∴ The domain = {–1, 0, 1}

6. Given : f (x) = x^2 , g(x) = 3x and h(x) = x −2 ; To prove : $(f\ o\ g)\ o\ h = f\ o\ (g\ o\ h)$

$$(f\ o\ g) = f\left(g\,(x)\right) = f\,(3x) = (3x)^2 = 9x^2$$

$$(f\ o\ g) o\ h = f\ o\ g\,(h(x)) = f\ o\ g\,(x-2) = 9(x-2)^2 \text{ -------- ①}$$

$$(g\ o\ h) = g\left(h\,(x)\right) = g\,(x-2) = 3(x-2)$$

$$f\ o\,(g\ o\ h) = f(goh) = (3(x-2))^2 = 9(x-2)^2 \text{ ------- ②}$$

since ① = ② , $(f\ o\ g)\ o\ h = f\ o\ (g\ o\ h)$ (Proved)

7. This question is also given as multiple choice no. ③

Given : A = {1,2} ; B = {1,2, 3, 4} ; C = {5,6} ; D = {5, 6, 7, 8}; To show : $A \times C \subset B \times D$

A x C = {1, 2} x {5, 6} = {(1, 5), (1, 6), (2, 5), (2, 6)} ------- ①

B x D = {1,2, 3, 4} x {5, 6, 7, 8} = {(1, 5), (1, 6), (2, 5), (2, 6)}

= {(1, 5), 1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)} ------- ②

Comparing ① & ② : $A \times C \subset B \times D$ (Proved)

8. Given : If $f(x) = \frac{x-1}{x+1}$ show that $f\left(f(x)\right) = -\frac{1}{x}$

$$f(x) = \frac{x-1}{x+1}$$

$$f\left(f(x)\right) = \frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1}$$

$$= \frac{\frac{x-1-x-1}{x+1}}{\frac{x-1+x+1}{x+1}} = \frac{-2}{x+1} \times \frac{x+1}{2x}$$

$$= \frac{-2}{2x} = -\frac{1}{x} \text{ (Proved)}$$

9. Given : $f(x) = 6x+8$; $g(x) = \frac{x-2}{3}$

(i) $gg(x) = g\left(g(x)\right) = \frac{\frac{x-2}{3}-2}{3}$

$$= \frac{x-2-6}{3\times 3} = \frac{x-8}{9}$$

$$gg(x) = \frac{x-8}{9}$$

$$gg\left(\frac{1}{2}\right) = \frac{\frac{1}{2}-8}{9} = \frac{1-16}{2\times 9} = -\frac{15}{2\times 9} = -\frac{5}{6}$$

(ii) $gf(x) = g(f(x))$

$$= g(6x+8) = \frac{6x+8-2}{3} = \frac{6x+6}{3} = \frac{6(x+1)}{3} = 2(x+1)$$

10. (i). $f(x) = \frac{2x+1}{x-9}$ By seeing the denominator except x = 9 , the other values x are defined

∴ Domain of f = ℝ – { 9 } (Note : ℝ means Real number) (ii). $p(x) = \frac{-5}{4x^2+1}$ By seeing the denominator, all values x is defined

∴ Domain of p = $\mathbb{R}$

(iii). $g(x) = \sqrt{x-2}$ By seeing the Square root , when x < 2, it will become an imaginary.

∴ Domain of g = $[2, \infty)$

(ii). $h(x) = x + 6$ $h(x)$ is defined all values x

∴ Domain of h = $\mathbb{R}$

Unit Exercise Chapter – 2

1. To Prove : $n^2 - n$ divisible by 2 for every positive integer n.

$$n^2 - n = n(n-1)$$

Here, when n = Odd, n – 1 becomes even

when n = Even, n – 1 becomes odd

The product of one odd and one even is always an even number which is divisible by 2

∴ $n^2 - n$ divisible by 2 for every positive integer n.

2. Cow's milk = 175 litres ; Buffalow's milk = 105 litres

The milkman wants to them separately with equal sizes of can

∴ The size can is the HCF 175, 105

175 = 5 x 5 x 7 ; 105 = 3 x 5 x 7 ∴ The HCF = 5 x 7 = 35

(i) Capacity of a can = 35 litre

(ii) Number of cans of cow's milk : $\frac{175}{35} = 5$

(iii) Number of cans of buffalow's milk : $\frac{105}{35} = 3$

3. As per given codition,

When a is divisible by 13, the remainder is 9

∴ a ≡ 9 (mod 13) ------ ①

Similarly b ≡ 7 (mod 13) ------ ②

Similarly c ≡ 10 (mod 13) ------ ③

② x 2 → 2b ≡ 14 (mod 13) (Multiplication of Modulo arithmetic)

2b ≡ 1 (mod 13)

③ x 3 → 3c ≡ 30 (mod 13)

3c ≡ 4 (mod 13)

a + 2b + 3c ≡ (9 +1+4) (mod 13) (Addition of Modulo arithmetic)

a + 2b + 3c ≡ 14 (mod 13)

a + 2b + 3c ≡ 1 (mod 13)

∴ When (a + b + c) is divisible by 13 , the remainder is 1.

4. Let 107 = 4q + 3

107 – 3 = 4q

104 = 4q

∵ 104 is divisible by 4 for any integer q, 107 is of the form 4q + 3.

5. Let a and d be the 1st term and common difference of an AP

It's nth term $t_n = a + (n-1)d$

(m+1)th term $t_{m+1} = a + (m+1-1)d$

$= a + md$ ------ ①

(n+1)th term $t_{n+1} = a + (n+1-1)d$

$= a + nd$ ------ ②

② x 2 → $2(t_{n+1}) = 2[a + nd]$ ------ ③

From the condition ① = ③

$a + md = 2[a + nd]$ ------ ④

$t_{3m+1} = a + (3m+1-1)d$

$= a + 3md$

$= a + md + 2md$

$= 2(a + nd) + 2md$ 〔As per ④〕

$= 2(a + md + nd)$

$= 2[a + (m+n)d]$

$= 2[a + (m+n+1-1)d]$

$= 2t_{m+n+1}$

∴ (3m+1)th term = 2 x (m+n+1)th term

6. Given A. P = –2, –4, –6,... –100 ; It's 1st term a = – 2 ; d = – 2

By reversing the A.P = –100, –98, –96, …, –2 ; Now a = –100, d = 2

$t_n = a + (n-1)d$

12th term $t_{12} = -100 + (12-1)2$

$t_{12} = -100 + 22 = -78$

7. Given : AP1 AP2

	AP_1	AP_2
1st term	2	7

Common difference is same for both AP's

Difference of 1st terms of two AP's = 2 – 7 = – 5

Since the common difference is same for both, then

The Difference of any corresponding terms two AP's = – 5

∴ t_{10} of AP_1 – t_{10} of AP_2 = – 5

t_{21} of AP_1 – t_{21} of AP_2 = – 5

∴ t_n of AP_1 – t_n of AP_2 = – 5

8. Given : S_{10} = 16500,

Let the 1st year savings = a

The 2nd year savings = a + 100

The 3rd year savings = a + 100 + 100 = a + 200

It forms an AP with a common difference d = 100

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}[2a + (10-1)100] = 16500$$

$$5[2a + 900] = 16500$$

$$2a + 900 = \frac{16500}{5} = 3300$$

$$2a = 3300 - 900 = 2400$$

$$\therefore a = \frac{2400}{2} = 1200$$

1st year he saved Rs 1200

9. Given : 2nd term of a GP i.e. $ar = \sqrt{6}$

6th term of a GP i.e. $ar^5 = 9\sqrt{6}$

$$\frac{ar^5}{ar} = \frac{9\sqrt{6}}{\sqrt{6}}$$

$$r^4 = 9$$

$$r^2 = 3$$

$$r = \sqrt{3},\ -\sqrt{3}$$

① When $r = \sqrt{3}$, $ar = \sqrt{6}$; $a = \frac{\sqrt{6}}{r} = \frac{\sqrt{3}\times\sqrt{2}}{\sqrt{3}} = \sqrt{2}$

② When $r = -\sqrt{3}$, $ar = \sqrt{6}$; $a = \frac{\sqrt{6}}{r} = \frac{\sqrt{3}\times\sqrt{2}}{-\sqrt{3}} = -\sqrt{2}$

GP : $a,\ ar,\ ar^2, \ldots$

GP as per ① : $\sqrt{2},\ \sqrt{2}\times\sqrt{3},\ \sqrt{2}\times\sqrt{3}^2, \ldots$

$\sqrt{2},\ \sqrt{6},\ 3\sqrt{2}, \ldots$

GP as per ② : $-\sqrt{2},\ (-\sqrt{2})\times(-\sqrt{3}),\ (-\sqrt{2})\times(-\sqrt{3})^2, \ldots$

$-\sqrt{2},\ \sqrt{6},\ -3\sqrt{2}, \ldots$

10. Given : Value motor cycle (a) = ₹ 45000

Depriciation = 15%

To find : Value of the motor cycle after 3 years : n = 3

Depriciated value after 1 year = 45000 x (100 -15)% = 45000 x 85%

After 1 year $= 45000 \times \frac{85}{100}$

After 2 year $= 45000 \times \frac{85}{100} \times \frac{85}{100}$

After 3 year $= 45000 \times \frac{85}{100} \times \frac{85}{100} \times \frac{85}{100}$

= 27635.625

Value of the motor cycle after 3 years = ₹ 27635

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Unit Exercise Chapter – 3

(1) Taking First two

$\frac{1}{3}(x+y-5) = y-z$

$x+y-5 = 3y-3z$

$x+y-3y+3z = 5$

$x - 2y + 3z = 5$ —— (1)

Taking middle two

$y - z = 2x - 11$

$y = 2x + z - 11$ —— (2)

Put (2) in (1)

$x - 2(2x+z-11) - 3z = 5$

$x - 4x - 2z + 22 - 3z = 5$

$-3x + z = -17$ —— (3)

Taking Last two

$2x - 11 = 9 - (x+2z)$

$2x + x + 2z = 9 + 11$

$3x + 2z = 20$ —— (4)

$-3x + z = -17$ —— (3)

$3x + 2z = 20$ —— (4)

Adding: $0 + 3z = 3$

$z = \frac{3}{3} = 1$

From (4) $3x + 2\times1 = 20$

$3x = 18$

$x = \frac{18}{3} = 6$

Put $x = 6$, $z = 1$ in (2)

$y = 2\times6 + 1 - 11$

$= 13 - 11 = 2$

$x = 6, y = 6, z = 1$

(2) Let x, y, z be the Students in A, B, C.

As per conditions.

$x + y + z = 150$ —— (1)

$x - 6 = z + 6$

$x = z + 12$ —— (2)

$4z - x = y$

$-x - y + 4z = 0$ —— (3)

Putting (2) in (1)

$z + 12 + y + z = 150$

$y + 2z = 138$ —— (4)

Putting (2) in (3)

$-(z+12) - y + 4z = 0$

$-z - 12 - y + 4z = 0$

$-y + 3z = 12$ —— (5)

Adding (4) & (5) $5z = 150$, $\therefore z = 30$

$x = z + 12 = 30 + 12 = 42$

$y = 150 - (30 + 42) = 78$

Students in A = 42, B = 78, C = 30

(4) $xy(k^2+1) + k(x^2+y^2)$

$= k^2xy + xy + kx^2 + ky^2$

$= k^2xy + kx^2 + ky^2 + xy$

$= kx(ky+x) + y(ky+x)$

$= (kx+y)(ky+x)$

$xy(k^2-1) + k(x^2-y^2)$

$= k^2xy - xy + kx^2 - ky^2$

$= k^2xy + kx^2 - ky^2 - xy$

$= kx(ky+x) - y(ky+x)$

$= (kx-y)(ky+x)$

LCM of Both $= (ky+x)(kx+y)(kx-y)$

$= (ky+x)(k^2x^2 - y^2)$

$LCM = (ky+x)(k^2x^2-y^2)$

(3) Let x, y, z be the 100^{th}, 10^{th} & unit place.

$\therefore$ The number $= 100x + 10y + z$.

If 100^{th} & 10^{th} changed, then it is 54 more than thrice of original.

$100y + 10x + z = 3(100x + 10y + z) + 54$

$100y + 10x + z - 300x - 30y - 3z = 54.$

$-290x + 70y - 2z = 54$

÷ by 2 → $-145x + 35y - z = 27$ —— (1)

If the digits are reversed then, it is 198 of the original.

$100z + 10y + x = 100x + 10y + z + 198$

$100z + 10y + x - 100x - 10y - z = 198$

$-99x + 99z = 198$

÷ by 99 $-x + z = 2$

$z = x + 2$ —— (2)

As per 3^{rd} condition.

$y - x = 2(y - z)$

$y - x - 2y + 2z = 0$

$-x - y + 2z = 0$ —— (3)

Putting (2) in (1)

$-145x + 35y - (x+2) = 27$

$-145x + 35y - x - 2 = 27$

$-146x + 35y = 29$ —— (4)

Putting (2) in (3)

$-x - y + 2x + 4 = 0$

$x - y = -4$ —— (5)

$y = x + 4$

Putting (5) in (4)

$-146x + 35(x+4) = 29$

$-146x + 35x + 140$

$-111x = -111$

$x = 1.$

From (2) $z = 1 + 2 = 3$

From (5) $y = 1 + 4 = 5.$

The Number $= 100\times1 + 10\times5 + 3 = 153.$

(5) Let $f(x) = 2x^4+13x^3+27x^2+23x+7$

$g(x) = x^3+3x^2+3x+1$

$h(x) = x^2+2x+1$.

(i) $f(x) \div g(x)$

$$\begin{array}{r|l} & 2x+7 \\ x^3+3x^2+3x+1 & 2x^4+13x^3+27x^2+23x+7 \\ & 2x^4+6x^3+6x^2+2x \\ \hline & 7x^3+21x^2+21x+7 \\ & 7x^3+21x^2+21x+7 \\ \hline & 0 \end{array}$$

Since the remainder is zero, $g(x)$ is GCD of $f(x)$ and $g(x)$.

(ii) $g(x) \div h(x)$

$$\begin{array}{r|l} & x+1 \\ x^2+2x+1 & x^3+3x^2+3x+1 \\ & x^3+2x^2+x \\ \hline & x^2+2x+1 \\ & x^2+2x+1 \\ \hline & 0 \end{array}$$

Here also remainder $=0$

$\therefore$ The GCD $= x^2+2x+1$

(6) (i) $\dfrac{x^{3a}+8}{x^{2a}+2x^a+4} = \dfrac{(x^a)^3+2^3}{(x^a)^2+2x^a+4}$

$= \dfrac{(x^a+2)\left[(x^a)^2+2x^a+2^2\right]}{\left[(x^a)^2+2x^a+4\right]}$

$= x^a+2$

(ii) $= \dfrac{10x^3-25x^2+4x-10}{-4-10x^2}$

$= \dfrac{5x^2(2x-5)+2(2x-5)}{-2(5x^2+2)}$

$= \dfrac{(5x^2+2)(2x-5)}{-2(5x^2+2)}$

$= \dfrac{2x-5}{-2} = -x+\dfrac{5}{2}$

8. Arul, Ravi, Ram together complete the work in } 6 hr.

Their workmanship per hour $= \frac{1}{6}$

Let Arul complete it alone in x hr.

$\therefore$ Ravi " " " " $2x$ hr.

Ram " " " " $3x$ hr.

Their individual workmanship } $= \frac{1}{x}, \frac{1}{2x}, \frac{1}{3x}$

$\therefore \frac{1}{x}+\frac{1}{2x}+\frac{1}{3x} = \frac{1}{6}$

$\frac{6+3+2}{6x} = \frac{1}{6}$

$\frac{11}{6x} = \frac{1}{6}$; $\therefore x = \frac{6\times 11}{6} = 11$ hrs.

$\therefore$ Arul complete it in 11 hrs

Ravi " " " 22 hrs

Ram " " " 33 hrs

9

$$\begin{array}{r|l} & 17x^2-18x+19 \\ 17x^2 & 289x^4-612x^3+970x^2+684x+361 \\ & 289x^4 \\ \hline 34x^2-18x & -612x^3+970x^2 \\ & -612x^3+324x^2 \\ \hline 34x^2-36x+19 & 646x^2+684x+361 \\ & 646x^2+684x+361 \\ \hline & 0 \end{array}$$

$\therefore$ The Square root: $17x^2+18x+19$

(10) $\sqrt{y+1}+\sqrt{2y-5} = 3$

Squaring both sides,

$(\sqrt{y+1}+\sqrt{2y-5})^2 = 3^2$

$(\sqrt{y+1})^2+(\sqrt{2y-5})^2+2\sqrt{y+1}\times\sqrt{2y-5} = 9$

$y+1+2y-5+2\sqrt{(y+1)(2y-5)} = 9$

$3y-4+2\sqrt{2y^2-5y+2y-5} = 9$

$2\sqrt{2y^2-3y-5} = 13-3y$

Again squaring on both sides.

$(2\sqrt{2y^2-3y-5})^2 = (13-3y)^2$

$4(2y^2-3y-5) = 169+9y^2-78y$

$8y^2-12y-20 = 169+9y^2-78y$

$8y^2-12y-20-9y^2+78y-169 = 0$

$-y^2+66y-189 = 0$

$y^2-66y+189 = 0$

$(y-63)(y-3) = 0$

$y = 63, 3$

(11) Speed $= \frac{\text{Dist}}{\text{Time}}$; Time $= \frac{\text{Dist}}{\text{Speed}}$.

Distance $= 36$ Km.

Speed of water current $= 4$ Km/hr

Let the speed of the boat $= x$ Km/hr

$\therefore$ Nett speed along the current $= x+4$

Net speed opposite to the current $= x-4$

Difference time $= 1.6$ hr $= \frac{16}{10} = \frac{8}{5}$

$t_2-t_1 = \frac{8}{5}$

$\frac{36}{x-4}-\frac{36}{x+4} = \frac{8}{5}$

$36\left[\frac{1}{x-4}-\frac{1}{x+4}\right] = \frac{8}{5}$

$36\left[\frac{x+4-x+4}{(x-4)(x+4)}\right] = \frac{8}{5}$

$\frac{36\times 8}{x^2-16} = \frac{8}{5}$; $\therefore x^2-16 = 36\times 5$

$x^2 = 180+16 = 196$

$x = \pm 14$

Speed can not be negative.

$\therefore$ Speed of the boat $= 14$ Km/hr

(12) Rectangular park
Perimeter = 320 m : Area = 4800 m^2
Let l, b be the length and breadth
$\therefore 2(l+b) = 320$
$l + b = 160$ — ①
$l \times b = 4800$; $l = \frac{4800}{b}$ — ②
Placing ② in ①
$\frac{4800}{b} + b = 160$
× by b, $4800 + b^2 = 160b$
$b^2 - 160b + 4800 = 0$
$(b-120)(b-40) = 0$
$b = 120$ or 40

4800: −120 | −40; $(-120)+(-40) = -160$; $(-120)\times(-40) = 4800$

$\therefore$ Taking Breadth = 40 m
Length = 120 m.

(13) The time past after 2 pm and the time need to 3 pm is
$t + \frac{t^2}{4} - 3 = 60$ mts.
× by 4 ⇒ $4t + t^2 - 12 = 240$
$t^2 + 4t - 12 - 240 = 0$
$t^2 + 4t - 252 = 0$
$(t+18)(t-14) = 0$
$t = -18$ or 14

−252: +18 | −14; $18 + (-14) = 4$; $18 \times (-14) = -252$

$\because$ Time can not be negative,
$t = 14$ mts.

(14) Let the number of rows = x
As per condition, the number seats in a row also = x
$\therefore$ Total seats in the hall = $x \times x = x^2$
when row doubled = $2x$
Seat reduced to 5 = $x - 5$
As per the II[nd] condition,
$2x(x-5) = x^2 + 375$
$2x^2 - 10x = x^2 + 375$
$2x^2 - x^2 - 10x - 375 = 0$
$x^2 - 10x - 375 = 0$
$(x-25)(x+15) = 0$
$x = 25$ or -15

−375: −25 | +15; $(-25)+15 = -10$; $(-25)\times 15 = -375$

$\because x$ can not be a negative.
$\therefore$ The number of rows = 25
seats = 25

(16) Let $f(x) = x^2 + px - 4$
It has a root of (-4)
$f(-4) = (-4)^2 + p(-4) - 4 = 0$
$16 - 4p - 4 = 0$
$-4p = -12$
$p = \frac{-12}{-4} = -3$.

$g(x) = x^2 + px + q = 0$
It has equal roots.
For equal roots $b^2 - 4ac = 0$
Here $a = 1, b = p, c = q$
$p^2 - 4 \times 1 \times q = 0$
$(-3)^2 - 4q = 0$
$9 - 4q = 0$
$4q = 9$
$q = \frac{9}{4}$

(15) $f(x) = x^2 - 2x + 3$
Here $a = 1, b = -2, c = 3$
Sum of roots $\alpha + \beta = \frac{-b}{a} = \frac{-(-2)}{1} = 2$
Product of the roots $\alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$.

(i) $\alpha + 2, \beta + 2$
Sum of the roots $= \alpha + 2 + \beta + 2$
$= (\alpha + \beta) + 4$
$= 2 + 4 = 6$
Product of the roots $= (\alpha+2)(\beta+2)$
$= \alpha\beta + 2\alpha + 2\beta + 4$
$= \alpha\beta + 2(\alpha+\beta) + 4$
$= 3 + 2 \times 2 + 4 = 11$
The required equation,
$x^2 - (\text{Sum of roots})x + (\text{Product of roots}) = 0$
$x^2 - 6x + 11 = 0$.

(ii) $\frac{\alpha-1}{\alpha+1}, \frac{\beta-1}{\beta+1}$.
Sum of roots $= \frac{\alpha-1}{\alpha+1} + \frac{\beta-1}{\beta+1}$
$= \frac{(\alpha-1)(\beta+1) + (\beta-1)(\alpha+1)}{(\alpha+1)(\beta+1)}$
$= \frac{\alpha\beta + \alpha - \beta - 1 + \alpha\beta + \beta - \alpha - 1}{\alpha\beta + \alpha + \beta + 1}$
$= \frac{2\alpha\beta - 2}{\alpha\beta + (\alpha+\beta) + 1} = \frac{2\times 3 - 2}{3 + 2 + 1} = \frac{4}{6} = \frac{2}{3}$
Product of roots $= \frac{(\alpha-1)}{(\alpha+1)} \times \frac{(\beta-1)}{(\beta+1)} = \frac{\alpha\beta - \alpha - \beta + 1}{\alpha\beta + \alpha + \beta + 1}$
$= \frac{\alpha\beta - (\alpha+\beta) + 1}{\alpha\beta + (\alpha+\beta) + 1} = \frac{3 - 2 + 1}{3 + 2 + 1} = \frac{2}{6} = \frac{1}{3}$
$\therefore$ The eqn. : $x^2 - \left(\frac{2}{3}\right)x + \frac{1}{3} = 0$
⊗ by 3, $3x^2 - 2x + 1 = 0$

(17) April Sales $A = \begin{bmatrix} 500 & 1000 & 1500 \\ 2500 & 1500 & 500 \end{bmatrix}$

It is doubled in May.

$\therefore$ May sales : $2A = 2\begin{bmatrix} 500 & 1000 & 1500 \\ 2500 & 1500 & 500 \end{bmatrix}$

$\begin{bmatrix} 1000 & 2000 & 3000 \\ 5000 & 3000 & 500 \end{bmatrix}$

Average Sales $= \dfrac{A+2A}{2} = \dfrac{3}{2} \times A$

$= \dfrac{3}{2}\begin{bmatrix} 500 & 1000 & 1500 \\ 2500 & 1500 & 500 \end{bmatrix}$

$= \begin{bmatrix} 750 & 1500 & 2250 \\ 3750 & 2250 & 750 \end{bmatrix}$

April = A, May = 2A, June = 4A, July = 8A

$\therefore$ August $= 16A = 16\begin{bmatrix} 500 & 1000 & 1500 \\ 2500 & 1500 & 500 \end{bmatrix}$

$= \begin{bmatrix} 8000 & 16000 & 21000 \\ 40000 & 21000 & 8000 \end{bmatrix}$

(18) $LHS = \cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} x & -\cos\theta \\ \cos\theta & x \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{bmatrix} + \begin{bmatrix} x\sin\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & x\sin\theta \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta + x\sin\theta & 0 \\ 0 & \cos^2\theta + x\sin\theta \end{bmatrix} = I_2.$

$\therefore \begin{bmatrix} \cos^2\theta + x\sin\theta & 0 \\ 0 & \cos^2\theta + x\sin\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\cos^2\theta + x\sin\theta = 1$

$x\sin\theta = 1 - \cos^2\theta$

$x\sin\theta = \sin^2\theta$

$x = \dfrac{\sin^2\theta}{\sin\theta}$

$x = \sin\theta$

(19) $A = \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$

$BA = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -2q \\ p & 0 \end{bmatrix}$

$C^2 = C \times C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2\times2+(-2)\times2 & 2\times(-2)+(-2)\times2 \\ 2\times2+2\times2 & 2\times(-2)+2\times2 \end{bmatrix}$

$= \begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix}$

$BA = C^2 ; \begin{bmatrix} 0 & -2q \\ p & 0 \end{bmatrix} = \begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix}$

$p = 8, \quad -2q = -8$

$q = \dfrac{-8}{-2} = 4$

$p = 8, \ q = 4$

(20) $A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 6 & 3 \\ 8 & 5 \end{bmatrix}, C = \begin{bmatrix} 3 & 6 \\ 1 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} 3\times6+0\times5 & 3\times3+0\times5 \\ 4\times6+5\times8 & 4\times3+5\times5 \end{bmatrix} = \begin{bmatrix} 18 & 9 \\ 64 & 37 \end{bmatrix}$

Let $D = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\therefore CD = \begin{bmatrix} 3 & 6 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$CD = \begin{bmatrix} 3a+6c & 3b+6d \\ a+c & b+c \end{bmatrix}$

$AB - CD = 0, \therefore CD = AB$

$\begin{bmatrix} 3a+6c & 3b+6d \\ a+c & b+d \end{bmatrix} = \begin{bmatrix} 18 & 9 \\ 64 & 37 \end{bmatrix}$

$\therefore 3a+6c = 18 \Rightarrow a+2c = 6$ —①

$3b+6d = 9 \rightarrow b+2d = 3$ —②

$a+c = 64$ —③

$b+d = 37$ —④

cont....

(20) Contd.

$a+2c = 6$ —①

$a+c = 64$ —③

① − ② → $c = -58$

$\therefore a = 64 - c = 64 - (-58)$

$= 122$

$b+2d = 3$ —②

$b+d = 37$ —④

② − ④ → $d = -34$

$\therefore b = 37 - d = 37 - (-34)$

$= 71$

$D = \begin{bmatrix} 122 & 71 \\ -58 & -34 \end{bmatrix}$

——x——

(7) $\dfrac{\frac{1}{p} + \frac{1}{q+r}}{\frac{1}{p} - \frac{1}{q+r}} = \dfrac{\frac{q+r+p}{p(q+r)}}{\frac{q+r-p}{p(q+r)}} = \dfrac{q+r+p}{q+r-p}$ ①

$1 + \dfrac{q^2+r^2-p^2}{2qr} = \dfrac{2qr+q^2+r^2-p^2}{2qr}$

$= \dfrac{(q+r)^2 - p^2}{2qr}$

$= \dfrac{(q+r+p)(q+r-p)}{2qr}$ ②

① × ② $= \dfrac{(q+r+p)}{(q+r-p)} \times \dfrac{(q+r+p)(q+r-p)}{2qr}$

$= \dfrac{(p+q+r)^2}{2qr}$

——x——