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# PHYSICS

11<sup>th</sup> Standard

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  - (iv) Choose the odd one out
  - (v) Choose the correct pair
  - (vi) Choose the incorrect pair
  - (vii) Assertion-Reason
  - (viii) Choose the correct or incorrect statements
- Govt. Model Question Paper (2018) [**Govt. MQP-2018**], First Mid-Term Test (2018) [**First Mid-2018**], Quarterly Exam - 2018 [**QY-2018**], Half Yearly Exam - 2018 [**HY-2018**], March - 2019 [**Mar.-2019**], June - 2019 [**Jun.-2019**] Quarterly Exam - 2019 [**QY-2019**], Half Yearly Exam - 2019 [**HY. 2019**], are incorporated at appropriate sections.
- Govt. Model Question Paper, Sura's Model Question Paper, Quarterly Common Examination - 2019, Half Yearly Examination - 2019 Question Papers are given.
- Public Examination March 2020 Question Paper with Answers are given.



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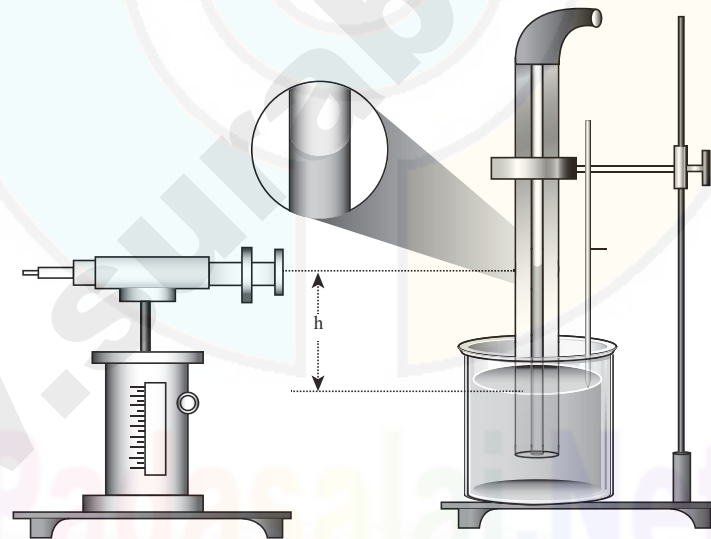
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# PHYSICS

## VOLUME - I

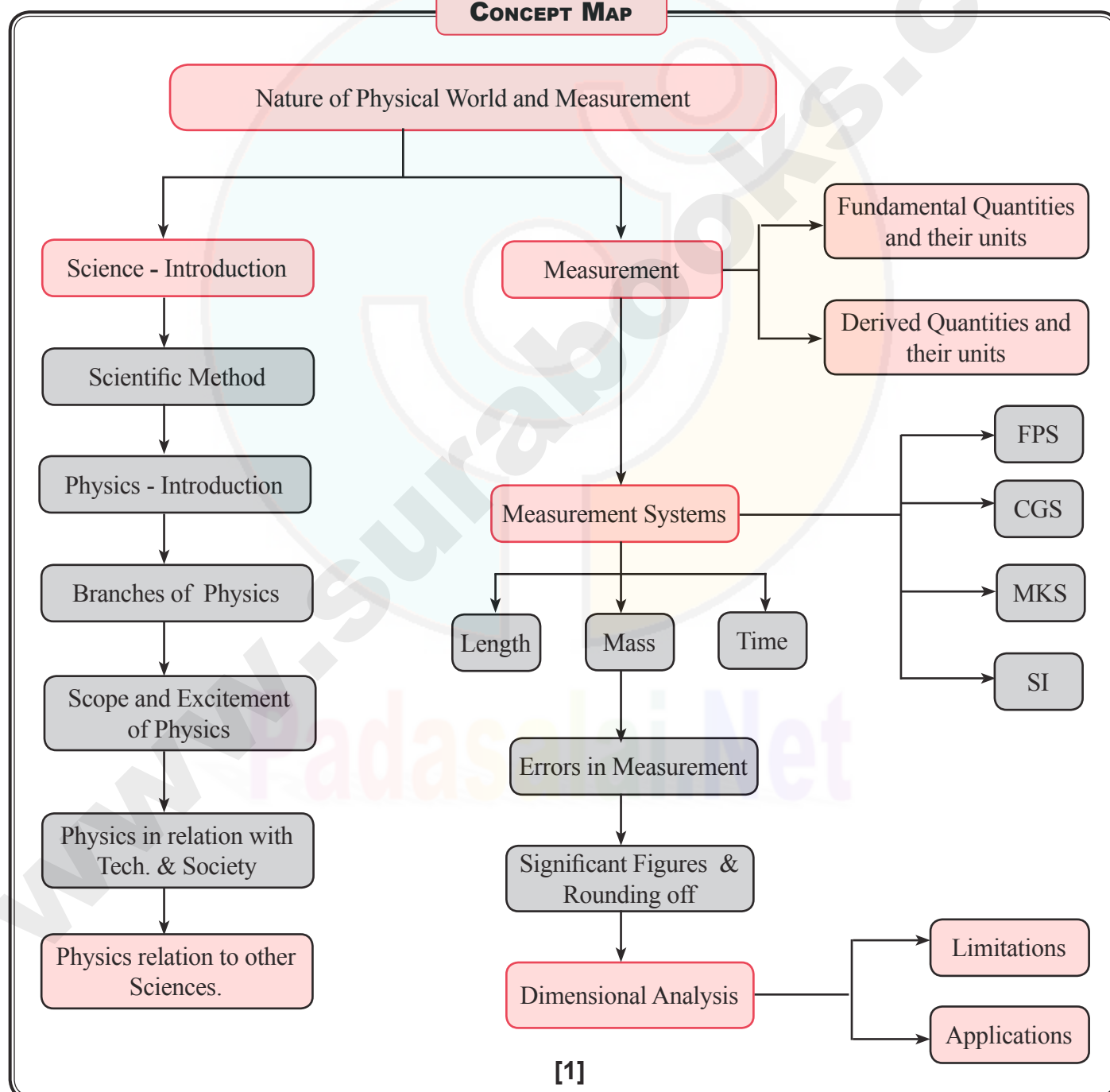


## UNIT

## 01

# NATURE OF PHYSICAL WORLD AND MEASUREMENT

## CONCEPT MAP



[1]

$$\therefore g = 4\pi^2 \frac{l}{T^2}$$

The errors in both  $l$  &  $T$  are least count errors

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T}$$

Length of simple pendulum  $l = 20$  cm

accuracy  $\Delta l = 2\text{mm} = 0.2$  cm

Time for 50 oscillations  $T = 40$  s

resolution  $\Delta T = 1$  s

$$\therefore \frac{\Delta g}{g} = \left(\frac{0.2}{20}\right) + 2\left(\frac{1}{40}\right) = \frac{0.2}{20} + \frac{2}{40} = \frac{1.2}{20}$$

Percentage error

$$\frac{\Delta g}{g} \times 100 = \frac{1.2}{20} \times 100 = \pm 6\%$$

% accuracy in  $g = 6\%$ .

### Government Exam Question & Answers

#### I. MULTIPLE CHOICE QUESTIONS :

— 1 Mark —

1. A substance whose mass is 4.27 g occupies 1.3 cm<sup>3</sup>. The number of significant figure in density is

[Govt. M.Q.P - 2018]

- (a) 1 (b) 2 (c) 3 (d) 4

[Ans. (d) 4]

2. Triple point of water is :

[QY. - 2018]

- (a) 273.16 k (b) 237.16 c  
(c) 273.16 c (d) 0 k

[Ans. (a) 273.16k]

3. Mass, temperature, electric current are

[QY. - 2018]

- (a) fundamental quantities  
(b) scalar quantities (c) vector quantities  
(d) both a and b

[Ans. (d) both a and b]

4. The amplitude and time period of a simple pendulum bob are 0.05 m and 2 s respectively. Then the maximum velocity of the bob is :

[Mar. - 2019]

- a) 0.157 ms<sup>-1</sup> b) 0.257 ms<sup>-1</sup>  
c) 0.10 ms<sup>-1</sup> d) 0.025 ms<sup>-1</sup>

[Ans. (a) 0.157 ms<sup>-1</sup>]

5. The significant figure of the number 0.003401 is:

[QY. - 2019]

- (a) 6 (b) 3  
(c) 5 (d) 4

[Ans. (d) 4]

#### II. VERY SHORT ANSWER QUESTIONS :

— 2 Marks —

1. Check the dimensional correctness for the given equations.

[First Mid-2018]

- (a)  $V = u + at$  (b)  $s = ut + \frac{1}{2}at$

Ans. (a)  $v = u + at$  Apply dimensional formula on both sides

$$[LT^{-1}] = [LT^{-1}] + [LT^{-2}][T]$$

$$[LT^{-1}] = [LT^{-1}] + [LT^{-1}]$$

(Quantities of same dimension only can be added)

Since dimensions on both sides are same, the given equation is dimensionally correct.

(b)  $[L] = [LT^{-1}][T] + [LT^{-2}][T^2]$

$$[L] = [LT^{-1+1}] + [LT^{-2+2}]$$

$$[L] = [LT^0] + [LT^0]$$

$$\therefore [L] = [L] + [L]$$

Since dimensions on both sides are same, the given equation is dimensionally correct.

2. Round off to required significant figures

[First Mid-2018]

(a)  $3.1 + 1.780 + 2.046$  (b)  $12.637 - 2.42$

(c)  $1.21 \times 36.72$  (d)  $36.72 \div 1.2$

Ans. (a) 6.96 (b) 10.22 (c) 44.4 (d) 31

3. What are random errors? How to minimise it?

[First Mid-2018]

Ans. (i) Random errors may arise due to random and unpredictable variations in experimental conditions like pressure, temperature, voltage supply, etc. Random errors are sometimes called “chance error”.

(ii) Random errors can be evaluated through statistical analysis and can be reduced by averaging over a large number of observations.

4. Write down the number of significant figures in the following: (i) 0.007 (ii) 400.

[Govt. MQP-2018]

Ans. (i) One (ii) One

5. What are the advantages of SI system?

[QY-2018]

Ans. (i) It is a rational system, in which only one unit is used for one physical quantity.

**Hint:**

$$1 \text{ AU} = 1.5 \times 10^{11} \text{ m.}$$

$$1 \text{ AU in } 1000 \text{ km} = \frac{1.5 \times 10^{11} \text{ m}}{10^6 \text{ m}} \\ [\because 1000 \text{ km} = 10^6 \text{ m}] \\ = 1.5 \times 10^5 \text{ m}$$

**2. How many AU present in one light year?**

- (a)  $6.30 \times 10^4 \text{ m}$  (b)  $9.46 \times 10^{15} \text{ m}$   
(c)  $6.2 \times 10^2 \text{ m}$  (d)  $9.4 \times 10^{16} \text{ m}$

**[Ans. (a)  $6.30 \times 10^4 \text{ m}$ ]****Hint:**

$$1 \text{ light year} = 9.45 \times 10^{15} \text{ m.}$$

$\therefore$  No. of AU in 1 light year

$$= \frac{9.45 \times 10^{15} \text{ m}}{1.5 \times 10^{11} \text{ m}} = 6.30 \times 10^4 \text{ m}$$

**3. How many  $\mu\text{m}$  present in one metre?**

- (a)  $10^{-6} \mu\text{m}$  (b)  $10^6 \mu\text{m}$   
(c)  $10^{-3} \mu\text{m}$  (d)  $10^{-2} \mu\text{m}$

**[Ans. (b)  $10^6 \mu\text{m}$ ]****Hint:**

$$1 \text{ micron} = 1 \mu\text{m} = 10^{-6} \text{ m.}$$

$$\text{No. of } \mu\text{m in } 1 \text{ m} = \frac{1}{10^{-6}} = 10^6 \mu\text{m}$$

**4. Express the derived unit of pressure.****Ans.**

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} \\ = \frac{\text{unit of force}}{\text{unit of Area}} = \frac{\text{N}}{\text{m}^2} = \text{Nm}^{-2}$$

**5. What is the formula representation of Mean Absolute error?**

$$\text{Ans. Mean Absolute error, } \Delta a_m = \frac{1}{n} \sum_{i=1}^n |\Delta a_i|$$

**6. The speed of an object  $v = 40 \text{ ms}^{-1}$ . The same quantity of speed in  $\text{kmh}^{-1}$  is,**

- (a) 60 (b) 160 (c) 40 (d) 144

**[Ans. (d) 144]****Hint:**

$$\text{Quantity of speed in m/s} = 40$$

$$\text{Quantity of speed in km/h} = 40 \times \frac{18}{5} \\ = 144 \text{ kmh}^{-1}.$$

**7. The speed of an object  $v = 90 \text{ km/h}$ . The same quantity of speed in m/s is,**

- (a) 90 (b) 25 (c) 45 (d) 180

**[Ans. (b) 25]****Hint:**

$$\text{Quantity of speed in km/h} = 90$$

$$\text{Quantity of speed in m/s} = 90 \times \frac{5}{18} = 25 \text{ m/s.}$$

**8. 3.5kg mass of a metal plate has the volume of  $1.5 \text{ m}^3$ . Find the density of metal plate.**

- (a)  $1.5 \text{ kg/m}^3$  (b)  $2.3 \text{ kg/m}^3$   
(c)  $3.4 \text{ kg/m}^3$  (d)  $4.8 \text{ kg/m}^3$

**[Ans. (b)  $2.3 \text{ kg/m}^3$ ]****Hint:**

Density is a derived unit.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{3.5 \text{ kg}}{1.5 \text{ m}^3} = 2.3 \text{ kg/m}^3$$

**9. The value of  $1^\circ$  is,**

- (a)  $1.745 \times 10^{-2} \text{ rad}$  (b)  $1.946 \times 10^{-11} \text{ rad}$   
(c) 3.6 rad (d) 3600 rad

**[Ans. (a)  $1.745 \times 10^{-2} \text{ rad}$ ]****Hint:**

$$1^\circ = \frac{2\pi}{360} = \frac{\pi}{180} = \frac{22}{7 \times 180} = 1.745 \times 10^{-2} \text{ rad}$$

**10. How many parsec are there in one kilometer?**

- (a)  $3.084 \times 10^{-16}$  (b)  $3.08 \times 10^8$   
(c)  $3.24 \times 10^{-14}$  (d) None

**Hint:****[Ans. (c)  $3.24 \times 10^{-14}$ ]**

$$1 \text{ parsec} = 3.08 \times 10^{16} \text{ m}$$

$$3.08 \times 10^{16} \text{ m} = 1 \text{ parsec}$$

$$1 \text{ km} = \frac{1 \times 10^3}{3.08 \times 10^{16}} \text{ parsec} = 3.24 \times 10^{-14} \text{ parsec}$$

**11. The angle of an object is  $18.2^\circ$ . What is the angular diameter of the object in radians?**

- (a) 36.4 rad (b)  $3.64 \times 10^{-2} \text{ rad}$   
(c)  $31.74 \times 10^{-2} \text{ rad}$  (d) 3.17 rad

**Hint:****[Ans. (c)  $31.74 \times 10^{-2} \text{ rad}$ ]**

$$\theta = 18.2^\circ = \frac{18.2}{60 \times 60} \times \frac{\pi}{180} = \frac{18.2}{60 \times 60} \times \frac{3.14}{180} \\ = 31.74 \times 10^{-2} \text{ rad}$$

**12. If a circle with 10 m radius and angle  $60^\circ$  at centre, then what will be the length of arc?**

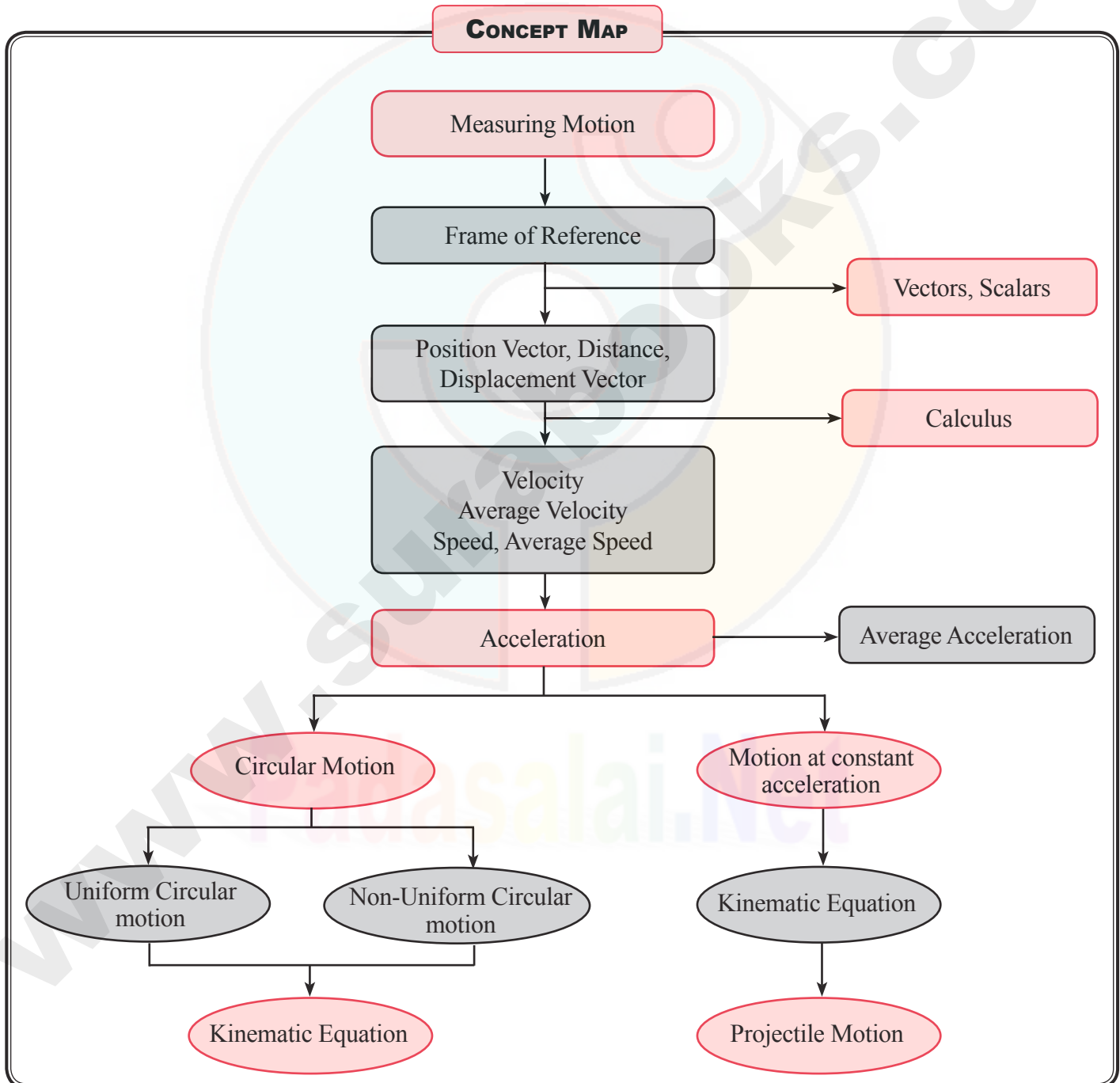
- (a) 5.24m (b) 6.21m  
(c) 7.1 mm (d) 10m

**[Ans. (a) 5.24m]**

## UNIT

## 02

## KINEMATICS



(ii) Maximum height,  $h = \frac{u^2 \sin^2 \theta}{2g}$

(iii) Horizontal range,  $R = \frac{u^2 \sin 2\theta}{g}$

(iv) Maximum horizontal range  $R_{\max} = \frac{u^2}{g}$

(17) Centripetal acceleration  $a_c = \omega^2 r$

(18) For motion along x axis,  $v_x = u_x + a_x t$  and  $x = x_0 + u_x t + \frac{1}{2} a_x t^2$

(19) For motion along y axis,  $v_y = u_y + a_y t$  and  $y = y_0 + u_y t + \frac{1}{2} a_y t^2$

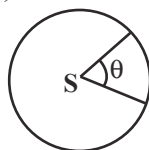
## IMPORTANT TERMS & DEFINITIONS

<b>Rest</b>	: An object or a particle is said to be in the state of rest when it does not change its position with time with respect to its surroundings. Depending upon the position of observer, the state of rest (i) Absolute state of rest (ii) Relative state of rest
<b>Motion</b>	: An object or a particle is said to be in the state of motion when it changes its position with time with respect to its surroundings. The motion of an object can be linear or curvilinear, circular or in a plane or in a space.
<b>Motion in 1 - Dimensional</b>	: Motion of a body in a straight line is called one dimensional motion.
<b>Motion in 2 – Dimensional</b>	: Motion of body in a plane is called two dimensional motion.
<b>Motion in 3 – Dimensional</b>	: Motion of body in a space is called three dimensional motion.
<b>Distance</b>	: It is the actual length of the path covered by a moving particle in a given interval of time, unit : metre (SI).
<b>Displacement</b>	: Displacement is the difference between the final and initial positions of the object in a given interval of time. It can also be defined as the shortest distance between these two positions of the object and its direction is from the initial to final position of the object, during the given interval of time. It is a vector quantity.
<b>Speed</b>	: The rate of the path length (or) the distance covered by an object to the time taken. It's SI unit is m/s.
<b>Uniform Speed</b>	: When a particle covers equal distances in equal intervals of time, then it is said to be moving with uniform speed.



- 21.** The Moon is orbiting the Earth approximately once in 27 days, what is the angle transversed by the Moon per day?

**Ans.** Duration taken by the moon in orbiting the earth = 27 days (approx)



The angle transversed by the moon per day

$$= \frac{360}{27} = 13^\circ 3'$$

- 22.** An object of mass  $m$  has angular acceleration  $\alpha = 0.2 \text{ rad s}^{-2}$ . What is the angular displacement covered by the object after 3 second? (Assume that the object started with angle zero with zero angular velocity).

**Ans.** Angular acceleration  $\alpha = 0.2 \text{ rad s}^{-2}$

The angular displacement covered by the object after 3 seconds is,

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\theta = 0 + \frac{1}{2} \times \frac{2}{10} \times 3 \times 3 = \frac{9}{10} \text{ rad}$$

$$1 \text{ rad} = 57.295^\circ = 0.9 \times 57.295^\circ \quad \theta = 51^\circ$$

### Government Exam Question & Answers

#### I. MULTIPLE CHOICE QUESTIONS :

== 1 Mark ==

- 1.** Which of the following physical quantities have same dimensional formula?

[Govt. M.Q.P - 2018; HY-2018]

- (a) Torque and Work done
- (b) Energy and Angular momentum
- (c) Force and Torque
- (d) Angular momentum and Linear momentum

[Ans. (a) Torque and Work done]

- 2.** The dimensions of physical quantity X in the equation  $\text{Force} = \frac{X}{\text{Density}}$  is given by

[First Mid - 2018]

- (a)  $M^1 L^4 T^{-2}$
- (b)  $M^2 L^{-2} T^{-1}$
- (c)  $M^2 L^{-2} T^{-2}$
- (d)  $M^1 L^{-2} T^{-1}$

[Ans. (d)  $M^2 L^{-2} T^{-2}$ ]

- 3.** A vector  $\vec{A}$  points vertically upward and  $\vec{B}$  points towards north. The vector product  $\vec{A} \times \vec{B}$  is

[First Mid - 2018]

- (a) along west
- (b) along east
- (c) zero
- (d) vertically downward

[Ans. (a) along west]

- 4.** The maximum value of fractional error in division of two quantities i.e.,  $x = \frac{A}{B}$  is

[Govt. M.Q.P - 2018]

- (a)  $\frac{\Delta x}{x} = \mp \left( \frac{\Delta A}{A} - \frac{\Delta B}{B} \right)$
- (b)  $\frac{\Delta x}{x} = \left( -\frac{\Delta A}{A} + \frac{\Delta B}{B} \right)$
- (c)  $\frac{\Delta x}{x} = \left( \frac{\Delta A}{A} + \frac{\Delta B}{B} \right)$
- (d)  $\frac{\Delta x}{x} = \left( \frac{A}{\Delta A} + \frac{B}{\Delta B} \right)$

[Ans. (a)  $\frac{\Delta x}{x} = \mp \left( \frac{\Delta A}{A} - \frac{\Delta B}{B} \right)$  ]

- 5.** The unit vector in the direction of  $\vec{A} = \hat{i} + \hat{j} + \hat{k}$  is

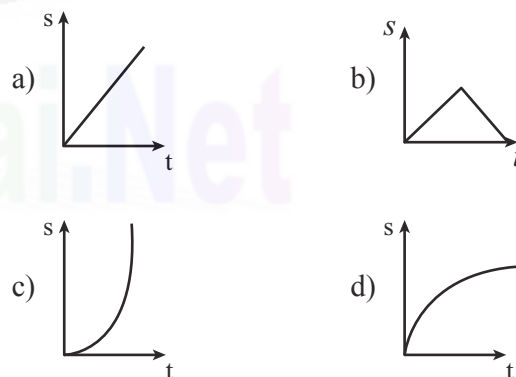
[Govt. M.Q.P - 2018]

- (a)  $\hat{i} + \hat{j} + \hat{k}$
- (b)  $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{2}}$
- (c)  $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$
- (d)  $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{6}}$

[Ans. (c)  $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$  ]

- 6.** Which graph represents uniform acceleration?

[Mar.-2019]



[Ans. (c) ]

5. A particle moves along the  $x$ -axis in such a way that its coordinates  $x$ -varies with time ' $t$ ' according to the equation  $x = 2 - 5t + 6t^2$ . What is the initial velocity of the particle? [HY-2019]

**Solution:**

$$x = 2 - 5t + 6t^2$$

$$\text{Velocity, } v = \frac{dx}{dt} = \frac{d}{dt} (2 - 5t + 6t^2)$$

$$\text{or } v = -5 + 12t$$

For initial velocity,  $t = 0$

$$\therefore \text{Initial velocity} = -5 \text{ ms}^{-1}$$

The negative sign implies that at  $t = 0$  the velocity of the particle is along negative  $x$  direction.

Average speed = total path length / total time period.

## ADDITIONAL QUESTIONS

### I. MULTIPLE CHOICE QUESTIONS :

— 1 Mark —

1. Which one of the following statement is true?

- (a) A scalar quantity is conserved in a process
- (b) A scalar quantity does not vary from one point to another in space
- (c) A scalar quantity can never take -ve values
- (d) A scalar quantity has only magnitude and no direction.

[Ans. (d) A scalar quantity has only magnitude and no direction.]

2. The angle between  $A = i + j$  and  $B = i - j$  is .

- (a)  $45^\circ$
- (b)  $90^\circ$
- (c)  $-45^\circ$
- (d)  $180^\circ$

[Ans. (b)  $90^\circ$ ]

3. The component of position vector  $\vec{r}$  along  $x$ -axis will maximum value if

- (a)  $\vec{r}$  is along the  $x$  - axis
- (b)  $\vec{r}$  makes an angle of  $45^\circ$  with  $x$  - axis
- (c)  $\vec{r}$  is along the  $y$  - axis
- (d)  $\vec{r}$  is along -ve  $y$  - axis

[Ans. (a)  $\vec{r}$  is along the  $x$  - axis]

4. Consider the quantities pressure, power, energy, impulse, charge. Out of these, the only vector quantity is

- (a) pressure
- (b) power
- (c) impulse
- (d) charge

[Ans. (c) impulse]

5. The length of a vector is \_\_\_\_\_

- (a) always a negative quantity
- (b) always a positive quantity
- (c) either positive or negative
- (d) denoted by ' $\lambda$ '

[Ans. (b) always a positive quantity]

6. Distance is a scalar quantity and \_\_\_\_\_ is a vector.

- (a) Speed
- (b) Length
- (c) Time
- (d) Displacement

[Ans. (d) Displacement]

7. The horizontal range of a projectile fired at an angle of  $15^\circ$  is 50 m. If it is fired with the same speed at angle of  $45^\circ$ , its range will be

- (a) 125 m
- (b) 75 m
- (c) 100 m
- (d) 150 m

[Ans. (c) 100 m]

8. Choose the motion in two dimension from the following.

- (a) Motion of a train along a straight railway track
- (b) An object falling freely under gravity close to the Earth.
- (c) A particle moving along a curved path in a plane.
- (d) Flying of a kite on a windy day.

[Ans. (c) A particle moving along a curved path in a plane]

9. Which one of the following physical quantities cannot be represented by a scalar?

- (a) Mass
- (b) Length
- (c) Momentum
- (d) Magnitude of acceleration

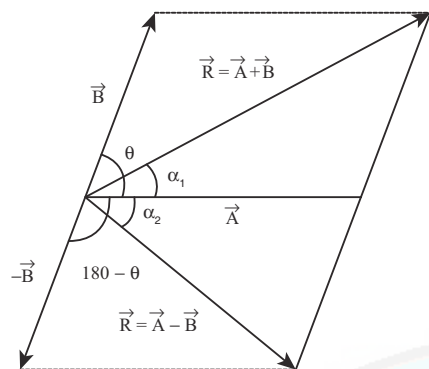
[Ans. (c) Momentum]

10. An object is dropped in an unknown planet from height 50m, it reaches the ground in 2s. The acceleration due to gravity in this unknown planet is

- (a)  $g = 20 \text{ ms}^{-2}$
- (b)  $g = 25 \text{ ms}^{-2}$
- (c)  $g = 15 \text{ ms}^{-2}$
- (d)  $g = 30 \text{ ms}^{-2}$

[Ans. (b)  $g = 25 \text{ ms}^{-2}$ ]





- (ii) The difference  $\vec{A} - \vec{B}$  is the same as the resultant of  $\vec{A}$  and  $-\vec{B}$ .

We can write  $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$  and using the equation

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}, \text{ we have}$$

$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos(180 - \theta)}$$

- (iii) Since,  $\cos(180 - \theta) = -\cos\theta$ , we get  
Magnitude of averting

$$\Rightarrow |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

- (iv) Again from the Figure and using an equation similar to equation

$$\tan\alpha = \frac{B\sin\theta}{A + B\sin\theta}$$

we have

$$\tan\alpha_2 = \frac{B\sin(180^\circ - \theta)}{A + B\cos(180^\circ - \theta)}$$

- (v) But  $\sin(180^\circ - \theta) = \sin\theta$ , hence we get

$$\Rightarrow \tan\alpha_2 = \frac{B\sin\theta}{A - B\cos\theta}$$

3. Mention important properties of the scalar product of two vectors. (or) Write the important properties of vector (cross) product.

**Ans.** This question answer is given in Exercise question (III) Long Answer Question No. 2.

## Numerical Problems

1. The displacement of a particle moving along  $x$ -axis is given by  $x = 6t + 12t^2$ , calculate the instantaneous velocity at  $t = 0$  and  $t = 2s$ .

### Solution:

Displacement of a particle  $x = 6t + 12t^2$   
Differentiate with respect to 't'

$$x = 6t + 12t^2$$

$$\frac{dx}{dt} = 6 + 24t \quad [\because x^n = nx^{n-1}]$$

$$\therefore \text{The instantaneous velocity } v_i = \frac{dx}{dt} = 6 + 24t.$$

Instantaneous velocity at  $t = 0$ ,

$$v_i = u + at = 6 + 24(0)$$

$$v_i = 6 \text{ m/s}$$

Instantaneous velocity at  $t = 2s$ .

$$v_i = u + at = 6 + 24(2) = 6 + 48$$

$$v_i = 54 \text{ ms}^{-1}$$

2. The displacement  $x$  of a particle varies with time 't' as,  $x = 3t^2 - 4t + 30$ . Find the position, velocity and acceleration of the particle at  $t = 0$ .

### Solution:

Position of a particle  $x = 3t^2 - 4t + 30$ .

$$\text{velocity } v = \frac{dx}{dt} = \frac{d}{dt}(3t^2 - 4t + 30) \\ = 6t - 4 \quad [\because x^n = nx^{n-1}]$$

$$\text{Acceleration } a = \frac{dv}{dt} = \frac{d}{dt}(6t - 4) \\ a = 6$$

At time  $t = 0$ , we have,

$$\begin{aligned} \text{Position } x &= 3t^2 - 4t + 30 \\ &= 3(0)^2 - 4(0) + 30 \\ x &= 30 \text{ m.} \end{aligned}$$

$$\begin{aligned} \text{Velocity } v &= 6t - 4 = 6(0) - 4 \\ &= -4 \text{ m/s} = -4 \text{ ms}^{-1} \end{aligned}$$

$$\text{Acceleration } a = 6 \text{ ms}^{-2}$$

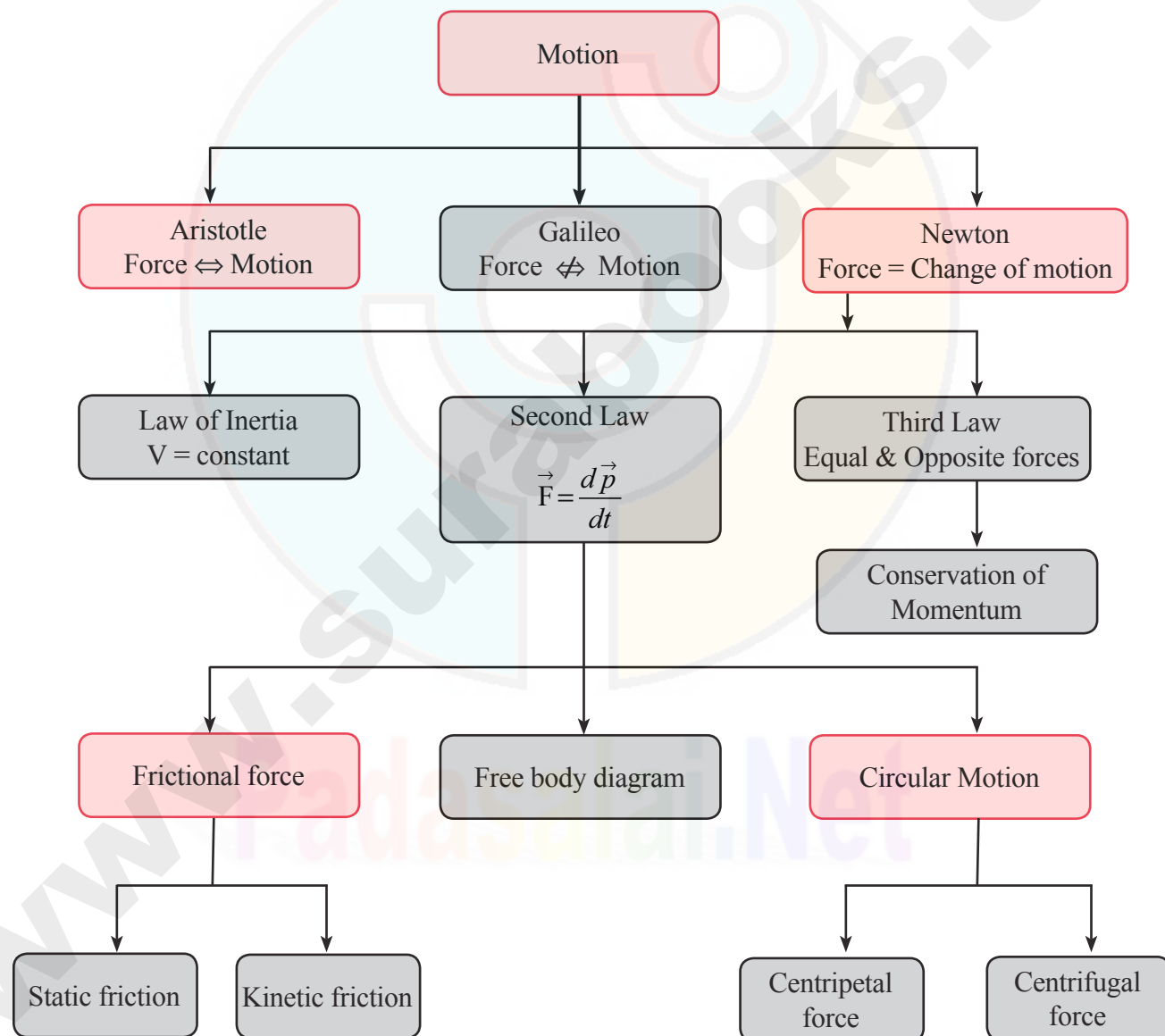
$$\therefore \text{position } x = 30 \text{ m, velocity } v = -4 \text{ ms}^{-1}, \\ \text{acceleration } = 6 \text{ ms}^{-2}$$

## UNIT

## 03

## LAWS OF MOTION

## CONCEPT MAP



10. Co-efficient of kinetic friction :  $\mu_k = \frac{f_k}{R}$   
 $\mu_k \rightarrow$  co-efficient of kinetic friction  
 $f_k \rightarrow$  kinetic friction  
 $R \rightarrow$  normal reaction.
11. Acceleration of a block which sliding down a rough inclined plane :  $a = g (\sin \alpha + \mu \cos \alpha)$
12. Retardation of a block which sliding up a rough inclined plane :  $a = g (\sin \alpha + \mu \cos \alpha)$
13. When the lift is at rest :  $N = mg$
14. When the lift is accelerating upwards :  $N = m (a + g)$
15. When the lift is accelerating downwards :  $N = m (g - a)$
16. The maximum speed of circulation is :  $V_{\max} = \sqrt{\mu_s r g}$   
 $r \rightarrow$  radius  
 $g \rightarrow$  acceleration due to gravity  
 $\mu_s \rightarrow$  co-efficient of static friction

## IMPORTANT TERMS & DEFINITIONS

<b>Newton's first law of motion</b>	: Every object continues to be in the state of rest or of uniform motion (Constant Velocity) unless there is external force acting on it.
<b>Inertia of a body</b>	: Inertia is the property of the body by virtue of which a body cannot change on its own state of rest or uniform motion in a straight line.
<b>Inertia of rest</b>	: It is the inability of an object to change its state of rest on its own.
<b>Inertia of motion</b>	: Inertia of motion is the inability of an object to change its state of uniform speed (Constant speed) on its own.
<b>Inertia of direction</b>	: It is the inability of an object to change its direction of motion on its own.
<b>Newton's second law of motion</b>	: The force acting on an object is equal to the rate of change of its momentum $\vec{F} = m \vec{a}$
<b>Concurrent forces</b>	: A number of forces, acting at the same point are called concurrent forces.
<b>Equilibrium of the body</b>	: If the resultant of a number of forces acting on a body is zero, the body will be in equilibrium
<b>Newton's third law</b>	: For every action there is an equal and opposite reaction. $\vec{F}_{12} = -\vec{F}_{21}$

$$f_k = \mu_k \cdot R \Rightarrow ma = \mu_c \cdot mg$$

$$a = \mu_k \cdot g = 0.7 \times 9.8$$

$$a = 6.86 \text{ ms}^{-2}$$

To find minimum velocity

$$v^2 = u^2 + 2as$$

$$u = \text{Initial velocity, } = 0$$

$$v_{\min}^2 = 2 \times 6.86 \times 10 = 137.2$$

$$v_{\min} = \sqrt{137.2} = 11.71 \text{ ms}^{-1}$$

### Government Exam Question & Answers

#### I. MULTIPLE CHOICE QUESTIONS :

===== 1 Mark =====

1. A car accelerates on a horizontal road due to the force exerted by [First Mid. - 2018]

- (a) the engine of the car  
(b) the driver of the car (c) the earth  
(d) the road [Ans. (d) the road]

2. When a fast moving bus suddenly stops, the passenger is thrown forward because of

[Govt. M.Q.P - 2018]

- (a) inertia of rest (b) inertia of direction  
(c) moment of inertia (d) inertia of motion

[Ans. (d) inertia of motion]

3. In studying motion of a body, the starting of motion is more difficult than maintaining it because, the coefficient of static friction and kinetic friction satisfy the relation

[Govt. M.Q.P - 2018]

- (a)  $\mu_s > \mu_k$  (b)  $\mu_s < \mu_k$   
(c)  $\mu_s = \mu_k$  (d)  $\mu_s = \frac{1}{2} \mu_k$

[Ans. (b)  $\mu_s < \mu_k$ ]

4. If two masses  $m_1$  and  $m_2$  are experiencing the same force, then the ratio of respective acceleration is [Govt. M.Q.P - 2018]

- (a)  $\frac{a_1}{a_2} = \frac{m_1}{m_2}$  (b)  $\frac{a_1}{a_2} = 1$   
(c)  $\frac{a_1}{a_2} = \frac{m_2}{m_1}$  (d)  $\frac{a_1}{a_2} = \sqrt{\frac{m_1}{m_2}}$

[Ans. (c)  $\frac{a_1}{a_2} = \frac{m_2}{m_1}$ ]

5. A bus is moving with a speed of  $10 \text{ ms}^{-1}$  on a straight road. A scooterist wishes to overtake the bus in 100s. If the bus is at a distance of 1 km from the scooterist, with what speed should the scooterist chase the bus? [QY. - 2018]

- (a)  $40 \text{ ms}^{-1}$  (b)  $25 \text{ ms}^{-1}$   
(c)  $10 \text{ ms}^{-1}$  (d)  $20 \text{ ms}^{-1}$

[Ans. (d)  $20 \text{ ms}^{-1}$ ]

6. Earth moving about sun is elliptical orbit is an example for [QY. - 2018]

- (a) force and motion is same direction  
(b) force and motion in different direction  
(c) force and motion in opposite direction  
(d) zero net force

[Ans. (b) force and motion in different direction]

7. For a satellite moving in an orbit around the earth, the ratio of kinetic energy to potential [Mar. - 2019]

- (a) 2 (b)  $\sqrt{2}$  (c)  $\frac{1}{2}$  (d)  $\frac{1}{\sqrt{2}}$

[Ans. (c)  $\frac{1}{2}$ ]

8. Consider a circular leveled road of radius 10 m having coefficient of static friction 0.81. Three cars (A, B and C) are travelling with speed  $7 \text{ ms}^{-1}$ ,  $8 \text{ ms}^{-1}$  and  $10 \text{ ms}^{-1}$  respectively, which car will skid when it moves in the circular level road? ( $g = 10 \text{ ms}^{-2}$ ) : [QY. - 2019]

- (a) A (b) B  
(c) C (d) Both B and C

[Ans. (c) C]

9. Which of the following force is pseudo force? [Jun.-2019]

- (a) viscous force (b) surface tension  
(c) centrifugal force (d) gravitational force

[Ans. (c) centrifugal force]

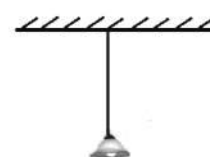
#### II. VERY SHORT ANSWER QUESTIONS :

===== 2 Marks =====

1. Consider a lamp (with holder) of mass 50 g (shown in the figure) Draw free body diagram and compute the tension in the string. (assume lamp with holder as a point mass)

[Govt. MQP-2018]

Ans. Tension,  $T = mg$   
 $m = 50\text{g} = 0.5\text{kg}$   
 $g = 9.8\text{ms}^{-2}$   
 $T = 0.05 \times 9.8$   
 $T = 0.49\text{N}$



## Numerical Problems

1. In a rocket, the fuel burns at the rate of  $1 \text{ kg s}^{-1}$ . This fuel burnt eject gases at a speed of  $90 \text{ km s}^{-1}$ . The force exerted on the rocket is,

(a) 45000 N (b) 90000 N  
(c) 60000 N (d) 10000 N

**Hint:**

**[Ans. (b) 90000 N]**

Force exerted on the rocket

$$F = V \frac{dm}{dt} = 90 \times 10^3 \times 1$$

$$F = 90000 \text{ N}$$

2. Two masses  $m_1 = 2 \text{ kg}$  and  $m_2 = 1 \text{ kg}$  are connected by a string which passes over a smooth frictionless pulley. The acceleration of the masses when they are released is,

(a)  $3.26 \text{ ms}^{-2}$  (b)  $2.26 \text{ ms}^{-2}$   
(c)  $1.26 \text{ ms}^{-2}$  (d)  $4.26 \text{ ms}^{-2}$

**Hint:**

**[Ans. (a)  $3.26 \text{ ms}^{-2}$ ]**

The common acceleration of the masses is

$$a = \frac{(m_1 - m_2)}{(m_1 + m_2)} \times g = \frac{(2 - 1)}{(2 + 1)} \times 9.8 = \frac{(1)}{3} \times 9.8 = 3.26 \text{ ms}^{-2}$$

3. A body of mass  $2 \text{ kg}$  rests on a horizontal plane. If the angle of friction is  $45^\circ$ . The least force required to move the body along the plane is,

(a) 20 N (b) 40 N  
(c) 30 N (d) 10 N **[Ans. (a) 20 N]**

**Hint:**  $F = mg \tan \theta = 2 \times 10 \times \tan 45^\circ = 2 \times 10 \times 1 = 20 \text{ N}$ .

4. A machine gun fires bullets each of mass  $2 \text{ g}$  with a velocity of  $20 \text{ ms}^{-1}$  by exerting a constant force of  $10 \text{ g}$  weight. Then the number of bullets fired per second is,

(a)  $n = 5$  (b)  $n = 15$   
(c)  $n = 10$  (d)  $n = 4$

**[Ans. (c)  $n = 10$ ]**

**Hint:**

$$m = \frac{10}{1000} \text{ kg} = 0.01 \text{ kg} \quad m = 0.01 \text{ kg}$$

$$= 0.01 \times 10 = \frac{(2 \times 10^{-3}) \times 10 \times n}{1} \quad t = 1 \text{ s}$$

$$n = 10. \quad g = 10 \text{ ms}^{-2}$$

5. A body of mass  $20 \text{ kg}$  moving with a uniform velocity of  $4 \text{ ms}^{-1}$ . Then the net force acting on a body is,

(a) constant (b) increase  
(c) decrease (d) zero

**Hint:**

**[Ans. (d) zero]**

In uniform velocity,

Acceleration  $a = 0$ .

According to Newton's second law of motion.

$$F = ma$$

$$F = 20 \times 0$$

$$F = 0.$$

6. A ball of  $300 \text{ g}$  mass moving with a speed of  $20 \text{ m/s}$  rebounds after striking normally a perfectly elastic wall. The change in momentum of a ball is.

(a)  $12 \text{ kg ms}^{-1}$  (b)  $-12 \text{ kg ms}^{-1}$   
(c)  $6 \text{ kg ms}^{-1}$  (d)  $-6 \text{ kg ms}^{-1}$

**Hint:**

**[Ans. (b)  $-12 \text{ kg ms}^{-1}$ ]**

$$\text{Change in linear momentum} = -mv - mv = -2mv$$

$$= -2 \times 0.3 \times 20 = -12 \text{ kg ms}^{-1}$$

$$\left[ m = \frac{300}{1000} \text{ kg} = 0.3 \text{ kg} \right]$$

7. A force of  $10 \text{ N}$  changes velocity of a body from  $20 \text{ ms}^{-1}$  to  $40 \text{ ms}^{-1}$  in  $8 \text{ seconds}$ . How much force is required to bring about the same change in  $4 \text{ s}$ .

**Hint:** Force,  $F = F_1 = F_2$

$$F_1 = \frac{dp}{dt_1}, F_2 = \frac{dp}{dt_2}$$

$$F_2 = F_1 \frac{dt_1}{dt_2} = 10 \times \frac{8}{4} = 20 \text{ N}$$

8. A constant retarding force of  $60 \text{ N}$  is applied to a body of mass  $25 \text{ kg}$  moving with a speed of  $20 \text{ ms}^{-1}$ . How long does the body take to stop?

(a)  $8.33 \text{ s}$  (b)  $7.58 \text{ s}$   
(c)  $4 \text{ s}$  (d)  $5.8 \text{ s}$

**[Ans. (a) 8.33 seconds]**

**Hint:** According to Newton's second law,

$$F = -60 \text{ N}, m = 25 \text{ kg}, u = 20 \text{ ms}^{-1}, v = 0$$

Now,

$$F = ma, a = \frac{F}{m} \quad [\because F = ma]$$

$$= -\frac{60}{25} = -2.4 \text{ ms}^{-2}.$$

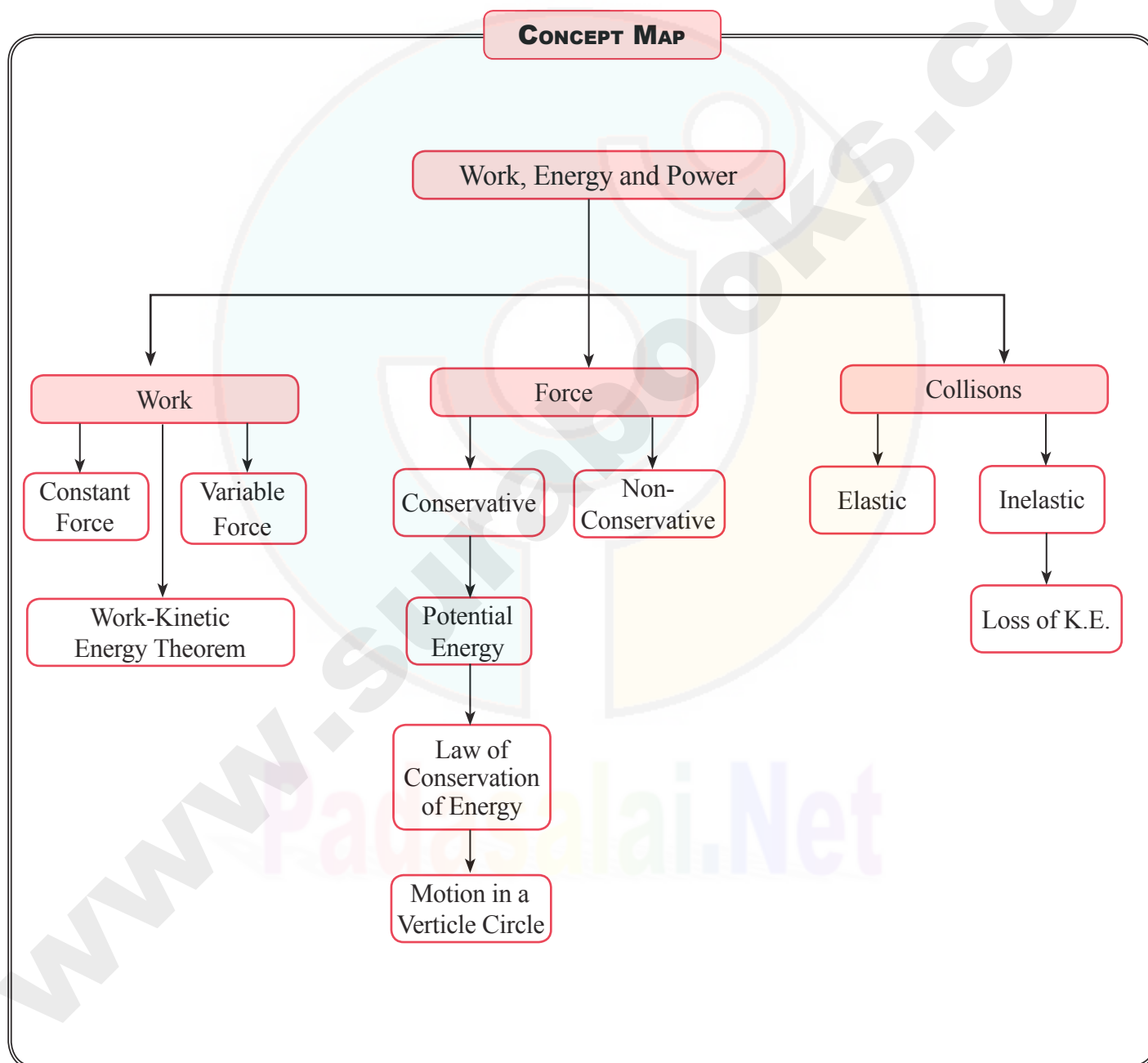


## UNIT

## 04

## WORK, ENERGY AND POWER

## CONCEPT MAP



## FORMULAE TO REMEMBER

- (1) Work done by the force  $F$  on the body is given by  $W = \vec{F} \cdot d\vec{r} = F dr \cos \theta$   
 $F \rightarrow$  Force.  
 $dr \rightarrow$  displacement.  
 $\theta \rightarrow$  angle between them (between  $F$  and  $dr$ ).
- (2) The kinetic energy of a body of mass  $m$  moving with velocity  $v$  is given  
 by  $KE = \frac{1}{2}mv^2$  or  $p^2/2m$   $p \rightarrow$  Linear Momentum;  $m \rightarrow$  mass
- (3) The work done by the net force acting on a body is equal to the change in kinetic energy of the body  
 $W = \text{change in KE} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$
- (4) The elastic potential energy,  $U = \frac{1}{2}kx^2$   
 $x \rightarrow$  distance  
 $k \rightarrow$  force constant of the spring
- (5) Power is the rate of doing work,  $P = \frac{\text{workdone}(W)}{\text{time}(t)}$
- (6) In such a case loss in kinetic energy of the system is  $\Delta KE = \frac{m_1 m_2 (u_1 - u_2)^2}{2(m_1 + m_2)}$
- (7) The final common velocity of the system is  $v = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$
- (8) If a particle of masses  $m_1$  and  $m_2$  moving with velocities  $u_1$  and  $u_2$  ( $u_2 > u_1$ ) in the same direction suffer elastic collision, such that  $v_1$  and  $v_2$  is,  

$$v_1 = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) u_1 + \left( \frac{2m_2}{m_1 + m_2} \right) u_2$$

$$v_2 = \left( \frac{2m_1}{m_1 + m_2} \right) u_1 + \left( \frac{m_2 - m_1}{m_1 + m_2} \right) u_2$$
- (9) If a particle of mass  $m_1$  moving with velocity  $v$ , collides with another particle of mass  $m_2$  at rest, then after the collision two particles move with velocities  $v_1$  and  $v_2$  making angles  $\theta_1$  and  $\theta_2$  with axis,  
 $m_1 v \sin \theta_2 = m_2 v_2 \sin \theta_2$  (along  $y$  axis)  
 $m_1 u_1 = m_1 v_1 \cos \theta_1 + m_1 v_2 \cos \theta_2$  (along  $x$  axis)
- (10) Nature of the work done:
  - (i) Positive work done:  $W = Fdr \cos \theta$ ;  $\theta = 0$   
 $W = Fdr \cos 0^\circ$   
 $W = Fdr$  [ $\cos 0^\circ = 1$ ]
  - (ii) Negative work done:  $W = Fdr \cos \theta$ ;  $\theta = 180^\circ$   
 $W = Fdr \cos 180^\circ$   
 $W = -Fdr$  [ $\cos 180^\circ = -1$ ]
  - (iii) Zero work done:  $W = Fdr \cos \theta$ ;  $\theta = 90^\circ$   
 $W = Fdr \cos 90^\circ$   
 $W = 0$  [ $\cos 90^\circ = 0$ ]

## ADDITIONAL QUESTIONS

### I. MULTIPLE CHOICE QUESTIONS :

== 1 Mark ==

1. The work done on an object does not depend upon the

(a) displacement (b) force applied  
(c) angle between force and displacement  
(d) initial velocity of the object

[Ans. (d) initial velocity of the object]

2. The force on a particle as the function of displacement  $x$  is given by  $F = 9 + 0.3x$ . The work done corresponding to displacement of particle from  $x = 0$  to  $x = 2$  unit is

(a) 18.6 J (b) 21 J  
(c) 25 J (d) 9.6 J

**Hint:**

[Ans. (a) 18.6 J]

$$W = \sum dW = \int_0^2 f dx = \int_0^2 (9 + 0.3x) dx = 9x + \frac{0.3x^2}{2} \bigg|_0^2 = 18.6 \text{ J.}$$

3. A boy carrying a box on his head is walking on a level road from one place to another on a straight road is doing no work. This statement is

(a) correct (b) incorrect  
(c) partly correct  
(d) insufficient information

[Ans. (a) correct]

**Hint:**  $[F \cos 90^\circ = 0 = W]$

4. A ball moves on a frictionless inclined table without slipping. The work done by the table surface on the ball is

(a) positive (b) negative  
(c) zero (d) none

[Ans. (c) zero]

5. A force  $\vec{F} = 3\hat{i} + c\hat{j} + 2\hat{k}$  acting on a particle causes a displacement  $\vec{S} = (2\hat{i} - 3\hat{j} + 4\hat{k})$  in its own direction. If the work done is 8J, then the value of  $c$  is

(a) 0 (b) 6  
(c) 2 (d) 1

[Ans. (c) 2]

**Hint:**

$$[W = \vec{F} \cdot \vec{S} = (3\hat{i} + c\hat{j} + 2\hat{k}) \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 6 - 3c + 8 = 8; c = \frac{6}{3} = 2]$$

6. A body is falling from a height  $h$ . After it has fallen a height  $\frac{h}{2}$  it will possess

(a) only Potential Energy  
(b) only Kinetic Energy  
(c) half potential and half kinetic energy  
(d) more kinetic and less potential energy

[Ans. (c) half potential and half kinetic energy]

7. Water stored in a dam possesses

(a) no energy (b) electrical energy  
(c) kinetic energy (d) potential energy

[Ans. (d) potential energy]

8. Which one of the following is not the unit of energy?

(a) joule (b) Nm  
(c) kW (d) kWh

[Ans. (c) kW]

9. A car is accelerated on a levelled road and attains a velocity 3 times of its initial velocity. In this process the potential energy of the car

(a) does not change  
(b) becomes twice to that of initial  
(c) becomes 4 times to initial  
(d) becomes 16 times to that of initial

[Ans. (a) does not change]

10. When a body falls freely towards the earth, then its T.E.

(a) increases (b) decreases  
(c) remains constant  
(d) first increases and then decreases

[Ans. (c) remains constant]

11. An iron sphere of mass 8 kg has the same diameter as a copper sphere of mass 4 kg. Both spheres are dropped simultaneously from a tower. When they are 10 m above the ground, they have the same

(a) acceleration (b) momenta  
(c) Potential Energy (d) Kinetic Energy

[Ans. (a) acceleration]

12. A boy is carrying a school bag of 5 kg mass on his back and moves 100 m on a levelled road. The work done against the gravitational force is ( $g = 10 \text{ ms}^{-2}$ )

(a) 5 J (b) 500 J  
(c) 0.5 J (d) zero

[Ans. (d) zero]



- (i) The capacity to do a work is called Energy.

Ramu has got more energy than Raju.

Work = Energy = Force  $\times$  displacement

- (ii) Rate of doing work (or) Rate at which work is done is Power.

$$(P) \text{ Power} = \frac{\text{Work}}{\text{time}} = \frac{W}{t}$$

Work (Energy) = Power  $\times$  time

$$W = P \times t$$

Unit of power is watt (W)

Unit of Energy is wat second (joule).

2. Ruvanjeni was told to switch on motor by her mother. Since the tank at the top of her building is empty, water is filled from the well. After the tank is full, she switched it off. She went near the tap opened it and washed her hands and legs with water from the tap for ten minutes. What concept is hidden in this scene.

- (i) What are the different energies at different position from the tank to the Tap.  
 (ii) Does Ruvanjeni waste water for a small job?  
 (iii) What is the Total Energy near the tap?

**Ans.** From this incident, when water rises from the well to the tank, Energy changes from Kinetic to Potential Energy. When water is stored fully in the tank, the total energy is wholly Potential (P.E =  $mgh$ ) 'h' is the height from the ground level.

- (i) From the Tank to the tap, the energy changes from potential to kinetic energy. Here Kinetic Energy, K.E =  $\frac{1}{2}mv^2$  'v' is the speed of water. At the tap, P.E = 0.  
 (ii) Yes. Ruvanjeni wastes more water for washing her hands and legs for a little job. Wasting water is not a good habit. Even a drop is precious. So we must use water only for essential needs.

- (iii) At the Top (Tank)

$$P.E = mgh, K.E = 0, T.E = P.E$$

At the Middle,

$$P.E \neq 0, K.E \neq 0$$

$$T.E = P.E + K.E$$

Near the tap (bottom)

$$P.E = 0, K.E \neq 0$$

$$T.E = K.E$$

### CONCEPTUAL QUESTIONS

1. A spring which is initially in un-stretched condition, is first stretched by a length  $x$  and again by a further length  $x$ . The work done in the first case  $W_1$  is one third of the work done in second case  $W_2$ . True or false?

**Ans.** Work done in the first case  $W_1 = \frac{1}{2} kx^2$ .

(i.e. P.E. stored)

In second case, work done  $W_2 = \text{Change in P.E.}$

$$= P.E_{(\text{final})} - P.E_{\text{initial}}$$

$$= \frac{1}{2} k (2x)^2 - \frac{1}{2} kx^2 = \frac{1}{2} k (4x^2 - x^2)$$

$$W_2 = \frac{1}{2} k 3x^2$$

$$W_2 = 3. W_1$$

$$\therefore W_1 = \frac{1}{3} W_2$$

= True.

2. Which is conserved in inelastic collision? Total energy (or) Kinetic energy?

**Ans.** Total energy is always conserved.

But K.E. is not conserved.

3. Is there any net work done by external forces on a car moving with a constant speed along a straight road?

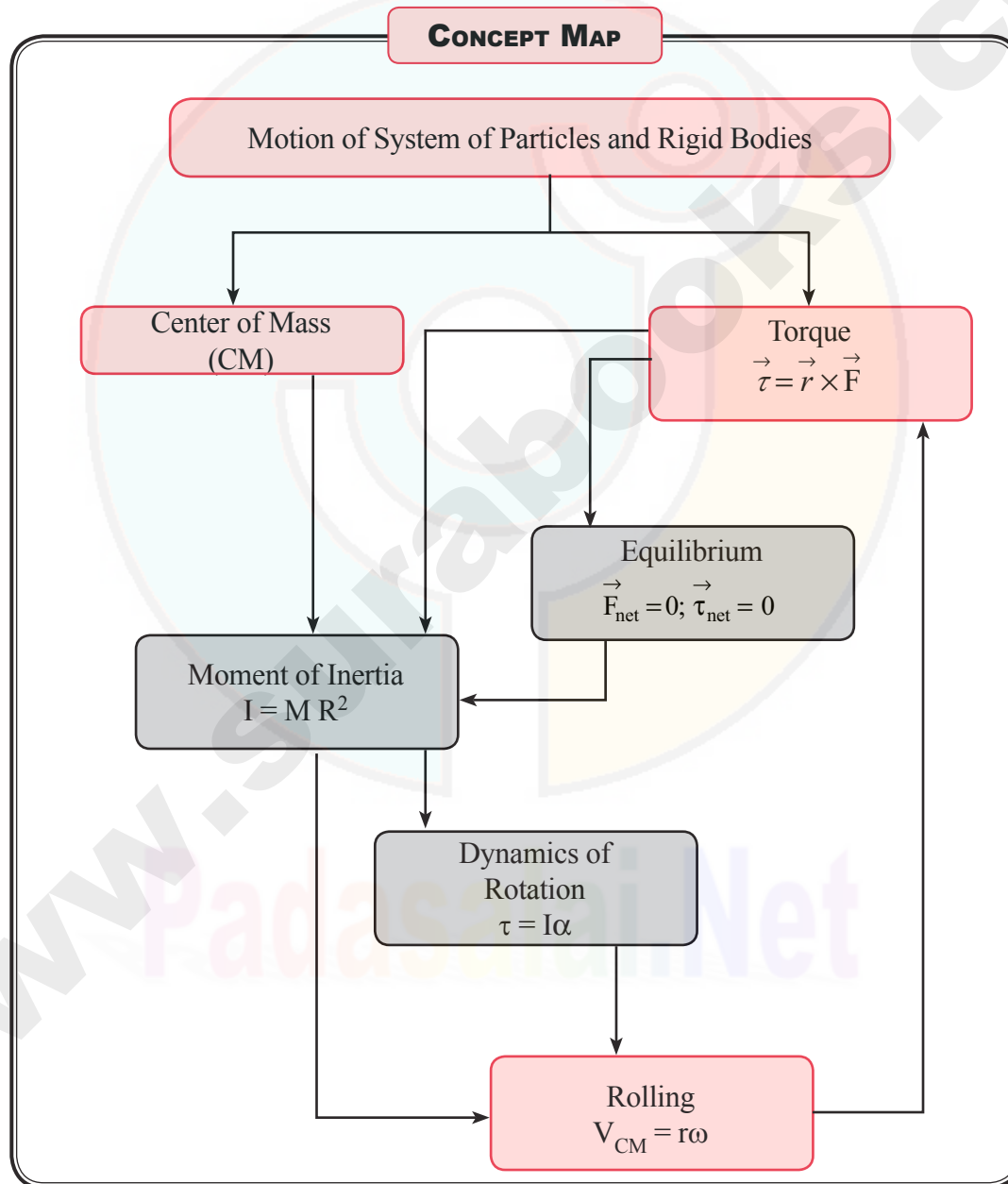
**Ans.** No.

If a car is moving at a constant speed, then net external force will be zero.

UNIT

05

# MOTION OF SYSTEM OF PARTICLES AND RIGID BODIES



(1) (2) (3) (4)

- a) c d b a  
 b) d c b a  
 c) b c d a  
 d) d a b c

[Ans:(a) c d b a]

2.	Rotational Motion about a fixed Axis	Expression
(1)	Power	(a) $\tau \cdot \Delta t$
(2)	Impulse	(b) $\tau \theta$
(3)	Torque	(c) $\tau \omega$
(4)	Work done	(d) $I \omega$

(1) (2) (3) (4)

- a) b c a d  
 b) d c a b  
 c) c a d b  
 d) b d c a

[Ans:(c) c a d b]

4.	(1)	Angular Momentum	(a)	$mr^2$
	(2)	Torque	(b)	$I\omega$
	(3)	Angular Acceleration	(c)	$\frac{dL}{dt}$
	(4)	Moment of Inertia	(d)	$\frac{d\omega}{dt}$

(1) (2) (3) (4)

- a) c d b a  
 b) b c d a  
 c) d c b a  
 d) c a b d

[Ans:(c) d c b a]

3.	Type of Equilibrium	Conditions
(1)	Dynamic Equilibrium	(a) Net force and Net torque are zero
(2)	Static Equilibrium	(b) Linear Momentum is constant
(3)	Rotational Equilibrium	(c) Linear Momentum and Angular momentum are constant
(4)	Translational Equilibrium	(d) Net torque is zero

(1) (2) (3) (4)

- a) b c d a  
 b) d c b a  
 c) b a d c  
 d) c a d b

[Ans:(d) c a d b]

## III. FILL IN THE BLANKS :

1. A \_\_\_\_\_ is the one in which the distances between different particles remain constant.

- (a) flexible body (b) Rigid body  
 (c) Spring body (d) Slim body

[Ans. (b) Rigid body]

2. In \_\_\_\_\_ the translational motion is more than rotational motion.

- (a) Slipping (b) frictional motion  
 (c) Circular motion (d) Sliding

[Ans. (d) Sliding]

3. \_\_\_\_\_ can also be treated as the momentary rotation about the point of contact.

- (a) Sliding (b) Rolling  
 (c) Slipping (d) Skating

[Ans. (b) Rolling]

4. If the external torque acting on the body zero, the component of \_\_\_\_\_ along the axis of rotation is constant.

- (a) Velocity (b) Force  
 (c) Angular Momentum (d) Momentum

[Ans. (c) Angular Momentum]

5. In \_\_\_\_\_ the rotational motion is more than translational motion.

- (a) Sliding (b) Slipping  
 (c) Rolling (d) Skating

[Ans. (b) Slipping]

6. The expression for Impulse in the case of Rotational motion about a fixed axis is \_\_\_\_\_

- (a)  $F \cdot \Delta t$  (b)  $\tau \cdot \Delta t$   
 (c)  $\tau \cdot \Delta m$  (d)  $\tau \cdot \Delta k$

[Ans. (b)  $\tau \cdot \Delta t$ ]

7.  $\frac{1}{2} I \omega^2$  is the expression for \_\_\_\_\_

- (a) Moment of Inertia  
 (b) Rotational kinetic Energy  
 (c) Elastic Potential Energy  
 (d) Kinetic Energy

[Ans. (b) Rotational kinetic Energy]

4. Find the moment of inertia of a mass of 5 kg and another mass 10 kg about an axis of rotation which is 0.2 m from the 5 kg mass and 0.4 m from the 10 kg mass. Find the radius of gyration for the system.

**Solution:**

$$\text{Moment of inertia } I = m_1 r_1^2 + m_2 r_2^2$$

$$I = 5 \times (0.2)^2 + 10 \times (0.4)^2$$

$$I = (5 \times 0.04) + (10 \times 0.16)$$

$$I = (0.20 + 1.6) = 1.8 \text{ kgm}^2$$

$$Mk^2 = 1.8 \text{ (k is the radius of gyration of the system)}$$

$$k^2 = \frac{1.8}{(10+5)} = \frac{1.8}{15} = 0.12$$

$$k = \sqrt{0.12} = 0.35 \text{ m (or) } 35 \text{ cm.}$$

5. Find the torque of a force  $7\hat{i} + 3\hat{j} - 5\hat{k}$  about the origin. The force acts on a particle whose position vector is  $\hat{i} - \hat{j} + \hat{k}$ .

**Solution:**

Given, position vector

$$\vec{r} = \hat{i} - \hat{j} + \hat{k}$$

$$\text{Force, } \vec{F} = 7\hat{i} + 3\hat{j} - 5\hat{k}; \text{ Torque, } \tau = ?$$

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 7 & 3 & -5 \end{vmatrix}$$

$$= (5-3)\hat{i} - (-5-7)\hat{j} + (3+7)\hat{k}$$

$$= 2\hat{i} + 12\hat{j} + 10\hat{k}$$

6. A hoop of radius 2m weights 100 kg. It rolls along a horizontal floor so that its centre of mass has speed of 20 cm/s. How much work has to be done to stop it?

**Solution:**

Moment of inertia of hoop about its centre

$$I = MR^2$$

and energy of loop = translational kinetic energy of CM and rotational kinetic energy about axis through CM.

$$= \frac{1}{2} mv_{CM}^2 + \frac{1}{2} I\omega^2 \quad \left[ I = mR^2 \text{ and } \omega = \frac{v}{R} \right]$$

$$= \frac{1}{2} mv_{CM}^2 + \frac{1}{2} mR^2 \times \frac{v_{CM}^2}{R^2} = mv_{CM}^2$$

Work done is stopping the hoop

$$= \text{total KE of hoop} = mv_{CM}^2 = 100 \times (0.2)^2 = 4 \text{ J.}$$

### Creative Questions (HOTS)

#### 1. Explain Torque in vector form.

**Ans. (i)** The torque of a force about an axis is independent of the choice of the origin as long as it is chosen on that axis itself.

**(ii)** Let O be the origin on the axis AB, which is the rotational axis of a rigid body. F is the force acting at the point P. Now, choose another point O' anywhere on the axis.

**(iii)** The torque of F about O' is,

$$\begin{aligned} \vec{O'P} \times \vec{F} &= (\vec{O'O} + \vec{OP}) \times \vec{F} \\ &= (\vec{O'O} \times \vec{F}) + (\vec{OP} \times \vec{F}) \end{aligned}$$

**(iv)** As  $\vec{O'O} \times \vec{F}$  is perpendicular to  $\vec{O'O}$ , this term will not have a component along AB. Thus, the component of  $\vec{O'P} \times \vec{F}$  is equal to that of  $\vec{OP} \times \vec{F}$ .

#### 2. Explain how the torque can be expressed as a vector product of two vectors? How the direction and magnitude of torque is determined?

**Ans. (i)** For direction, the vector rule or right hand rule is used.

$$\tau = r F \sin \theta$$

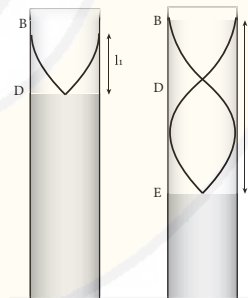
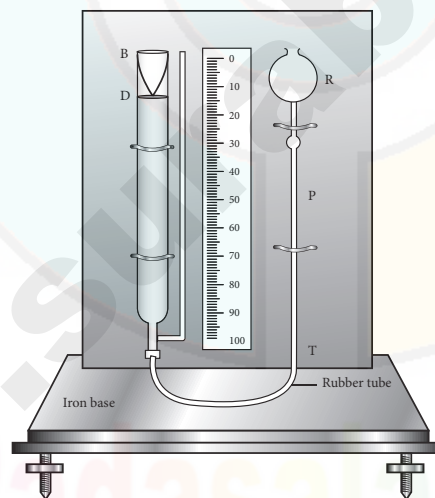
**(ii)** The expression for the magnitude of torque can be written in two different ways by associating  $\sin \theta$  either with  $r$  or  $F$  in the following manner.

$$\tau = r (F \sin \theta) = r \times (F \perp)$$

$$\tau = (r \sin \theta) F = (r \perp) \times F$$

# PHYSICS

## VOLUME - II

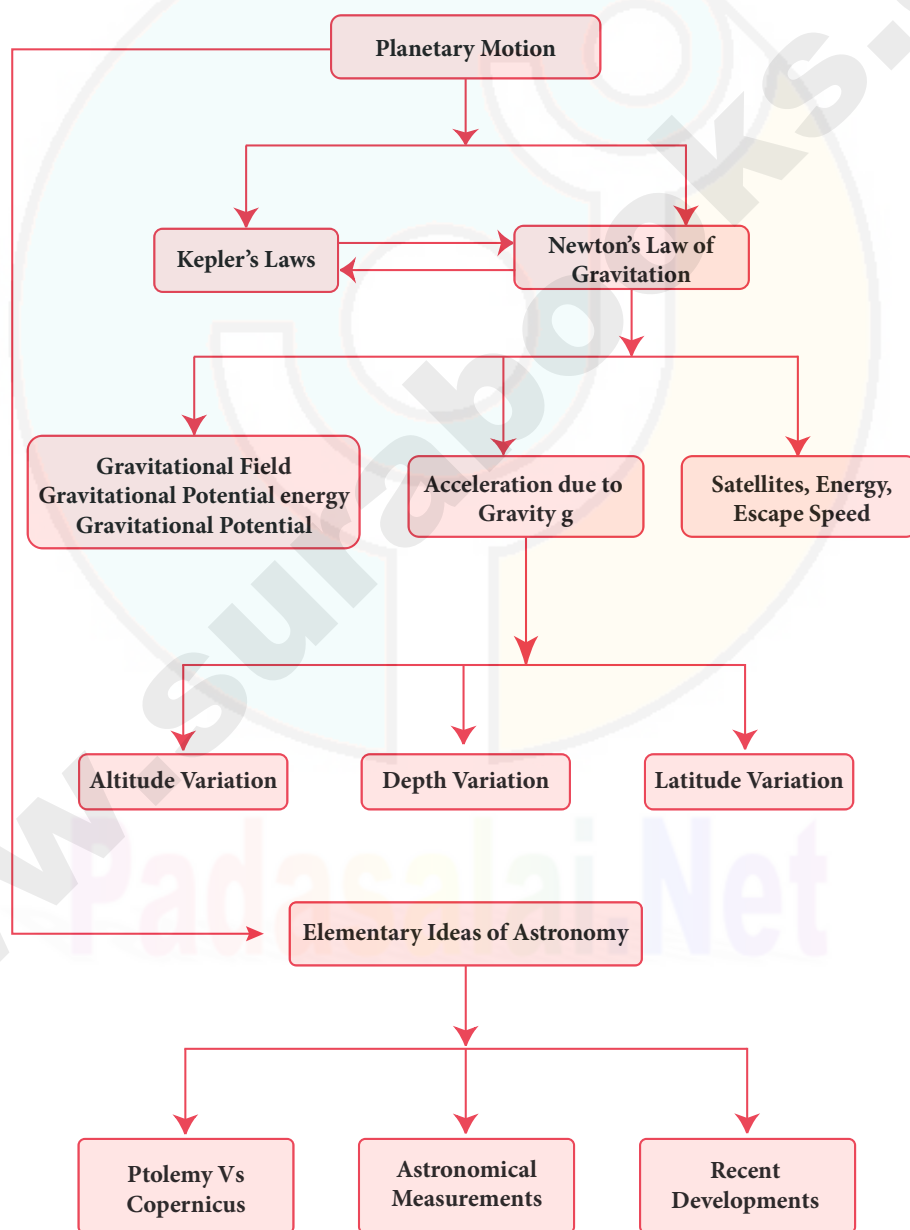


## UNIT

## 06

## GRAVITATION

## CONCEPT MAP





**Government Exam Question & Answers****I. MULTIPLE CHOICE QUESTIONS :****===== 1 Mark =====**

1. If the radius of Earth is 'R' at what height acceleration due to gravity becomes zero?

[Jun.-2019]

- (a) R      (b)  $\frac{R}{4}$       (c) 2R      (d)  $\frac{R}{2}$

[Ans. (★  $h = \infty$ ) ]**II. SHORT ANSWER QUESTIONS :****===== 3 Marks =====**

1. Define orbital velocity and establish an expression for it. [Mar.-2019]

**Ans. (i)** Definition : It is the horizontal velocity that has to be imparted to a satellite at the determined height so that it makes a circular orbit around the planet is called orbital velocity.

- (ii) Let us assume that a satellite of mass 'm' moves around the earth in a circular orbit of radius 'r' with uniform speed 'v'. Let the satellite be at a height 'h' from the surface of the earth. Here  $r = R + h$ , where 'R' is the radius of the earth.

- (iii) The centripetal force required to keep the

$$\begin{aligned} \text{satellite in circular orbit is } F &= \frac{mv^2}{r} \\ &= \frac{mv_0^2}{R+h} \end{aligned}$$

- (iv) The gravitational force between the earth and the satellite is  $F = \frac{GMm}{r^2} = \frac{GMm}{(R+h)^2}$  for stable motion.

$$\begin{aligned} \frac{mv_0^2}{R+h} &= \frac{GMm}{(R+h)^2} \\ v_0 &= \sqrt{\frac{GM}{R+h}} \\ \therefore g &= \frac{GM}{R^2} \Rightarrow v_0 = \sqrt{\frac{gR^2}{R+h}} \end{aligned}$$

If satellite is at a height of few hundred kilometres (i.e. say 200 km)  $R + h$  will be R.

$$\therefore v_0 = \sqrt{gR}$$

2. Calculate the value of orbital velocity for an artificial satellite of earth orbiting at a height of 1000 km (Mass of the earth =  $6 \times 10^{24}$  kg, radius of the earth = 6400 km). [Mar.-2019]

Given : height  $h = 1000$  km

Mass of Earth  $M = 6 \times 10^{24}$  kg

Radius of Earth  $R = 6400$  km

Orbital velocity  $v_0 = ?$

$$\text{Formula : } v_0 = \sqrt{\frac{GM}{R+h}}$$

$$v_0 = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6400 + 1000}}$$

$$= \sqrt{\frac{40.02 \times 10^{13}}{7400}}$$

$$= \sqrt{\frac{40.02 \times 10^{13}}{7.4 \times 10^3}} = \sqrt{\frac{40.02 \times 10^{10}}{7.4}}$$

$$v_0 = \sqrt{\frac{40.02}{7.4}} \times 10^5 = \frac{6.32613 \times 10^5}{2.7202}$$

$$= 232561.209 = 2.325 \times 10^5 \text{ km/s}$$

$$= 2.325 \times 10^5 \times 10^3 \text{ m/s}$$

$$= 2.325 \times 10^8 \text{ m/s.}$$

3. What do you mean by the term weightlessness?

Explain the state of weightlessness of a freely falling body. [Mar.-2019]

**Ans. (i)** Freely falling objects experience only gravitational force. As they fall freely, they are not in contact with any surface (by neglecting air friction).

The normal force acting on the object is zero.

- (ii) The downward acceleration is equal to the acceleration due to the gravity of the Earth. i.e ( $a = g$ ).

- (iii)  $a = g \therefore N = m(g - g) = 0$ .

- (iv) This is called the state of weightlessness.

- (v) Eg. When the lift falls (when the lift wire cuts) with downward acceleration  $a = g$ , the person inside the elevator is in the state of weightlessness.

**III. LONG ANSWER QUESTIONS :****===== 5 Marks =====**

1. Define orbital velocity and establish an expression for it. [Mar.-2019]

**Ans. (i)** Definition : It is the horizontal velocity that has to be imparted to a satellite at the determined height so that it makes a circular orbit around the planet is called orbital velocity.

7. The time period of the satellite of the earth is 5h. If the separation between the earth and the satellite is increased to 4 times the previous value, then what will be the new time period of the satellite acceleration to Kepler's law of periods.

**Solution:**

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3$$

$$\text{(or)} \left(\frac{T_2}{5}\right)^2 = \left(\frac{4R_1}{R_1}\right)^3$$

$$T_2 = \sqrt{64 \times 25} = 40\text{h}$$

### Creative Questions (HOTS)

1. A mass  $M$  is broken into two parts,  $m$  &  $(M-m)$ . How is  $m$  related to  $M$  so that the gravitational force between two parts is maximum?

**Solution:**

$$\text{Let } m_1 = m; m_2 = M - m$$

$$F = \frac{G(M-m)m}{r^2} = \frac{G(Mm - m^2)}{r^2}$$

Differentiating with respect to  $m$ ,

$$\frac{dF}{dm} = \frac{G}{r^2} (M - 2m)$$

$$\text{For } F \text{ is maximum } \frac{dF}{dm} = 0$$

$$\frac{G}{r^2} (M - 2m) = 0$$

$$M = 2m \Rightarrow m = \frac{M}{2}$$

$$m_1 = m_2 = \frac{M}{2}$$

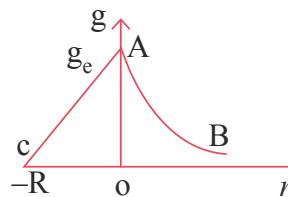
2. Draw graphs showing the variation of acceleration due to gravity with (i) height above the earth's surface (ii) depth below the earth's surface.

**Solution:**

- (i) The value of  $g$  varies with height  $h$  as

$$g \propto \frac{1}{(R+h)^2} \quad \text{or} \quad g \propto \frac{1}{r^2}$$

Thus the graph of  $g$  versus  $V$  is the parabolic curve AB



- (ii) The value of  $g$  varies with depth  $d$  as

$$g = g \left(1 - \frac{d}{R}\right) \quad \text{i.e.} \quad g \propto (R - d)$$

Thus the graph of  $g$  versus depth  $d$  is the straight line AC.

3. Suppose the gravitational force varies inversely as the  $n^{\text{th}}$  power of distance then, find the expression for the time period of a planet in a circular orbit of radius  $r$  around the sun.

**Solution:**

As the gravitational force varies inversely as the  $n^{\text{th}}$  power of the distance, so the gravitational force on the planet is given by

$$F = \frac{GMm}{r^n}$$

This force provides the centripetal force  $mr\omega^2$  to the planet

$$\frac{GMm}{r^n} = mr\omega^2$$

$$= mr \left(\frac{2\pi}{T}\right)^2$$

$$\text{or } T^2 = \frac{r \times 4\pi^2 \times r^n}{GM} = \frac{4\pi^2 r^{(n+1)}}{GM}$$

$$T = \frac{2\pi}{\sqrt{GM}} \cdot r^{\frac{(n+1)}{2}}$$

$$\therefore T \propto r^{\frac{(n+1)}{2}}$$

4. A simple pendulum has a time period  $T_1$  when on the earth's surface &  $T_2$  when taken to a height & above the earth's surface, where  $R$  is the radius of the earth. What is the value of  $T_2/T_1$ ?

**Solution:**

Let  $g$  &  $g'$  be the acceleration due to gravity on the earth's surface at a height  $R$  above the earth's surface.

$$g' = \frac{g}{\left(1 + \frac{n}{R}\right)} = \frac{g}{\left(1 + \left(\frac{R}{R}\right)^2\right)} = \frac{g}{4}$$

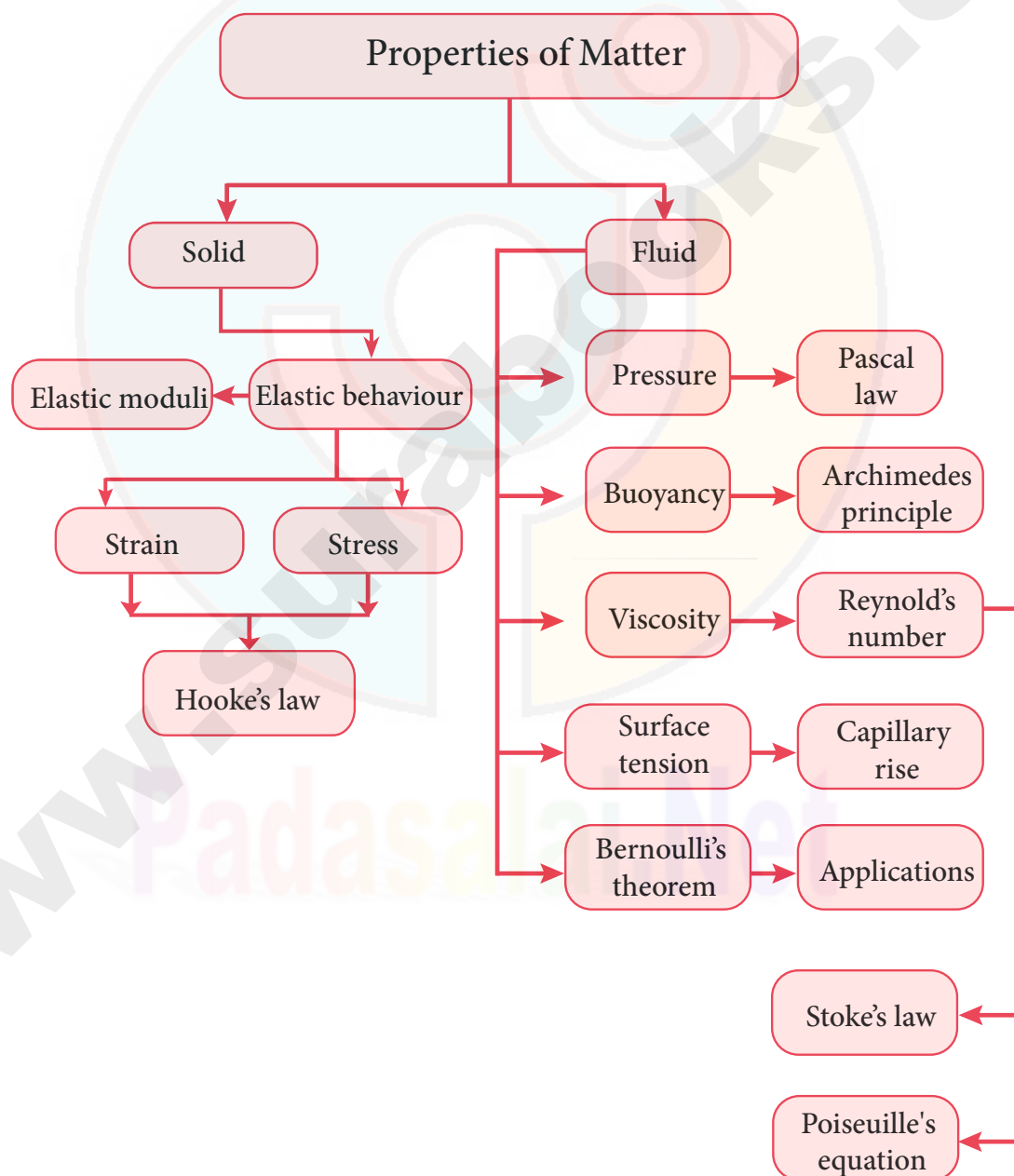


## UNIT

## 07

## PROPERTIES OF MATTER

## CONCEPT MAP



## IV. NUMERICAL PROBLEMS

1. A capillary of diameter  $d$  mm is dipped in water such that the water rises to a height of 30 mm. If the radius of the capillary is made  $\left(\frac{2}{3}\right)$  of its previous value, then compute the height up to which water will rise in the new capillary?

**Solution:**

$$h \propto \frac{1}{r} \Rightarrow hr = a \text{ constant}$$

$$h_1 r_1 = h_2 r_2 = \text{constant}$$

$$h_2 = \frac{h_1 r_1}{r_2}$$

$$\text{Height of water rises } h_1 = 30 \text{ mm}$$

$$\text{Radius of capillary } r_1 = \frac{d}{2} \text{ mm}$$

$$\text{New radius of capillary } r_2 = \frac{2}{3} \left( \frac{d}{2} \right) \text{ mm}$$

$$h_2 = \frac{30 \times \frac{d}{2}}{\frac{2}{3} \left( \frac{d}{2} \right)} = 45 \text{ mm}$$

$$h_2 = 45 \text{ mm}$$

2. A cylinder of length 1.5 m and diameter 4 cm is fixed at one end. A tangential force of  $4 \times 10^5$  N is applied at the other end. If the rigidity modulus of the cylinder is  $6 \times 10^{10} \text{ Nm}^{-2}$  then, calculate the twist produced in the cylinder.

**Solution:**

$$\text{Length of a cylinder} = 1.5 \text{ m}$$

$$\text{Diameter} = 4 \text{ cm; Tangential force } F = 4 \times 10^5 \text{ N}$$

$$\text{Rigidity modulus } \eta = 6 \times 10^{10} \text{ Nm}^{-2}$$

$$\text{Twist produced } \theta = ? \quad \left( \frac{\text{Tangential force}}{\text{Area}} \right)$$

$$\text{Rigidity Modulus } \eta_2 = \frac{\text{Shear Angle (Twist Angle)}}{\text{Angle Twist } \theta}$$

$$\text{Angle Twist } \theta = \frac{F/A}{\eta_R} = \frac{\frac{4 \times 10^5}{\pi \times r^2}}{6 \times 10^{10}}$$

$$= \frac{4 \times 10^5}{3.14 \times (2 \times 10^{-2})^2 \times 6 \times 10^{10}} = \frac{4 \times 10^5 \times 10^4 \times 10^{-10}}{24 \times 3.14}$$

$$\theta = 0.053 \times 10^{-1} = 53 \times 10^{-4}$$

3. A spherical soap bubble A of radius 2 cm is formed inside another bubble B of radius 4 cm. Show that the radius of a single soap bubble which maintains the same pressure difference as inside the smaller and outside the larger soap bubble is lesser than radius of both soap bubbles A and B.

**Solution:**

Excess pressure create with S.T of spherical surface of the liquid =  $\Delta P = \frac{2T}{R}$

T - surface tension

In case of soap bubbles,

The excess pressure of air inside them is doubled due to the presence of two interfaces are inside and one outside.

$$\Delta P_b = \frac{4T}{R}$$

Excess pressure of air inside the bigger bubble

$$\Delta P_b = \frac{4T}{4} = T$$

Excess pressure of air inside the smaller bubble

$$\Delta P_s = \frac{4s}{R} = \frac{4T}{2} = 2T$$

Air pressure difference between the smaller bubble and the atmosphere will be equal to the sum of excess pressure inside the bigger and smaller bubbles.

$$\begin{aligned} \text{Pressure different } \Delta P &= \Delta P_b + \Delta P_s \\ &= T + 2T = 3T \end{aligned}$$

Excess pressure inside a single soap bubble

$$= \frac{4T}{R} = \frac{4T}{4} = T$$

$\therefore$  Pressure difference of single soap bubble less than radius of both  $T < 3T$

4. A block of Ag of mass  $x$  kg hanging from a string is immersed in a liquid of relative density 0.72. If the relative density of Ag is 10 and tension in the string is 37.12 N then compute the mass of Ag block.

**Solution:**

$$\text{Relative density of liquid } \rho_{\text{liquid}} = 0.72$$

$$\text{Relative density of Ag } \rho_{\text{Ag}} = 10$$

$$\text{Mass of the Ag block} = ?$$

$$\text{Tension in the string } T = 37.12 \text{ N}$$

$$\text{Apparent weight } W_{\text{app}} = \rho_{\text{Ag}} - \rho_{\text{liquid}}$$

- (iii) On the other hand, as the wood a bad conductor, heat does not flow to the wooden tray from our body, on touching it.

#### IV. LONG ANSWER QUESTIONS :

==5 Marks==

##### 1. Write the Applications of elasticity.

- Ans. (i)** Elastic behavior is one such property which especially decides the structural design of the columns and beams of a building.
- (ii)** As the **structural engineering is concerned**, the amount of **stress that the design** could withstand is a primary safety factor.
- (iii)** A bridge has to be designed in such a way that it should have the capacity to withstand the load of the flowing traffic, the force of winds, and even its own weight.
- (iv)** The elastic behavior or in other words the bending of beams is a major concern over the stability of the buildings or bridges.
- (v)** To reduce the bending of a beam for a given load, one should use the material with a higher Young's modulus of elasticity (Y).
- (vi)** The Young's modulus of steel is greater than aluminium or copper.
- (vii)** Iron comes next to steel.
- (viii)** This is the reason why steel is mostly preferred in the design of heavy duty machines and iron rods in the construction of buildings.

##### 2. Write the applications of surface tension?

- Ans. (i)** Mosquitoes lay their eggs on the surface of water. To reduce the surface tension of water, a small amount of oil is poured. This breaks the elastic film of water surface and eggs are killed by drowning.
- (ii)** Chemical engineers must finely adjust the surface tension of the liquid, so it forms droplets of designed size and so it adheres to the surface without smearing. This is used in desktop printing, to paint automobiles and decorative items.
- (iii)** Specks of dirt get removed when detergents are added to hot water while washing clothes because surface tension is reduced.

- (iv)** A fabric can be made waterproof, by adding suitable waterproof material (wax) to the fabric. This increases the angle of contact.

##### 3. Write any two applications of Bernoulli's Theorem.

###### **Ans. (a) Blowing off roofs during wind storm**

- (i)** In olden days, the roofs of the huts or houses were designed with a slope.
- (ii)** One important scientific reason is that as per the Bernoulli's principle, it will be safeguarded except roof during storm or cyclone.
- (iii)** During cyclonic condition, the roof is blown off without damaging the other parts of the house.
- (iv)** In accordance with the Bernoulli's principle, the high wind blowing over the roof creates a low-pressure  $P_1$ .
- (v)** The pressure under the roof  $P_2$  is greater. Therefore, this pressure difference ( $P_2 - P_1$ ) creates an up thrust and the roof is blown off.

###### **(b) Aerofoil lift**

- (i)** The wings of an airplane (aerofoil) are so designed that its upper surface is more curved than the lower surface and the front edge is broader than the rear edge.
- (ii)** As the aircraft moves, the air moves faster above the aerofoil than at the bottom.
- (iii)** According to Bernoulli's Principle, the pressure of air below is greater than above, which creates an upthrust called the dynamic lift to the aircraft.

##### 4. What is the effect of pressure on the boiling point of a liquid. Describe a simple experiment to demonstrate the boiling of $H_2O$ at a temperature much lower than $100^\circ C$ . Give a practical application of this phenomenon.

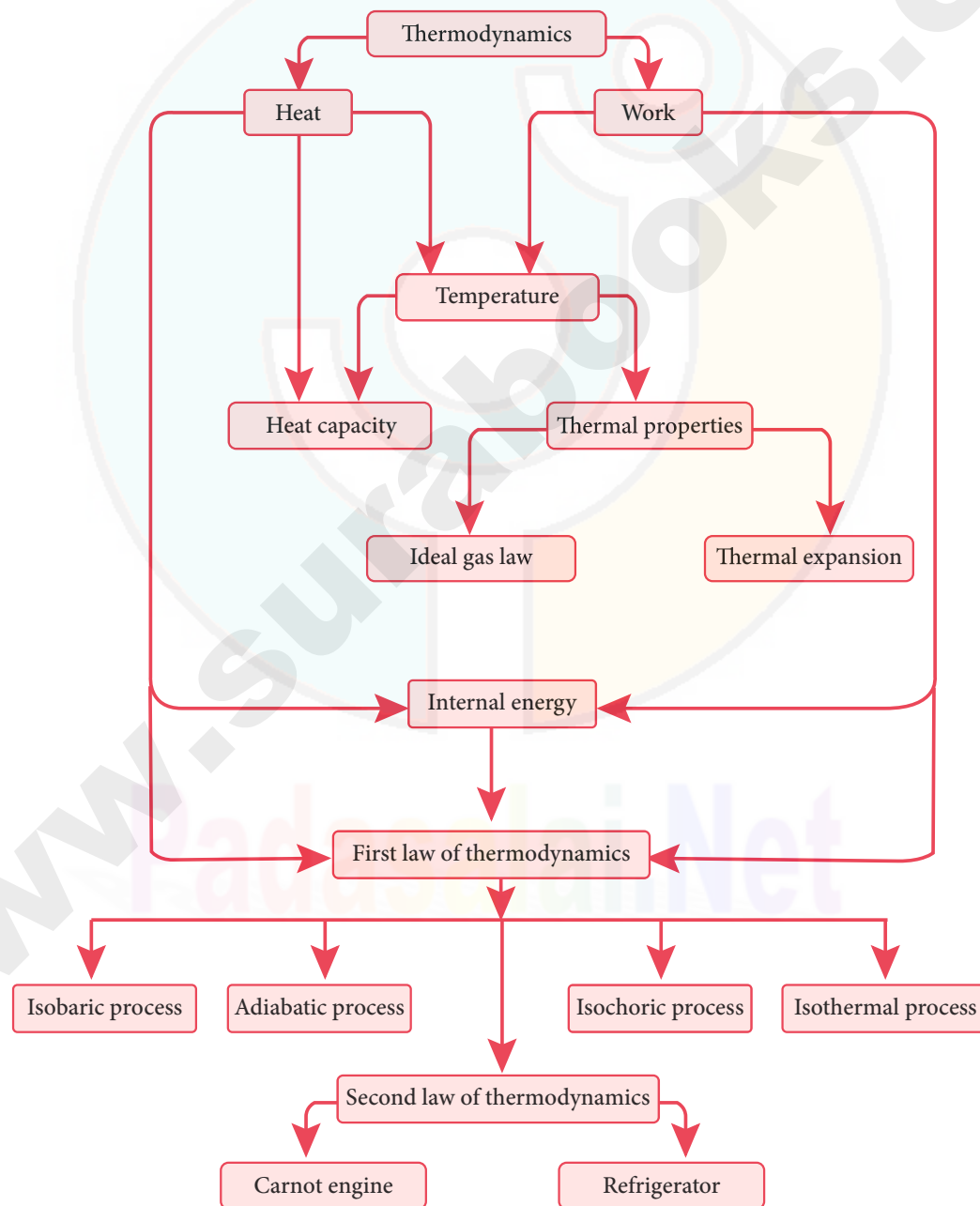
- Ans. (i)** Effect of pressure on the boiling point of a liquid:

## UNIT

## 08

HEAT AND  
THERMODYNAMICS

## CONCEPT MAP



### III. LONG ANSWER QUESTIONS :

1. Explain the meaning of heat and work with suitable examples.

**Ans. Meaning of heat and work :**

- (i) When hands are rubbed against each other the temperature of the hands increases. Some work is done on hands by rubbing.
- (ii) The temperature of the hands increases due to this work. Now if the hands are placed on the chin, the temperature of the chin increases. This is because the hands are at higher temperature than the chin.
- (iii) The temperature of hands is increased due to work and temperature of the chin is increased due to heat transfer from the hands to the chin.



**Difference between work and heat**

- (iv) By doing work on the system, the temperature in the system will increase and sometimes may not. Like heat, work is also not a quantity and through the work energy is transferred to the system.
- (v) Either the system can transfer energy to the surrounding by doing work on surrounding or the surrounding may transfer energy to the system by doing work on the system.
- (vi) For the transfer of energy from one body to another body through the process of work, they need not be at different temperatures.

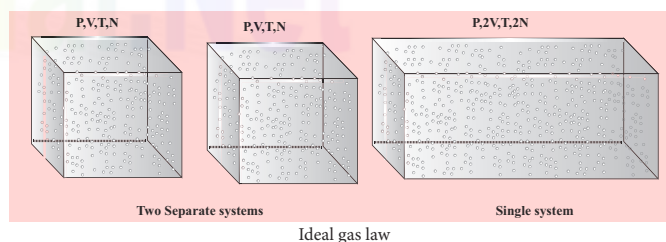
2. Discuss the ideal gas laws.

[Jun-2019]

**Ans. Boyle's law, Charles' law and ideal gas law :**

For a given gas at low pressure (density) kept in a container of volume  $V$ , experiments revealed the following information.

- (i) When the gas is kept at constant temperature, the pressure of the gas is inversely proportional to the volume. According to Boyle's law  $P \propto \frac{1}{V}$ . It was discovered by Robert Boyle (1627-1691) and is known as Boyle's law.
- (ii) When the gas is kept at constant pressure, the volume of the gas is directly proportional to absolute temperature. According to Charles' law  $V \propto T$ . By combining these two equations we have  $PV = CT$ . Here  $C$  is a positive constant.
- (iii)  $C$  is proportional to the number of particles in the gas container by considering the following argument.
- (iv) If we take two containers of same type of gas with same volume  $V$ , same pressure  $P$  and same temperature  $T$ , then the gas in each container obeys the equation,  $PV = CT$ .
- (v) If the two containers of gas is considered as a single system, then the pressure and temperature of this combined system will be same but volume will be twice and number of particles will also be double as shown in Figure.



- (vi) For this combined system,  $V$  becomes  $2V$ , so  $C$  should also double to match with the ideal gas equation  $\frac{P(2V)}{T} = 2C$ .



## ADDITIONAL QUESTIONS

### I. MULTIPLE CHOICE QUESTIONS :

== 1 Mark ==

1. The molar specific heats of an ideal gas at a constant pressure & volume are denoted by

$C_p$  &  $C_v$  if  $r = \frac{C_p}{C_v}$  &  $R$  the universal gases

constant then  $C_v$  is equal

(a)  $\frac{1+r}{1-r}$  (b)  $\frac{R}{(1-r)}$

(c)  $\frac{r-1}{R}$  (d)  $rR$

[Ans. (b)  $\frac{R}{(r-1)}$ ]

2. Steam at  $100^\circ\text{C}$  is passed into 20g of water at  $10^\circ\text{C}$  when water acquires a temp of  $80^\circ\text{C}$ , the mass of water present will be (take specific heat of water  $1 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$  & latent heat of steam  $= 540 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$ )

(a) 42.5 g (b) 22.5g  
(c) 24g (d) 31.5g

[Ans. (b) 22.5g]

3. The value of coefficient of volume expansion of glycerine is  $5 \times 10^{-4} \text{ K}^{-1}$  the fractional change in the density of glycerine for a rise of  $40^\circ\text{C}$  in its temp is

(a) 0.020 (b) 0.025 (c) 0.010 (d) 0.015

[Ans. (a) 0.020]

4. Two metal wires of identical dimensions are connected in series if  $\sigma_1$  and  $\sigma_2$  are the conductivity of the metal wire respectively, the effective conductivity of the combination is

(a)  $\frac{\sigma_1 + \sigma_2}{2\sigma_1\sigma_2}$  (b)  $\frac{\sigma_1 + \sigma_2}{\sigma_1\sigma_2}$

(c)  $\frac{\sigma_1\sigma_2}{(\sigma_1 + \sigma_2)}$  (d)  $\frac{2\sigma_1\sigma_2}{\sigma_1 + \sigma_2}$

[Ans. (b)  $\frac{\sigma_1 + \sigma_2}{\sigma_1\sigma_2}$ ]

5. Coefficient of linear expansion of brass and steel rods are  $\sigma_1$  and  $\sigma_2$ , length of brass and steel rods are  $l_1$  &  $l_2$  if  $(l_2 - l_1)$  is maintained same at all temperature which one of the following relations hold good?

(a)  $\alpha_1^2 l_2 = \alpha_2^2 l_1$  (b)  $\alpha_1 l_1 = \alpha_2 l_2$   
(c)  $\alpha_1 l_2 = \alpha_2 l_1$  (d)  $\alpha_1 l_2^2 = \alpha_2 l_1^2$

[Ans. (b)  $\alpha_1 l_1 = \alpha_2 l_2$ ]

6. A black body is continuously radiating energy at a temperature of  $2880 \text{ K}$  if  $U_1$ ,  $U_2$  and  $U_3$  are the amount of radiation measured between the wavelengths  $599 \text{ nm}$  &  $600 \text{ nm}$ ,  $999 \text{ nm}$  &  $1000 \text{ nm}$  &  $1499 \text{ nm}$  &  $1500 \text{ nm}$  respectively then (wien's constant  $b = 2.88 \times 10^{-6} \text{ K m}$ )

(a)  $U_2 < U_3$  (b)  $U_1 = U_2 = U_3$   
(c)  $U_1 < U_2 < U_3$  (d)  $U_2 > U_1 > U_3$

[Ans. (a)  $2.4 \pi \text{ mJ}$ ]

7. Aluminium has specific heat capacity of

(a)  $450 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$  (b)  $900 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$   
(c)  $1350 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$  (d)  $1800 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$

[Ans. (b)  $900 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$ ]

8. Internal energy comprises of two types of energies, those are

(a) mechanical and electrical energy  
(b) magnetic and electrical energy  
(c) kinetic and potential energy  
(d) kinetic and magnetic energy

[Ans. (c) kinetic and potential energy]

9. A pure substance would freeze or solidify at its

(a) boiling point (b) condensation point  
(c) melting point (d) sublimation point

[Ans. (c) melting point]

10. Amount of energy required to change liquid to gas and vice versa without any change in temperature is termed as

(a) Latent Heat and Fusion  
(b) Latent Heat of Vaporisation  
(c) Heat capacity  
(d) Specific heat capacity

[Ans. (b) Latent Heat of Vaporisation]

### VALUE BASED QUESTIONS

1. Gopi, a small boy does mischievous things at home. He one day took a spoon and kept in the flame of stove by holding it at the edge. His mom went to pluck flowers in the garden. When she came to kitchen, Gopi was found to have swellings due to burns in his finger. She asked what he did, he cried and said the truth. He asked how the heat wounded his finger, as he was holding the spoon only at edge. What is the explanation given by his mother?

(i) Why birds swell there feathers in winter season?

(ii) Why Ice is packed in sawdust?

**Ans.** Gopi's mom told him, this is due to a process called conduction. In solids, the heat will be transmitted from the hotter region to the colder region. This is due to the agitation of atoms or molecules of the solid which is transferred to the neighboring atoms or molecules in the whole body. His mother should have taken Gopi along with her to the Garden, knowing that he should not be left alone at home, as he is mischievous. Parental guidance is must for every child.

(i) Birds often swell their feathers in winter to enclose air between their body and the feathers. Air prevents the loss of heat from the body of the bird to the coldest surroundings.

(ii) Ice is packed in saw dust because air trapped in the saw dust prevents the transfer of heat from the surroundings to the Ice. Hence Ice does not melt.

2. Sudeep and Karthikeyan were room mates. Sudeep was boiling water in a vessel for bathing purpose. Karthikeyan said Sudeep, you know when you touch the middle part of the vessel containing water, you can feel less heat than what you can feel at the top. Sudeep was shocked to hear. Karthikeyan took a thermometer and kept it at the top surface of water. The temperature was rising. Again he inserted it deep into the vessel containing water, now the temperature has fallen down than in the previous condition. What is the truth behind it?

(i) It is good to keep our hand deep into the vessel to check this while the liquid is boiling?

(ii) What are convection currents?

**Ans.** The truth behind this magic is convection process. It is a phenomenon of heat transfer in a fluid with the actual movement of the particles of the fluid. i.e. when a fluid is heated the hot part expands and becomes less dense, so it rises up and replaces the colder part of fluid. Again this fluid is heated, rises up of replaces the colder part of the fluid. This process goes on. That is why, when we boil water, heat will be from top layer of the liquid to the bottom layer.

(i) No, it is not good to check like this. As the top layer of fluid is not, our hands must cross this layer first, to move it deep into the vessel containing water. So you may get burns or swells.

(ii) Convection currents are the constant cyclical motion of air water, and other substances to distribute heat. As heated air rises, for example, it pulls cooler air into its place where it can be heated, rise and pull in most cool air. Some geologists believe convection currents within the Earth's molten rocks (in semi-liquid state) are a contributing cause of volcanoes, earthquakes and continental drift.

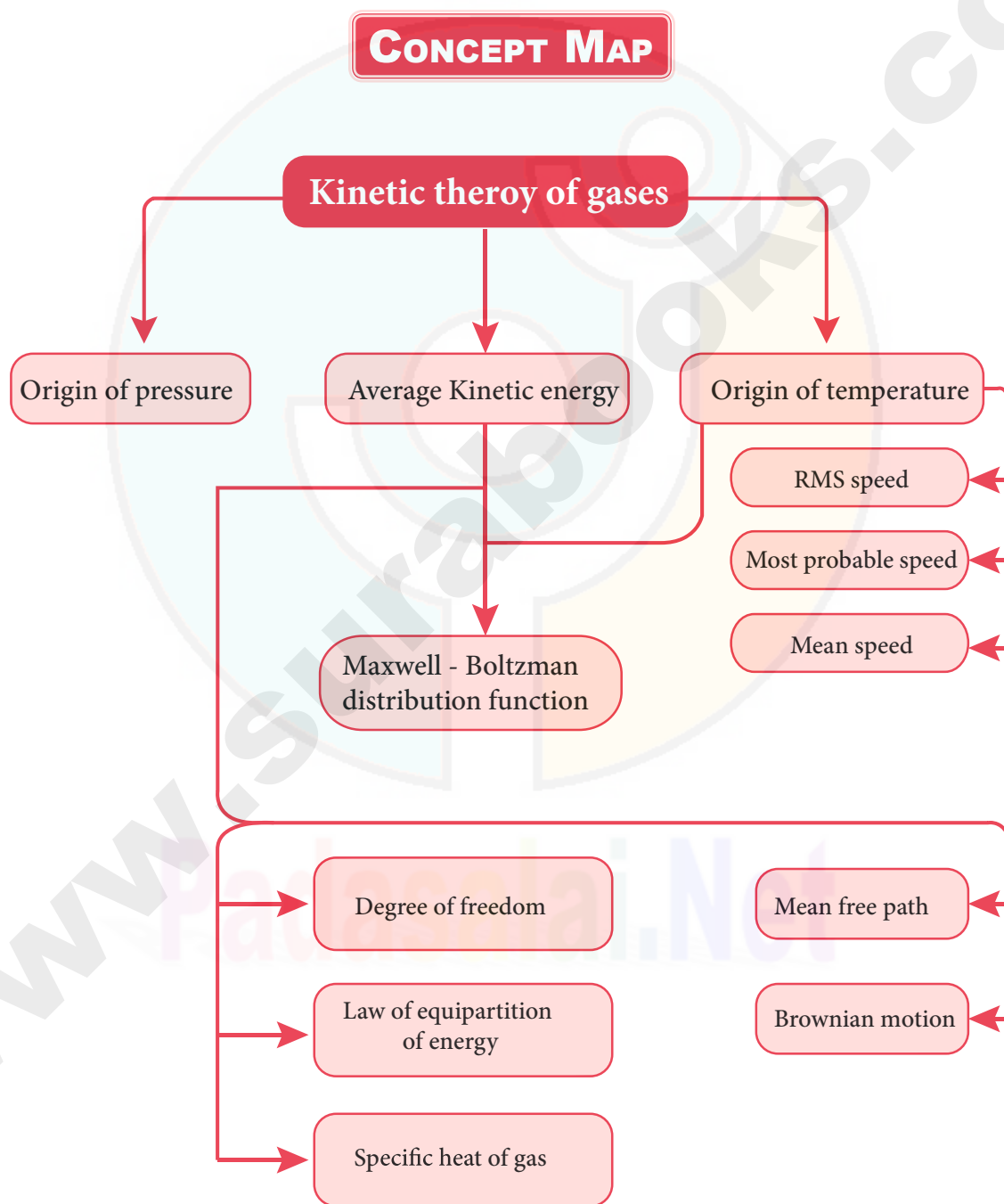


## UNIT

## 09

KINETIC THEORY  
OF GASES

## CONCEPT MAP

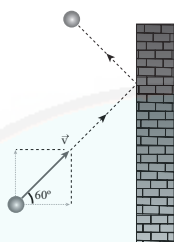




## EVALUATION

## I. MULTIPLE CHOICE QUESTIONS:

1. A particle of mass  $m$  is moving with speed  $u$  in a direction which makes  $60^\circ$  with respect to  $x$  axis. It undergoes elastic collision with the wall. What is the change in momentum in  $x$  and  $y$  direction?



- (a)  $\Delta p_x = -mu, \Delta p_y = 0$   
 (b)  $\Delta p_x = -2mu, \Delta p_y = 0$   
 (c)  $\Delta p_x = 0, \Delta p_y = mu$   
 (d)  $\Delta p_x = mu, \Delta p_y = 0$

[Ans. (a)  $\Delta p_x = -mu, \Delta p_y = 0$ ]

2. A sample of ideal gas is at equilibrium. Which of the following quantity is zero?

- (a) rms speed (b) average speed  
 (c) average velocity (d) most probable speed

[Ans. (c) average velocity]

3. An ideal gas is maintained at constant pressure. If the temperature of an ideal gas increases from 100K to 10000K then the rms speed of the gas molecules

- (a) increases by 5 times (b) increases by 10 times  
 (c) remains same (d) increases by 7 times

[Ans. (b) increases by 10 times]

4. Two identically sized rooms A and B are connected by an open door. If the room A is air conditioned such that its temperature is  $4^\circ\text{C}$  lesser than room B, which room has more air in it?

- (a) Room A (b) Room B  
 (c) Both room has same air  
 (d) Cannot be determined

[Ans. (a) Room A]

5. The average translational kinetic energy of gas molecules depends on

- (a) number of moles and  $T$   
 (b) only on  $T$   
 (c)  $P$  and  $T$   
 (d)  $P$  only

[Ans. (a) number of moles and  $T$ ]

6. If the internal energy of an ideal gas  $U$  and volume  $V$  are doubled, then the pressure

- (a) doubles (b) remains same  
 (c) halves (d) quadruples

[Ans. (b) remains same]

7. The ratio  $\gamma = \frac{C_p}{C_v}$  for a gas mixture consisting of 8 g of helium and 16 g of oxygen is

(Physics Olympiad -2005) [Jun.-2019]

- (a)  $\frac{23}{15}$  (b)  $\frac{15}{23}$  (c)  $\frac{27}{17}$  (d)  $\frac{17}{27}$

[Ans. (c)  $\frac{27}{17}$ ]

8. A container has one mole of monoatomic ideal gas. Each molecule has  $f$  degrees of freedom.

What is the ratio of  $\gamma = \frac{C_p}{C_v}$ ?

- (a)  $f$  (b)  $\frac{f}{2}$  (c)  $\frac{f}{f+2}$  (d)  $\frac{f+2}{f}$

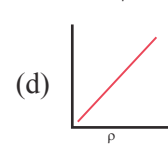
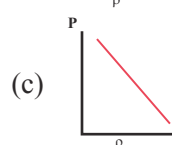
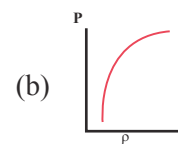
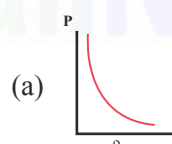
[Ans. (d)  $\frac{f+2}{f}$ ]

9. If the temperature and pressure of a gas is doubled the mean free path of the gas molecules

- (a) remains same (b) doubled  
 (c) tripled (d) quadrupled

[Ans. (a) remains same]

10. Which of the following shows the correct relationship between the pressure and density of an ideal gas at constant temperature? [HY-2019]



[Ans. (d)]

## VERY SHORT ANSWER QUESTIONS :

===== 2 Mark =====

1. What are the factors which effect Brownian motion? [Mar.-2019; HY - 2019]

Ans. The Brownian motion increases

- (i) with the decrease in single of the suspended particle
- (ii) with the increases in temperature of the fluid
- (iii) with the decrease in density of the fluid.
- (iv) with the decrease in viscosity of the fluid

## NUMERICAL PROBLEMS :

===== 3 Mark =====

1. An oxygen molecule is travelling in air at 300 K and 1 atm, and the diameter of oxygen molecule is  $1.2 \times 10^{-10}$  m. Calculate the mean free path of oxygen molecule. [HY - 2019]

**Solution:**

$$\text{From } \lambda = \frac{1}{\sqrt{2}n\pi d^2}$$

We have to find the number density n By using ideal gas law

$$n = \frac{N}{V} = \frac{P}{kT} = \frac{101.3 \times 10^3}{1.381 \times 10^{-23} \times 300}$$

$$= 2.449 \times 10^{25} \text{ molecules/m}^3$$

$$\lambda = \frac{1}{\sqrt{2} \times \pi \times 2.449 \times 10^{25} \times (1.2 \times 10^{-10})^2}$$

$$= \frac{1}{15.65 \times 10^5}$$

$$\lambda = 0.63 \times 10^{-6} \text{ m.}$$

## ADDITIONAL QUESTIONS

## I. MULTIPLE CHOICE QUESTIONS :

===== 1 Mark =====

1. Kinetic theory explains the behavior

- (a) of solids and liquid Based on the idea that the gas consists of rapidly vibrating atoms or molecules.
- (b) of gases based on the idea that the gas consists of rapidly moving atoms or molecules.
- (c) of solids based on the idea that the solid consists of rapidly vibrating atoms or molecules.
- (d) of liquid based on the idea that the liquid consist of rapidly moving atoms or molecules.

[Ans. (b) of gases based on the idea that the gas consists of rapidly moving atoms or molecules.]

2. Kinetic theory

- (a) Correctly explains specific heat capacities of many liquid
- (b) Correctly explains specific heat capacities of many gases
- (c) Correctly explains specific heat capacities of many solids

- (d) Correctly explains specific heat capacities of many super cooled liquid

[Ans. (b) Correctly explains specific heat capacities of many gases]

3. The average distance a molecule can travel without colliding is called

- (a) mean free distance      (b) mean free path
- (c) mean free length      (d) mean free motion

[Ans. (b) mean free path]

4. In dynamic equilibrium, molecules of gas collide and change their speeds during the collision

- (a) but the average properties vary
- (b) but the average properties constant
- (c) but the peak properties constant
- (d) but the rms properties constant [Ans. (b) but the average properties constant]

5. The perfect gas equation can be written as

- (a)  $PV = \mu RT$       (b)  $PV = \mu R$
- (c)  $PV = RT$       (d)  $P = \mu RTV$

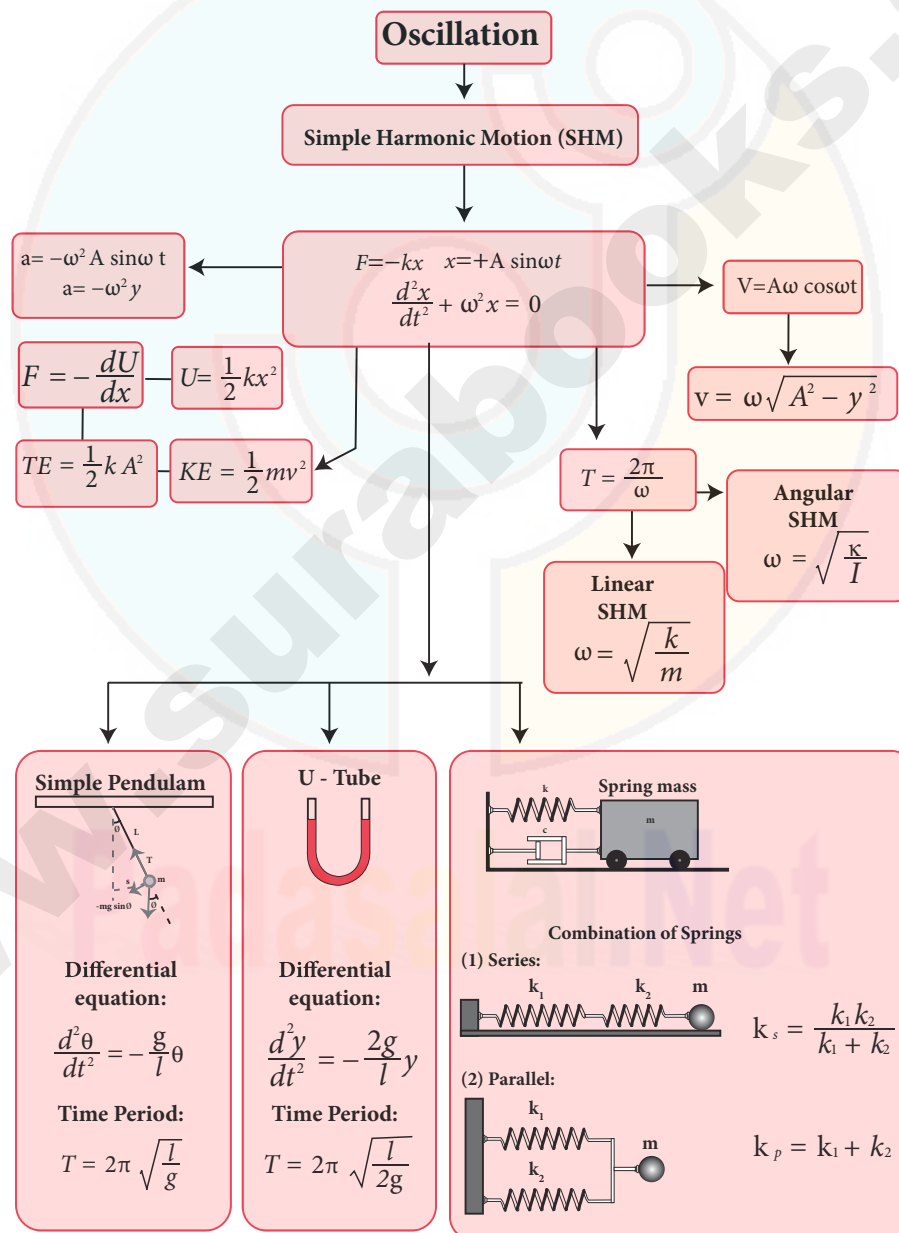
[Ans. (a)  $PV = \mu RT$ ]

## UNIT

## 10

## OSCILLATIONS

## CONCEPT MAP



### III. LONG ANSWER QUESTIONS

1. What is meant by simple harmonic oscillation? Give examples and explain why every simple harmonic motion is a periodic motion whereas the converse need not be true.

**Ans. (i)** Simple harmonic motion is a special type of oscillatory motion in which the acceleration or force on the particle is directly proportional to its displacement from a fixed point and is always directed towards that fixed point.

- (ii) In one dimensional case, let  $x$  be the displacement of the particle and  $a_x$  be the acceleration of the particle, then

$$a_x \propto x; a_x = -bx$$

where  $b$  is a constant which measures acceleration per unit displacement and dimensionally it is equal to  $T^{-2}$ .

- (iii) By multiplying by mass of the particle on both sides of equation and from Newton's second law, the force is

$$F_x = -kx$$

where  $k$  is a force constant which is defined as force per unit length.

- (iv) The negative sign indicates that displacement and force (or acceleration) are in opposite directions.

- (v) This means that when the displacement of the particle is taken towards right of equilibrium position ( $x$  takes positive value), the force (or acceleration) will point towards equilibrium (towards left) and similarly, when the displacement of the particle is taken towards left of equilibrium position ( $x$  takes negative value), the force (or acceleration) will point towards equilibrium (towards right).

- (vi) This type of force is known as restoring force because it always directs the particle executing simple harmonic motion to restore to its original (equilibrium or mean) position. This force (restoring force) is central and attractive whose center of attraction is the equilibrium position.

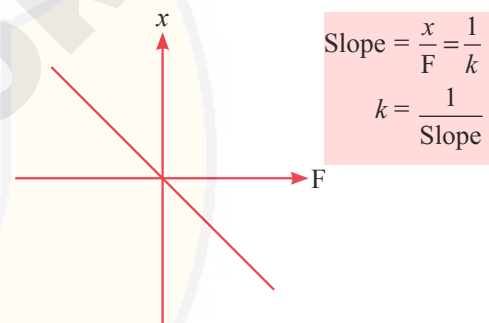
- (vii) In order to represent in two or three dimensions, we can write using vector notation

$$\vec{F} = -k\vec{r}$$

- (viii) where  $\vec{r}$  is the displacement of the particle from the chosen origin. Note that the force and displacement have a linear relationship.

This means that the exponent of force  $\vec{F}$  and the exponent of displacement  $\vec{r}$  are unity.

- (ix) The sketch between cause (magnitude of force) and effect (magnitude of displacement  $|\vec{r}|$ ) is a straight line passing through second and fourth quadrant by measuring slope  $\frac{1}{k}$ , one can find the numerical value of force constant  $k$ .



Force versus displacement graph

2. Describe Simple Harmonic Motion as a projection of uniform circular motion.

**Ans. (i)** Consider a particle of mass  $m$  moving with uniform speed  $v$  along the circumference of a circle whose radius is  $r$  in anti-clockwise direction.

- (ii) Let us assume that the origin of the coordinate system coincides with the center  $O$  of the circle.

- (iii) If  $\omega$  is the angular velocity of the particle and  $\theta$  the angular displacement of the particle at any instant of time  $t$ , then  $\theta = \omega t$ .

- (iv) By projecting the uniform circular motion on its diameter gives a simple harmonic motion. This means that we can associate a map (or a relationship) between uniform circular (or revolution) motion to vibratory motion.





**Motion of spring mass (or simple pendulum) related to uniform circular motion**

- (v) Conversely, any vibratory motion or revolution can be mapped to uniform circular motion. In other words, these two motions are similar in nature.
- (vi) Let us first project the position of a particle moving on a circle, on to its vertical diameter or on to a line parallel to vertical diameter. Similarly, we can do it for horizontal axis or a line parallel to horizontal axis.
- (vii) As a specific example, consider a spring mass system (or oscillation of pendulum). When the spring moves up and down (or pendulum moves to and fro), the motion of the mass or bob is mapped to points on the circular motion.
- (viii) Thus, if a particle undergoes uniform circular motion then the projection of the particle on the diameter of the circle (or on a line parallel to the diameter) traces straightline motion which is simple harmonic in nature.
- (ix) The circle is known as reference circle of the simple harmonic motion. The simple harmonic motion can also be defined as the motion of the projection of a particle on any diameter of a circle of reference.

**3. What is meant by angular harmonic oscillation? Compute the time period of angular harmonic oscillation.**

- Ans. (i)** When a body is allowed to rotate freely about a given axis then the oscillation is known as the angular oscillation.
- (ii)** The point at which the resultant torque acting on the body is taken to be zero is called mean position. If the body is displaced from the mean position, then the resultant torque acts such that it is

proportional to the angular displacement and this torque has a tendency to bring the body towards the mean position.



- (iii)** Let  $\vec{\theta}$  be the angular displacement of the body and the resultant torque  $\vec{\tau}$  acting on the body is

$$\vec{\tau} \propto \vec{\theta} \quad \dots(1)$$

$$\vec{\tau} = -\kappa \vec{\theta} \quad \dots(2)$$

$\kappa$  is the restoring torsion constant, which is torque per unit angular displacement. If  $I$  is the moment of inertia of the body and  $\vec{\alpha}$  is the angular acceleration then

$$\vec{\tau} = I \vec{\alpha} = -\kappa \vec{\theta}$$

But  $\vec{\alpha} = \frac{d^2 \vec{\theta}}{dt^2}$  and therefore,

$$\vec{\alpha} = \frac{d^2 \vec{\theta}}{dt^2} = -\frac{\kappa}{I} \vec{\theta} \quad \dots(3)$$

$$\vec{\alpha} = \omega^2 \vec{\theta} \quad \dots(4)$$

- (iv)** This differential equation resembles simple harmonic differential equation. So, comparing equations (3) & (4) we get

$$\omega = \sqrt{\frac{\kappa}{I}} \text{ rad s}^{-1} \quad \dots(4)$$

- (v)** The frequency of the angular harmonic motion is

$$f = \frac{1}{2\pi} \sqrt{\frac{\kappa}{I}} \text{ Hz} \quad \dots(5)$$

- (vi)** The time period is

$$T = 2\pi \sqrt{\frac{I}{\kappa}} \text{ second} \quad \dots(6)$$

4. Write down the difference between simple harmonic motion and angular simple harmonic motion.

Ans.

S. No	Simple Harmonic Motion	Angular Harmonic Motion
1.	The displacement of the particle is measured in terms of linear displacement $\vec{r}$	The displacement of the particle is measured in terms of angular displacement $\vec{\theta}$ (also known as angle of twist).
2.	Acceleration of the particle is $\vec{a} = -\omega^2 \vec{r}$	Angular acceleration of the particle is $\vec{\alpha} = -\omega^2 \vec{\theta}$
3.	Force, $\vec{F} = m \vec{a}$ where $m$ is called mass of the particle.	Torque, $\vec{\tau} = I \vec{\alpha}$ where $I$ is called moment of inertia of a body.
4.	The restoring force $\vec{F} = -k \vec{r}$ , where $k$ is restoring force constant.	The restoring torque $\vec{\tau} = -\kappa \vec{\theta}$ where the symbol $\kappa$ ('kappa') is called restoring torsion constant. It depends on the property of a particular torsion fiber.
5.	Angular frequency $\omega = \sqrt{\frac{k}{m}} \text{ rad s}^{-1}$	Angular frequency, $\omega = \sqrt{\frac{\kappa}{I}} \text{ rad s}^{-1}$

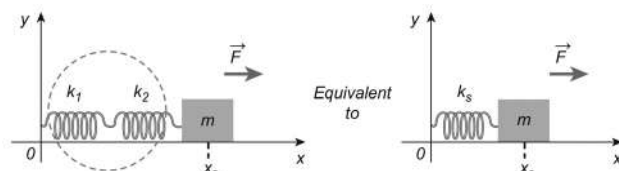
5. Discuss the simple pendulum in detail.

- Ans. (i) A pendulum is a mechanical system which exhibits periodic motion.
- (ii) It has a bob with mass  $m$  suspended by a long string (assumed to be massless and inextensible string) and the other end is fixed on a stand.
- (iii) When a pendulum is displaced through a small displacement from its equilibrium position and released, the bob of the pendulum executes to and fro motion.
- (iv) Let  $l$  be the length of the pendulum. The gravitational force acting on the body  $(\vec{F} = m \vec{g})$  which acts vertically downwards.
- (v) The tension in the string  $\vec{T}$  which acts along the string to the point of suspension. Resolving the gravitational force into its components:
- a. **Normal component:** The component along the string but in  $F_{as} = mg \cos\theta$  opposition to the direction of tension.
- b. **Tangential component:** The component  $F_{ps} = mg \sin\theta$  perpendicular to the string, i.e., along tangential direction of arc of swing.

6. Explain the horizontal oscillations of a spring.

[Mar.-2019]

- Ans. (i) Consider only two springs whose spring constant are  $k_1$  and  $k_2$  and which can be attached to a mass  $m$ . The results thus obtained can be generalized for any number of springs in series.
- (ii) Let  $F$  be the applied force towards right as shown in figure.
- (iii) Since the spring constants for different spring are different and the connection points between them is not rigidly fixed, the strings can stretch in different lengths.



- (iv) Let  $x_1$  and  $x_2$  be the elongation of springs from their equilibrium position (un-stretched position) due to the applied force  $F$ . Then, the net displacement of the mass point is



$$x = x_1 + x_2$$

From Hooke's law, the net force

$$F = -k_s (x_1 + x_2) \Rightarrow x_1 + x_2 = -\frac{F}{k_s} \quad \dots(1)$$

For springs in series connection

$$-k_1 x_1 = -k_2 x_2 = F \quad \dots(A)$$

$$\Rightarrow x_1 = -\frac{F}{k_1} \text{ and } x_2 = -\frac{F}{k_2} \quad \dots(2)$$

- (v) Therefore, substituting equation in (1), the effective spring constant can be calculated as

$$-\frac{F}{k_1} - \frac{F}{k_2} = -\frac{F}{k_s}$$

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} \quad \text{Or } k_s = \frac{k_1 k_2}{k_1 + k_2} \text{ Nm}^{-1}$$

Suppose we have  $n$  springs connected in series, the effective spring constant in series is

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \dots + \frac{1}{k_n} = \sum_{i=1}^n \frac{1}{k_i}$$

If all spring constants are identical i.e.,

$$k_1 = k_2 = \dots = k_n = k \text{ then}$$

$$\frac{1}{k_s} = \frac{n}{k} \Rightarrow k_s = \frac{k}{n}$$

- (vi) This means that the effective spring constant reduces by the factor  $n$ . Hence, for springs in series connection, the effective spring constant is lesser than the individual spring constants.

- (vii) From equation (A), we have,

$$k_1 x_1 = k_2 x_2$$

Then the ratio of compressed distance or elongated distance  $x_1$  and  $x_2$  is

$$\frac{x_2}{x_1} = \frac{k_1}{k_2}$$

- (viii) The elastic potential energy stored in first and second springs are  $U_1 = \frac{1}{2} k_1 x_1^2$ , and  $U_2 = \frac{1}{2} k_2 x_2^2$  = respectively. Then, their ratio is

$$\frac{U_1}{U_2} = \frac{\frac{1}{2} k_1 x_1^2}{\frac{1}{2} k_2 x_2^2} = \frac{k_1}{k_2} \left( \frac{x_1}{x_2} \right)^2 = \frac{k_2}{k_1}$$

## 7. Describe the vertical oscillations of a spring.

**Ans. (i)** Consider a massless spring with stiffness constant or force constant  $k$  attached to a ceiling.

- (ii) Let the length of the spring before loading mass  $m$  be  $L$ . If the block of mass  $m$  is attached to the other end of spring, then the spring elongates by a length  $l$ .

- (iii) Let  $F_1$  be the restoring force due to stretching of spring. Due to mass  $m$ , the gravitational force acts vertically downward. We can draw free-body diagram for this system as shown in Figure. When the system is under equilibrium,

$$F_1 + mg = 0 \quad \dots(1)$$

- (iv) But the spring elongates by small displacement  $l$ , therefore,

$$F_1 \propto l \Rightarrow F_1 = -kl \quad \dots(2)$$

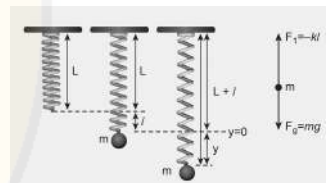
Substituting equation (2) in equation (1), we get

$$-kl + mg = 0$$

$$mg = kl$$

or

$$\frac{m}{k} = \frac{l}{g} \quad \dots(3)$$



- (v) Suppose we apply a very small external force is applied on the mass such that the mass further displaces downward by a displacement  $y$ , then it will oscillate up and down. Now, the restoring force due to this stretching of spring (total extension of spring is  $y + l$ ) is

$$F_2 \propto (y + l)$$

$$F_2 = -k(y + l) = -ky - kl$$

Since, the mass moves up and down with acceleration  $\frac{d^2 y}{dt^2}$ , by drawing the free

body diagram for this case, we get

$$-ky - kl + mg = m \frac{d^2 y}{dt^2}$$

- (vi) The net force acting on the mass due to this stretching is

$$F = F_2 + mg$$

$$F = -ky - kl + mg \quad \dots(4)$$

The gravitational force opposes the restoring force. Substituting equation (3) in equation (4), we get

$$F = -ky - kl + kl = -ky$$

Applying Newton's law, we get

$$m \frac{d^2 y}{dt^2} = -ky; \quad \frac{d^2 y}{dt^2} = -\frac{k}{m} y$$

- (vii) The above equation is in the form of simple harmonic differential equation. Therefore, we get the time period as

$$T = 2\pi \sqrt{\frac{m}{k}} \text{ second}$$

- (viii) The time period can be rewritten using equation

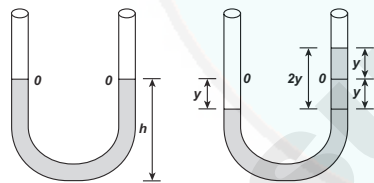
$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{l}{g}} \text{ second}$$

The acceleration due to gravity  $g$  can be computed from the formula

$$g = 4\pi^2 \left( \frac{l}{T^2} \right) \text{ms}^{-1}.$$

### 8. Write short notes on the oscillations of liquid column in U-tube.

Ans.



U-shaped glass tube

- (i) Consider a U-shaped glass tube which consists of two open arms with uniform cross-sectional area. Pour a non-viscous uniform incompressible liquid of density  $\rho$  in the U-shaped tube to a height  $h$ .
- (ii) If the liquid and tube are not disturbed then the liquid surface will be in equilibrium position O.
- (iii) It means the pressure as measured at any point on the liquid is the same and also at the surface on the arm (edge of the tube on either side), which balances with the atmospheric pressure.

- (iv) Due to this the level of liquid in each arm will be the same. By blowing air one can provide sufficient force in one arm, and the liquid gets disturbed from equilibrium position O, which means, that the pressure at blown arm is higher than the other arm.

- (v) This creates difference in pressure which will cause the liquid to oscillate for a very short duration of time about the mean or equilibrium position and finally comes to rest. Time period of the oscillation is

$$T = 2\pi \sqrt{\frac{l}{2g}} \text{ second.}$$

$l \rightarrow$  length of the liquid column in U-tube.

### 9. Discuss in detail the energy in simple harmonic motion. [HY - 2019]

Ans. (a) **Expression for Potential Energy** For the simple harmonic motion, the force and the displacement are related by Hooke's law

$$\vec{F} = -k \vec{r}$$

Since Force is a vector quantity it has three components.

- (i) The force in the above equation is a conservative force field; such a force can be derived from a scalar function which has only one component. In one dimensional case

$$F = -kx \quad \dots(1)$$

- (ii) The work done by the conservative force field is independent of path. The potential energy  $U$  can be calculated from the following expression.

$$F = -\frac{dU}{dx} \quad \dots\dots(2)$$

Comparing (1) and (2), we get

$$-\frac{dU}{dx} = -kx$$

$$dU = kx dx$$

- (iii) This work done by the force  $F$  during a small displacement  $dx$  stores as potential energy

$$U(x) = \int_0^x kx' dx'$$

$$= \frac{1}{2} k(x')^2 \Big|_0^x = \frac{1}{2} Kx^2 \quad \dots(3)$$

From equation  $\omega = \sqrt{\frac{k}{m}}$ , we can substitute the value of force constant  $k = m \omega^2$  in equation (3):

$$U(x) = \frac{1}{2} m \omega^2 x^2$$

where  $\omega$  is the natural frequency of the oscillating system. For the particle executing simple harmonic motion from equation  $y = A \sin \omega t$ ,

we get  $x = A \sin \omega t$

$$U(t) = \frac{1}{2} m \omega^2 A^2 \sin^2 \omega t \quad \dots(4)$$

This variation of  $U$  is shown below.



Variation of potential energy with time  $t$

### (b) Expression for Kinetic Energy

$$\text{Kinetic Energy} = \frac{1}{2} m v_x^2 = \frac{1}{2} m \left( \frac{dx}{dt} \right)^2$$

- (i) Since the particle is executing simple harmonic motion,  $x = A \sin \omega t$

Therefore, velocity is

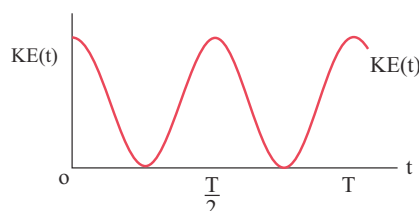
$$v_x = \frac{dx}{dt} = A \omega \cos \omega t = A \omega \sqrt{1 - \left( \frac{x}{A} \right)^2}$$

$$v_x = \omega \sqrt{A^2 - x^2} \quad \dots(5)$$

Hence,

$$\text{KE} = \frac{1}{2} m v_x^2 = \frac{1}{2} m \omega^2 (A^2 - x^2) \quad \dots(6)$$

$$\text{KE} = \frac{1}{2} m \omega^2 A^2 \cos^2 \omega t \quad \dots(7)$$



Variation of kinetic energy with time  $t$

### (c) Expression for Total Energy

- (i) Total energy is the sum of kinetic energy and potential energy

$$E = \text{KE} + U \quad \dots(8)$$

$$E = \frac{1}{2} m \omega^2 (A^2 - x^2) + \frac{1}{2} m \omega^2 x^2$$

Hence canceling  $x^2$  term,

$$E = \frac{1}{2} m \omega^2 A^2 = \text{constant} \quad \dots(9)$$

- (ii) Alternatively, from equation (4) and equation (7), we get the total energy as

$$E = \frac{1}{2} m \omega^2 A^2 \sin^2 \omega t + \frac{1}{2} m \omega^2 A^2 \cos^2 \omega t$$

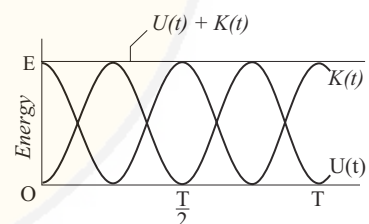
$$= \frac{1}{2} m \omega^2 A^2 (\sin^2 \omega t + \cos^2 \omega t)$$

- (iii) From trigonometry identity,

$$(\sin^2 \omega t + \cos^2 \omega t) = 1$$

$$E = \frac{1}{2} m \omega^2 A^2 = \text{constant}$$

which gives the law of conservation of total energy.



- (iv) Thus the amplitude of simple harmonic oscillator, can be expressed in terms of total energy.

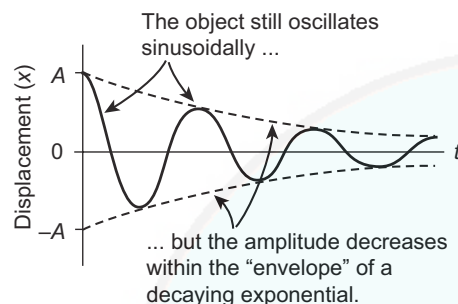
$$A = \sqrt{\frac{2E}{m\omega^2}} = \sqrt{\frac{2E}{k}}$$

### 10. Explain in detail the four different types of oscillations. [Jun.-2019]

**Ans. Damped oscillations :**

- (i) Due to the presence of friction and air drag, the amplitude of oscillation decreases as time progresses.
- (ii) It implies that the oscillation is not sustained and the energy of the SHM decreases gradually indicating the loss of energy.

- (iii) The energy lost is absorbed by the surrounding medium. This type of oscillatory motion is known as damped oscillation.
- (iv) The motion of the oscillator is said to be damped the resistive force (or damping force) is proportional to the velocity of the oscillator.



### Examples

- (i) The oscillations of a pendulum.
- (ii) Electromagnetic oscillations in a tank circuit.
- (iii) Oscillations in a dead beat and ballistic galvanometers.

### Maintained oscillations :

- (i) While playing in swing, the oscillations will stop after a few cycles, this is due to damping.
- (ii) By supplying energy from an external source, the amplitude of the oscillation can be made constant. Such vibrations are known as maintained vibrations.

### Example:

The vibration of a tuning fork getting energy from a battery or from external power supply.

### Forced oscillations :

- (i) Any oscillator driven by an external periodic agency to overcome the damping is known as forced oscillator or driven oscillator.
- (ii) In this type of vibration, the body executing vibration initially vibrates with its natural frequency and due to the presence of external periodic force, the body later vibrates with the frequency of the applied periodic force.
- (iii) Such vibrations are known as forced vibrations.

### Example:

Sound boards of stringed instruments.

### Resonance :

- (i) It is a special case of forced vibrations where the frequency of external periodic force (or driving force) matches with the natural frequency of the vibrating body (driven).
- (ii) As a result the oscillating body begins to vibrate such that its amplitude increases at each step and ultimately it has a large amplitude. Such a phenomenon is known as resonance and the corresponding vibrations are known as resonance vibrations.

### Example :

The breaking of glass due to sound.

## IV. NUMERICAL PROBLEMS :

1. Consider the Earth as a homogeneous sphere of radius  $R$  and a straight hole is bored in it through its centre. Show that a particle dropped into the hole will execute a simple harmonic motion such that its time period is

$$T = 2\pi \sqrt{\frac{R}{g}}$$

### Solution:

Earth is assumed to be a homogeneous sphere. Its centre is at  $O$  and Radius =  $R$

The hole is bored straight through the centre along its diameter.

The acceleration due to gravity at the surface of the earth =  $g$

Mass of the body dropped inside the hole =  $m$

After time  $t$ , the depth it reached (inside the earth) =  $d$

The value of ' $g$ ' decreases with deportation.

So acceleration due to gravity at deportation =  $g'$

$$\text{i.e., } g' = g(1 - d/R) = g\left(\frac{R-d}{R}\right) \quad \dots(1)$$

Let  $y$  be the distance from the centre of the earth

Then  $y$  = Radius – distance =  $R - d$

Substitute  $y$  in (1)

$$g' = g y/R$$



Now, force on the body of mass  $m$  due to this new acceleration  $g'$  will be

$$F = mg' = mgy/R$$

and this force is directed towards the mean position  $O$ .

The body dropped in the hole will execute S.H.M

Spring factor  $k = mg/\text{Radius}$

$$T = 2\pi \sqrt{\frac{\text{Inertial factor}}{\text{spring factor}}} = 2\pi \sqrt{\frac{m}{mg/R}} = 2\pi \sqrt{\frac{R}{g}}$$

2. Consider a simple pendulum of length  $l = 0.9$  m which is properly placed on a trolley rolling down on a inclined plane which is at  $\theta = 45^\circ$  with the horizontal. Assuming that the inclined plane is frictionless, calculate the time period of oscillation of the simple pendulum.

**Solution:**

Length of simple pendulum  $l = 0.9$  m

Inclined plane with the horizontal plane  $\alpha = 45^\circ$

Time period of oscillation of simple pendulum  $= T = ?$

$$T = 2\pi \sqrt{\frac{l}{g \cos \alpha}} = 2 \times 3.14 \sqrt{\frac{0.9}{9.8 \times \cos 45^\circ}}$$

$$= 2 \times 3.14 \sqrt{\frac{0.9}{9.8 \times .7071}}$$

$$T = 2.263 \text{ s}$$

3. A piece of wood of mass  $m$  is floating erect in a liquid whose density is  $\rho$ . If it is slightly pressed down and released, then executes simple harmonic motion. Show that its time

period of oscillation is  $T = 2\pi \sqrt{\frac{m}{A\rho g}}$

**Solution:**

Spring factor of liquid  $= A\rho g$

Inertra factor of wood piece  $= m$

$$\text{Timeperiod } T = 2\pi \sqrt{\frac{\text{Inertra factor}}{\text{Spring factor}}}$$

$$T = 2\pi \sqrt{\frac{m}{A\rho g}}$$

4. Consider two simple harmonic motion along  $x$  and  $y$ -axis having same frequencies but different amplitudes as  $x = A \sin(\omega t + \phi)$  (along  $x$  axis) and  $y = B \sin \omega t$  (along  $y$  axis). Then show that

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB} \cos = \sin^2 \phi$$

and also discuss the special cases when

a.  $\phi = 0$       b.  $\phi = \pi$       c.  $\phi = \frac{\pi}{2}$

d.  $\phi = \frac{\pi}{2}$  and  $A = B$       e.  $\phi = \frac{\pi}{4}$

**Note:** when a particle is subjected to two simple harmonic motion at right angle to each other the particle may move along different paths. Such paths are called Lissajous figures.

**Ans.** a.  $y = \frac{B}{A}x$ , equation is a straight line passing through origin with positive slope.

b.  $y = -\frac{B}{A}x$ , equation is a straight line passing through origin with negative slope.

c.  $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$ , equation is an ellipse whose center is origin.

d.  $x^2 + y^2 = A^2$ , equation is a circle whose center is origin.

e.  $\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB} \frac{1}{\sqrt{2}} = \frac{1}{2}$  equation is an ellipse (oblique ellipse which means tilted ellipse).

5. Show that for a particle executing simple harmonic motion

a. the average value of kinetic energy is equal to the average value of potential energy.

b. average potential energy = average kinetic

energy  $= \frac{1}{2}$  (total energy) (**Hint** : average kinetic energy  $= \langle \text{kinetic energy} \rangle$ )

$$= \frac{1}{T} \int_0^T (\text{Kinetic energy}) dt \text{ and}$$

average Potential energy  $= \langle \text{Potential energy} \rangle$

$$= \frac{1}{T} \int_0^T (\text{Potential energy}) dt$$



**Solution:**

$$\begin{aligned} a). (E_k)_{\text{ave}} &= \frac{1}{T} \int_0^T E_k dt = \frac{1}{T} \int_0^T \frac{1}{2} m A^2 \omega^2 \cos^2 \omega t dt \\ &= \frac{1}{4T} m A^2 \omega^2 T \end{aligned}$$

$$(E_k)_{\text{ave}} = \frac{1}{4} m A^2 \omega^2 \quad \dots(1)$$

$$\begin{aligned} (E_p)_{\text{ave}} &= \frac{1}{T} \int_0^T E_p dt = \frac{1}{T} \int_0^T \frac{1}{2} m \omega^2 A^2 \sin^2 \omega t dt \\ &= \frac{1}{2T} \cdot m \omega^2 A^2 \int_0^T \frac{(1 - \cos 2\omega t)}{2} dt \\ &= \frac{1}{4T} \cdot m \omega^2 A^2 \left[ t - \frac{\sin 2\omega t}{2\omega} \right]_0^T \end{aligned}$$

$$\begin{aligned} (E_p)_{\text{ave}} &= \frac{1}{4T} m \omega^2 A^2 T \\ &= m \omega^2 A^2 / 4 \quad \dots(2) \end{aligned}$$

From equations (1) and (2) it can show that the average K.E. is equal to average P.E.

$$(\text{Total Energy}) TE = \frac{1}{2} m \omega^2 A^2$$

$$(E_k)_{\text{ave}} = (E_p)_{\text{ave}} = \frac{1}{2} \left( \frac{1}{2} m \omega^2 A^2 \right) = \frac{1}{2} (TE)$$

b. average kinetic energy = <kinetic energy>

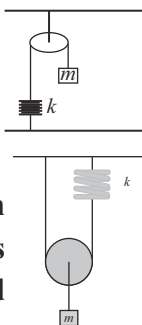
$$= \frac{1}{T} \int_0^T (\text{Kinetic energy}) dt$$

and

average Potential energy = <Potential energy>

$$= \frac{1}{T} \int_0^T (\text{Potential energy}) dt.$$

6. Compute the time period for the following system if the block of mass  $m$  is slightly displaced vertically down from its equilibrium position and then released. Assume that the pulley is light and smooth, strings and springs are light.

**Solution:**

**Case (a) :** Pulley is fixed rigidly here. When the mass displace by  $y$  and the spring will also stretch by  $y$ . Therefore,  $F = T = ky$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

**Case (b) :** Mass displace by  $y$ , pulley also displaces by  $y$ .  $T = 4ky$ .

$$T = 2\pi \sqrt{\frac{m}{4k}}$$

**Government Exam Question & Answers****MULTIPLE CHOICE QUESTIONS :**

===== 1 Mark =====

1. Which one of the following represents simple harmonic motion? [Jun.-2019]

- (a) acceleration =  $kx$
- (b) acceleration =  $k_0 x + k_1 x^2$
- (c) acceleration =  $-k(x+a)$
- (d) acceleration =  $k(x+a)$

[Ans. (c) acceleration =  $-k(x+a)$ ]

**Creative Questions (HOTS)**

===== 2 Mark =====

1. "Soldiers are not allowed to march on a bridge". Give reason. [Mar.-2019]

- Ans. (i)** Soldiers are not allowed to march on a bridge. The reason behind this is resonance.
- (ii)** If the frequency of soldiers marching on a bridge, equals the frequency of vibration of bridge, then the displacement of the bridge movement will be maximum resulting what we call as Resonance.
- (iii)** Due to this resonance the bridge may get broken or get damaged.

**NUMERICAL PROBLEMS : ===== 2 Mark =====**

1. If the length of the simple pendulum is increased by 44% from its original length, calculate the percentage increase in time period of the pendulum. [HY - 2019]

**Solution:**

$$\text{Since, } T \propto \sqrt{l} ; T = \text{constant } \sqrt{l}$$

$$\frac{T_f}{T_i} = \sqrt{\frac{l + \frac{44}{100}l}{l}} = \sqrt{1.44} = 1.2$$

## ADDITIONAL QUESTIONS

### I. MULTIPLE CHOICE QUESTIONS :

== 1 Mark ==

1. A particle excites simple harmonic motion between  $x = -A$  &  $x = +A$ , the time taken for it to go from O to  $\frac{A}{2}$  is  $T_1$  & to go from  $\frac{A}{2}$  to A is  $T_2$  then

- (a)  $T_1 < T_2$  (b)  $T_1 > T_2$   
(c)  $T_1 = T_2$  (d)  $T_1 = 2T_2$

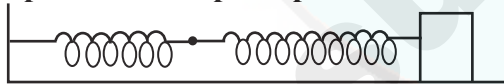
[Ans. (a)  $T_1 < T_2$ ]

2. The  $x$ - $t$  graph of a particle undergoing simple harmonic motion is shown. The acceleration of the particle at  $t = \frac{4}{3}$  is

- (a)  $\frac{\sqrt{3}}{32} \pi \text{ cm/s}^2$  (b)  $-\frac{\pi^2}{32} \text{ cm/s}^2$   
(c)  $\frac{\pi^2}{32} \text{ cm/s}^2$  (d)  $-\frac{\sqrt{3}}{32} \pi^2 \text{ cm/s}^2$

[Ans. (d)  $-\frac{\sqrt{3}}{32} \pi^2 \text{ cm/s}^2$ ]

3. The mass  $m$  shown oscillates in a simple harmonic motion with amplitude  $A$ , the amplitude of the point p is



- (a)  $\frac{k_1 A}{k_2}$  (b)  $\frac{k_2 A}{k_1}$   
(c)  $\frac{k_1 A}{k_1 + k_2}$  (d)  $\frac{k_2 A}{k_1 + k_2}$

[Ans. (d)  $\frac{k_2 A}{k_1 + k_2}$ ]

4. The function  $x = A \sin^2 \omega t + B \cos^2 \omega t + C \sin \omega t \cos \omega t$  represents simple harmonic motion for which of the option?

- (a) for all values of  $A$ ,  $B$  &  $C$  ( $C \neq 0$ ) (b)  $\frac{r}{10}$   
(c)  $CA = -B$ ,  $C = 2B$   
(d) all of the above

[Ans. (d) all of the above]

5. The displacement of an object attached to a spring & executing SAM is given by  $x = 2 \times 10^{-2} \cos t$  the time at which the speed first occurs is

- (a)  $0.55 \mu$  (b)  $0.75 \text{ s}$   
(c)  $0.125 \text{ s}$  (d)  $0.25 \text{ s}$

[Ans. (a)  $0.55 \mu$ ]

6. The maximum velocity of a particle, executing simple harmonic motion with an amplitude  $7 \text{ mm}$  is  $4.4 \text{ m s}^{-1}$ . The period of oscillation is

- (a)  $0.01 \text{ s}$  (b)  $0.1 \text{ s}$  (c)  $10 \text{ s}$  (d)  $100 \text{ s}$

[Ans. (a)  $0.01 \text{ s}$ ]

7. If  $x$ ,  $v$  &  $a$  denote the displacement, the velocity & acceleration of a particle executing simple harmonic motion of time period  $T$ , then which of the following does not change with time

- (a)  $a^2 T^2 + 4\pi^2 v^2$  (b)  $aT/x$   
(c)  $aT + 2\pi v$  (d)  $aT/v$

[Ans. (b)  $aT/x$ ]

8. A particle executing simple harmonic motion along  $y$ -axis has its motion described by the equation  $y = A \sin(\omega t) + B$ , the amplitude of the example harmonic motion is

- (a)  $A$  (b)  $B$   
(c)  $A + B$  (d)  $\sqrt{A+B}$

[Ans. (a)  $A$ ]

9. When the maximum k.E of a simple pendulum is  $k$ , then what is its displacement in terms of amplitude  $a$  when its K.E. is  $k/2$

- (a)  $a/\sqrt{2}$  (b)  $a/2$   
(c)  $a/\sqrt{3}$  (d)  $a/3$

[Ans. (a)  $a/\sqrt{2}$ ]

10. A particle is executing SHM at mid point of mean position & extremity. What is the potential energy in terms of total energy ( $E$ )

- (a)  $\frac{E}{4}$  (b)  $\frac{E}{16}$   
(c)  $\frac{E}{2}$  (d)  $\frac{C}{8}$

[Ans. (a)  $\frac{E}{4}$ ]

- 11.** A spring is cut into 4 equal parts & 2 parts are connected in parallel. What is the effective in parallel. What is the effective spring constant.  
 (a) 4k (b) 16k  
 (c) 8k (d) 6k **[Ans. (c) 8k]**
- 12.** If the length of simple pendulum is tripled, what will its new time period is terms of original period T?  
 (a) 0.7 T (b) 1.73 T  
 (c) T/2 (d) T **[Ans. (b) 1.73 T]**
- 13.** The ratio of frequencies of 2 pendulums are 2 : 3, then their lengths are in ratio,  
 (a)  $\sqrt{\frac{2}{3}}$  (b)  $\sqrt{\frac{3}{2}}$  (c)  $\frac{4}{9}$  (d)  $\frac{9}{4}$   
**[Ans. (d)  $\frac{9}{4}$ ]**
- 14.** What is time period of a pendulum hanged in a satellite? (T is time period on earth)  
 (a) zero (b) T  
 (c) infinite (d)  $\frac{T}{\sqrt{6}}$   
**[Ans. (c) infinite]**
- 15.** If a simple pendulum of length 'L' has maximum angular displacement. Then the maximum kinetic energy of bob mass m is  
 (a)  $\frac{1}{2} \frac{ML}{9}$  (b)  $\frac{Mg}{2L}$   
 (c)  $MgL(1 - \cos \alpha)$  (d)  $MgL \sin \alpha/2$   
**[Ans. (c)  $MgL(1 - \cos \alpha)$ ]**
- 16.** Which of the following function represents a simple harmonic oscillation?  
 (a)  $\sin \omega t - \cos \omega t$  (b)  $\sin \omega t + \sin 2 \omega t$   
 (c)  $\sin \omega t - \sin 2 \omega t$  (d)  $\sin^2 \omega t$   
**[Ans. (a)  $\sin \omega t - \cos \omega t$ ]**
- 17.** The phase difference between the instantaneous velocity & acceleration of a particle executing simple harmonic motion is  
 (a)  $0.5\pi$  (b)  $\pi$   
 (c)  $0.707\pi$  (d)  $0.61\pi$   
**[Ans. (a)  $0.5\pi$ ]**
- 18.** Two simple pendulums of time periods 2.0 s & 2.1 s are made to vibrate simultaneously. They are in phase initially, after how may vibrations are there in the same phase?  
 (a) 21 (b) 25  
 (c) 30 (d) 35 **[Ans. (a) 21]**
- 19.** The magnitude of acceleration of particle executing SHM at the position of maximum displacement is  
 (a) zero  
 (b) minimum  
 (c) maximum  
 (d) none of these **[Ans. (c) maximum]**
- 20.** The SHMs are represented by the equation  $y_1 = 0.1 \sin(100\pi t + \frac{\pi}{3})$  &  $y_2 = 0.1 \cos \pi t$ . The phase difference of the velocity of particle is  
 (a)  $-\frac{\pi}{6}$  (b)  $\frac{\pi}{3}$  (c)  $-\frac{\pi}{3}$  (d)  $\frac{\pi}{6}$   
**[Ans. (a)  $-\frac{\pi}{6}$ ]**

## II. MATCH THE FOLLOWING :

1.	(1) Free oscillation	(a)	Electromagnetic oscillation in a tank circuit
	(2) Damped oscillation	(b)	A swing motion
	(3) Maintained oscillation	(c)	Sound boards of stringed instruments
	(4) Forced oscillation	(d)	Oscillations of simple pendulum

- (1) (2) (3) (4)  
 (a) d a b c  
 (b) d b c a  
 (c) b c d a  
 (d) c d b a

**[Ans: (a) d a b c]**

2.	(1)	Displacement	(a)	$A \sin \omega t + B \cos \omega t$
	(2)	Velocity	(b)	$-\omega^2 y$
	(3)	Acceleration	(c)	$A \sin \omega t$
	(4)	SHM	(d)	$A \omega \cos \omega t$

- (1) (2) (3) (4)  
 (a) b d c a  
 (b) b c d a  
 (c) c d b a  
 (d) d c a b

[Ans: (c) c d b a]

3.		Angular Harmonic motion		Expression
	(1)	Angular acceleration	(a)	$I \propto$
	(2)	Restoring torque	(b)	$\sqrt{\frac{\kappa}{I}}$
	(3)	Torque	(c)	$-K\theta$
	(4)	Angular frequency	(d)	$-\omega^2 \theta$

- (1) (2) (3) (4)  
 (a) c d a b  
 (b) b c d a  
 (c) d a b c  
 (d) d c a b

[Ans: (d) d c b a]

## III. FILL IN THE BLANKS :

1. The time period for U-tube Oscillation is \_\_\_\_.

- (a)  $T = \sqrt{\frac{l}{2g}}$  (b)  $T = 2\pi \sqrt{\frac{2g}{l}}$   
 (c)  $T = 2\pi \sqrt{\frac{l}{g}}$  (d)  $T = 2\pi \sqrt{\frac{l}{2g}}$

[Ans. (d)  $T = 2\pi \sqrt{\frac{l}{2g}}$ ]

2. In SHM, kinetic energy is \_\_\_\_.

- (a)  $\frac{1}{2} m \omega^2 x^2$  (b)  $\frac{1}{2} m \omega^2 A^2$   
 (c)  $\frac{1}{2} m \omega^2 (A^2 - x^2)$  (d)  $\frac{1}{2} m \omega^2 x^2 A^2$

[Ans. (c)  $\frac{1}{2} m \omega^2 (A^2 - x^2)$ ]

3. \_\_\_\_\_ is a special case of forced oscillations.

- (a) Resonance (b) SHM  
 (c) Angular harmonic motion  
 (d) Torsional motion

[Ans. (a) Resonance]

4. Force unit length is called \_\_\_\_.

- (a) Torsional constant (b) acceleration  
 (c) force constant (d) Resonance

[Ans. (c) force constant]

5. The motion which has single frequency and constant amplitude is called \_\_\_\_.

- (a) SHM (b) resonance  
 (c) frequency (d) damping

[Ans. (a) SHM]

6. The maximum displacement of the particle is called \_\_\_\_.

- (a) SHM (b) Resonance  
 (c) Amplitude (d) Altitude

[Ans. (c) Amplitude]

7. \_\_\_\_\_ of oscillation is large for resonance.

- (a) Frequency (b) Time period  
 (c) Amplitude (d) Phase

[Ans. (c) Amplitude]

8. Electromagnetic oscillations in a tank circuit is an example for \_\_\_\_.

- (a) free oscillations (b) damped oscillations  
 (c) maintained oscillations  
 (d) forced oscillations

[Ans. (b) damped oscillations]

9. For a conservative system in one dimension, the force field can be derived for a scalar \_\_\_\_.

- (a) potential energy (b) total energy  
 (c) kinetic energy (d) surface energy

[Ans. (a) potential energy]

10. Frequency of a given tuning fork is determined with a sonometer using \_\_\_\_.

- (a) maintained oscillation  
 (b) resonance  
 (c) forced oscillation  
 (d) damped oscillation

[Ans. (b) resonance]

## IV. CHOOSE THE ODD ONE OUT:

1. (a)  $-\omega^2 r$  (b)  $ma$  (c)  $-kr$  (d)  $\sqrt{\frac{\kappa}{I}}$

[Ans. (d)  $\sqrt{\frac{\kappa}{I}}$ ]

Hint:

$\sqrt{\frac{\kappa}{I}}$  is angular harmonic motion all the three are SHM,



2. (a) Oscillation (b) Vibration  
(c) Rotation (d) SHM

[Ans. (d) SHM]

**Hint:**

Other three can have many cycles (frequencies), but SHM has single frequency.

3. (a) Driver (b) Driven  
(c) Resonance (d) Force constant

[Ans. (d) Force constant]

**Hint:**

All the three are not related with force constant but those three are interrelated.

**V. CHOOSE THE CORRECT PAIR :**

1. (a) Scalar potential energy -  $\left(-\frac{dU}{dx}\right)$   
(b) Potential energy -  $\frac{1}{2}m\omega^2 A^2$   
(c) Kinetic energy -  $\frac{1}{2}m\omega^2 x^2$   
(d) Total energy -  $\frac{1}{2}m\omega^2 A^2 x^2$

[Ans. (a) Scalar potential energy -  $\left(-\frac{dU}{dx}\right)$ ]

2. (a) Angular SHM -  $\sqrt{\frac{K}{I}}$   
(b) Linear SHM -  $\sqrt{\frac{2k}{m}}$   
(c) U - tube -  $2\pi\sqrt{\frac{2l}{2g}}$   
(d) Simple Pendulum -  $2\pi\sqrt{\frac{l}{2g}}$

[Ans. (a) Angular SHM -  $\sqrt{\frac{K}{I}}$ ]**VI. CHOOSE THE INCORRECT PAIR :**

1. (a) Amplitude - Displacement  
(b) Resonance - forced oscillation  
(c) Free oscillation - tuning fork vibration  
(d) Maintained oscillation- swing movement

[Ans. (a) Amplitude - Displacement]

2. (a) K.E. -  $\frac{3}{4}$  T.E.  
(b) P.E. -  $\frac{1}{4}$  T.E.  
(c) T.E. - (K.E. + P.E.)  
(d) T.E. -  $\frac{1}{2}m\omega^2 x^2$

[Ans. (d) T.E. -  $\frac{1}{2}m\omega^2 x^2$ ]**VII. ASSERTION & REASON :****Directions :**

- (a) Assertion and Reason are correct and Reason is correct explanation of Assertion  
(b) Assertion and Reason are true but Reason is the false explanation of the Assertion  
(c) Assertion is true but Reason is false  
(d) Assertion is false but Reason is true

1. **Assertion :** The projection of uniform circular motion on a diameter is SHM.

**Reason :** A motion which has single frequency and constant amplitude is called SHM.

[Ans. (a) Assertion and Reason are correct and Reason is correct explanation of Assertion]

2. **Assertion :** The differential equation of SHM is  $\frac{d^2 y}{dt^2} + \omega^2 y = 0$

**Reason :**  $\mp a\omega^2$  is called acceleration amplitude.

[Ans. (b) Assertion and Reason are true but Reason is the false explanation of the Assertion]

**VIII. CHOOSE THE CORRECT OR INCORRECT STATEMENTS :**

1. (I) If the frequency of driver (external periodic force) is equal to the frequency of driven (natural frequency) then resonance occurs.  
(II) A singer maintaining a note at a frequency of a glass and cause it to shatter into pieces is an example of resonance.

Which one is correct statement?

- (a) I only (b) II only  
(c) Both are correct (d) None

[Ans. (c) Both are correct]



2. (I) Air blown gently across the mouth of a bottle is an example for forced vibrations.  
(II) Oscillations of a coil in a galvanometer is an example for free oscillation.

Which one is correct statement?

- (a) I only (b) II only  
(c) Both are correct (d) None

[Ans. (d) None]

3. (I) S.I. unit of frequency is hertz (Hz).  
(II) S.I. unit of force constant is Nm.

Which one is correct statement?

- (a) I only (b) II only  
(c) Both are correct (d) None

[Ans. (a) I only]

4. (I) Combination of springs connected in series  $K_s = K_1 + K_2$   
(II) Frequency of oscillation in a U - tube is  $n = \frac{1}{2\pi} \sqrt{\frac{2g}{l}}$ .

Which one is incorrect statement?

- (a) I only (b) II only  
(c) Both are correct (d) None

[Ans. (a) I only]

### VERY SHORT ANSWER QUESTIONS :

==2 Marks==

#### 1. What is Oscillatory motion?

**Ans.** When an object or a particle moves back and forth repeatedly for some duration of time its motion is said to be oscillatory (or vibratory).

**Examples:** our heart beat, swinging motion of the wings of an insect,

#### 2. What is Phase of SHM?

**Ans. (i)** The phase of a vibrating particle at any instant completely specifies the state of the particle.

**(ii)** It expresses the position and direction of motion of the particle at that instant with respect to its mean position.

**(iii)** where  $\omega t + \phi_0 = \phi$  is called the phase of the vibrating particle.

$$y = A \sin (\omega t + \phi_0)$$

**(iv)** The displacement acquired by an oscillating body when time  $t = 0$  s (initial time), the phase  $\phi = \phi_0$  is called epoch (initial phase) where  $\phi_0$  is called the angle of epoch.

#### 3. What is oscillatory motion? Give any two examples.

**Ans.** If a body moves to and fro repeatedly about a mean position it is called oscillatory motion.

**Examples:**

- (i)** Oscillations of a simple pendulum.  
**(ii)** Vibrations of a mass attached to spring.

#### 4. What are periodic motion? Give any two examples.

**Ans.** A phenomenon, process or motion, which repeats itself after equal intervals of time, is called periodic motion

**Examples:**

- (i)** Heart beat of a person; period about 0.83s for a normal person.  
**(ii)** Motion of earth around the sun; period is one year.

#### 5. Define simple harmonic motion (S.H.M.)

**Ans.** S.H.M is the motion in which the restoring force is proportional to the displacement from the mean position and opposes its increase. Such a motion the displacement varies harmonically with time.

#### 6. Why the amplitude of the vibrating pendulum should be small?

**Ans. (i)** When amplitudes of the vibrating pendulum is small then pendulum is small. Here the restoring force  $F = mg \sin \theta = mg \theta = mg x/l$ .

**(ii)** Where  $x$  is the displacement of the bob and  $l$  is the length of pendulum. Hence  $F \propto x$ . Since  $F$  is directed towards mean position.

**(iii)** Therefore the motion of the bob of simple pendulum will be S.H.M. if  $\theta$  is small.

#### 7. When a pendulum clock gains time, what adjustments should be made?

**Ans. (i)** When a pendulum clock gains time, it means it has gone fast, it makes more vibrations per day than required.

- (ii) This shows that the time period of oscillations has decreased. Therefore, to correct it, the length of pendulum should be properly increased.

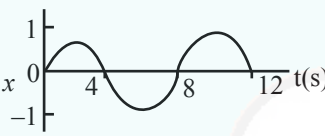
8. The displacement of harmonic oscillator is given by  $x = \alpha \sin \omega t + \beta \cos \omega t$ . what is the amplitude of the oscillation?

Ans.  $x = \alpha \sin \omega t + \beta \cos \omega t$   
 $\alpha = r \cos \theta$  and  $\beta = r \sin \theta$   
 $x = r \cos \theta \sin \omega t + r \sin \theta \cos \omega t$   
 $\omega t = r \sin (\omega t + \theta)$

### SHORT ANSWER QUESTIONS :

== 3 Marks ==

1. State five characteristics of SHM.

Ans. (i) **Displacement:**   
 T h e displacement of a particle

executing linear SHM, at an instant is defined as the distance of the particle from the mean position at that instant.

(ii) **Velocity:**

Is defined as the rate of change of the displacement of the particle at the given instant.

(iii) **Amplitude:**

The maximum displacement on either side of mean position.

(iv) **Acceleration:**

It is defined as the rate of change of the velocity of the particle at the given instant.

(v) **Time period:**

It is defined as the time taken by the particle executing S.H.M to complete one full vibration.

2. Tabulate, the displacement, velocity and acceleration in SHM.

Ans.

Time	$\omega t$	Displacement $a \sin \omega t$	Velocity $a\omega \cos \omega t$	Acceleration $-\omega^2 a \sin \omega t$
$t = 0$	0	0	$a\omega$	0
$t = \frac{T}{4}$	$\frac{\pi}{2}$	$+a$	0	$-a\omega^2$
$t = \frac{T}{2}$	$\pi$	0	$-a\omega$	0
$t = \frac{3T}{4}$	$\frac{3\pi}{2}$	$-a$	0	$+a\omega^2$
$t = T$	$2\pi$	0	$+a\omega$	0

3. Derive an expression for PE and KE in SHM, and deduced the graphical representation.

Ans. Let the displacement of particle of mass  $m$  executing simple harmonic motion at any instant  $t$  be  $x$ , then in that position of potential energy and kinetic energy of particle are given as

$$\text{Potential energy } U = \frac{1}{2} m \omega^2 x^2$$

$$\text{Kinetic energy } K = \frac{1}{2} m \omega^2 (a^2 - x^2)$$

(i) When  $x = 0$  (i.e., at mean position) potential energy  $U = 0$  kinetic energy  $K = \frac{1}{2} m \omega^2 a^2 = E$

(ii) When  $x = \pm \frac{a}{2}$ , potential energy

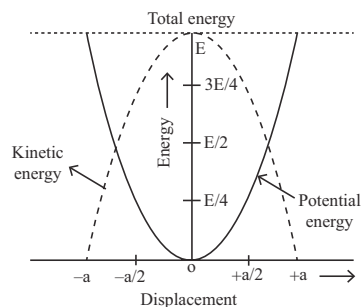
$$U = \frac{1}{2} m \omega^2 \left( \frac{a}{2} \right)^2$$

$$= \frac{1}{4} \times \frac{1}{2} m \omega^2 a^2 = \frac{E}{4}$$

Kinetic energy

$$K = \frac{1}{2} m \omega^2 \left[ a^2 - \left( \frac{a}{2} \right)^2 \right]$$

$$= \frac{3}{4} \times \frac{1}{2} m \omega^2 a^2 = \frac{3}{4} E$$



Displacement

Graphical representation of pE and kE in SHM

(iii) When  $x = \pm a$  P.E,  $U = \frac{1}{2} m \omega^2 - a^2 = E$

KE = 0.

Now taking the displacement  $x$  on  $x$ -axis and the energy on  $y$ -axis, the graph plotted, the dotted line drawn parallel to  $x$ -axis represents the total energy of the particle, which remains constant during the entire motion.

4. The bob of vibrating simple pendulum is made of ice. How will the period of swing will change when the ice starts melting?

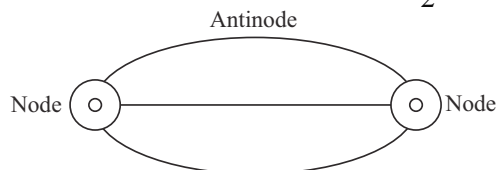
Ans. (i) The period of swing of simple pendulum will remain unchanged till the location of the center of gravity of the bob left after melting the ice remains at a fixed distance from the point of suspension.

- (ii) If the centre of gravity of ice bob after melting is raised upwards, then the effective length of pendulum decreases and hence the time period of swing decreases. If the centre of gravity of ice shifts on lower side, the time period of swing increases.

5. Discuss strings stretched between fixed points.

Ans. (i) Let a spring be stretched between two points. Vibrations will set up in this string when it is disturbed, say from middle.

- (ii) The figure given below shows that vibration set up in this string appears to be similar to a half wave having wavelength  $\frac{\lambda}{2}$ .



$$\text{Length of the spring } l = \frac{\lambda}{2}$$

$$\lambda = 2l$$

If  $T$  is tension in the string and  $M$  is mass per unit length of the string, then velocity of disturbance is given by,

$$v = \sqrt{\frac{T}{M}} \quad (M \rightarrow \text{Linear Density})$$

$$v = \frac{\text{wavelength}}{\text{time}} = \text{wavelength} \times \text{frequency} = \lambda f$$

$$v = 2l \times f$$

$$f = \frac{v}{2l} = \frac{1}{2l} \sqrt{\frac{T}{M}} \quad \left[ \because v = \sqrt{\frac{T}{M}} \right]$$

6. At what displacement, (i) the P. E of a simple harmonic oscillator is maximum, (ii) the k. E is maximum?

Ans. The P.E of a particle executing S.H.M is given by,

$$E_p = \frac{1}{2} m \omega^2 y^2$$

$E_p$  is maximum when  $y = r =$  amplitude of vibration, i.e., the particle is passing from the extreme position and is minimum when  $y = 0$ , i.e., the particle is passing from the mean position.

The kinetic energy of a particle executing S.H.M. is given by;

$$E_k = m \omega^2 (r^2 - y^2)$$

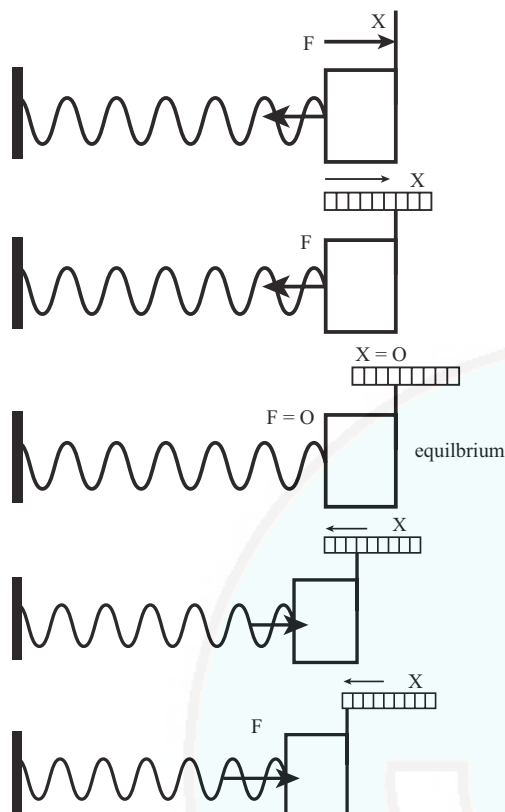
$E_k$  is maximum when  $y = 0$ , i.e., the particle is passing from the mean position and  $E_k$  is minimum when  $y = r$ , is the particle is passing from the extreme position.

### LONG ANSWER QUESTIONS :

—5 Marks—

1. Write a short note on Simple Harmonic Motion.

Ans. We shall first discuss the general dynamics of an SHM. We know that work has to be done on a system to displace it from its position of equilibrium. The restoring force  $F$  obviously depends on the work done to give a displacement  $x$ .



- (i) Thus,  $F$  is some general function of  $x$ . For system oscillating violently (large  $x$ ) the dependence of  $F$  on  $x$  is very complex.
- (ii) This is called small oscillation approximation. For such system, the restoring force is proportional to the displacement and opposes its increase.
- $$F = -kx \quad (1)$$

- (iii) The negative sign indicates that  $F$  opposes increase in  $x$ .  $k$ , the constant of proportionality, is called the force constant.

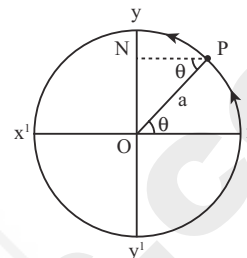
The Mks unit for  $k$  is  $\frac{N}{m}$ . The magnitude

of  $k$  depends on the elastic properties of the system. In the specific examples, in the system consisting of a mass and a spring,  $k$  will depend on the stiffness or strength of the spring.

- (iv) The equation (1) is a statement of Hooke's law for elastic forces. The general definition of SHM is the motion in which the restoring force is proportional to the displacement from the mean position and opposes its increase.
- (v) That in such a motion the displacement varies harmonically with time.

## 2. Show that the projection of uniform circular motion on a diameter is SHM.

**Ans.** Consider a particle moving along the  $y$  circumference of a circle of radius  $a$  and centre  $O$ , with uniform speed  $v$ , in anticlockwise direction.



### Projection of uniform circular motion

Let  $xx'$  and  $yy'$  be the two perpendicular  $x$  diameters. Suppose the particle is at  $p$  after a time  $t$ . If  $\omega$  is the angular velocity then the angular displacement  $\theta$  in time  $t$  is given by  $\theta = -\omega t$ .

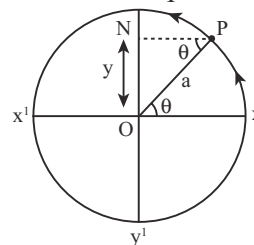
From  $P$  draw  $PN$  perpendicular to  $yy'$ . As the particle moves from  $x$  to  $y$ , foot of the perpendicular  $N$  moves from  $0$  to  $y$ . As it moves further from  $y$  to  $x'$ , then from  $x'$  to  $y'$  and back again to  $x$ , the point  $N$  moves from  $y$  to  $0$ , from  $0$  to  $y'$  and back again to  $0$ . When the particle completes one revolution along the circumference, the point  $N$  completes one vibration about the mean position  $0$ . The motion of the point  $N$  along the diameter  $yy'$  is simple harmonic.

Hence the projection of a uniform circular motion on a diameter of side in simple harmonic motion.

## 3. Explain Displacement, velocity in SHM, and derive special cases.

**Ans. Displacement in SHM:**

The distance travelled by the vibrating particle at any instant of time  $t$  from its mean position is known as displacement. When the particle is at  $P$ , the displacement of the particle along  $y$  axis is  $y$ .



### Displacement in SHM.



The in  $\Delta OPN$ ,  $\sin \theta = \frac{ON}{OP}$

$$ON = y = OP \sin \theta$$

$$y = OP \sin \omega t \quad (\because \theta = \omega t)$$

Since  $OP = a$ , the radius of the circle, the displacement of the vibrating particle is

$$y = a \sin \omega t. \quad (1)$$

The amplitude of the vibrating particle is defined as its maximum displacement from the mean position.

### Velocity in SHM:

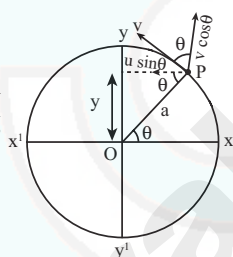
The rate of change of displacement is the velocity of the vibrating particle.

Differentiating equation (1) with respect to time  $t$ .

$$\frac{dy}{dt} = \frac{d}{dt}(a \sin \omega t) \quad (2)$$

$$\therefore v = a \omega \cos \omega t$$

The velocity  $v$  of the particle moving along the circle can also be obtained by resolving it into two components.



Velocity in SHM

(i)  $V \cos \theta$  in a direction parallel to  $OY$ .

(ii)  $V \sin \theta$  in a direction perpendicular to  $OY$ .  
The component  $v \sin \theta$  has no effect along  $YOY'$  since it is perpendicular to  $OY$ .

$$\text{Velocity} = V \cos \theta = V \cos \omega t$$

We know that,

Linear Velocity = radius  $\times$  angular velocity

$$\therefore v = a \omega$$

$$\text{Velocity} = a \omega \cos \omega t$$

$$\text{Velocity} = a \omega \sqrt{1 - \sin^2 \omega t}$$

$$\therefore \text{Velocity} = \omega \sqrt{(a^2 - y^2)}$$

### Special cases:

When the particle is at mean position,

(i.e.)  $y = 0$ . Velocity is  $a \omega$  and is maximum.

$v = \pm a \omega$  is called velocity amplitude.

When the particle is in the extreme position,

(i.e.)  $y = \pm a$ , the velocity is zero.

### 4. Explain acceleration is S.H.M, and discuss its special cases.

#### Ans. Acceleration in SHM:

The rate of change of velocity is the acceleration of the vibrating particle.

$$\begin{aligned} \frac{d^2 y}{dt^2} &= \frac{d}{dt} \left( \frac{dy}{dt} \right) = \frac{d}{dt} (a \omega (\cos \omega t)) \\ &= -\omega^2 a \sin \omega t \end{aligned}$$

$$\text{Acceleration} = \frac{d^2 y}{dt^2} = -\omega^2 y \quad (1)$$

The acceleration of the particle can also be obtained by component method.

The centripetal acceleration of the particle acting along  $PO$  is  $\frac{v^2}{a}$ .

#### Acceleration in SHM:

This acceleration is resolved into two components.

$\frac{v^2}{a} \cos \theta$  along  $PN$   
perpendicular to  $OY$ .

$\frac{v^2}{a} \sin \theta$  in a direction

parallel to  $YO$ .

The component  $\frac{v^2}{a} \cos \theta$  has no effect along  $YOY'$  since it is perpendicular to  $OY$ .

$$\text{Hence acceleration} = -\frac{v^2}{a} \sin \theta$$

$$= -a \omega^2 \sin \omega t$$

$$= -\omega^2 y$$

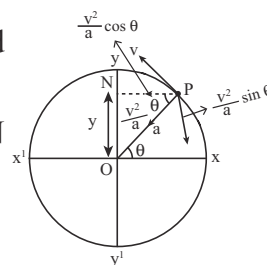
$$\text{Acceleration} = -\omega^2 y$$

The negative sign indicates that the acceleration is always opposite to the direction of displacement and is directed toward the centre.

### Special cases:

When particle is at the mean position (i.e.),  $y = 0$ , the acceleration is zero.

When the particle is at the extreme position (i.e.)  $y = \pm a$  acceleration is  $\mp a \omega^2$  which is called as acceleration amplitude.





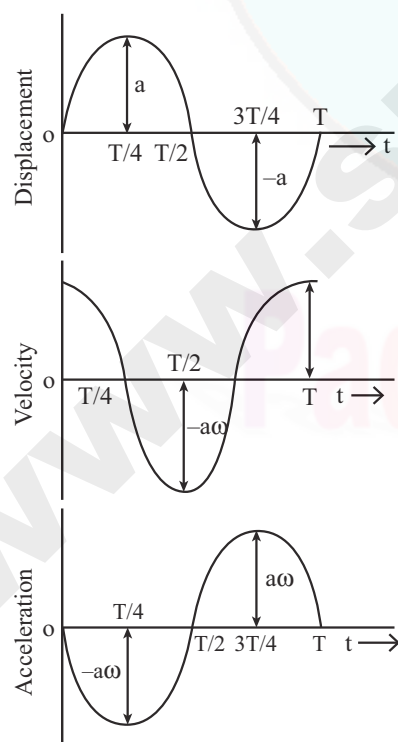
**5. Explain briefly about the graphical representation of Displacement, velocity and acceleration in SHM.**

**Ans.** Graphical representation of displacement, velocity and acceleration of a particle vibrating simple harmonically with respect to time  $t$ .

- (i) Displacement graph is a sine curve. Maximum displacement of the particle is  $y = +a$ .
- (ii) The velocity of the vibrating particle is maximum at the mean position i.e.,  $v = +a\omega$  and it is zero at the extreme position.
- (iii) The acceleration of the vibrating particle is zero at the mean position and maximum at the extreme position, (i.e.)  $\mp a\omega^2$ .

The velocity is ahead of displacement by a phase angle of  $\frac{\pi}{2}$ . The acceleration is ahead of the

velocity by a phase angle of  $\frac{\pi}{2}$  or by phase  $\pi$  ahead of displacement. (i.e.) When the displacement has its greatest positive value, acceleration has its negative maximum value or vice versa.

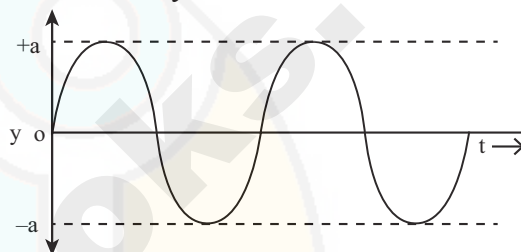


Graphical representation.

**6. Explain briefly about oscillations**

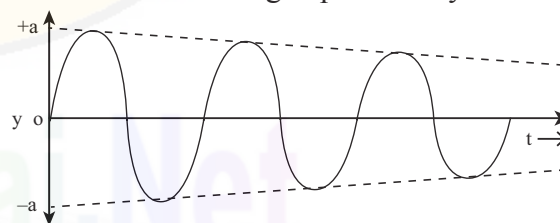
**Ans. (1) Free oscillation:**

- (i) The oscillation of a particle with fundamental frequency under the influence of restoring force are defined as free oscillation.
- (ii) The amplitude, frequency, and energy of oscillation remains constant.
- (iii) Frequency of oscillation is called natural frequency because it depends upon the nature and structure of the body.



**(2) Damped oscillation:**

- (i) The oscillation of a body whose amplitude goes on decreasing with time are defined as damped oscillation.
- (ii) In these oscillation the amplitude of oscillation decreases exponentially due to damping forces like frictional force, viscous force, etc.
- (iii) Due to decrease in amplitude the energy of the oscillation also goes on decreasing exponentially.



- (iv) The force produces a resistance to the oscillation is called damping force. If the velocity of oscillation is  $v$  then Damping force  $F_d = -bv$ ,  $b$  = damping constant.
- (v) Resultant force on a damped oscillator is given by

$$F = F_R + F_d = -kx - kv \Rightarrow \frac{md^2x}{dt^2} + b \frac{dx}{dt} + kx = 0$$

- (vi) Displacement of damped oscillation is given by

$$x = x_m e^{-\frac{bt}{2m}} \sin(\omega' t + \psi)$$

where  $\omega'$  = angular frequency of the

$$\text{damped oscillation} = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$$

The amplitude decreases continuously

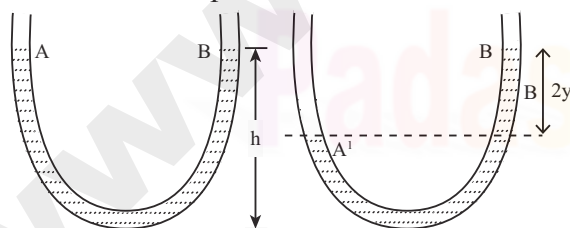
with time according to  $x = x_m e^{-\left(\frac{b}{2m}\right)t}$ .

- (vii) For a damped oscillation if the damping is small then the mechanical energy decreases exponentially with time as

$$E = \frac{1}{2} k x_m^2 e^{-\frac{bt}{m}}$$

- 7. One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that, when the suction pump is removed, the liquid column of mercury in the U-tube executes simple harmonic motion.**

- Ans. (i)** The suction pump creates the pressure difference. Hence mercury rises in one limb of the U-tube.
- (ii)** When it is removed a net force acts on the liquid column due to the difference in level of mercury in the two limbs and therefore the liquid column executes S.H.M. Which can be explained as follows.



- (iii)** The mercury contained in a vertical U-tube upto the level A and B in its two limbs.

Suppose  $\rho$  = density of the mercury.

Where  $L$  is total length of the mercury column in both the limbs.

$A$  is internal cross-sectional area of U-tube

$M$  is mass of mercury in U-tube  $M = L\rho A$

- (iv) Suppose the mercury be depressed in left limb to  $A'$  by the small distance  $y$ , then it rises by the same amount in the right limb to position  $B'$ .

$\therefore$  Difference in levels in the two limbs.

$$= A'B' = 2y$$

- (v) Volume of mercury contained in the column of length,  $2y$

$$V_y = A \times 2y$$

$$m = A \times 2y \times \rho$$

- (vi) When  $W$  = weight of liquid contained in the column of length  $2y$

$$\text{Then } W = mg = A \times 2y \times \rho \times g$$

This weight produced the restoring force  $F$  which tends to bring back the mercury to its equilibrium position.

$$F = -2A\rho y g$$

- (vii) When  $a$  = acceleration produced in the liquid column, then

$$a = \frac{F}{M} = -\frac{(2A\rho g)y}{L\rho A}$$

$$\text{or } a = \frac{-2\rho y g}{L\rho} = \frac{-2\rho g y}{2h\rho} \quad (1)$$

$$\Rightarrow \frac{y}{a} = \frac{2h\rho}{2\rho g} \quad [\because L = 2h]$$

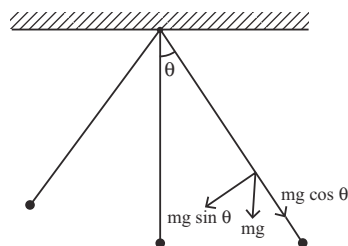
Where,  $h$  is height of mercury in each limb. Now from eqn (1), it is clear that  $a \propto y$  and Negative sign shows that it acts opposite to  $y$ , so the motion of mercury in U-tube is simple harmonic in nature having time period ( $T$ ) given by,

$$T = 2\pi \sqrt{\frac{y}{a}} = 2\pi \sqrt{\frac{2h\rho}{2\rho g}}$$

$$T = 2\pi \sqrt{\frac{h}{g}}$$

- 8. Consider a simple pendulum, having a bob attached to a string, that oscillates under the action of the force of gravity. Suppose that the period of oscillation of the simple pendulum depends on its length ( $l$ ), mass of the bob ( $m$ ), and acceleration due to gravity ( $g$ ). Derive the expression for its time period using method of dimension.**

- Ans. (i)** Let  $\theta$  be the angle made by the string with vertical. When the bob is at the mean position  $\theta = 0$ .



- (ii) There are two forces acting on the bob. The tension  $T$  along the vertical force due to gravity ( $= mg$ ).
- (iii) The force can be resolved into  $mg \cos \theta$  along a circle of length  $L$  and centre at this support point.
- (iv) Its radial acceleration ( $\omega^2 L$ ) and also tangential acceleration. So net radial force  $= T - mg \cos \theta$ , while the tangential acceleration provided by  $mg \sin \theta$ . Since the radial force gives zero torque.
- (v) So torque provided by the tangential component.

$$\tau = Lmg \sin \theta$$

$\tau = I\alpha$  (by Newton's law of rotational motion)

$$I\alpha = mg \sin \theta L$$

$$\alpha = \frac{mgL}{I} \sin \theta.$$

Where  $I$  is the moment of inertia,  $\alpha$  is angular acceleration,

- (vi) The  $\theta$  is so small,  
 $\sin \theta = \theta$

$$\alpha = -\frac{mgL}{I} \theta ; \omega = \sqrt{\frac{I}{mgL}}$$

[for simple harmonic  $\alpha(t) = -\omega^2 t \times \theta$  is

$$\text{small}] \text{ and } T = 2\pi \sqrt{\frac{I}{mgL}} \quad \left[ \because \omega = \frac{2\pi}{T} \right].$$

## Numerical Problems

== 1 Mark ==

1. Find the period of a simple pendulum 1.20 m long.

- (a) 1.4 s (b) 3.2 s  
 (c) 4.1 s (d) 2.2 s

[Ans. (d) 2.2 s]

**Solution:**

$$\begin{aligned} \text{Time period, } T_p &= 2\pi \sqrt{\frac{\rho}{g}} = 2\pi \sqrt{\frac{1.20}{9.8}} \\ &= 2 \times 3.14 \times \sqrt{\frac{1.20}{9.8}} \\ &= 2 \times 3.14 \times \sqrt{0.1224} \\ &= 2 \times 3.14 \times 0.3498 \text{ s} = 2.1967 \text{ s} \\ T_p &= 2.2 \text{ s} \end{aligned}$$

2. Find the length of a simple pendulum whose period is 2.00 s.

- (a) 2 m (b) 0.4 m  
 (c) 0.1 m (d) 3 m

[Ans. (c) 0.1 m]

**Solution:**

The length of the pendulum is,

$$\begin{aligned} \rho &= \frac{T_p^2}{4\pi^2} g \\ &= \frac{(2.00)^2 \times (9.8)}{4 \times (3.14)^2} = \frac{400 \times 9.8}{4 \times 9.85} \\ &= \frac{4.00 \times 9.8}{4 \times 9.85} = \frac{39.2}{39.4} = 0.994 \text{ m} = 0.1 \text{ m} \end{aligned}$$

3. A pendulum 1.20 m long is observed to have 1 m Long is observed to have a period of 2.00 s at a certain location then the acceleration due to gravity is,

- (a) 9.71 m/s<sup>2</sup> (b) 9.85 m/s<sup>2</sup>  
 (c) 9.79 m/s<sup>2</sup> (d) 10.1 m/s<sup>2</sup>

[Ans. (b) 9.85 m/s<sup>2</sup>]

**Solution:**

The acceleration due to gravity is,

$$\begin{aligned} g &= \frac{4\pi^2}{T_p^2} l = \frac{4\pi^2}{(2.00)^2} = \frac{4 \times (3.14)^2}{(4.00 \text{ s}^2)} \times 1 \\ &= \frac{4 \times 9.85}{4.00} = \frac{39.4}{4} = 9.85 \text{ ms}^{-2} \end{aligned}$$

4. The three springs with force constant  $k_1 = 7.5 \frac{\text{N}}{\text{m}}$ ,  $k_2 = 10.0 \frac{\text{N}}{\text{m}}$ ,  $k_3 = 12.5 \frac{\text{N}}{\text{m}}$ , are connected

in parallel to a mass of 0.500 kg. The mass is then pulled to the right and released. Then the period of the motion is.

- (a) 0.6 s (b) 0.8 s  
(c) 0.5 s (d) 0.4 s [Ans. (b) 0.8s]

**Solution:**

The period of motion, is,

$$T = 2\pi \sqrt{\frac{m}{k_1 + k_2 + k_3}}$$

$$T = 2\pi \sqrt{\frac{0.500}{7.5 + 10.0 + 12.5}}$$

$$= 2 \times 3.14 \times \sqrt{\frac{0.5}{30}} = 2 \times 3.14 \times \sqrt{0.0167}$$

$$= 2 \times 3.14 \times 0.1290 \text{ s}$$

5. The three springs with force constant  $k_1 = 8 \frac{\text{N}}{\text{m}}$ ,  $k_2 = 10 \frac{\text{N}}{\text{m}}$ ,  $k_3 = 12 \frac{\text{N}}{\text{m}}$ , are connected

in series to a mass of 0.500. The mass is then pulled to the right and released. Then the period of the motion is,

- (a) 2s (b) 2.2s (c) 2.5s (d) 3.1s  
[Ans. (c) 2.5s]

**Solution:**

The time period of motion is,

$$T = 2\pi \sqrt{m \left( \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \right)}$$

$$= 2 \times 3.14 \times \sqrt{0.5 \left( \frac{1}{8} + \frac{1}{10} + \frac{1}{12} \right)}$$

$$= 2 \times 3.14 \times \sqrt{0.5 \times 0.308} = 2 \times 3.14 \times \sqrt{0.154}$$

$$= 2 \times 3.14 \times 0.393 \text{ s} = 2.468 \text{ s}$$

$$T = 2.5 \text{ s}$$

6. A particle executing a SHM has a maximum displacement of 2 cm its acceleration at a distance of 0.5 cm from its mean position is  $2 \text{ cm/s}^2$ . What will be its velocity when it is at a distance of 1 cm from its mean position?

- (a) 4 cm/s (b)  $2\sqrt{3}$   
(c) 11.2 cm/s (d)  $4\sqrt{7}$

[Ans. (b)  $2\sqrt{3}$ ]

**Solution:**

Acceleration at a point =  $\omega^2 y$ ,  $a = 2 \text{ cm s}^{-2}$ ,  
 $y = 0.5 \text{ cm}$ .

$$\therefore a = \omega^2 y$$

$$2 = \omega^2 \times 0.5$$

$$\omega^2 = \frac{2}{0.5} = 4$$

$$\omega = 2 \text{ rad s}^{-1}$$

Velocity at any point,  $v = \omega \sqrt{a^2 - y^2}$

$$\omega = 2, a = 2, y = 1$$

$$\text{velocity, } v = 2\sqrt{2^2 - 1^2} = 2\sqrt{4 - 1} = 2\sqrt{3} \text{ cm s}^{-1}$$

7. A body of mass 1 kg is executing SHM given by,  $x = 4 \cos \left( 100t + \frac{\pi}{2} \right) \text{ cm}$ . What is the velocity?

- (a)  $200 \sin t \left( 100t + \frac{\pi}{2} \right)$   
(b)  $-200 \sin t \left( 100t + \frac{\pi}{2} \right)$   
(c)  $400 \sin t \left( 100t + \frac{\pi}{2} \right)$   
(d)  $-400 \sin t \left( 100t + \frac{\pi}{2} \right)$

[Ans. (d)  $-400 \sin t \left( 100t + \frac{\pi}{2} \right)$ ]

**Solution:**

$$\text{Given, } x = 4 \cos \left( 100t + \frac{\pi}{2} \right)$$

$$\text{Velocity} = \frac{dx}{dt} = -400 \sin t \left( 100t + \frac{\pi}{2} \right)$$

8. The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with S.H.M with an angular frequency of  $200 \text{ rad min}^{-1}$ . Then the maximum speed will be,

- (a) 50 m/min (b) 100 m/min  
(c) 150 m/min (d) 200 m/min

[Ans. (b) 100 m min<sup>-1</sup>]

**Solution:**

Acceleration  $a = \frac{1}{2} m \text{ min}^{-2}$ ,  $\omega = 200 \text{ rad/min}$

$$V_{\max} = r\omega = \frac{1}{2} \times 200$$

$$\therefore V_{\max} = 100 \text{ m min}^{-1}.$$

9. A spring balance has a scale that reads from 0 to 50 kg the length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. Then the weight of the body will be

- (a) 200 N (b) 208 N  
(c) 219.3 N (d) 272.1 N

[Ans. (c) 219.3 N]

**Solution:**

Force,  $F = mg = 50 \times 9.8 \text{ N}$

$$k = \frac{F}{y} = \frac{50 \times 9.8}{0.2} = 2450 \text{ Nm}^{-1}$$

$$T = 2\pi \sqrt{\frac{m}{k}} \Rightarrow m = \frac{T^2 k}{4\pi^2}$$

$$= \frac{(0.6)^2 \times 2450}{4 \times (3.14)^2} = \frac{0.36 \times 2450}{4 \times 9.85} = \frac{882}{39.4} = 22.38 \times 9.8$$

$$= 219.3 \text{ N}.$$

10. A small body of mass 50 g is undergoing SHM of amplitude 100 cm and period 0.2 s. What is the maximum value of the force acting on the body?

- (a) 86.4 N (b) 98.5 N  
(c) 102.1 N (d) 71.2 N

[Ans. (b) 98.5N]

**Solution:**

Maximum force,

$$F = m\omega^2 = [\text{mass} \times \text{maximum acceleration}]$$

$$F = ma \left( \frac{2\pi}{T} \right)^2 = \frac{4\pi^2 ma}{T^2}$$

$$F = \frac{4 \times (3.14)^2 \times 0.1 \times 1}{(0.20)^2} = \frac{4 \times 9.85 \times 0.1 \times 1}{(0.04)^2}$$

$$F = \frac{3.94}{0.04} = 98.5 \text{ N}.$$

**==2 Marks ==**

1. A mass M attached to a spring oscillates with a period of 2 sec. if the mass is increased by 2 kg, the period increases by the second. Find the initial mass M assuming that Hooke's law is obeyed.

**Solution:**

We know that  $T = 2\pi \sqrt{\frac{M}{k}}$ , where  $k$  = spring constant

$$\text{In first case, } 2 = 2\pi \sqrt{\frac{M}{k}} \quad (1)$$

$$\text{In second case, } 3 = 2\pi \sqrt{\frac{M+2}{k}} \quad (2).$$

Squaring equation (1) and (2) and then dividing (2) by (1), we have,

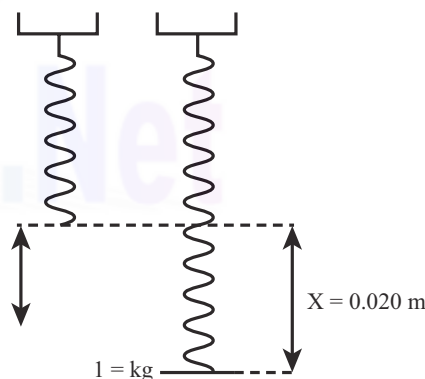
$$\frac{3^2}{2^2} = \frac{M+2}{M}$$

$$\frac{9}{4} = \frac{M+2}{M} = 1 + \frac{2}{M}$$

$$\frac{9}{4} = 1 + \frac{2}{M}$$

Solving we get, initial mass  $M = 1.6 \text{ kg}$ .

2. A spring stretches by 0.020 m when a 1.5 kg object is suspended from its end. How much mass should be attached to the spring so that its frequency of vibration is  $f = 3.1 \text{ Hz}$ ?

**Solution:**

$$\sum F_y = ma_y = 0$$

$$kx - mg = 0 \Rightarrow k = \frac{mg}{x}$$



The force constant is,

$$k = \frac{(1.5)(9.8)}{(0.020)} = \frac{14.7}{0.020} = 735 \text{ N/m.}$$

$$\text{Frequency} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$\Rightarrow 4\pi^2 f^2 = \frac{k}{m}$$

$$\begin{aligned} m &= \frac{k}{4\pi^2 f^2} \Rightarrow m = \frac{(735)}{4\pi^2 (3.0)^2} \\ &= \frac{(735)}{4 \times (3.75)^2 \times (9.0)} \\ &= \frac{735}{4 \times 9.85 \times 9.0} \\ &= \frac{735}{354.6} = 2.07 \text{ kg} \end{aligned}$$

- 3.** An oscillating block-spring system has a mechanical energy of 1.00 J, an amplitude of 10.0 cm and a maximum speeding of 1.20 m/s. Find the spring constant, the mass of the block, and the frequency of oscillation.

**Solution:**

$$E = 1.00 \text{ J}, A = 10.0 \text{ cm} = 0.10 \text{ m}, v_m = 1.20 \text{ m/s.}$$

$$k = ?, m = ?, f = ?$$

The total mechanical energy of a block spring system is related at A and  $v_m$ .

$$E = \frac{1}{2} m v_m^2 \quad (\text{occurs when } x = 0 \text{ so P.E} = 0)$$

$$E = \frac{1}{2} k A^2 \quad (\text{occurs when } v = 0 \text{ so K.E} = 0)$$

$$E = \frac{1}{2} k A^2 \Rightarrow k = \frac{2E}{A^2} = \frac{2(1.00 \text{ J})}{(0.10)^2} \Rightarrow k = 200 \text{ Nm}^{-1}$$

$$E = \frac{1}{2} m v_m^2 \Rightarrow m = \frac{2E}{v_m^2} = \frac{2(1.00 \text{ J})}{(1.20)^2} \Rightarrow m = 1.39 \text{ kg}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{(200)}{1.39}} \Rightarrow f = 1.90 \text{ Hz.}$$

- 4.** The acceleration dual to gravity on the surface of moon is  $1.7 \text{ ms}^{-2}$ . What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 3.5 s?

**Solution:**

On earth time period

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$3.5 = 2\pi \sqrt{\frac{l}{9.8}} \quad \dots(1)$$

$\therefore$  Acceleration due to gravity of the moon's is  $1.7 \text{ ms}^{-2}$

$$T' = 2\pi \sqrt{\frac{l}{1.7}} \quad \dots(2)$$

Dividing (2) by (1),

$$\text{Dividing } \frac{T'}{3.5} = 2\pi \sqrt{\frac{1}{1.7}} ; / 2\pi \sqrt{\frac{l}{9.8}}$$

$$T' = \sqrt{\frac{9.8}{1.7}} \times 3.5 = 8.4 \text{ s}$$

- 5.** A 0.950 kg mass hangs vertically from a spring that has a spring constant of 8.50 N/m. The mass is set into vertical oscillation and after 600 s, you find that the amplitude of the oscillation is  $\frac{1}{10}$  that of initial amplitude.

What is the damping constant associated with this motion?

**Solution:**

The amplitude is  $A = A_0 e^{-\alpha t}$

Divide by  $A_0$ , take natural logarithms and solve for  $\alpha$

$$\frac{A}{A_0} = e^{-\alpha t} \Rightarrow \ln \left( \frac{A}{A_0} \right) = -\alpha t$$

$$\Rightarrow \alpha = -\frac{1}{t} \ln \left( \frac{A}{A_0} \right)$$

Since  $\alpha = \frac{\beta}{2m}$ , the  $\beta = 2m\alpha$ . Thus,

$$\beta = -\frac{2m}{t} \ln \left( \frac{A}{A_0} \right)$$

Now substituting  $0.950 \text{ kg}$  for  $m$ ,  $600 \text{ s}$ , for  $t$ , and

$$0.10 \text{ for } \left( \frac{A}{A_0} \right)$$

$$\beta = -\frac{2m}{t} l_n \left( \frac{A}{A_0} \right) = -\frac{2(0.950)}{600} l_n 0.10$$

$$= 7.3 \times 10^{-3} \text{ kg s}^{-1}$$

### == 3 Marks ==

1. A uniform disk of radius  $r = 0.6 \text{ m}$  and mass  $M = 2.5 \text{ kg}$  is freely suspended from a horizontal pivot located a radial distance  $d = 0.30 \text{ m}$  from its centre. Find the angular frequency of small amplitude oscillations of the disk.

#### Solution:

The moment of inertia of the disk about a perpendicular axis passing through its centre is

$$I = \frac{1}{2} m r^2.$$

From the parallel axis theorem, the moment of inertia of the disk about the pivot point is

$$I' = I + M d^2 = \frac{2.5 \times 0.6 \times 0.6}{2} + 2.5 \times 0.30 \times 0.30$$

$$= \frac{0.9}{2} + 0.225$$

$$= 0.45 + 0.225$$

$$= 0.675 \text{ kg m}^2$$

The angular frequency of small amplitude oscillations of a compound pendulum is given by

$$\omega = \sqrt{\frac{Mgd}{I'}} = \sqrt{\frac{2.5 \times 9.8 \times 0.30}{0.675}}$$

$$= \sqrt{\frac{7.35}{0.675}} = \sqrt{\frac{0.675}{10.9}} = 3.3 \text{ rad/s}$$

2. A body oscillates with SHM along with  $x$ -axis.

Its displacement varies with time according to the equation  $x = (4.00 \text{ m}) \cos \left( \pi t + \frac{\pi}{4} \right)$  calculate

at  $t = 1.00 \text{ s}$ : (a) displacement (b) velocity (c) acceleration (d) Also calculate the maximum speed and maximum acceleration and (e) phase at  $t = 2.00 \text{ s}$ .

#### Solution:

By comparing the given equation with the general equation for SHM along  $x$ -axis,

$x = A \cos (\omega t + \phi_0)$ , we get

$$A = 4.00 \text{ m}, \omega = \pi \text{ rad/s}, \phi_0 = \frac{\pi}{4}$$

Displacement at  $t = 1.00 \text{ s}$ ,

$$x = (4.00 \text{ m}) \cos \left( \pi \times 1 + \frac{\pi}{4} \right) = (4.00) \left( -\cos \frac{\pi}{4} \right)$$

$$= (4.00) (-0.707) = -2.83 \text{ m}.$$

Velocity at  $t = 1.00 \text{ s}$ ,  $v = -\omega A \sin (\omega t + \phi_0)$

$$v = -\left( \frac{\pi}{s} \right) (4.00 \text{ m}) \sin \left[ \pi \times 1 + \frac{\pi}{4} \right]$$

$$= -(4.00 \pi) \left( -\sin \frac{\pi}{4} \right) \text{ ms}^{-1}$$

$$= -(4.00 \times 3.14) (0.707) \text{ ms}^{-1}$$

$$= 8.89 \text{ m/s}.$$

#### Acceleration:

$$a = -\omega^2 A \cos (\omega t + \phi_0)$$

$$= -\pi^2 \times 4.00 \cos \left( \pi \times 1 + \frac{\pi}{4} \right)$$

$$= -(4.00 \pi^2) \left( -\cos \frac{\pi}{4} \right) \text{ ms}^{-2}$$

$$= 4.00 \times (3.14)^2 \times 0.707 \text{ ms}^{-2}$$

$$= 27.9 \text{ ms}^{-2}$$

$$\text{Maximum Velocity, } v_{\max} = \omega A \times \pi \times 4.00$$

$$= 12.6 \text{ ms}^{-1}$$

$$\text{Maximum acceleration, } a_{\max} = \omega^2 A = \pi^2 \times 4.00$$

$$= 39.5 \text{ ms}^{-2}$$

$$\text{Phase } (\omega t + \phi_0) = \left( \frac{\pi}{s} \right) \times 2s + \frac{\pi}{4} = 2\pi + \frac{\pi}{4} = \frac{9\pi}{4}$$

3. A particle executes SHM with a time period of  $16 \text{ s}$ . At time  $t = 2 \text{ s}$ , the particle crosses the mean position while at  $t = 4 \text{ s}$ , its velocity is  $4 \text{ ms}^{-1}$ . Find its amplitude of motion.

#### Solution:

Here,

$$T = 16 \text{ s}, \text{ At } t = 2 \text{ s}, y = 0 \text{ and at } t = 4 \text{ s } v = 4 \text{ ms}^{-1},$$

$$a = ?$$

For simple harmonic motion,  $y = a \sin \omega t = a \sin \frac{2\pi}{T}t$  when  $t = 4$  s, the time taken by particle to travel from the mean position to a given position  $= 4 - 2 = 2$  s the displacement.

$$y = a \sin \left( \frac{2\pi}{16} \times 2 \right) = a \sin \left( \frac{\pi}{4} \right) = \frac{a}{\sqrt{2}}$$

$$\text{Velocity } v = \omega \sqrt{a^2 - y^2}$$

$$4 = \left( \frac{2\pi}{16} \right) \sqrt{a^2 - \frac{a^2}{2}}$$

$$= \frac{\pi}{8} \times \frac{a}{\sqrt{2}}$$

$$a = \frac{32\sqrt{2}}{\pi} = 14.4 \text{ m}$$

4. A particle moving in a straight line has velocity  $v$  given by  $v^2 = \alpha - \beta y^2$ , where  $\alpha$  and  $\beta$  are constant and  $y$  is its distance from a fixed point in the line. Show that the motion of the particle is SHM. Find its time period and amplitude.

**Solution:**

$$v^2 = \alpha - \beta y^2 \quad (1)$$

Differentiating it with respect to time,  $t$  we have

$$2v \frac{dv}{dt} = -\beta y \frac{dy}{dt} \quad (2)$$

$$\frac{dv}{dt} = -\beta y$$

It means acceleration  $a = \frac{dv}{dt} = -\beta y$

As  $a$  and  $y$  have negative sign shows that acceleration is directed towards mean position. So, if the particle is left force, it will executes S.H.M

$$\omega^2 = \beta$$

$$\omega = \sqrt{\beta}$$

$$\therefore \text{Time period, } T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\beta}}$$

We know that,  $v = 0$ , when  $y = r$  from equation (2).

$$0 = \alpha - \beta r^2$$

$$A = \sqrt{\frac{\alpha}{\beta}}$$

$$\text{amplitude, } A = \sqrt{\frac{\alpha}{\beta}}$$

5. A spring compressed by 10 cm develops a restoring force of 10 N. A body of mass 9 kg is placed on it. What is the force constant of the spring? What is the depression in the spring under the weight of the body? What is the period of oscillation if the body is disturbed from its equilibrium position?

**Solution:**

$$F = 10 \text{ N, } y = 10 \text{ cm} = 0.1 \text{ m}$$

$$\text{Force constant, } k = \frac{F}{y} = \frac{10}{0.1} = 100 \text{ N/m}$$

$$\text{Further, when } F = \text{weight of the body} \\ = 9 \text{ kg wt} = 90 \text{ N}$$

$$y = \frac{F}{k} = \frac{90}{100} = 0.9 \text{ m}$$

$$\text{Time period, } T = 2\pi \sqrt{\frac{m}{k}} \\ = 2\pi \sqrt{\frac{9}{100}} \text{ s} = 2\pi \left( \frac{3}{10} \right) \text{ s}$$

### Creative Questions (HOTS)

1. Can a motion be oscillatory, but not simple harmonic. If your answer is yes, give an explanation and if not explain why?

**Ans.** Yes, when a ball is dropped from a height on a perfectly elastic surface, the motion is oscillatory but not simple harmonic as restoring force  $F = mg = \text{constant}$  and not  $F \propto -x$ , which is an essential condition for S.H.M.

2. Every simple harmonic motion is periodic motion but every periodic motion need not be simple harmonic motion. Do you agree? Give example.

**Ans.** Yes, every periodic motion need not be simple harmonic motion. The motion of the earth round the sun is a periodic motion, but not simple harmonic motion as the back and forth motion is not taking place.

**3. What is the basic condition for the motion of a particle to be S.H.M?**

**Ans.** The motion of a particle will necessarily be simple harmonic if the restoring force acting on it is proportional to its displacement from the mean position i.e.  $F = -kx$ .

**4. The maximum acceleration of a simple harmonic oscillator is  $a_0$  and the maximum velocity is  $V_0$ . What is the displacement amplitude?**

**Ans.** Let  $A$  be the displacement amplitude and  $\omega$  be the angular frequency of S.H.M then maximum

$$\text{velocity, } V_0 = \omega A \quad \therefore \omega = \frac{V_0}{A}$$

$$\text{Maximum acceleration, } a_0 = \omega^2 A = \left( \frac{V_0}{A} \right)^2 A = \frac{V_0^2}{A}$$

$$\therefore \text{Displacement amplitude, } A = \frac{V_0^2}{a_0}$$

**5. A girl is surging in the sitting position. How will the period of the swing change if she stands up?**

**Ans.** The girl and the swing together constitute a pendulum of time period,

$$T = 2\pi \sqrt{\frac{l}{g'}}$$

As the girl stands up, her E.G is raised the distance between the point of suspension and the C.G decreases i.e length  $l$  decreases. Hence the time period  $T$  decreases.

**6. Will a pendulum clock lose or gain time when taken to the top of a mountain?**

**Ans.** On the top of the mountain, the value of  $g$  is less than that on the surface of the earth the decrease in the value of  $g$  increases the time period of the pendulum on the top of the top of the mountain. So the pendulum clock loses time.

**7. How would the time period of a spring mass system change, when it is made to oscillate horizontally and then vertically?**

**Ans.** Time period  $T$  will remain the same for both the cases.

**8. Alcohol in a U tube executes S.H.M of time period  $T$ . Now, alcohol is replaced by water upto the same height in the U-tube. What will be the effect on the time period?**

**Ans.** Time period  $T$  remains same, this is because the period of oscillation of a liquid in a U-tube does not depend on the density of the liquid.

**9. What is the ratio between the potential energy the total energy of a particle executing S.H.M, when its displacement is half of its amplitude?**

$$\text{Ans. } \frac{\text{Potential energy}}{\text{Total energy}} = \frac{\frac{1}{2} m \omega^2 y^2}{\frac{1}{2} m \omega^2 a^2} = \frac{y^2}{a^2} = \left( \frac{a}{2} \right)^2 \frac{1}{a^2} = \frac{1}{4}$$

$$= 1 : 4$$

**10. Why are army troops not allowed to march in steps while crossing the ridge?**

**Ans.** Army troops are not allowed to march in steps because it is quite likely that the frequency of the foot steps may match with the natural frequency of the bridge and due to resonance the bridge may pick up large amplitude and break.

**11. How can earthquakes cause disaster sometimes?**

**Ans.** The resonance may cause disaster during the earthquake, if the frequency of oscillation present within the earth per chance coincides with natural frequency of some building, which may start vibrating with large amplitude due to resonance and may get damaged.

**12. Sometimes a wire glass is broken by the powerful voice of a celebrated singer why?**

**Ans.** When the natural frequency of the wire glass becomes equal to that of the singers voice, the resulting resonance due to the powerful voice of the singer may break the glass.



**13. Glass windows may be broken by a far away explosion. Explain why?**

**Ans. (i)** A large amplitude in all directions. As these sound waves strike the glass windows, they set them into forced oscillations.

**(ii)** Since glass is brittle, so the glass windows break as soon as they start oscillating due to forced oscillations.

**14. The body of a bus begins to rattle sometimes, when the bus picks up a certain speed, why?**

**Ans.** At a particular speed, the frequency of the engine of the bus becomes equal to the natural frequency of the body of the bus so the frame of the bus begins to vibrate strongly due to resonance.

**15. What will be the change in time period of a loaded spring, when taken to moon?**

**Ans.** Time period of a loaded spring,

$$T = 2\pi \sqrt{\frac{m}{k}}$$

As  $T$  is independent of  $g$ , will not be affected when the loaded spring is taken to the moon.

**16. In forced oscillation of a particle, the amplitude is maximum for a frequency  $\omega_1$  of the force, while the energy is maximum for a frequency  $\omega_2$  of the force, what is relation between  $\omega_1$  and  $\omega_2$ ?**

**Ans.** Only in the case of resonance, both amplitudes and energy of oscillation are maximum.

$$\omega_1 = \omega_2$$

**17. The maximum velocity of a particle, executing simple harmonic motion with an amplitude of 7mm is  $4.4\text{ms}^{-1}$ . What is the period of oscillation?**

**Ans.**  $V_{\max} = \omega_A = \frac{2\pi}{T} A$

$$T = \frac{2\pi A}{V_{\max}} = \frac{2 \times 22 \times 7 \times 10^{-3}}{7 \times 4.4} = 0.01\text{s}$$

**18. The bob of a simple pendulum is a hollow sphere filled with water. How will the period of oscillation change if the water begins to drain out of the hollow sphere from a fine hole at its bottom?**

**Ans.**  $T = 2\pi \sqrt{\frac{l}{g'}}$

**(i)** As the water flows out of the sphere, the time period first increases and then decreases.

**(ii)** Initially when the sphere is completely filled with water, its C.G lies at its center.

**(iii)** As water flows out, the C.G begins to shift below the centre of the sphere the effective length of the pendulum increases & hence its time period increase when the sphere becomes more than half empty, its C.G begins to rise up the effective length of the pendulum increases and true period  $T$  decreases.

**(iv)** When the entire water is drained out of the sphere, the C.G is once again shifted to centre of the sphere and the time period  $T$  attains its initial value.

**19. Two simple harmonic moles are represented by  $y_1 = 0.1 \sin(100\pi t + \pi/3)$  and  $y_2 = 0.1 \cos \pi t$ . What is the phase difference of the velocity of the particle 1 with respect to the velocity of particle 2?**

**Ans.** Velocity of particle 1

$$V_1 = \frac{dy_1}{dt} = 0.1 \cos(100\pi t + \frac{\pi}{3}) \times 100\pi$$

$$= 10\pi \cos(100\pi t + \frac{\pi}{3})$$

Velocity of particle 2

$$V_2 = \frac{dy_2}{dt} = 0.1 (-\sin \pi t) \times \pi$$

$$= -0.1 \pi \sin \pi t$$

$$= 0.1 \cos(\pi t + \frac{\pi}{2})$$

Phase difference of the of particle 1 velocity with respect to the velocity of particle 2 is

$$\Delta\phi = \phi_1 - \phi_2 = \frac{\pi}{3} - \frac{\pi}{2} = -\frac{\pi}{6}$$

**20. A simple pendulum has time period  $T$ , the point of suspension is now moved upward acceleration to the relation  $y = kt^2$  ( $k = 1\text{ms}^{-2}$ ) where  $y$  is the vertical displacement the time period now becomes  $T_2$ . What is the ratio  $\frac{T_1^2}{T_2^2}$  ? Given  $g = 10\text{ms}^{-2}$ .**

**Ans.** In first case

$$T_1 = 2\pi \sqrt{\frac{l}{g'}}$$



in second case, displacement  $y = kt^2$

upward velocity,  $v = \frac{dy}{dt} = 2kt$ .

upward acceleration  $a = \frac{d^2y}{dt^2} = 2k$ ,

$$k = 2 \times 1 \text{ ms}^{-2} = 2 \text{ ms}^{-2} \quad [\because k = 1 \text{ ms}^{-2}]$$

$$T_2 = 2\pi \sqrt{\frac{l}{g+a}} = 2\pi \sqrt{\frac{l}{g+2}}$$

$$\text{hence } \frac{T_1^2}{T_2^2} = \frac{4\pi^2 l}{g} \times \frac{g+2}{4\pi^2 l} = \frac{g+2}{g} = \frac{10+2}{10} = \frac{6}{5}$$

- 21.** The bob of simple pendulum executes S.H.M in water with a period  $t$ , while the period of oscillation of the bob is  $t_0$  in air, neglecting frictional force of water and given that the density of the bob is  $\frac{4000}{3} \text{ kg m}^{-3}$ , find the relationship between  $t$  and  $t_0$ ?

**Ans.** In air  $t_0 = 2\pi \sqrt{\frac{l}{g'}}$

Let  $V$  be the volume of the bob. Then apparent weight of bob in water = weight of bob in air – up thrust

$$V\rho g' = V\rho g - V\sigma g$$

$$V\rho g' = V(\rho g - \sigma g)$$

$$g' = \left(g - \frac{\sigma}{\rho}g\right)$$

$$g' = g\left(1 - \frac{\sigma}{\rho}\right)$$

$$g' = \left(1 - \frac{\sigma}{\rho}\right)g$$

$$\text{Density of bob, } \rho = \frac{4000}{3} \text{ kg m}^{-3}$$

$$\text{Density of water, } \sigma = 1000 \text{ kg m}^{-3}$$

$$g' = \left(1 - \frac{1000 \times 3}{4000}\right)g$$

$$= \frac{g}{4}$$

Time period of the pendulum in water

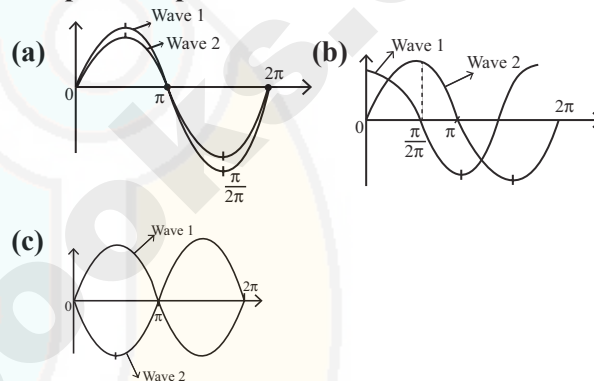
$$t = 2\pi \sqrt{\frac{l}{g'}} = 2\pi \sqrt{\frac{l}{\frac{g}{4}}} = 2 \times 2\pi \sqrt{\frac{l}{g}}$$

$$\therefore t_0 = 2t.$$

### VALUE BASED QUESTIONS

- 1.** Students in a class were asked by their science teacher about different types of motions, with examples. But few students were confused with explanations, for example. Periodic motion. SHM, Oscillatory motion, Rotational motion etc. How would have the teacher explained, so that students understood easily?

- (i) What are path difference & phase difference?  
(ii) For these waves, given below find out the phase & path differences.



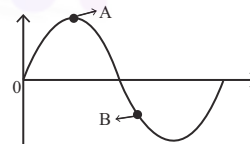
**Ans. Periodic Motion :** Motion which is repeated after regular interval of time. Ex: Earth's rotation or revolution.

**SHM :** A motion which has single frequency and constant amplitude. Ex. : Motion of a string for one full vibration.

**Rotational Motion :** The motion of a body which takes place about its own axis. Ex. : Movement of a Top.

**Vibratory Motion :** An upward and downward movement of a body or a string is called vibratory motion. Ex. : Earth quake, guitar string etc.

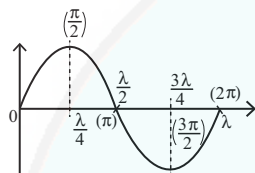
- (i) **Phase :** Phase is the state or position of the particle in a medium or a wave.



Here in the above diagram, the particle at A is at an angle of  $\frac{\pi}{2}$  ( $90^\circ$ ). So the phase is  $\frac{\pi}{2}$  ( $90^\circ$ ). The phase of the particle at B is  $225^\circ$ . i.e. ( $90^\circ + 45^\circ$ )

**Phase difference :** Consider two waves, which are the disturbances produced by two particles at different instants of time. The difference in phase between the two particles (making two different waves) is called phase difference if phase difference is zero, then the two waves are said to be in phase if the two waves have a phase difference then two waves are out of phase.

**Phase difference :** For one full cycle of a wave, the wavelength is ' $\lambda$ '



wavelength can also be defined as the distance between two consecutive crests or troughs in a wave.

The difference in wavelengths (paths) between the two waves is called path difference.

**(ii) In the diagram 1 :**

The phase difference is zero. i.e. the two waves are in phase. All particles between these two waves are moving with the same time and same angle at different points.

The path difference is also zero.

**In the diagram 2 :**

Wave - 1 starts before wave-2 and they meet at a point in line with each other at an angle of  $\frac{\pi}{2}$ .

So the phase difference between waves 1 & 2 is  $\frac{\pi}{2}$ . The path difference is  $\frac{\lambda}{4}$ .

**In the diagram 3 :**

Wave - 1 and wave 2 are moving in opposite direction and meet at an angle of  $\pi$  initially. So the phase difference is  $\pi$  and the path difference is  $\frac{\lambda}{2}$ .

- 2. Shiva and Arun went for a magic show. In that show a lady was singing a song with high pitch and a glass kept before her shattered into pieces. Arun was admiring at this and asked Shiva how this happened?**

**(i) What was shiva's explanation?**

- Ans. (i)** Shiva explained this phenomenon is due to a physics concept called resonance. When the singer sings with a high pitch, her frequency is also high, which has some value. Due to her songs vibration, the glass also vibrates with a frequency. If the frequency of vibration of the glass matches the frequency of the singer then amplitude will be maximum. This results in a Phenomenon named Resonance, due to which the glass shatter in to pieces.

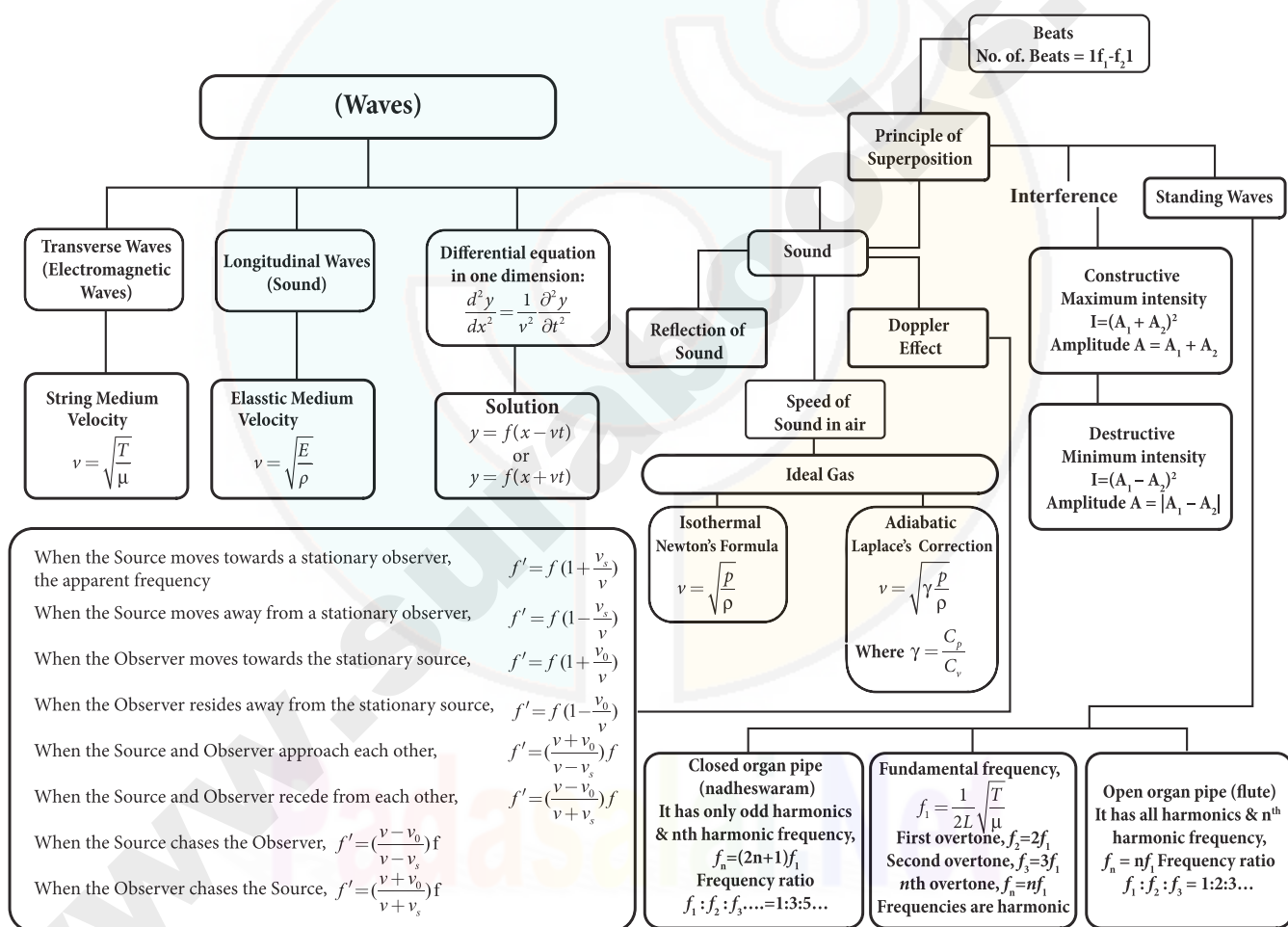


## UNIT

## 11

## WAVES

## CONCEPT MAP



## FORMULAE TO REMEMBER

- (1) Velocity of the wave is  $v = \lambda f$ .
- (2) Velocity of transverse wave on a string is  $v = \sqrt{\frac{T}{\mu}} \text{ ms}^{-1}$
- (3) Velocity of longitudinal wave in an elastic medium is  $v = \sqrt{\frac{E}{\rho}} \text{ ms}^{-1}$
- (4) Wave number  $k = \frac{2\pi}{\lambda} \text{ rad m}^{-1}$ .
- (5) For constructive interference,  $I_{\text{maximum}} = (\sqrt{I_1} + \sqrt{I_2})^2 = (A_1 + A_2)^2$
- (6) For destructive interference,  $I_{\text{minimum}} = (\sqrt{I_1} - \sqrt{I_2})^2 = (A_1 - A_2)^2$
- (7) Sound intensity level,  $\Delta L = 10 \log_{10} \left| \frac{I_1}{I_0} \right| \text{ decibel}$ .
- (8) Frequency of the (Closed organ pipe)  $n^{\text{th}}$  harmonic is  $f_n = (2n + 1)f_1$ .
- (9) Frequency of the (Open organ pipe)  $n^{\text{th}}$  harmonic is  $f_n = n f_1$ .
- (10) When the Source moves towards a stationary observer, the apparent frequency  $f' = f \left( 1 + \frac{v_s}{v} \right)$
- (11) When the Source moves away from the observer  $f' = f \left( 1 - \frac{v_s}{v} \right)$
- (12) When the Observer moves towards the stationary source,  $f' = f \left( 1 + \frac{v_0}{v} \right)$
- (13) When the Observer recedes away from the stationary source,  $f' = f \left( 1 - \frac{v_0}{v} \right)$
- (14) When the Source and Observer approach each other,  $f' = \left( \frac{v + v_0}{v - v_s} \right) f$
- (15) When the Source and Observer recede from each other,  $f' = \left( \frac{v - v_0}{v + v_s} \right) f$
- (16) When the Source chases the Observer,  $f' = \left( \frac{v - v_0}{v - v_s} \right) f$
- (17) When the Observer chases the Source,  $f' = \left( \frac{v + v_0}{v + v_s} \right) f$
- (18) To find end correction,  $e = \frac{L_2 - 3L_1}{2}$   
 $L_2 \rightarrow$  Length at which second resonance Occur  
 $L_1 \rightarrow$  Length at which first resonance Occur

## IMPORTANT TERMS & DEFINITIONS

<b>wave</b>	: A disturbance which carries energy and momentum from one point in space to another point in space without the transfer of medium is known as a wave.
<b>Mechanical waves</b>	: The waves which require medium for their propagation are known as mechanical waves.
<b>Non-mechanical waves</b>	: The waves which do not require medium for their propagation are known as non-mechanical waves.
<b>Wavelength</b>	: The distance between two consecutive crests or troughs is known as wavelength, $\lambda$ .
<b>Frequency</b>	: The number of waves or cycles per second is known as frequency, $f$ .
<b>Time period</b>	: The time taken by one wave to complete one full cycle is known as time period, $T$ .
<b>Velocity of a transverse</b>	: The velocity of a transverse wave produce in a stretched string depends on tension in the string and mass per unit length. It does not depend on shape of the wave form.
<b>Harmonics</b>	: If natural frequencies are written as integral multiples of fundamental frequency, then the frequencies are said to be in harmonics. Thus, the first harmonic is $\nu_1 = \nu_1$ , (the fundamental frequency is called first harmonics), the second harmonics is $\nu_2 = 2 \nu_1$ , the third harmonics is $\nu_3 = 3 \nu_1$ , and soon.
<b>Loudness of sound</b>	: Loudness of sound is defined as “the degree of sensation of sound produced in the ear or the perception of sound by the listener”.
<b>Intensity of sound</b>	: The intensity of sound is defined as “the sound power transmitted per unit area placed normal to the propagation of sound wave”.
<b>Doppler Effect</b>	: When the source and the observer are in relative motion with respect to the medium in which sound propagates, the frequency of the sound wave observed is different from the frequency of the source. This phenomenon is called Doppler Effect.
<b>Apparent frequency</b>	: The frequency received by the observer is known as apparent frequency.
<b>Beat frequency</b>	: When we superimpose two or more waves with slightly different frequencies then a sound of periodically varying amplitude at a point is observed. This phenomenon is known as beats. The number of amplitude maxima per second is called beat frequency.



## EVALUATION

### I. MULTIPLE CHOICE QUESTIONS:

1. A student tunes his guitar by striking a 120 Hertz with a tuning fork, and simultaneously plays the 4<sup>th</sup> string on his guitar. By keen observation, he hears the amplitude of the combined sound oscillating thrice per second. Which of the following frequencies is the most likely the frequency of the 4<sup>th</sup> string on his guitar?

(a) 130 (b) 117 (c) 110 (d) 120

[Ans. (b) 117]

2. A transverse wave moves from a medium A to a medium B. In medium A, the velocity of the transverse wave is  $500 \text{ ms}^{-1}$  and the wavelength is 5 m. The frequency and the wavelength of the wave in medium B when its velocity is  $600 \text{ ms}^{-1}$ , respectively are

(a) 120 Hz and 5 m  
(b) 100 Hz and 5 m  
(c) 120 Hz and 6 m  
(d) 100 Hz and 6 m

[Ans. (d) 100 Hz and 6 m]

3. For a particular tube, among six harmonic frequencies below 1000 Hz, only four harmonic frequencies are given : 300 Hz, 600 Hz, 750 Hz and 900 Hz. What are the two other frequencies missing from this list?

(a) 100 Hz, 150 Hz  
(b) 150 Hz, 450 Hz  
(c) 450 Hz, 700 Hz  
(d) 700 Hz, 800 Hz

[Ans. (b) 150 Hz, 450 Hz]

4. Which of the following options is correct?

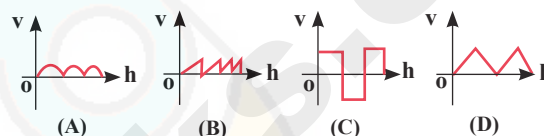
A	B
(1) Quality	(A) Intensity
(2) Pitch	(B) Waveform
(3) Loudness	(C) Frequency

Options for (1), (2) and (3), respectively are

(a) (B), (C) and (A) (b) (C), (A) and (B)  
(c) (A), (B) and (C) (d) (B), (A) and (C)

[Ans. (a) (B), (C) and (A)]

5. Compare the velocities of the wave forms given below, and choose the correct option.



where,  $v_A$ ,  $v_B$ ,  $v_C$  and  $v_D$  are velocities given in (A), (B), (C) and (D), respectively.

(a)  $v_A > v_B > v_D > v_C$   
(b)  $v_A < v_B < v_D < v_C$   
(c)  $v_A = v_B = v_D = v_C$   
(d)  $v_A > v_B = v_D > v_C$

[Ans. (c)  $v_A = v_B = v_D = v_C$ ]

6. A sound wave whose frequency is 5000 Hz travels in air and then hits the water surface. The ratio of its wavelengths in water and air is

(a) 4.30 (b) 0.23 (c) 5.30 (d) 1.23

[Ans. (a) 4.30]

7. A person standing between two parallel hills fires a gun and hears the first echo after  $t_1$  sec and the second echo after  $t_2$  sec. The distance between the two hills is

[HY-2019]

(a)  $\frac{v(t_1 - t_2)}{2}$  (b)  $\frac{v(t_1 t_2)}{2(t_1 + t_2)}$   
(c)  $v(t_1 t_2)$  (d)  $\frac{v(t_1 + t_2)}{2}$

[Ans. (d)  $\frac{v(t_1 + t_2)}{2}$ ]

8. An air column in a pipe which is closed at one end, will be in resonance with the vibrating body of frequency 83 Hz. Then the length of the air column is

(a) 1.5 m (b) 0.5 m (c) 1.0 m (d) 2.0 m

[Ans. (c) 1.0 m]

9. The displacement  $y$  of a wave travelling in the  $x$  direction is given by  $y = (2 \times 10^{-3}) \sin(300t - 2x + \frac{\pi}{4})$ , where  $x$  and  $y$  are measured in metres and  $t$  in second. The speed of the wave is

(a)  $150 \text{ ms}^{-1}$  (b)  $300 \text{ ms}^{-1}$   
(c)  $450 \text{ ms}^{-1}$  (d)  $600 \text{ ms}^{-1}$

[Ans. (a)  $150 \text{ ms}^{-1}$ ]

10. Consider two uniform wires vibrating simultaneously in their fundamental notes. The tensions, densities, lengths and diameter of the two wires are in the ratio  $8 : 1$ ,  $1 : 2$ ,  $x : y$  and  $4 : 1$  respectively. If the note of the higher pitch has a frequency of  $360 \text{ Hz}$  and the number of beats produced per second is  $10$ , then the value of  $x : y$  is

(a)  $36 : 35$  (b)  $35 : 36$  (c)  $1 : 1$  (d)  $1 : 2$

[Ans. (a)  $36 : 35$ ]

11. Which of the following represents a wave?

(a)  $(x - vt)^3$  (b)  $x(x + vt)$   
(c)  $\frac{1}{(x + vt)}$  (d)  $\sin(x + vt)$

[Ans. (d)  $\sin(x + vt)$ ]

12. A man sitting on a swing which is moving to an angle of  $60^\circ$  from the vertical is blowing a whistle which has a frequency of  $2.0 \text{ kHz}$ . The whistle is  $2.0 \text{ m}$  from the fixed support point of the swing. A sound detector which detects the whistle sound is kept in front of the swing. The maximum frequency the sound detector detected is

(a)  $2.027 \text{ kHz}$  (b)  $1.974 \text{ kHz}$   
(c)  $9.74 \text{ kHz}$  (d)  $1.011 \text{ kHz}$

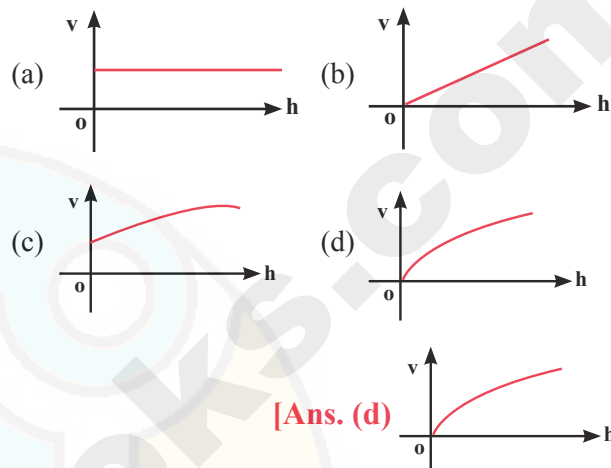
[Ans. (a)  $2.027 \text{ kHz}$ ]

13. Let  $y = \frac{1}{1+x^2}$  at  $t = 0 \text{ s}$  be the amplitude of the wave propagating in the positive  $x$ -direction. At  $t = 2 \text{ s}$ , the amplitude of the wave propagating becomes  $y = \frac{1}{1+(x-2)^2}$ . Assume that the shape of the wave does not change during propagation. The velocity of the wave is

(a)  $0.5 \text{ m s}^{-1}$  (b)  $1.0 \text{ m s}^{-1}$   
(c)  $1.5 \text{ m s}^{-1}$  (d)  $2.0 \text{ m s}^{-1}$

[Ans. (b)  $1.0 \text{ m s}^{-1}$ ]

14. A uniform rope having mass  $m$  hangs vertically from a rigid support. A transverse wave pulse is produced at the lower end. Which of the following plots shows the correct variation of speed  $v$  with height  $h$  from the lower end?



[Ans. (d)]

15. An organ pipe A closed at one end is allowed to vibrate in its first harmonic and another pipe B open at both ends is allowed to vibrate in its third harmonic. Both A and B are in resonance with a given tuning fork. The ratio of the length of A and B is

(a)  $\frac{8}{3}$  (b)  $\frac{3}{8}$  (c)  $\frac{1}{6}$  (d)  $\frac{1}{3}$   
[Ans. (c)  $\frac{1}{6}$ ]

## II. SHORT ANSWER QUESTIONS:

1. What is meant by waves?

**Ans.** The disturbance which carries energy and momentum from one point in space to another point in space without the transfer of the medium is known as a wave.

2. Write down the types of waves.

**Ans. (a) Mechanical wave:** Waves which require a medium for propagation are known as mechanical waves.

**Examples :** sound waves, ripples formed on the surface of water, etc.

**(b) Non-mechanical wave:** Waves which do not require any medium for propagation are known as non-mechanical waves.

**Example :** light

Further, waves can be classified into two types

- a. Transverse waves
- b. Longitudinal waves

**3. What are transverse waves? Give one example.**

**Ans.** In transverse wave motion, the constituents of the medium oscillate or vibrate about their mean positions in a direction perpendicular to the direction of propagation (direction of energy transfer) of waves.

**Example:** light (electromagnetic waves)

**4. What are longitudinal waves? Give one example.**

**Ans.** In longitudinal wave motion, the constituent of the medium oscillate or vibrate about their mean positions in a direction parallel to the direction of propagation (direction of energy transfer) of waves.

**Example:** Sound waves travelling in air.

**5. Define wavelength.**

**Ans.** For transverse waves, the distance between two neighbouring crests or troughs is known as the **wavelength**. For longitudinal waves, the distance between two neighbouring compressions or rarefactions is known as the wavelength. The SI unit of wavelength is **meter**.

**6. Write down the relation between frequency, wavelength and velocity of a wave.**

**Ans.** Dimension of wavelength is,  $[\lambda] = L$

Frequency  $f = \frac{1}{\text{Time period}}$ , which implies that the dimension of frequency is,

$$[f] = \frac{1}{[T]} = T^{-1}$$

$$\Rightarrow [\lambda f] = [\lambda][f] = LT^{-1} = [\text{velocity}]$$

Therefore,

$$\text{Velocity, } \lambda f = v$$

where  $v$  is known as the **wave velocity** or **phase velocity**. This is the velocity with which the wave propagates. **Wave velocity** is the distance travelled by a wave in one second.

**7. What is meant by interference of waves?**

**Ans.** **Interference** is a phenomenon in which two waves superimpose to form a resultant wave of greater, lower or the same amplitude.

**8. Explain the beat phenomenon.**

**Ans.** When two or more waves superimpose each other with slightly different frequencies, then a sound of periodically varying amplitude at a point is observed. This phenomenon is known as beats. The number of amplitude maxima per second is called beat frequency.

If we have two sources, then their difference in frequency gives the beat frequency.

Number of beats per second

$$n = |f_1 - f_2| \text{ per second}$$

**9. Define intensity of sound and loudness of sound.**

**Ans.** "The sound power transmitted per unit area taken normal to the propagation of the sound wave".

The loudness of sound is defined as "the degree of sensation of sound produced in the ear or the perception of sound by the listener".

**10. Explain Doppler effect.**

**Ans.** When the source and the observer are in relative motion with respect to each other and to the medium in which sound propagates, the frequency of the sound wave observed is different from the frequency of the source. This phenomenon is called Doppler effect.

**11. Explain red shift and blue shift in Doppler effect.**

**Ans.** The spectral lines of the star are found to shift towards red end of the spectrum (called as red shift) then the star is receding away from the Earth. Similarly, if the spectral lines of the star are found to shift towards the blue end of the spectrum (called as blue shift) then the star is approaching Earth.

**12. What is meant by end correction in resonance air column apparatus?**

**Ans.** The antinodes is not exactly formed at the open end, but a small distance above the open end. This is called end correction.

To compute the end correction,

$$L_1 + e = \frac{\lambda}{4} \text{ and } L_2 + e = \frac{3\lambda}{4}$$

**13. Sketch the function  $y = x + a$ . Explain your sketch.**

- Ans.** (i) A combination of constant and direct.  
(ii) A fixed amount is added at regular intervals.  
(iii)  $y = x + a$ , a suitable conclusion statement would be that,
1.  $y$  is linear with  $x$
  2.  $y$  varies linearly with  $x$
  3.  $y$  is a linear function of  $x$   $y$  is the intercept.

**14. Write down the factors affecting velocity of sound in gases.**

- Ans.** (i) Pressure  
(ii) Temperature  
(iii) Density  
(iv) humidity and  
(v) wind

**15. What is meant by an echo? Explain.**

**Ans.** An echo is a repetition of sound produced by the reflection of sound waves from a wall, mountain or other obstructing surfaces. The speed of sound in air at  $20^\circ\text{C}$  is  $344 \text{ m s}^{-1}$ . If we shout at a wall which is at 344 m away, then the sound will take 1 second to reach the wall. After reflection, the sound will take one more second to reach us. Therefore, we hear the echo after two seconds. Scientists have estimated that we can hear two sounds properly if the time gap or time interval between each sound is  $\left(\frac{1}{10}\right)^{\text{th}}$  of a second (persistence of hearing) i.e., 0.1 s. Then,

$$\text{Velocity} = \frac{\text{Distance travelled}}{\text{time taken}} = \frac{2d}{t}$$

$$2d = 344 \times 0.1 = 34.4 \text{ m}$$

$$d = 17.2 \text{ m}$$

The minimum distance from a sound reflecting wall to hear an echo at  $20^\circ\text{C}$  is 17.2 meter.

**III. LONG ANSWER QUESTIONS****1. Discuss how ripples are formed in still water.**

**Ans.** A stone is dropped in a trough of still water, we can see a disturbance produced at the place where the stone strikes the water surface is seen.

This disturbance spreads out (diverges out) in the form of concentric circles of ever increasing radii (ripples) and strike the boundary of the trough. This is because **some of the kinetic energy of the stone is transmitted to** the water molecules on the surface. Actually the particles of the water (medium) themselves do not move outward with the disturbance. This can be observed by keeping a paper strip on the water surface. The strip moves up and down when the disturbance (wave) passes on the water surface. This shows that the water molecules only undergo vibratory motion about their mean positions.

**2. Briefly explain the difference between travelling waves and standing waves.**

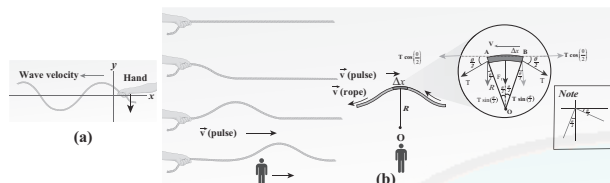
**Ans.**

S. No	Travelling Waves (or) Progressive Waves	Standing Waves (or) Stationary Waves
1.	Crests and troughs are formed in transverse progressive waves, and compression and rarefaction are formed in longitudinal progressive waves. These waves move forward or backward in a medium i.e., they will advance in a medium with a definite velocity.	Crests and troughs are formed in transverse stationary waves, and compression and rarefaction are formed in longitudinal stationary waves. These waves neither move forward nor backward in a medium i.e., they will not advance in a medium.
2.	All the particles in the medium vibrate such that the amplitude of the vibration for all particles is same.	Except at nodes, all other particles of the medium vibrate such that amplitude of vibration is different for different particles. The amplitude is minimum or zero at nodes and maximum at anti-nodes.
3.	These wave carry energy while propagating.	These waves do not transport energy.



- 3. Show that the velocity of a travelling wave produced in a string is  $v = \sqrt{\frac{T}{\mu}}$ .**

**Ans.** Consider an elemental segment in the string as shown in the Figure (a).



**(a) Transverse waves in a stretched string.**

**(b) Elemental segment in a stretched string is zoomed and the pulse seen from an observer frame who moves with velocity  $v$ .**

Let A and B be two points on the string at an instant of time. Let  $dl$  and  $dm$  be the length and mass of the elemental string, respectively. By definition, linear mass density,  $\mu$  is

$$\mu = \frac{dm}{dl} \quad \text{..... (1)}$$

$$dm = \mu dl \quad \text{..... (2)}$$

The elemental string AB has a curvature which looks like an arc of a circle with centre at O, radius  $R$  and the arc subtending an angle  $\theta$  at the origin O as shown in Figure (b). The angle  $\theta$  can be written in terms of arc length and radius as  $\theta = \frac{dl}{R}$ . The centripetal acceleration supplied by the tension in the string is

$$a_{cp} = \frac{v^2}{R} \quad \text{..... (3)}$$

Then, centripetal force can be obtained when mass of the string ( $dm$ ) is included in equation (3)

$$F_{cp} = \frac{(dm)v^2}{R} \quad \text{..... (4)}$$

The centripetal force experienced by elemental string can be calculated by substituting equation (2) in equation (4) we get

$$\frac{(dm)v^2}{R} = \frac{\mu v^2 dl}{R} \quad \text{..... (5)}$$

The tension  $T$  acts along the tangent of the elemental segment of the string at A and B. Since the arc length is very small, variation in the tension force can be ignored.  $T$  is resolved into horizontal component

$T \cos\left(\frac{\theta}{2}\right)$  and vertical component  $T \sin\left(\frac{\theta}{2}\right)$ .

The horizontal components at A and B are equal in magnitude but opposite in direction; therefore, they cancel each other. Since the elemental arc length AB is taken to be very small, the vertical components at A and B appears to act vertically towards the centre of the arc and hence, they add up. The net radial force  $F_r$  is

$$F_r = 2T \sin\left(\frac{\theta}{2}\right) \quad \text{..... (6)}$$

Since the amplitude of the wave is very small when it is compared with the length of the string, the sine of small angle is approximated as  $\sin\left(\frac{\theta}{2}\right) \approx \frac{\theta}{2}$ . Hence equation (6) can be written as

$$F_r = 2T \times \frac{\theta}{2} = T\theta \quad \text{..... (7)}$$

But  $\theta = \frac{dl}{R}$ , therefore substituting in equation (7), we get

$$F_r = T \frac{dl}{R} \quad \text{..... (8)}$$

Applying Newton's second law to the elemental string in the radial direction, under equilibrium, the radial component of the force is equal to the centripetal force. Hence equating equation (5) and equation (8), we have

$$T \frac{dl}{R} = \mu v^2 \frac{dl}{R} \quad \text{..... (9)}$$

$$v = \sqrt{\frac{T}{\mu}} \text{ measured in ms}^{-1}$$

- 4. Describe Newton's formula for velocity of sound waves in air and also discuss the Laplace's correction.** [HY-2019]

**Ans.** Newton assumed that when sound propagates in air, the formation of compression and rarefaction takes place in a very slow manner so that the process is isothermal in nature. That is, the heat produced during compression (pressure increases, volume decreases), and heat lost during rarefaction (pressure decreases, volume increases) occur over a period of time such that the temperature of the medium remains constant. Therefore, by treating the air molecules to form an ideal gas, the changes in pressure and volume obey Boyle's law, Mathematically



$$PV = \text{Constant} \quad \dots (1)$$

Differentiating equation (1), we get

$$PdV + VdP = 0$$

$$\text{or, } P = -V \frac{dP}{dV} = B_T \quad \dots (2)$$

where,  $B_T$  is an isothermal bulk modulus of air.

Substituting equation (2) in equation  $v = \sqrt{\frac{B}{\rho}}$ , the speed of sound in air is

$$v_T = \sqrt{\frac{B_T}{\rho}} = \sqrt{\frac{P}{\rho}} \quad \dots (3)$$

Since  $P$  is the pressure of air whose value at NTP (Normal Temperature and Pressure) is 76 cm of mercury, we have

$$P = (0.76 \times 13.6 \times 10^3 \times 9.8) \text{ N m}^{-2}$$

$$\rho = 1.293 \text{ kg m}^{-3}. \text{ here } \rho \text{ is density of air}$$

Then the speed of sound in air at Normal Temperature and Pressure (NTP) is

$$v_T = \sqrt{\frac{0.76 \times 13.6 \times 10^3 \times 9.8}{1.293}}$$

$$= 279.80 \text{ ms}^{-1} \approx 280 \text{ ms}^{-1} \text{ (theoretical value)}$$

But the speed of sound in air at  $0^\circ\text{C}$  is experimentally observed as  $332 \text{ ms}^{-1}$  which is close upto 16% more than theoretical value

$$\text{(Percentage error is } \frac{(332 - 280)}{332} \times 100\% = 15.6\%).$$

This error is not small.

Laplace assumed that when the sound propagates through a medium, the particles oscillate very rapidly such that the compression and rarefaction occur very fast. Hence the exchange of heat produced due to compression and cooling effect due to rarefaction do not take place, because, air (medium) is a bad conductor of heat. Since, temperature is no longer considered as a constant here, sound propagation is an adiabatic process. By adiabatic considerations, the gas obeys Poisson's law (not Boyle's law as Newton assumed), which is

$$PV^\gamma = \text{constant} \quad \dots (4)$$

where,  $\gamma = \frac{C_p}{C_v}$ , which is the ratio between specific heat at constant pressure and specific heat at constant volume.

Differentiating equation (4) on both the sides, we get

$$V^\gamma dP + P (\gamma V^{\gamma-1} dV) = 0$$

$$\text{or, } \gamma P = -V \frac{dP}{dV} B_A \quad \dots (5)$$

where,  $B_A$  is the adiabatic bulk modulus of air. Now, substituting equation (5) in equation

$v = \sqrt{\frac{B}{\rho}}$ , the speed of sound in air is

$$v_A = \sqrt{\frac{B_A}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma} v_T \quad \dots (6)$$

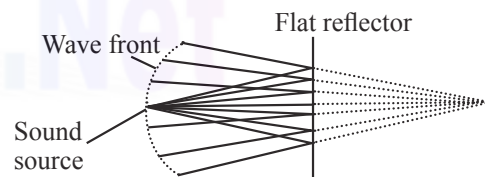
$$v_A = 331 \text{ ms}^{-1}.$$

### 5. Write short notes on reflection of sound waves from plane and curved surfaces.

**Ans.** Sound reflects from a harder flat surface, is called as **specular reflection**.

Specular reflection is observed only when the wavelength of the source is smaller than dimensions of the reflecting surface, as well as smaller than surface irregularities.

When the sound waves hit the plane wall, they bounce off in a manner similar to that of light. Suppose a loudspeaker is kept at an angle with respect to a wall (plane surface), then the waves coming from the source (assumed to be a point source) can be treated as spherical wave fronts (say, compressions moving like a spherical wave front). Therefore, the reflected wave front on the plane surface is also spherical, such that its centre of curvature (which lies on the other side of plane surface) can be treated as the image of the sound source (virtual or imaginary loud speaker) which can be assumed to be at a position behind the plane surface.



**Reflection of sound through the plane surface**

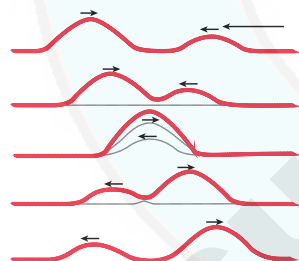
### Reflection of sound through the curved surface

The behaviour of sound is different when it is reflected from different surfaces-convex or concave or plane. The sound reflected from a

convex surface is spread out and so it is easily attenuated and weakened. Whereas, if it is reflected from the concave surface it will converge at a point and this can be easily amplified. The parabolic reflector (curved reflector) which is used to focus the sound precisely to a point is used in designing the parabolic mics which are known as high directional microphones. We know that any surface (smooth or rough) can absorb sound.

### 6. Briefly explain the concept of superposition principle.

When a jerk is given to a stretched string which is tied at one end, a wave pulse is produced and the pulse travels along the string. Suppose two persons holding the stretched string on either side give a jerk simultaneously, then these two wave pulses move towards each other, meet at some point and move away from each other with their original identity. Their behaviour is very different only at the crossing/meeting points; this behaviour depends on whether the two pulses have the same or different shape as shown in Figure.s



**Superposition of two waves**

When the pulses have the same shape, at the crossing, the total displacement is the algebraic sum of their individual displacements and hence its net amplitude is higher than the amplitudes of the individual pulses. Whereas, if the two pulses have same amplitude but shapes are  $180^\circ$  out of phase at the crossing point, the net amplitude vanishes at that point and the pulses will recover their identities after crossing. Only waves can possess such a peculiar property and it is called **superposition of waves**. This means that the principle of superposition explains the net behaviour of the waves when they overlap. Generalizing to any number of waves i.e, if two or more waves in a medium move simultaneously, when they overlap, their total displacement is the vector sum of the individual displacements.

To express mathematically, consider two functions which characterize the displacement of the waves, for example,

$$y_1 = A_1 \sin(kx - \omega t)$$

and

$$y_2 = A_2 \cos(kx - \omega t)$$

Since, both  $y_1$  and  $y_2$  satisfy the wave equation (solutions of wave equation) then their algebraic sum

$$y = y_1 + y_2$$

also satisfies the wave equation. This means, the displacements are additive. Suppose we multiply  $y_1$  and  $y_2$  with some constant then their amplitude is scaled by that constant. Further, if  $C_1$  and  $C_2$  are used to multiply the displacements  $y_1$  and  $y_2$ , respectively, then, their net displacement  $y$  is

$$y = C_1 y_1 + C_2 y_2$$

This can be generalized to any number of waves. In the case of  $n$  such waves in more than one dimension the displacements are written using vector notation.

Here, the net displacement  $\vec{y}$  is

$$\vec{y} = \sum_{i=1}^n C_i \vec{y}_i$$

The principle of superposition can explain the following :

- Space (or spatial) Interference (also known as Interference)
- Time (or Temporal) Interference (also known as Beats)
- Concept of stationary waves

Waves that obey principle of superposition are called linear waves (amplitude is much smaller than their wavelengths). In general, if the amplitude of the wave is not small then they are called non-linear waves.

### 7. Explain how the interference of waves is formed.

**Ans.** Consider two harmonic waves having identical frequencies, constant phase difference  $\phi$  and same wave form (can be treated as coherent source), but having amplitudes  $A_1$  and  $A_2$ , then

$$y_1 = A_1 \sin(kx - \omega t) \quad \dots (1)$$

$$y_2 = A_2 \sin(kx - \omega t + \phi) \quad \dots (2)$$

Suppose they move simultaneously in a particular direction, then interference occurs (i.e., overlap of these two waves). Mathematically

$$y = y_1 + y_2 \quad \dots (3)$$

Therefore, substituting equation (1) and equation (2) in equation (3), we get

$$y = A_1 \sin(kx - \omega t) + A_2 \sin(kx - \omega t + \phi)$$

Using trigonometric identity  $\sin(\alpha + \beta) = (\sin \alpha \cos \beta + \cos \alpha \sin \beta)$ , we get

$$y = A_1 \sin(kx - \omega t) + A_2 [\sin(kx - \omega t) \cos \phi + \cos(kx - \omega t) \sin \phi]$$

$$y = \sin(kx - \omega t)(A_1 + A_2 \cos \phi) + A_2 \sin \phi \cos(kx - \omega t) \quad \dots (4)$$

Let us re-define

$$A \cos \theta = (A_1 + A_2 \cos \phi) \quad \dots (5)$$

$$\text{and } A \sin \theta = A_2 \sin \phi \quad \dots (6)$$

then equation (4) can be rewritten as

$$y = A \sin(kx - \omega t) \cos \theta + A \cos(kx - \omega t) \sin \theta$$

$$y = A [\sin(kx - \omega t) \cos \theta + \sin \theta \cos(kx - \omega t)]$$

$$y = A \sin(kx - \omega t + \theta) \quad \dots (7)$$

By squaring and adding equation (5) and equation (6), we get

$$A^2 = A_1^2 + A_2^2 + 2A_1 A_2 \cos \phi \quad \dots (8)$$

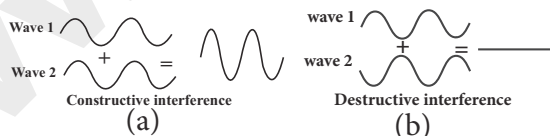
Since, intensity is square of the amplitude ( $I = A^2$ ), we have

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi \quad \dots (9)$$

This means the resultant intensity at any point depends on the phase difference at that point.

#### (a) For constructive interference :

When crests of one wave overlap with crests of another wave, their amplitudes will add up and constructive interference is obtained. The resultant wave has a larger amplitude than the individual waves as shown in Figure (a).



The constructive interference at a point occurs if there is maximum intensity at that point, which means that

$$\cos \phi = +1 \Rightarrow \phi = 0, 2\pi, 4\pi, \dots = 2n\pi,$$

where  $n = 0, 1, 2, \dots$

This is the phase difference in which two waves overlap to give constructive interference.

Therefore, for this resultant wave,

$$I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2 = (A_1 + A_2)^2$$

Hence the resultant amplitude,  $A = A_1 + A_2$

**(b) For destructive interference :** When the trough of one wave overlaps with the crest of another wave, their amplitudes “cancel” each other and we get destructive interference is obtained. The resultant amplitude is nearly zero. The destructive interference occurs if there is minimum intensity at that point, which means  $\cos \phi = -1 \Rightarrow \phi = \pi, 3\pi, 5\pi, \dots = (2n-1)\pi$ , where  $n = 0, 1, 2, \dots$  i.e. This is the phase difference in which two waves overlap to give destructive interference. Therefore,

$$I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2 = (A_1 - A_2)^2$$

Hence, the resultant amplitude

$$A = |A_1 - A_2|$$

#### 8. Describe the formation of beats.

**Ans.** When two or more waves superimpose each other with slightly different frequencies, then a sound of periodically varying amplitude at a point is observed. This phenomenon is known as beats. The number of amplitude maxima per second is called beat frequency. If we have two sources, then their difference in frequency gives the beat frequency.

Number of beats per second

$$n = |f_1 - f_2| \text{ per second.}$$

#### 9. What are stationary waves? Explain the formation of stationary waves and also write down the characteristics of stationary waves.

[Jun.-2019]

**Ans.** When the wave hits the rigid boundary it bounces back to the original medium and can interfere with the original waves. A pattern is formed, which are known as standing waves or stationary waves. Consider two harmonic progressive waves (formed by strings) that have the same amplitude and same velocity but move in opposite directions. Then the displacement of the first wave (incident wave) is

$$y_1 = A \sin(kx - \omega t) \quad \dots (1)$$

(waves move toward right)



and the displacement of the second wave (reflected wave) is

$$y_2 = A \sin(kx + \omega t) \quad \dots (2)$$

(waves move toward left)

both will interfere with each other by the principle of superposition, the net displacement is

$$y = y_1 + y_2 \quad \dots (3)$$

Substituting equation (1) and equation (2) in equation (3), we get

$$y = A \sin(kx - \omega t) + A \sin(kx + \omega t) \quad \dots (4)$$

Using trigonometric identity, we rewrite equation (4) as

$$y(x, t) = 2A \cos(\omega t) \sin(kx) \quad \dots (5)$$

This represents a stationary wave or standing wave, which means that this wave does not move either forward or backward, whereas progressive or travelling waves will move forward or backward. Further, the displacement of the particle in equation (5) can be written in more compact form,

$$y(x, t) = A' \cos(\omega t)$$

where,  $A' = 2A \sin(kx)$ , implying that the particular element of the string executes simple harmonic motion with amplitude equals to  $A'$ . The maximum of this amplitude occurs at positions for which

$$\sin(kx) = 1 \Rightarrow kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots = m\pi$$

where  $m$  takes half integer or half integral values. The position of maximum amplitude is known as **antinode**.

### Characteristics of stationary waves

- (i) Stationary waves are characterised by the confinement of a wave disturbance between two rigid boundaries. This means, the wave does not move forward or backward in a medium (does not advance), it remains steady at its place. Therefore, they are called "stationary waves or standing waves".
- (ii) Certain points in the region in which the wave exists have maximum amplitude, called as anti-nodes and at certain points the amplitude is minimum or zero, called as nodes.
- (iii) The distance between two consecutive nodes (or) anti-nodes is  $\frac{\lambda}{2}$ .

(iv) The distance between a node and its neighbouring anti-node is  $\frac{\lambda}{4}$ .

(v) The transfer of energy along the standing wave is zero.

### 10. Discuss the law of transverse vibrations in stretched strings. [HY-2019]

**Ans.** There are three laws of transverse vibrations of stretched strings which are given as follows:

(i) **The law of length :** For a given wire with tension  $T$  (which is fixed) and mass per unit length  $\mu$  (fixed) the frequency varies inversely with the vibrating length. Therefore,

$$f \propto \frac{1}{l} \Rightarrow f = \frac{C}{l}$$

$$\Rightarrow l \times f = C, \text{ where } C \text{ is a constant.}$$

(ii) **The law of tension :** For a given vibrating length  $l$  (fixed) and mass per unit length  $\mu$  (fixed) the frequency varies directly with the square root of the tension  $T$ ,

$$f \propto \sqrt{T}$$

$$\Rightarrow f = A\sqrt{T}, \text{ where } A \text{ is a constant}$$

(iii) **The law of mass :** For a given vibrating length  $l$  (fixed) and tension  $T$  (fixed) the frequency varies inversely with the square root of the mass per unit length  $\mu$ ,

$$f \propto \frac{1}{\sqrt{\mu}}$$

$$\Rightarrow f = \frac{B}{\sqrt{\mu}}, \text{ where } B \text{ is a constant}$$

### 11. Explain the concepts of fundamental frequency, harmonics and overtones in detail.

**Ans.** Keep the rigid boundaries at  $x = 0$  and  $x = L$  and produce a standing waves by wiggling the string (as in plucking strings in a guitar). Standing waves with a specific wavelength are produced. Since, the amplitude must vanish at the boundaries, therefore, the displacement at the boundary.

$$y(x = 0, t) = 0 \text{ and } y(x = L, t) = 0 \quad \dots (1)$$

Since the nodes formed are at a distance  $\frac{\lambda_n}{2}$

apart, we have  $n\left[\frac{\lambda_n}{2}\right] = L$ , where  $n$  is an integer,

$L$  is the length between the two boundaries and  $\lambda_n$  is the specific wavelength that satisfy the specified boundary conditions. Hence,

$$\lambda_n = \left(\frac{2L}{n}\right) \quad \dots (2)$$

For  $n = 1$ , the first mode of vibration has specific wavelength  $\lambda_1 = 2L$ . Similarly for  $n = 2$ , the second mode of vibration has specific wavelength

$$\lambda_2 = \left(\frac{2L}{2}\right) = L$$

For  $n = 3$ , the third mode of vibration has specific wavelength

$$\lambda_3 = \left(\frac{2L}{3}\right)$$

and so on.

The frequency of each mode of vibration (called natural frequency) can be calculated.

We have,

$$f_n = \frac{v}{\lambda_n} = n\left(\frac{v}{2L}\right) \quad \dots (3)$$

The lowest natural frequency is called the fundamental frequency.

$$f_1 = \frac{v}{\lambda_1} = \left(\frac{v}{2L}\right) \quad \dots (4)$$

The second natural frequency is called the first over tone.

$$f_2 = 2\left(\frac{v}{2L}\right) = \frac{1}{L}\sqrt{\frac{T}{\mu}}$$

The third natural frequency is called the second over tone.

$$f_3 = 3\left(\frac{v}{2L}\right) = 3\left(\frac{1}{2L}\sqrt{\frac{T}{\mu}}\right)$$

and so on.

Therefore, the  $n$ th natural frequency is equal to integral (or integer) multiple of fundamental frequency, i.e.,

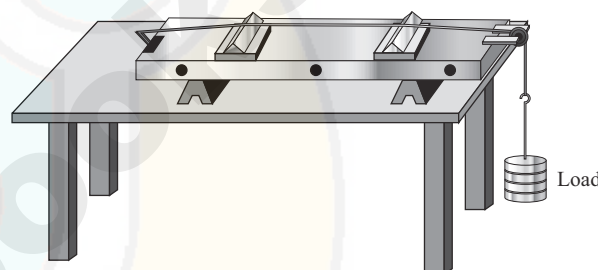
$$f_n = nf_1, \text{ where } n \text{ is an integer} \quad \dots (5)$$

If natural frequencies are written as integral multiple of fundamental frequencies, then the frequencies are called harmonics. Thus, the first harmonic is  $f_1 = f_1$  (the fundamental frequency is called first harmonic), the second harmonic is  $f_2 = 2f_1$ , the third harmonic is  $f_3 = 3f_1$  etc.

## 12. What is a sonometer? Give its construction and working. Explain how to determine the frequency of tuning fork using sonometer.

**Ans.** **Sono** means sound related, and sonometer implies sound-related measurements. It is a device for demonstrating the relationship between the frequency of the sound produced in the transverse standing wave in a string, and the tension, length and mass per unit length of the string. Therefore, using this device, we can determine the following quantities:

- the frequency of the tuning fork or frequency of alternating current
- the tension in the string
- the unknown hanging mass



### Construction :

The sonometer is made up of a hollow box which is one meter long with a uniform metallic thin string attached to it. One end of the string is connected to a hook and the other end is connected to a weight hanger through a pulley as shown in Figure. Since only one string is used, it is also known as monochord. The weights are added to the free end of the wire to increase the tension of the wire. Two adjustable wooden knives are put over the board, and their positions are adjusted to change the vibrating length of the stretched wire.

### Working :

A transverse stationary or standing wave is produced and hence, at the knife edges P and Q, nodes are formed. In between the knife edges, anti-nodes are formed.

If the length of the vibrating element is  $l$  then

$$l = \frac{\lambda}{2} \Rightarrow \lambda = 2l$$

Let  $f$  be the frequency of the vibrating element,  $T$  the tension of in the string and  $\mu$  the mass per unit length of the string. Then using equation



$v = \sqrt{\frac{T}{\mu}}$ , we get

$$f = \frac{v}{\lambda} = \frac{1}{2l} \sqrt{\frac{T}{\mu}} \text{ in Hertz} \quad \dots (1)$$

Let  $\rho$  be the density of the material of the string and  $d$  be the diameter of the string. Then the mass per unit length  $\mu$ ,

$$\mu = \text{Area} \times \text{density} = \pi r^2 \rho = \frac{\pi \rho d^2}{4}$$

$$\text{frequency } f = \frac{v}{\lambda} = \frac{1}{2l} \sqrt{\frac{T}{\frac{\pi \rho d^2}{4}}}$$

$$\therefore f = \frac{1}{ld} \sqrt{\frac{T}{\pi \rho}}$$

### 13. Write short notes on intensity and loudness.

**Ans. Intensity of sound :** When a sound wave is emitted by a source, the energy is carried to all possible surrounding points. The average sound energy emitted or transmitted per unit time or per second is called sound power. Therefore, the intensity of sound is defined as “the sound power transmitted per unit area taken normal to the propagation of the sound wave”.

For a particular source (fixed source), the sound intensity is inversely proportional to the square of the distance from the source.

$$I = \frac{\text{power of the source}}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2}$$

This is known as inverse square law of sound intensity.

**Intensity of sound :** Two sounds with same intensities need not have the same loudness. For example, the sound heard during the explosion of balloons in a silent closed room is very loud when compared to the same explosion happening in a noisy market. Though the intensity of the sound is the same, the loudness is not. If the intensity of sound is increased then loudness also increases. But additionally, not only does intensity matter, the internal and subjective experience of “how loud a sound is” i.e., the sensitivity of the listener also matters here. This is often called loudness. That is, loudness depends on both intensity of sound wave and sensitivity of the ear (It is purely observer dependent quantity which varies from person to person) whereas the intensity of sound

does not depend on the observer. The loudness of sound is defined as “the degree of sensation of sound produced in the ear or the perception of sound by the listener”.

### 14. Explain how overtones are produced in a

(a) Closed organ pipe

(b) Open organ pipe

**Ans. (a) Closed organ pipe :** If one end of a pipe is closed, the wave reflected at this closed end is  $180^\circ$  out of phase with the incoming wave. Thus there is no displacement of the particles at the closed end. Therefore, nodes are formed at the closed end and anti-nodes are formed at open end.

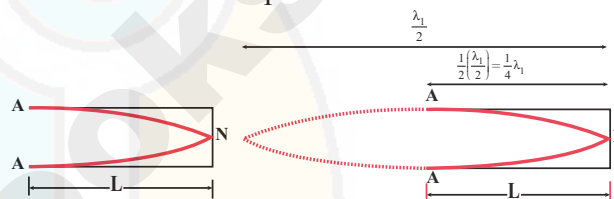


Figure 1 : No motion of particles which leads to nodes at closed end and antinodes at open end (fundamental mode) (N-node, A-antinode)

Consider the simplest mode of vibration of the air column called the fundamental mode. Anti-node is formed at the open end and node at closed end. From the Figure 1, let  $L$  be the length of the tube and the wavelength of the wave produced. For the fundamental mode of vibration, we have,

$$L = \frac{\lambda_1}{4} \text{ (or) } \lambda_1 = 4L$$

The frequency of the note emitted is

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$$

which is called the fundamental note.

The frequencies higher than fundamental frequency can be produced by blowing air strongly at open end. Such frequencies are called overtones.

The figure (2) shows the second mode of vibration having two nodes and two anti-nodes.

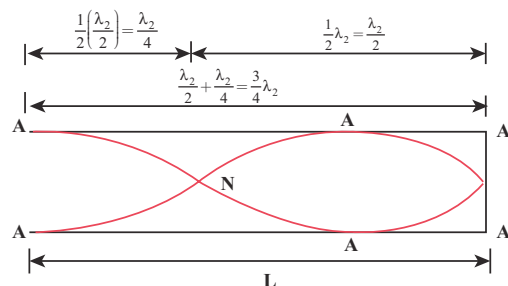


Figure 2 : Second mode of vibration having two nodes and two anti-nodes

$$4L = 3\lambda_2$$

$$L = \frac{3\lambda_2}{4} \text{ or } \lambda_2 = \frac{4L}{3}$$

The frequency for this,

$$f_1 = \frac{v}{\lambda_2} = \frac{3v}{4L} = 3f_1$$

is called first over tone, since here, the frequency is three times the fundamental frequency it is called third harmonic.

The Figure (3) shows third mode of vibration having three nodes and three anti-nodes.

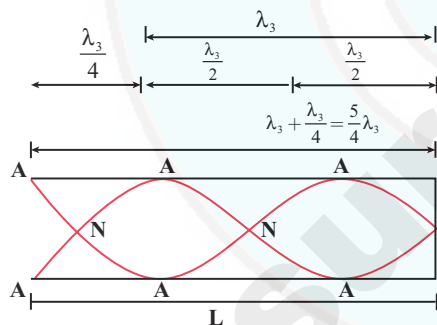


Figure 3 : Third mode of vibration having three nodes and three anti-nodes

We have,  $4L = 5\lambda_3$

$$L = \frac{5\lambda_3}{4} \text{ or } \lambda_3 = \frac{4L}{5}$$

The frequency

$$f_3 = \frac{v}{\lambda_3} = \frac{5v}{4L} = 5f_1$$

is called second over tone, and since  $n = 5$  here, this is called fifth harmonic. Hence, the closed organ pipe has only odd harmonics and frequency of the  $n^{\text{th}}$  harmonic is  $f_n = (2n+1)f_1$ . Therefore, the frequencies of harmonics are in the ratio

$$f_1 : f_2 : f_3 : f_4 : \dots = 1 : 3 : 5 : 7 : \dots$$

(b) **Open organ pipe :** Consider the picture of a flute, shown in Figure (4).

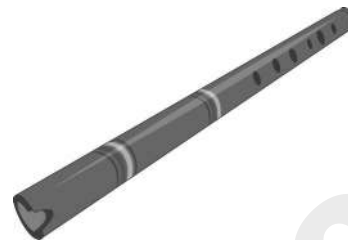


Figure 4 : Flute is an example of open organ pipe

It is a pipe with both the ends open. At both open ends, anti-nodes are formed. Let us consider the simplest mode of vibration of the air column called fundamental mode. Since anti-nodes are formed at the open end, a node is formed at the mid-point of the pipe.

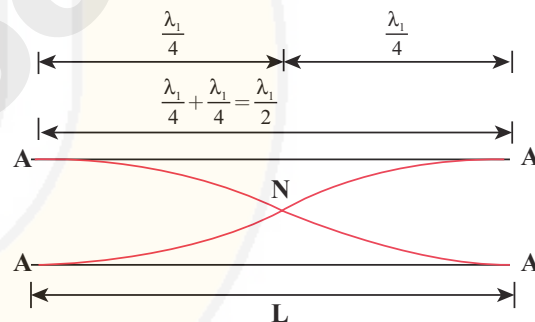


Figure 5 : Antinodes are formed at the open end and a node is formed at the middle of the pipe.

From Figure (5), if  $L$  be the length of the tube, the wavelength of the wave produced is given by

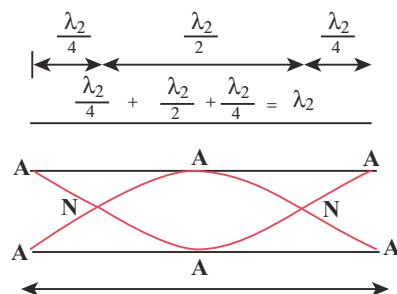
$$L = \frac{\lambda_1}{2} \text{ or } \lambda_1 = 2L$$

The frequency of the note emitted is

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

which is called the fundamental note.

The frequencies higher than fundamental frequency can be produced by blowing air strongly at one of the open ends. Such frequencies are called overtones.



**Figure 6 : Second mode of vibration in open pipes having two nodes and three anti-nodes**

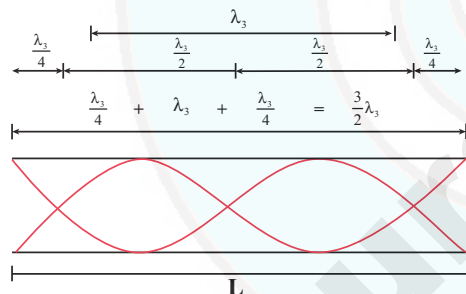
The Figure (6) shows the second mode of vibration in open pipes. It has two nodes and three anti-nodes, and therefore,

$$L = \lambda_2 \text{ or } \lambda_2 = L$$

The frequency

$$f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2 \times \frac{v}{2L} = 2f_1$$

is called **first over tone**. Since  $n = 2$  here, it is called the **second harmonic**.



**Figure 7 : Third mode of vibration having three nodes and four anti-nodes**

The Figure (7) above shows the third mode of vibration having three nodes and four anti-nodes

$$L = \frac{3}{2} \lambda_3 \text{ or } \lambda_3 = \frac{2L}{3}$$

$$f_3 = \frac{v}{\lambda_3} = \frac{3v}{2L} = 3f_1$$

is called **second over tone**. Since  $n = 3$  here, it is called the **third harmonic**.

Hence, the open organ pipe has all the harmonics and frequency of  $n^{\text{th}}$  harmonic is  $f_n = nf_1$ . Therefore, the frequencies of harmonics are in the ratio

$$f_1 : f_2 : f_3 : f_4 : \dots = 1 : 2 : 3 : 4 : \dots$$

### 15. How will you determine the velocity of sound using resonance air column apparatus?

[Mar.-2019]

**Ans.** The resonance air column apparatus is one of the simplest techniques to measure the speed of sound in air at room temperature. It consists of a cylindrical glass tube of one meter length whose one end A is open and another end B is connected to the water reservoir R through a rubber tube as shown in Figure. This cylindrical glass tube is mounted on a vertical stand with a scale attached to it. The tube is partially filled with water and the water level can be adjusted by raising or lowering the water in the reservoir R. The surface of the water will act as a closed end and other as the open end. Therefore, it behaves like a closed organ pipe, forming nodes at the surface of water and antinodes at the closed end. When a vibrating tuning fork is brought near the open end of the tube, longitudinal waves are formed inside the air column. These waves move downward as shown in Figure, and reach the surfaces of water and get reflected and produce standing waves. The length of the air column is varied by changing the water level until a loud sound is produced in the air column. At this particular length the frequency of waves in the air column resonates with the frequency of the tuning fork (natural frequency of the tuning fork). At resonance, the frequency of sound waves produced is equal to the frequency of the tuning fork. This will occur only when the

length of air column is proportional to  $\left(\frac{1}{4}\right)^{\text{th}}$  of the wavelength of the sound waves produced.

Let the first resonance occur at length  $L_1$ , then

$$\frac{1}{4} \lambda = L_1$$

But since the antinodes are not exactly formed at the open end, a correction called end correction  $e$ , is included by assuming that the antinode is formed at some small distance above the open end. Including this end correction, the first resonance is

$$\frac{1}{4} \lambda = L_1 + e \quad \dots(1)$$

Now the length of the air column is increased to get the second resonance. Let  $L_2$  be the length at which the second resonance occurs. Again taking end correction into account,

$$\frac{3}{4}\lambda = L_2 + e \quad \dots(2)$$

In order to avoid end correction, let us take the difference of equation  $\frac{1}{4}\lambda = L_1$  and equation

$f_1 : f_2 : f_3 : f_4 : \dots = 1 : 2 : 3 : 4 : \dots$ , we get

$$\frac{3}{4}\lambda - \frac{1}{4}\lambda = (L_2 + e) - (L_1 + e)$$

$$\Rightarrow \frac{1}{2}\lambda = L_2 - L_1 = \Delta L$$

$$\Rightarrow \lambda = 2\Delta L$$

The speed of the sound in air at room temperature can be computed by using the formula

$$v = f\lambda = 2f\Delta L$$

Further, to compute the end correction, we use equation (1) and (2), we get  $e = \frac{L_2 - 3L_1}{2}$

**16. What is meant by Doppler effect? Discuss the following cases.**

- (1) **Source in motion and Observer at rest**
  - (a) **Source moves towards observer**
  - (b) **Source moves away from the observer**
- (2) **Observer in motion and Source at rest.**
  - (a) **Observer moves towards Source**
  - (b) **Observer recedes away from the Source**
- (3) **Both are in motion**
  - (a) **Source and Observer approach each other**
  - (b) **Source and Observer resides from each other**
  - (c) **Source chases Observer**
  - (d) **Observer chases Source**

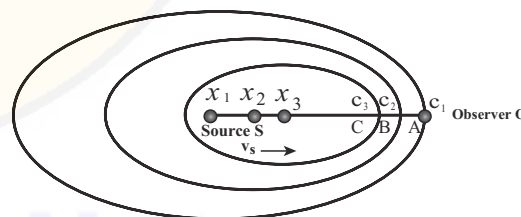
**Ans. Doppler Effect :** Often we have noticed that the siren sound coming from a police vehicle or ambulance increases when it comes closer to us and decreases when it moves away from us. When we stand near any passing train the train whistle initially increases and then it will decrease. This is known as Doppler Effect, named after Christian Doppler (1803 – 1853). Suppose a source produces sound with some frequency, we call it the as source frequency  $f_s$ . If the source and an observer are at a fixed distance then the observer observes the sound with frequency  $f_o$ . This is the same as the sound frequency produced by the source  $f_s$ , i.e.,

$f_o = f_s$ . Hence, there is no difference in frequency, implying no Doppler effect is observed.

**(1) Source in motion and Observer at rest :**

**(a) Source moves towards observer:**

A source S moves to the right (as shown in Figure 1) with a velocity  $v_s$  and let the frequency of the sound waves produced by the source be  $f_s$ . Velocity of sound in a medium is  $v$ . The compression (sound wave front) produced by the source S at three successive instants of time are shown in the Figure 1. When S is at position  $x_1$  the compression is at  $C_1$ . When S is at position  $x_2$ , the compression is at  $C_2$  and similarly for  $x_3$  and  $C_3$ . Assume that if  $C_1$  reaches the observer's position A then at that instant  $C_2$  reaches the point B and  $C_3$  reaches the point C as shown in the Figure 1. The distance between compressions  $C_2$  and  $C_3$  is shorter than distance between  $C_1$  and  $C_2$ . This means the wavelength decreases when the source S moves towards the observer O (since sound travels longitudinally and wavelength is the distance between two consecutive compressions). But frequency is inversely related to wavelength and therefore, frequency increases.



**Figure 1 Source S moves towards an Observer O (right) with velocity**

Let  $\lambda$  be the wavelength of the source S as measured by the observer when S is at position  $x_1$  and  $\lambda'$  be wavelength of the source observed by the observer when S moves to position  $x_2$ . Then the change in wavelength is  $\Delta\lambda = \lambda - \lambda' = v_s t$ , where  $t$  is the time taken by the source to travel between  $x_1$  and  $x_2$ . Therefore,

$$\lambda' = \lambda - v_s t$$

$$\text{But } t = \frac{\lambda}{v}$$



On substituting equation  $t = \frac{\lambda}{v}$  in equation  $\lambda' = \lambda - v_s t$ , we get

$$\lambda' = \lambda \left( 1 - \frac{v_s}{v} \right)$$

Since frequency is inversely proportional to wavelength, we have

$$f' = \frac{v_s}{\lambda'} \text{ and } f = \frac{v_s}{\lambda}$$

$$\text{Hence, } f' = \frac{f}{\left( 1 - \frac{v_s}{v} \right)}$$

Since,  $\frac{v_s}{v} \ll 1$ , we use the binomial expansion and retaining only first order in  $\frac{v_s}{v}$ , we get

$$f' = f \left( 1 + \frac{v_s}{v} \right)$$

**(b) Source moves away from the observer:**

Since the velocity here of the source is opposite in direction when compared to case (a), therefore, changing the sign of the velocity of the source in the above case i.e., by substituting  $(v_s \rightarrow -v_s)$  in equation  $\lambda' = \lambda - v_s t$ , we get

$$f' = \frac{f}{\left( 1 - \frac{v_s}{v} \right)}$$

Using binomial expansion again, we get,

$$f' = f \left( 1 - \frac{v_s}{v} \right)$$

**(2) Observer in motion and Source at rest:**

**(a) Observer moves towards Source :**

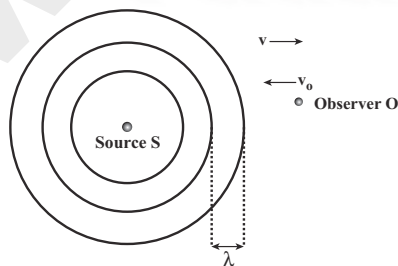


Figure 2 : Observer moves towards source

Assume that the observer O moves towards the source S with velocity  $v_o$ . The source S is at rest and the velocity of sound waves (with respect to the medium) produced by the source is  $v$ . From the figure (2), we observe that both  $v_o$  and  $v$  are in opposite direction. Then, their relative velocity is  $v_r = v + v_o$ . The wavelength of the sound wave is  $\lambda = \frac{v}{f}$ , which means the frequency observed by the observer O is  $f' = \frac{v_r}{\lambda}$ .

$$\text{Then } f' = \frac{v_r}{\lambda} = \left( \frac{v + v_o}{v} \right) f = f \left( 1 + \frac{v_o}{v} \right)$$

**(b) Observer recedes away from the Source :**

If the observer O is moving away (receding away) from the source S, then velocity  $v_o$  and  $v$  moves in the same direction. Therefore, their relative velocity is  $v_r = v - v_o$ . Hence, the frequency observed by the observer O is

$$f' = \frac{v_r}{\lambda} = \left( \frac{v - v_o}{v} \right) f = f \left( 1 - \frac{v_o}{v} \right)$$

**(3) Both are in motion :**

**(a) Source and Observer approach each other :**



Figure 3 Source and Observer approach towards each other.

Let  $v_s$  and  $v_o$  be the respective velocities of source and observer approaching each other as shown in Figure 3. In order to calculate the apparent frequency observed by the observer, as a simple calculation, let us have a dummy (behaving as observer or source) in between the source and observer. Since the dummy is at rest, the dummy (observer) observes the apparent frequency due to approaching source as given in

$$\text{equation } f' = \frac{f}{\left( 1 - \frac{v_s}{v} \right)} \text{ as}$$

$$f_d = \frac{f}{\left( 1 - \frac{v_s}{v} \right)} \quad \dots (1)$$



At that instant of time, the true observer approaches the dummy from the other side. Since the source (true source) comes in a direction opposite to true observer, the dummy (source) is treated as stationary source for the true observer at that instant. Hence, apparent frequency when the true observer approaches the stationary source (dummy source), from equation  $f' = \frac{v_r}{\lambda} = \left( \frac{v + v_0}{v} \right) f = f \left( 1 + \frac{v_0}{v} \right)$  is

$$f' = f_d \left( 1 + \frac{v_0}{v} \right) \Rightarrow f_d = \frac{f'}{\left( 1 + \frac{v_0}{v} \right)} \quad \dots (2)$$

Since this is true for any arbitrary time, therefore, comparing equation (1) and equation (2), we get

$$\begin{aligned} \frac{f}{\left( 1 - \frac{v_s}{v} \right)} &= \frac{f'}{\left( 1 + \frac{v_0}{v} \right)} \\ \Rightarrow \frac{vf'}{(v + v_0)} &= \frac{vf}{(v - v_s)} \end{aligned}$$

Hence, the apparent frequency as seen by the observer is

$$f' = \left( \frac{v + v_0}{v - v_s} \right) f \quad \dots (3)$$

**(b) Source and Observer recedes from each other :**



**Fig.4 Source and Observer recedes from each other**

Here, we can derive the result as in the previous case. Instead of a detailed calculation, by inspection from Figure 4, we notice that the velocity of the source and the observer each point in opposite directions with respect to the case in (a) and hence, we substitute ( $v_s \rightarrow -v_s$ ) and ( $v_0 \rightarrow -v_0$ ) in equation (3), and therefore, the apparent frequency observed by the observer when the source and observer recede from each other is

$$f' = \left( \frac{v - v_0}{v + v_s} \right) f \quad \dots (4)$$

**(c) Source chases Observer**



**Fig.5 Source chases observer**

Only the observer's velocity is oppositely directed when compared to case (a). Therefore, substituting ( $v_0 \rightarrow -v_0$ ) in equation (3), we get

$$f' = \left( \frac{v - v_0}{v - v_s} \right) f \quad \dots (5)$$

**(d) Observer chases Source**



**Fig.6 Observer chases Source**

Only the source velocity is oppositely directed when compared to case (a). Therefore, substituting ( $v_s \rightarrow -v_s$ ) in equation (3), we get

$$f' = \left( \frac{v + v_0}{v + v_s} \right) f$$

## IV. NUMERICAL PROBLEMS

- The speed of a wave in a certain medium is 900 m/s. If 3000 waves pass over a certain point of the medium in 2 minutes, then compute its wavelength.

**Solution:**

The speed of a wave in medium  $v = 900 \text{ ms}^{-1}$

Freq. of wave = no. of waves passing per sec ( $n$ ) =  $3000 \text{ waves} / 2 \text{ min} = \frac{3000}{2 \times 60} = 25 \text{ s}$

Wave length =  $\lambda = ?$

$$v = n \lambda \quad \lambda = \frac{v}{n}$$

$$\lambda = \frac{900}{25} = 36 \text{ m}$$

$$\lambda = 36 \text{ m}$$

- Consider a mixture of 2 mol of helium and 4 mol of oxygen. Compute the speed of sound in this gas mixture at 300 K.

**Solution:**

Helium - 2 mole, oxygen - 4 mol

He & O<sub>2</sub> are mixed, hence molecular weight of the mixture of gases

$$M_{\text{mix}} = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2} = \frac{2 \times 4 + 4 \times 32}{2 + 4}$$

$$= \frac{8 + 128}{6} = \frac{136}{6} = 22.6 \times 10^{-3} \text{ kg mol}^{-1}$$

Further as He is monoatomic

$$C_{V1} = 3R/2$$

and  $O_2$  is diatomic  $C_{V2} = 5R/2$ , hence for the mixture

$$(C_V)_{\text{mix}} = \frac{n_1 C_{V1} + n_2 C_{V2}}{n_1 + n_2} = \frac{2 \times \frac{3}{2}R + 4 \times \frac{5}{2}R}{2 + 4} = \frac{13R}{6}$$

$$(C_P)_{\text{mix}} = (C_V)_{\text{mix}} + R = \frac{13R}{6} + R = \frac{19R}{6}$$

$$\therefore \gamma_{\text{mix}} = \frac{C_P}{C_V} = \frac{19R/6}{13R/6} = \frac{19}{13}$$

Acceleration to Laplace, be speed of sound

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$= \sqrt{\frac{19}{13} \times \frac{8.31 \times 300 \times 6}{136 \times 10^{-3}}} = \sqrt{\frac{284202}{1768}} \times 10^3$$

$$= \sqrt{\frac{28420}{1768}} \times 10^4$$

$$v = 400.9 \text{ ms}^{-1}$$

- 3.** A ship in a sea sends SONAR waves straight down into the seawater from the bottom of the ship. The signal reflects from the deep bottom bed rock and returns to the ship after 3.5 s. After the ship moves to 100 km it sends another signal which returns back after 2s. Calculate the depth of the sea in each case and also compute the difference in height between two cases.

**Solution:**

Velocity of SONAR waves in water  $C = 1500 \text{ ms}^{-1}$

Time taken by the wave after reflection from the bottom of sea

$$2t = 3.5 \text{ s}$$

$$t = 1.75 \text{ s}$$

Distance covered (d) = ?

$$C = \frac{d}{t} \Rightarrow d = c.t = 1500 \times 1.75 = 2625 \text{ m}$$

After moving 100km

The time taken by the wave =  $2t = 2 \text{ s}$

$$t = \frac{2}{2} = 1 \text{ s}$$

$$d = ?$$

$$d = 1500 \times 1$$

$$= 1500$$

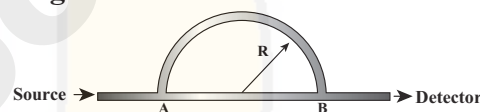
The different between these

two heights

$$= 2625 - 1500$$

$$= 1125 \text{ m}$$

- 4.** A sound wave is transmitted into a tube as shown in figure. The sound wave splits into two waves at the point A which recombine at point B. Let R be the radius of the semi-circle which is varied until the first minimum. Calculate the radius of the semi-circle if the wavelength of the sound is 50.0 m.



**Solution:**

Consider the sound waves start from the pt A. The sound waves will meet at the picture B & interfere and can be detected.

The path length of sound waves passing through the curve is equal to half the circle having radius centered at c.

Thus, the path length of the sound waves in curve part is

$$L_1 = \pi r$$

The path length of the sound in tube ABC is equal to the diameter of the circle having radius  $\gamma$  centered at e.

So the path length of the sound waves travelling in it =  $L_2 = 2r$

$\therefore$  The path difference of the sound waves at the picture is

$$\Delta P = L_1 - L_2$$

$$= \pi r - 2r$$

$$\Delta P = r(\pi - 2) \quad \dots(1)$$

For the sound to be heard minimum at the detected the difference in path length of the sound waves is

$$\Delta P = \frac{\lambda}{2} \quad \dots(2)$$

From equation (1)

$$r(\pi - 2) = \frac{\lambda}{2}$$

wavelength of sound =  $\lambda = 50\text{m}$

radius of be semicircle  $r = ?$

$$r = \frac{50}{2 \times 3.14 - 4} = \frac{50}{2.28}$$

$$r = 21.9\text{m}$$

5. **N** tuning forks are arranged in order of increasing frequency and any two successive tuning forks give **n** beats per second when sounded together. If the last fork gives double the frequency of the first (called as octave), Show that the frequency of the first tuning fork is  $f = (N-1)n$ .

**Solution:**

No. of tuning forks = **N**

The frequency of the first is double to be last

The difference of frequency for two successive tuning forks **NHz**

Let be frequency of the first tuning fork is **f**

Then be frequency of the last tuning fork is  $f(N-1)n$

$$f_{\text{last}} = \frac{f}{2} \Rightarrow$$

$$an = a + (n-1)d$$

$$(i.e) 2f = f + (N-1)n$$

$$= (N-1)n$$

$$f = (N-1)n$$

6. Let the source propagate a sound wave whose intensity at a point (initially) be **I**. Suppose we consider a case when the amplitude of the sound wave is doubled and the frequency is reduced to one-fourth. Calculate now the new intensity of sound at the same point?

**Solution:**

Intensity of sound wave =  $I_{\text{old}}$

Amplitude is doubled =  $2A = A_{\text{new}}$

frequency is reduced to are fourth

$$f_{\text{new}} = \frac{1}{4} f$$

Intensity  $I_{\text{new}} = ?$

$$I = 2\pi^2 \rho v f^2 A^2$$

$$I_{\text{old}} \propto f^2 A^2$$

$$I_{\text{new}} \propto \frac{1}{6} f^2 A^2$$

$$I_{\text{new}} \propto \frac{1}{4} I_{\text{old}}$$

$$I \propto \frac{A^2}{f^2}$$

$$I_{\text{new}} \propto \frac{(2A)^2}{\left(\frac{1}{4}f\right)^2}$$

$$I_{\text{new}} \propto \frac{4A^2}{\frac{1}{16}f^2}$$

$$I_{\text{new}} \propto I_{\text{old}}$$

7. Consider two organ pipes of same length in which one organ pipe is closed and another organ pipe is open. If the fundamental frequency of closed pipe is 250 Hz. Calculate the fundamental frequency of the open pipe.

**Solution:**

Fundamental frequency of closed organ pipe

$$\gamma_{\text{c}} = \frac{V}{4l} = 250\text{Hz}$$

Fundamental frequency of closed organ pipe

$$\gamma_0 = \frac{V}{2l} = ?$$

$$\frac{\gamma_{\text{c}}}{\gamma_0} = \frac{V}{4l} \times \frac{2l}{V} = \frac{1}{2}$$

$$\gamma_0 = 2 \gamma_{\text{c}} = 2 \times 250 = 500\text{Hz}$$

$$\gamma_0 = 500\text{Hz}$$

8. A police in a siren car moving with a velocity  $20 \text{ ms}^{-1}$  chases a thief who is moving in a car with a velocity  $v_0 \text{ ms}^{-1}$ . The police car sounds at frequency 300Hz, and both of them move towards a stationary siren of frequency 400Hz. Calculate the speed in which thief is moving. (Assume the thief does not observe any beat)

**Solution:**

$$\text{The velocity of car} = 20\text{ms}^{-1}$$

The frequency of car = 300Hz

The frequency stationary siren = 400Hz

The speed in which thief moving = ?

$$V_1 = 300 \left( \frac{330 - V_o}{330 - 20} \right)$$

$$V_2 = 400 \left( \frac{330 + V_o}{330} \right)$$

Both are moving towards stationary siren

$$\therefore V_1 = V_2$$

$$300 \left( \frac{330 - V_o}{310} \right) = 400 \left( \frac{330 + V_o}{330} \right)$$

Simplifying, we get  $V_o = 36.99 \text{ ms}^{-1}$ .

### 9. Consider the following function

(a)  $y = x^2 + 2 \alpha t x$

(b)  $y = (x + vt)^2$

which among the above function can be characterized as a wave ?

#### Solution:

(a)  $y = x^2 + 2 \alpha t x$

It is not describing wave.

(b)  $y = (x + vt)^2$

It satisfies wave equation.

### V. CONCEPTUAL QUESTIONS :

#### 1. Why is it that transverse waves cannot be produced in a gas? Can the transverse waves be produced in solids and liquids?

**Ans.** Transverse waves travel in the form of crests and troughs and so involve change in shape. As gas has no elasticity of shape, transverse waves cannot be produced in it.

Yes. solids and liquid have elasticity so transverse wave can be produced.

#### 2. Why is the roar of our national animal different from the sound of a mosquito?

**Ans.** Roaring of a national animal (tiger) produces a sound of low pitch and high intensity or loudness, whereas the buzzing of mosquito produces a sound of high pitch and low intensity or loudness.

#### 3. A sound source and listener are both stationary and a strong wind is blowing. Is there a Doppler effect?

**Ans.** Yes. It does not matter whether there is sound source or transmission media in motion.

#### 4. In an empty room why is it that a tone sounds louder than in the room having things like furniture etc.

**Ans.** Sound is a form of energy. The furniture which act as obstacles absorbs most of the energy. So the intensity of sound become low but in empty room, due to the absence of obstacles the intensity of sound remain mostly same and we feel it louder.

#### 5. How do animals sense impending danger of hurricane?

**Ans.** Some animals are believed to be sensitive to low frequency sound waves emitted by hurricanes. They can also detect the slight fall in air pressure and water pressure that signal a storm's approach.

#### 6. Is it possible to realize whether a vessel kept under the tap is about to fill with water?

**Ans.** The frequency of the note produced by an air column is inversely proportional to its length. As the level of water in the vessel rises, the length of the air column above it decreases. It produces sound of decreasing frequency. i.e., the sound becomes shorter. From the shrillness of sound, it is possible to realize whether the vessel is filled with water.

$$v_{\min} = 11.71 \text{ ms}^{-1}.$$

### Government Exam Question & Answers

#### MULTIPLE CHOICE QUESTIONS :

===== 1 Mark =====

#### 1. Which of the following represents a wave?

[Mar. HY. -2019]

a)  $\frac{1}{x+vt}$

b)  $\sin(x + vt)$

c)  $(x - vt)^3$

d)  $x(x + vt)$

[Ans. (b)  $\sin(x + vt)$ ]

#### 2. The waves produced by a motor boat sailing in water are

[HY. -2019]

(a) transverse

(b) longitudinal

(c) stationary

(d) longitudinal and transverse

[Ans. (d) longitudinal and transverse]



**VERY SHORT ANSWER QUESTIONS :****===== 2 Mark =====**

1. Distinguish between transverse and longitudinal waves. [HY-2019]

Ans.

S. No	Transverse waves	Longitudinal waves
1.	The direction of vibration of particles of the medium is perpendicular to the direction of propagation of waves.	The direction of vibration of particles of the medium is parallel to the direction of propagation of waves.
2.	The disturbances are in the form of crests and troughs.	The disturbances are in the form of compressions and rarefactions.
3.	Transverse waves are possible in elastic medium.	Longitudinal waves are possible in all types of media (solid, liquid and gas).

**SHORT ANSWER QUESTIONS :===== 3 Mark =====**

1. How are sound waves classified? [Jun.-2019]

Ans. Sound waves can be classified in three groups according to their range of frequencies:

- (1) **Infrasonic waves** : Sound waves having frequencies below 20 Hz are called infrasonic waves. These waves are produced during earthquakes. Human beings cannot hear these frequencies. Snakes can hear these frequencies.

- (2) **Audible waves** : Sound waves having frequencies between 20 Hz to 20,000 Hz (20kHz) are called audible waves. Human beings can hear these frequencies.

- (3) **Ultrasonic waves** : Sound waves having frequencies greater than 20 kHz are known as ultrasonic waves. Human beings cannot hear these frequencies. Bats can produce and hear these frequencies.

**NUMERICAL PROBLEMS :===== 2 Mark =====**

1. Two waves of wavelength 99 cm and 100 cm both travelling with the velocity of 396 ms<sup>-1</sup> are made to interfere. Calculate the number of beats produced by them per sec. [Mar.-2019]

**Solution:**

$$V = \lambda f$$

$$f = \frac{v}{\lambda}$$

$$f_1 = \frac{v}{\lambda_1} = \frac{396}{0.99} = 400 \text{ Hz.}$$

$$f_2 = \frac{v}{\lambda_2} = \frac{396}{1} = 396 \text{ Hz.}$$

$$n = f_2 - f_1 = 400 - 396 = 4 \text{ beats per second.}$$

**ADDITIONAL QUESTIONS****I. MULTIPLE CHOICE QUESTIONS :****===== 1 Mark =====**

1. Sound waves of wavelength travelling in a medium with a speed of  $v$  m/s enter into the medium where its speed is  $2v$  m/s, wavelength of sound waves in the second medium is?

- (a)  $\lambda$       (b)  $\frac{\lambda}{2}$       (c)  $2\lambda$       (d)  $4\lambda$

[Ans. (c)  $2\lambda$ ]

2. Speed of sound wave in air.

- (a) Independent of temp  
(b) Increase with pressure  
(c) Increase with increase in humidity  
(d) Decreases with increase in humidity

[Ans. (c) increase with increase in humidity]

3. Change in temperature of the medium changes

- (a) frequency of sound waves  
(b) amplitude of sound waves  
(c) wavelength of sound waves  
(d) loudness of sound waves

[Ans. (c) wavelength of sound waves]

4. With propagation of longitudinal waves through a medium the quantity transmitted is

- (a) matter      (b) energy  
(c) energy and matter  
(d) energy, matter and momentum

[Ans. (b) energy]



- 5. During propagation of a plane progressive mechanical wave**  
 (a) amplitude of all particles is equal  
 (b) particles of the medium execute S.H.M.  
 (c) wave velocity depends upon the nature of the medium  
 (d) all the above **[Ans. (d) all the above]**
- 6. Speed of sound wave in air.**  
 (a) independent of temperature  
 (b) increase with pressure  
 (c) increase with increase in humidity  
 (d) decreases with increase in humidity  
**[Ans. (c) increase with increase in humidity]**
- 7. Which of the following statement are true for a stationary wave?**  
 (a) Every particle has a forced amplitude which is different from the amplitude of its nearest particle.  
 (b) All the particles cross their mean position at the same time.  
 (c) There is no transfer of energy across any plane.  
 (d) All the above **[Ans. (d) All the above]**
- 8. The displacement  $y$  of a wave travelling in the  $x$ -direction is given by  $y = 10^{-4} \sin(600t - 2x + \pi/3)$  Where  $x$  is expressed in metres  $t$  is seconds, the speed of the wave motion(in  $\text{ms}^{-1}$ ) is**  
 (a) 300 (b) 600  
 (c) 1200 (d) 200 **[Ans. (a) 300]**
- 9. A wave travelling along the  $x$ -axis described by the equation  $y(x,t) = 0.005 \cos(\alpha x - \beta t)$ . of the wavelength and time period of the wave are 0.08 and 2.0s then in appropriate units are**  
 (a)  $\alpha = 12.50 \pi$ ;  $\beta = \frac{\pi}{2.0}$   
 (b)  $\alpha = 25.00 \pi$ ;  $\beta = \frac{\pi}{2.0}$   
 (c)  $\alpha = \frac{0.08}{\pi} \pi$ ;  $\beta = \frac{2.0}{\pi}$   
 (d)  $\alpha = \frac{0.04}{\pi} \pi$ ;  $\beta = \frac{4.0}{\pi}$   
**[Ans. (b)  $\alpha = 25.00 \pi$   $\beta = \frac{\pi}{2}$ ]**
- 10. The speed of sound in oxygen ( $\text{O}_2$ ) at a certain temp is  $460 \text{ ms}^{-1}$  the speed of sound in helium at the same temperature will be**  
 (a)  $460 \text{ ms}^{-1}$  (b)  $500 \text{ ms}^{-1}$   
 (c)  $650 \text{ ms}^{-1}$  (d)  $1420 \text{ ms}^{-1}$   
**[Ans. (d)  $1420 \text{ ms}^{-1}$ ]**
- 11. Length of a string tied to two rigid supports is 40cm maximum length of a stationary wave produced on it is**  
 (a) 20 (b) 80  
 (c) 40 (d) 120 **[Ans. (b) 80]**
- 12. Tube A has both ends open, while tube B has one end closed, otherwise they are identical the ratio of fundamental frequency of tubes A & B is**  
 (a) 1:2 (b) 1:4  
 (c) 2:1 (d) 4:1 **[Ans. (c) 2:1]**
- 13. When temperature increases, the frequency of a tuning fork**  
 (a) increases (b) decreases  
 (c) increases or-decreases depending (d) remains the same **[Ans. (b) decreases]**
- 14. An observer moves towards a stationary source of sound with a velocity on fifth of the velocity of sound, what is the percentage increase in the apparent frequency?**  
 (a) Zero (b) 0.5%  
 (c) 5% (d) 20% **[Ans. (d) 20%]**
- 15. A sound absorber attenuates the sound level by 20dB the intensity decreases by a factor of**  
 (a) 100 (b) 1000  
 (c) 10000 (d) 10 **[Ans. (a) 100]**
- 16. The disc of a siren containing 60 holes rotates at a constant speed of 360 rpm. The emitted sound is in unison with a tuning fork of frequency**  
 (a) 10Hz (b) 360Hz  
 (c) 216 Hz (d) 60z  
**[Ans. (b) 360Hz]**
- 17. The equation of a simple harmonic wave is given  $y = 5 \sin \frac{\pi}{2} (100t - x)$ ; where  $x$  and  $y$  are in meter and time is in seconds. The period of the wave in second will be**  
 (a) 0.04 (b) 0.01  
 (c) 1 (d) 5 **[Ans. (a) 0.04]**
- 18. If wave  $y = A \cos(\omega t + kx)$  moving alongs  $x$ -axis the shapes of pulse at  $t = 0$  and  $t = 2s$**   
 (a) are different (b) are same  
 (c) may not be same (d) none of these  
**[Ans. (b) are same]**

19.  $y_1 = 4 \sin(\omega t + kx)$ ,  $y_2 = -4 \cos(\omega t + kx)$ , the phase difference is

(a)  $\frac{\pi}{2}$  (b)  $\frac{3\pi}{2}$   
(c)  $\pi$  (d) Zero [Ans. (b)  $\frac{3\pi}{2}$ ]

20. A wave equation is  $y = 0.01 \sin(100\pi t - kx)$  of wave velocity is 100m/s, its number is equal to

(a)  $1\text{m}^{-1}$  (b)  $2\text{m}^{-1}$   
(c)  $\pi\text{m}^{-1}$  (d)  $2\pi\text{m}^{-1}$  [Ans. (c)  $\pi\text{m}^{-1}$ ]

21. A particle on the trough of a wave at any instant will come to the mean position after a time ( $T$  = time period)

(a)  $\frac{T}{2}$  (b)  $\frac{T}{4}$   
(c)  $T$  (d)  $2T$  [Ans. (b)  $\frac{T}{4}$ ]

22. An organ pipe of length  $l$  vibrates in the fundamental mode, the pressure variation is maximum

(a) at the 2 ends  
(b) at the distance  $l/2$  inside the ends  
(c) at the distance  $l/4$  inside the ends  
(d) at the distance  $l/6$  inside the ends  
[Ans. (b) at the distance  $l/2$  inside the ends]

23. If an experiment with sonometer, a tuning fork of frequency 256 Hz resonates with a length of 25cm and another tuning fork resonates with constants the frequency of the second tuning fork is

(a) 163.84Hz (b) 400 Hz  
(c) 320 Hz (d) 204.8 Hz  
[Ans. (b) 400 Hz]

24. An organ pipe open at one end is vibrating in first overtone and is in resonance with another pipe open at both ends and vibrating in third harmonic. The ratio of length of 2 pipes is

(a) 1:2 (b) 4:1  
(c) 8:3 (d) 3:8 [Ans. (a) 1:2]

25. The fractional change in wavelength of light coming from a star is 0.014%. What is its velocity?

(a)  $4.2 \times 10^3 \text{ m/s}$  (b)  $3.8 \times 10^8 \text{ m/s}$   
(c)  $3.5 \times 10^3 \text{ m/s}$  (d)  $4.2 \times 10^4 \text{ m/s}$   
[Ans. (d)  $4.2 \times 10^4 \text{ m/s}$ ]

## II. MATCH THE FOLLOWING :

1.	(1) Transverse waves	(a)	$\sqrt{\frac{E}{\rho}}$
	(2) Longitudinal waves	(b)	$\sqrt{\frac{T}{\mu}}$
	(3) Velocity of Transverse wave	(c)	Sounds
	(4) Velocity of longitudinal wave	(d)	Light

(1) (2) (3) (4)  
(a) b c d a  
(b) d c b a  
(c) c a d b  
(d) c b a d [Ans:(b) d c b a]

2.	(1) Constructive interference	(a)	$(A_1 - A_2)^2$
	(2) Destructive Interference	(b)	1 : 2 : 3 : 4
	(3) Closed organ pipe	(c)	$(A_1 + A_2)^2$
	(4) Open organ pipe	(d)	1 : 3 : 5 : 7

(1) (2) (3) (4)  
(a) b c d a  
(b) d c b a  
(c) c a d b  
(d) d a b c [Ans:(c) c a d b]

3.	(1) When source and observer approach each other	(a)	$f' = f \left( 1 - \frac{v_s}{v} \right)$
	(2) When source and observer recede from each other	(b)	$f' = f \left( 1 + \frac{v_s}{v} \right)$
	(3) When the source moves towards a stationary observer	(c)	$f' = \left( \frac{v + v_0}{v - v_s} \right) f$
	(4) When the source moves away from a stationary observer	(d)	$f' = \left( \frac{v - v_0}{v + v_s} \right) f$

(1) (2) (3) (4)  
(a) b c d a  
(b) c a b d  
(c) d c b a  
(d) c d b a [Ans:(d) c d b a]

**III. FILL IN THE BLANKS :**

- Wave number is given by  $k =$  \_\_\_\_\_  
 (a)  $\frac{1}{\lambda}$  (b)  $\frac{2\pi}{T}$  (c)  $\frac{2\pi}{\lambda}$  (d)  $\frac{\lambda}{2\pi}$   
**[Ans. (c)  $\frac{2\pi}{\lambda}$ ]**
- The unit of wave number is \_\_\_\_\_  
 (a) rad.m (b) m / rad  
 (c) rad  $m^{-1}$  (d) rad  
**[Ans. (c) rad  $m^{-1}$ ]**
- The waves which requires medium for their propagation are known as \_\_\_\_\_  
 (a) Non - mechanical waves  
 (b) Mechanical waves  
 (c) Electro magnetic waves  
 (d) Tidal waves **[Ans. (b) Mechanical waves]**
- If the vibration of particles in a medium is perpendicular to the direction of propagation of the wave, then it is \_\_\_\_\_  
 (a) longitudinal wave (b) transverse wave  
 (c) mechanical wave  
 (d) non-mechanical wave  
**[Ans. (b) transverse wave]**
- \_\_\_\_\_ is a source dependent  
 (a) Wave velocity (b) Wave length  
 (c) Frequency (d) Time  
**[Ans. (c) Frequency]**
- \_\_\_\_\_ is a medium dependent  
 (a) Wave velocity (b) Wavelength  
 (c) Frequency (d) Time  
**[Ans. (a) Wave velocity]**
- Velocity of a wave is \_\_\_\_\_  
 (a)  $\lambda f$  (b)  $f/\lambda$  (c)  $\lambda/f$  (d)  $\lambda+f$   
**[Ans. (a)  $\lambda f$ ]**
- The waves which do not require medium for their propagation are known as \_\_\_\_\_  
 (a) mechanical waves (b) electrical waves  
 (c) sea waves (d) Non-mechanical waves  
**[Ans. (d) Non- mechanical waves]**
- The distance between two consecutive crests or troughs is known as \_\_\_\_\_  
 (a) Wave velocity (b) Wavelength  
 (c) Frequency (d) Time period  
**[Ans. (b) Wavelength]**
- Number of cycles per second is called \_\_\_\_\_  
 (a) Wavelength (b) Frequency  
 (c) Wave velocity (d) Angular velocity  
**[Ans. (b) frequency]**

**IV. CHOOSE THE ODD ONE OUT :**

- (a) Transverse wave (b) Light wave  
 (c) Microwave (d) Sound wave  
**[Ans. (d) Sound wave]**  
**Hint:** Sound wave is longitudinal
  - (a) intensity of a wave (b) frequency of a wave  
 (c) wavelength of a wave (d) density of a wave  
**[Ans. (d) Density]**  
**Hint:** Density is not related with a wave
  - (a) node (b) antinode  
 (c) harmonics (d) end correction  
**[Ans. (d) end correction]**  
**Hint:** End correction is a correction in organ pipes.
- V. CHOOSE THE CORRECT PAIR :**
- (a) Beats - Waxing and waning of sound  
 (b) Wave number - Number of cycles per second  
 (c) Intensity - Inversely Proportional to frequency  
 (d) Loudness - Independent on sensitivity of the ear.  
**[Ans. (a) Beats - Waxing and waning of sound]**
  - (a) Frequency - medium dependent  
 (b) echo - 17.2 m  
 (c) end correction -  $\frac{L_2 - 2L_1}{3}$   
 (d) Law of g mass -  $\frac{B}{\mu}$   
**[Ans. (b) echo - 17.2m]**

**VI. CHOOSE THE INCORRECT PAIR :**

- (a) Loudness - Intensity  
 (b) Doppler Effect - Apparent frequency  
 (c) Organ pipes - Vibration of air columns  
 (d) Resonance Air Column Apparatus - Speed of sound in water  
**[Ans. (d) Resonance Air Column Apparatus - Speed sound in water]**

2. (a) Closed organ pipe -  $(f_n = nf_1)$   
 (b) Open Organ pipe -  $1 : 2 : 3 : 4$   
 (c) Constructive Interference -  $(A_1 + A_2)^2$   
 (d) Intensity - Square of Amplitude

[Ans. (a) closed organ pipe -  $(f_n = nf_1)$ ]

## VII. ASSERTION & REASON :

Directions :

- (a) Assertion and Reason are correct and Reason is the correct explanation of Assertion  
 (b) Assertion and Reason are true but Reason is the false explanation of the Assertion  
 (c) Assertion is true but Reason is false  
 (d) Assertion is false but Reason is true

1. **Assertion :** The frequencies of harmonics are in the ratio  $1 : 3 : 5 : 7$ , in the case of open organ pipe.

**Reason :** The second harmonic for an open organ pipe is  $2f_1$

[Ans. (d) Assertion is false but Reason is true]

2. **Assertion :** If the observer chases the source, then the apparent frequency is  $f'$

$$= \left[ \frac{\gamma + \gamma_o}{\gamma + \gamma_s} \right] f$$

**Reason :** Doppler effect in sound is asymmetrical whereas Doppler effect in light is symmetrical.

[Ans. (b) Assertion and Reason are true, but Reason is the false explanation of Assertion.]

## VIII. CHOOSE THE INCORRECT OR CORRECT STATEMENTS :

1. (I) Repetition of sound produced by the reflection of sound waves from a wall is echo.  
 (II) In a closed room, the sound is repeatedly reflected from the walls and it is heard even after the sound source is ceased. it is called reverberation?

Which one is correct?

- (a) I only (b) II only  
 (c) both are correct (d) None

[Ans. (c) both are correct]

2. (I) Time taken by a particle to complete one full cycle is called time period.

(II) For propagation of waves, the medium should possess elasticity alone.

Which one is incorrect?

- (a) I only (b) II only  
 (c) both are correct (d) None

[Ans. (b) II only]

3. (I) The number of waves per unit second is called Wave number.

(II)  $\frac{\text{Phasedifference}}{\text{Path difference}} = \frac{2\pi}{\lambda}$

Which one is Incorrect?

- (a) I only (b) II only  
 (c) both are correct (d) None

[Ans. (a) I only]

4. (I) Longitudinal waves are possible in all types of media (solid, liquid and gas).

(II) Transverse waves are also possible in all types of media.

Which one is correct?

- (a) I only (b) II only  
 (c) both are correct (d) None

[Ans. (a) I only]

## VERY SHORT ANSWER QUESTIONS :

—2 Marks—

1. Discuss about the formation of waves on stretched string?

**Ans.** Take a long string and tie one end of the string to the wall as shown in Figure 11.4 (a). A quick jerk, is 92 a bump (like pulse) is produced in the string as shown in Figure 11.4 (b). Such a disturbance is sudden and it lasts for a short duration, hence it is known as a wave pulse. If jerks are given continuously then the waves produced are standing waves. Similar waves are produced by a plucked string in a guitar.

2. Mention the important properties which medium should possess for propagation of waves.

**Ans.** The medium possesses both inertia and elasticity for propagation of waves.

3. Define frequency and time period.

**Ans.** Frequency is defined as "the number of waves crossing a point per second" It is measured in hertz. Frequency and time period are inversely related i.e.,

$$T = \frac{1}{f}$$

Time period is defined as the time taken by one wave to cross a point.



- 4. Give the relation between velocity  $v$ , angular frequency  $\omega$  and wave number  $\lambda$ ?**

**Ans.** Velocity,  $v = \lambda f = \frac{\lambda}{2\pi} (2\pi f) = \frac{2\pi f}{2\pi / \lambda} = \omega / k$

- 5. Define amplitude of the wave.**

**Ans.** An amplitude of the wave is defined as the maximum displacement of the medium with respect to a reference axis (for example in this case  $x$ -axis). Here, it is denoted by  $A$ .

- 6. What is the effect of pressure on velocity of sound in gas?**

**Ans.** For a fixed temperature, when the pressure varies, correspondingly density also varies such that the ratio  $\left(\frac{P}{\rho}\right)$  becomes constant. This means that the speed of sound is independent of pressure for a fixed temperature.

- 7. What is Echo?**

**Ans.** An echo is a repetition of sound produced by the reflection of sound waves from a wall, mountain or other obstructing surfaces.

- 8. What is persistence of hearing?**

**Ans.** Velocity =  $\frac{\text{Distance travelled}}{\text{time taken}} = \frac{2d}{t}$

$$2d = 344 \times 0.1 = 34.4 \text{ m}$$

$$d = 17.2 \text{ m.}$$

- 9. What is Reverberation?**

**Ans.** In a closed room the sound is repeatedly reflected from the walls and it is even heard long after the sound source ceases to function. The residual sound remaining in an enclosure and the phenomenon of multiple reflections of sound is called reverberation.

- 10. What is meant by reverberation time?**

**Ans.** The duration for which the sound persists is called reverberation time.

- 11. Define particle velocity.**

**Ans.** When a particle is in motion, the rate of change of displacement at any instant of time is defined as velocity of the particle at that instant of time. This is known as particle velocity.

- 12. Write the relation between path difference and phase difference?**

**Ans.** Phase difference =  $\frac{2\pi}{\lambda}$  (path difference)

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta r \quad \text{or} \quad \Delta r = \frac{\lambda}{2\pi} \Delta\phi$$

### SHORT ANSWER QUESTIONS :

== 3 Marks ==

- 1. Write about the formation of waves in a tuning fork.**

**Ans. (i)** A tuning fork is struck on a rubber pad, the prongs of the tuning fork vibrate about their mean positions.  
**(ii)** The prong vibrating about a mean position means moving outward and inward.  
**(iii)** When a prong moves outward, it pushes the layer of air in its neighbourhood which means there is more accumulation of air molecules in this region.  
**(iv)** Hence, the density and also the pressure increase. These regions are known as compressed regions or compressions.  
**(v)** This compressed air layer moves forward and compresses the next neighbouring layer in a similar manner. Thus a wave of compression advances or passes through air.  
**(vi)** When the prong moves inwards, the particles of the medium are moved to the right. In this region both density and pressure are low. It is known as a rarefaction or elongation.

- 2. Write the characteristics of wave motion.**

**Ans. (i)** For the propagation of the waves, the medium must possess both inertia and elasticity, which decide the velocity of the wave in that medium.  
**(ii)** In a given medium, the velocity of a wave is a constant whereas the constituent particles in that medium move with different velocities at different positions. Velocity is maximum at their mean position and zero at extreme positions.  
**(iii)** Waves undergo reflections, refraction, interference, diffraction and polarization.



### 3. Define angular frequency, wave number and wave vector.

**Ans. (i)** The number of cycles (or revolutions) per unit time is called **angular frequency**.

**Angular frequency**,  $\omega = \frac{2\pi}{T} = 2\pi f$  (unit is radians/second)

**(ii)** The number of cycles per unit distance or number of waves per unit distance is called **wave number**.

wave number,  $k = \frac{2\pi}{\lambda}$  (unit is radians/meter)

**(iii)** In two, three or higher dimensional case, the wave number is the magnitude of a vector called wave vector. The points in space of wave vectors are called reciprocal vectors,  $\vec{k}$ .

Dimensions of  $\vec{k}$  is  $L^{-1}$ .

### 4. What is the relation between the velocity and temperature?

**Ans.** Let us consider an ideal gas whose equation of state is

$$PV = nRT \quad \dots (1)$$

where, P is pressure, V is volume, T is temperature, n is number of mole and R is universal gas constant. For a given mass of a molecule, equation (1) can be written as

$$\frac{PV}{T} = \text{Constant} \quad \dots (2)$$

For a fixed mass m, density of the gas inversely varies with volume. i.e.,

$$\rho \propto \frac{1}{V}, \quad V = \frac{m}{\rho} \quad \dots (3)$$

Substituting equation (3) in equation (2), we get

$$\frac{P}{\rho} = cT \quad \dots (4)$$

where c is constant.

The speed of sound in air given in equation

$$v_A = \sqrt{\frac{B_A}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma} v_T \quad \text{can be written as}$$

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma cT} \quad \dots (5)$$

Since  $v \propto \sqrt{T}$ , the speed of sound varies directly to the square root of temperature in kelvin.

Let  $v_0$  be the speed of sound at temperature at  $0^\circ\text{C}$  or  $273\text{ K}$  and  $v$  be the speed of sound at any arbitrary temperature T (in kelvin), then

$$\frac{v}{v_0} = \sqrt{\frac{T}{273}} = \sqrt{\frac{273+t}{273}}$$

$$v = v_0 \sqrt{1 + \frac{t}{273}} \cong v_0 \left(1 + \frac{t}{546}\right)$$

(using binomial expansion)

Since  $v_0 = 331\text{ ms}^{-1}$  at  $0^\circ\text{C}$ ,  $v$  at any temperature in  $t^\circ\text{C}$  is

$$v = (331 + 0.60t)\text{ m s}^{-1}$$

Thus the speed of sound in air increases by  $0.61\text{ ms}^{-1}$  per degree celcius rise in temperature. Note that when the temperature is increased, the molecules will vibrate faster due to gain in thermal energy and hence, speed of sound increases.

### 5. Write the application of reflection of sound though the be curved surface.

**Ans.** The sound produced in a big hall or auditorium or theatre is absorbed by the walls, ceilings, floor, seats, etc. To avoid such losses, a curved sound board (concave board) is kept in front of the speaker, so that the board reflects the sound waves of the speaker towards the audience. This method will minimize the spreading of sound waves in all possible direction in that hall and also enhances the uniform distribution of sound throughout the hall. That is why a person sitting at any position in that hall can hear the sound without any disturbance.

### 6. Write a not about Stethoscope.

**Ans.** It works on the principle of multiple reflections. It consists of three main parts:

- (i) Chest piece
- (ii) Ear piece
- (iii) Rubber tube

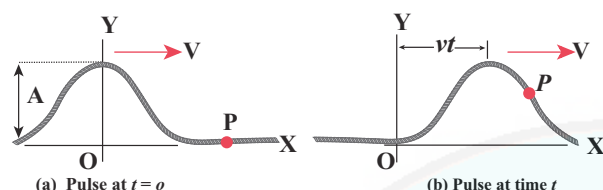
**(i) Chest piece :** It consists of a small disc-shaped resonator (diaphragm) which is very sensitive to sound and amplifies the sound it detects.

**(ii) Ear piece :** It is made up of metal tubes which are used to hear sounds detected by the chest piece.

**(iii) Rubber tube :** This tube connects both chest piece and ear piece. It is used to transmit the sound signal detected by the diaphragm, to the ear piece. The sound of heart beats (or lungs) or any sound produced by internal organs can be detected, and it reaches the ear piece through this tube by multiple reflections.

**7. Derive the Equation of a plane progressive wave.**

**Ans.** A jerk is given on a stretched string at time  $t = 0$  s. Assume that the wave pulse created during this disturbance moves along positive  $x$  direction with constant speed  $v$  as shown in Figure (a).



Represent the shape of the wave pulse, mathematically as  $y = y(x, 0) = f(x)$  at time  $t = 0$  s. Assume that the shape of the wave pulse remains the same during the propagation. After some time  $t$ , the pulse moving towards the right and any point on it can be represented by  $x'$  (read it as  $x$  prime) as shown in Figure (b). Then,

$$y(x, t) = f(x') = f(x - vt)$$

Similarly, if the wave pulse moves towards left with constant speed  $v$ , then  $y = f(x + vt)$ . Both waves  $y = f(x + vt)$  and  $y = f(x - vt)$  will satisfy the following one dimensional differential equation known as the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

where the symbol  $\partial$  represent partial derivative (read  $\frac{\partial y}{\partial x}$  as partial  $y$  by partial  $x$ ). Not all the solutions satisfying this differential equation can represent waves, because any physical acceptable wave must take finite values for all values of  $x$  and  $t$ . But if the function represents a wave then it must satisfy the differential equation. Since, in one dimension (one independent variable), the partial derivative with respect to  $x$  is the same as total derivative in coordinate  $x$ , we write So it can be written as

$$\frac{d^2 y}{dx^2} = \frac{1}{v^2} \frac{d^2 y}{dt^2}$$

**8. Derive the relation between Intensity and loudness.**

**Ans.** According to Weber-Fechner's law, "loudness ( $L$ ) is proportional to the logarithm of the actual intensity ( $I$ ) measured with an accurate non-human instrument". This means that

$$L \propto \ln I$$

$$L = k \ln I$$

where  $k$  is a constant, which depends on the unit of measurement. The difference between two loudnesses,  $L_1$  and  $L_0$  measures the relative loudness between two precisely measured intensities and is called as sound intensity level. Mathematically, sound intensity level is

$$\Delta L = L_1 - L_0 = k \ln I_1 - k \ln I_0 = k \ln \left[ \frac{I_1}{I_0} \right]$$

If  $k = 1$ , then sound intensity level is measured in bel, Therefore,

$$\Delta L = \ln \left[ \frac{I_1}{I_0} \right] \text{ bel}$$

However, this to express smaller unit, decibel.

Thus,  $\left[ \text{decibel} = \frac{1}{10} \text{ bel} \right]$  by multiplying and dividing by 10,

$$\Delta L = 10 \left( \ln \left[ \frac{I_1}{I_0} \right] \right) \frac{1}{10} \text{ bel}$$

$$\Delta L = 10 \ln \left[ \frac{I_1}{I_0} \right] \text{ decibel with } k = 10$$

For practical purposes,

$$\Delta L = 10 \log_{10} \left[ \frac{I_1}{I_0} \right] \text{ decibel.}$$

**LONG ANSWER QUESTIONS :****—5 Marks—****1. Write the expression for the velocity of longitudinal waves in an elastic medium.**

**Ans.** Consider an elastic medium (here we assume air) having a fixed mass contained in a long tube (cylinder) whose cross sectional area is  $A$  and maintained under a pressure  $P$ . One can generate longitudinal waves in the fluid either by displacing the fluid using a piston or by keeping a vibrating tuning fork at one end of the tube. Assume that the direction of propagation of waves coincides with the axis of the cylinder. Let  $\rho$  be the density of the fluid which is initially at rest. At  $t = 0$ , the piston at left end of the tube is set in motion toward the right with a speed  $u$ .

Let  $u$  be the velocity of the piston and  $v$  be the velocity of the elastic wave. In time interval  $\Delta t$ , the distance moved by the piston  $\Delta d = u \Delta t$ . Now, the distance moved by the elastic disturbance is  $\Delta x = v \Delta t$ . Let  $\Delta m$  be the mass of the air that has attained a velocity  $v$  in a time  $\Delta t$ . Therefore,

$$\Delta m = \rho A \Delta x = \rho A (v \Delta t)$$

Then, the momentum imparted due to motion of piston with velocity  $u$  is

$$\Delta p = [\rho A (v \Delta t)]u$$

But the change in momentum is impulse.

The net impulse is

$$I = (\Delta P A) \Delta t$$

$$\text{or } (\Delta P A) \Delta t = [\rho A (v \Delta t)]u$$

$$\Delta P = \rho v u \quad \dots (1)$$

When the sound wave passes through air, the small volume element ( $\Delta V$ ) of the air undergoes regular compressions and rarefactions. So, the change in pressure can also be written as

$$\Delta P = B \frac{\Delta V}{V}$$

where,  $V$  is original volume and  $B$  is known as bulk modulus of the elastic medium.

$$\text{But } V = A \Delta x = A v \Delta t \text{ and}$$

$$\Delta V = A \Delta d = A u \Delta t$$

Therefore,

$$\Delta P = B \frac{A u \Delta t}{A v \Delta t} = B \frac{u}{v} \quad \dots (2)$$

Comparing equation (1) and equation (2), we get

$$\rho v u = B \frac{u}{v} \text{ or } v^2 = \frac{B}{\rho}$$

$$\Rightarrow v = \sqrt{\frac{B}{\rho}} \quad \dots (3)$$

In general, the velocity of a longitudinal wave in elastic medium is  $v = \sqrt{\frac{E}{\rho}}$ , where  $E$  is the modulus of elasticity of the medium.

**Cases: For a solid :**

**(i) one dimension rod (1D)**

$$v = \sqrt{\frac{Y}{\rho}} \quad \dots (4)$$

where  $Y$  is the Young's modulus of the material of the rod and  $\rho$  is the density of the rod. The 1D rod will have only Young's modulus.

**(ii) Three dimension rod (3D)** The speed of longitudinal wave in a solid is

$$v = \sqrt{\frac{K + \frac{4}{3}\eta}{\rho}} \quad \dots (5)$$

where  $\eta$  is the modulus of rigidity,  $K$  is the bulk modulus and  $\rho$  is the density of the rod.

**Cases: For liquids:**

$$v = \sqrt{\frac{K}{\rho}} \quad \dots (6)$$

where,  $K$  is the bulk modulus and  $\rho$  is the density of the rod.

**2. Discuss the effect of density, humidity and wind.**

**Ans. Effect of density :** Consider two gases with different densities having same temperature and pressure. Then the speed of sound in the two gases are

$$v_1 = \sqrt{\frac{\gamma_1 P}{\rho_1}} \quad \dots (1)$$

and

$$v_2 = \sqrt{\frac{\gamma_2 P}{\rho_2}} \quad \dots (2)$$

Taking ratio of equation (1) and equation (2), we get

$$\frac{v_1}{v_2} = \frac{\sqrt{\frac{\gamma_1 P}{\rho_1}}}{\sqrt{\frac{\gamma_2 P}{\rho_2}}} = \sqrt{\frac{\gamma_1 \rho_2}{\gamma_2 \rho_1}}$$

For gases having same value of  $\gamma$ ,

$$\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}} \quad \dots (3)$$

Thus the velocity of sound in a gas is inversely proportional to the square root of the density of the gas.

**Effect of moisture (humidity) :** The density of moist air is 0.625 of that of dry air, which means the presence of moisture in air (increase in humidity) decreases its density. Therefore, speed of sound increases with rise in humidity.

$$\text{From equation } v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma c T}$$

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

Let  $\rho_1$ ,  $v_1$  and  $\rho_2$ ,  $v_2$  be the density and speeds of sound in dry air and moist air, respectively. Then

$$\frac{v_1}{v_2} = \frac{\sqrt{\frac{\gamma_1 P}{\rho_1}}}{\sqrt{\frac{\gamma_2 P}{\rho_2}}} = \sqrt{\frac{\rho_2}{\rho_1}} \text{ if } \gamma_1 = \gamma_2$$



Since  $P$  is the total atmospheric pressure, According to Dalton's law of partial pressure it can be shown that

$$\frac{P_2}{P_1} = \frac{P}{P_1 + 0.625P_2}$$

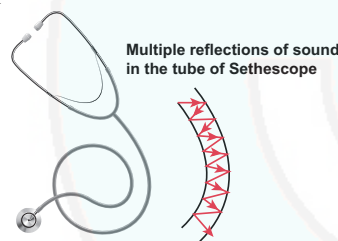
where  $P_1$  and  $P_2$  are the partial pressures of dry air and water vapour respectively. Then

$$v_1 = v_2 \sqrt{\frac{P}{P_1 + 0.625P_2}}$$

**Effect of wind :** The speed of sound is also affected by blowing of wind. In the direction along the wind blowing, the speed of sound increases whereas in the direction opposite to wind blowing, the speed of sound decreases.

### 3. Write the applications of reflection of sound waves.

**Ans. (a) Stethoscope :** It works on the principle of multiple reflections.



**Stethoscope**

It consists of three main parts:

- (i) Chest piece    (ii) Ear piece
- (iii) Rubber tube

**(i) Chest piece :** It consists of a small disc-shaped resonator (diaphragm) which is very sensitive to sound and amplifies the sound it detects.

**(ii) Ear piece :** It is made up of metal tubes which are used to hear sounds detected by the chest piece.

**(iii) Rubber tube :** This tube connects both chest piece and ear piece. It is used to transmit the sound signal detected by the diaphragm, to the ear piece. The sound of heart beats (or lungs) or any sound produced by internal organs can be detected, and it reaches the ear piece through this tube by multiple reflections.

**(b) Echo :** An echo is a repetition of sound produced by the reflection of sound waves from a wall, mountain or other obstructing surfaces. The speed of sound in air at  $20^\circ\text{C}$

is  $344 \text{ ms}^{-1}$ . If we shout at a wall which is at 344 m away, then the sound will take 1 second to reach the wall. After reflection, the sound will take one more second to reach us. Therefore, we hear the echo after two seconds.

Scientists have estimated that we can hear two sounds properly if the time gap or time interval between each sound is  $\left(\frac{1}{10}\right)^{\text{th}}$  of a

second (persistence of hearing) i.e., 0.1 s. Then,

$$\text{velocity} = \frac{\text{Distance travelled}}{\text{time taken}} = \frac{2d}{t}$$

$$2d = 344 \times 0.1 = 34.4 \text{ m}$$

$$d = 17.2 \text{ m.}$$

The minimum distance from a sound reflecting wall to hear an echo at  $20^\circ\text{C}$  is 17.2 meter.

**(c) SONAR:** Sound Navigation and Ranging. Sonar systems make use of reflections of sound waves in water to locate the position or motion of an object. Similarly, dolphins and bats use the sonar principle to find their way in the darkness.

**(d) Reverberation:** In a closed room the sound is repeatedly reflected from the walls and it is even heard long after the sound source ceases to function. The residual sound remaining in an enclosure and the phenomenon of multiple reflections of sound is called reverberation. The duration for which the sound persists is called reverberation time. It should be noted that the reverberation time greatly affects the quality of sound heard in a hall. Therefore, halls are constructed with some optimum reverberation time.

### 4. Write Characteristics of progressive waves.

**Ans. (i)** Particles in the medium vibrate about their mean positions with the same amplitude.

**(ii)** The phase of every particle ranges from 0 to  $2\pi$ .

**(iii)** No particle remains at rest permanently. During wave propagation, particles come to the rest position only twice at the extreme points.

**(iv)** Transverse progressive waves are characterized by crests and troughs whereas longitudinal progressive waves are characterized by compressions and rarefactions.



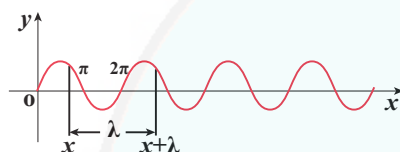
- (v) When the particles pass through the mean position they always move with the same maximum velocity.
- (vi) The displacement, velocity and acceleration of particles separated from each other by  $n\lambda$  are the same, where  $n$  is an integer, and  $\lambda$  is the wavelength.

### 5. Explain the Graphical representation of the wave.

**Ans.** Let us graphically represent the two forms of the wave variation

- (a) Space (or Spatial) variation graph  
(b) Time (or Temporal) variation graph

#### (a) Space variation graph



**Figure 1 :** Graph of sinusoidal function  $y = A \sin(kx)$

By keeping the time fixed, the change in displacement with respect to  $x$  is plotted. Consider a sinusoidal graph,  $y = A \sin(kx)$  as shown in the figure 1, where  $k$  is a constant. Since the wavelength  $\lambda$  denotes the distance between any two points in the same state of motion, the displacement  $y$  is the same at both the ends

$$\begin{aligned} y &= x \text{ and } y = x + \lambda, \text{ i.e.,} \\ y &= A \sin(kx) = A \sin(k(x + \lambda)) \\ &= A \sin(kx + k\lambda) \quad \dots (1) \end{aligned}$$

The sine function is a periodic function with period  $2\pi$ . Hence,

$$y = A \sin(kx + 2\pi) = A \sin(kx) \quad \dots (2)$$

Comparing equation (1) and equation (2), we get

$$kx + k\lambda = kx + 2\pi$$

This implies

$$k = \frac{2\pi}{\lambda} \text{ rad } m^{-1} \quad \dots (3)$$

where  $k$  is called wave number. This measures how many wavelengths are present in  $2\pi$  radians.

The spatial periodicity of the wave is

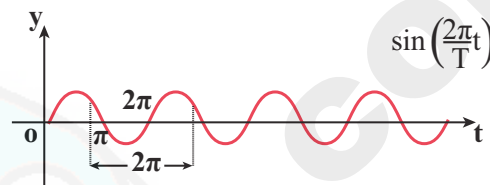
$$\lambda = \frac{2\pi}{k} \text{ m}$$

Then,

$$\text{At } t = 0 \text{ s} \quad y(x, 0) = y(x + \lambda, 0) \text{ and}$$

$$\text{At any time } t, \quad y(x, t) = y(x + \lambda, t)$$

- (b) **Time variation graph :** By keeping the position fixed, the change in displacement with respect to time is plotted. Let us consider a sinusoidal graph,  $y = A \sin(\omega t)$  as shown in the figure 2, where  $\omega$  is angular frequency of the wave which measures how quickly wave oscillates in time or number of cycles per second.



Graph of sinusoidal function  $y = A \sin(\omega t)$

The temporal periodicity or time period is

$$T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T}$$

The angular frequency is related to frequency  $f$  by the expression  $\omega = 2\pi f$ , where the frequency  $f$  is defined as the number of oscillations made by the medium particle per second. Since inverse of frequency is time period, we have,

$$T = \frac{1}{f} \text{ in seconds}$$

This is the time taken by a medium particle to complete one oscillation. Hence, we can define the speed of a wave (wave speed,  $v$ ) as the distance traversed by the wave per second

$$v = \frac{\lambda}{T} = \lambda f \text{ in } ms^{-1}$$

### Numerical Problems

1. Audible frequencies have a range of 20 Hz to  $20 \times 10^3$  Hz. Express 't' is range in terms of (i) period  $T$ , (ii) wavelength  $\lambda$  at, (iii) angular frequency  $\omega$ . (Given velocity of sound in  $0^\circ\text{C} = 331 \text{ m/s}$ .)

#### Solution:

Velocity of sound in air at  $0^\circ\text{C} = 331 \text{ m/s}$ .

Audible frequencies,  $\gamma_1 = 20 \text{ Hz}$ ,

$\gamma_2 = 20 \times 10^3 \text{ Hz}$ .

#### (i) Period ( $T$ ):

Time period  $T = \frac{1}{\gamma}$ , we get,

$$\therefore T_1 = \frac{1}{\gamma_1} = \frac{1}{20} = 0.05 \text{ s}$$

$$T_2 = \frac{1}{\gamma_2} = \frac{1}{20 \times 10^3} = 0.055 \times 10^{-3} \text{ s}$$

$\therefore$  Time period range is 0.05 to  $0.055 \times 10^{-3} \text{ s}$ .

(ii) Wavelength  $\lambda$ :

From the formula,  $\lambda = \frac{v}{\gamma}$

$$\therefore \lambda_1 = \frac{v}{\gamma_1} = \frac{331}{20} = 16.55 \text{ m}$$

and

$$\lambda_2 = \frac{v}{\gamma_2} = \frac{331}{20 \times 10^3} = 0.0165 \text{ m}$$

$\therefore$  Wavelength range is 16.55 m to 0.0165 m.

(iii) Angular frequency  $\omega$ :

From the formula,  $\omega = 2\pi \gamma$

$$\therefore \omega = 2\pi \gamma_1 = 2\pi \times 20 = 40\pi \text{ rad s}^{-1}$$

$$\text{and } \omega_2 = 2\pi \gamma_2 = 2\pi \times 20 \times 10^3$$

$$= 40\pi \times 10^3 \text{ rad/s.}$$

$\therefore$  Angular frequency range is  $40\pi$  to  $40\pi \times 10^3 \text{ rad/s}$ .

2. A fruit dropped from the top of a tree of height 200 m high splashes into the water of a pond near the base of the tree. When is the splash heard at the top given that the speed of sound in air is  $340 \text{ ms}^{-1}$ .

**Solution:**

Let total time  $t = t_1 + t_2$ , where  $t_1$  is the time take from top to surface of water and  $t_2$  is the time taken by sound to reach the top.

**Calculation of  $t_1$ :**

Using  $s = ut_1 + \frac{1}{2}at_1^2$

Putting,  $u = 0$ ,  $a = g = 9.8 \text{ m/s}^2$

$$S = 200 \text{ m, we get } 200 = \frac{1}{2} \times 9.8 \times t_1^2$$

$$200 = 4.9 \times t_1^2; t_1^2 = 40.816$$

$$t_1 = 6.3887 \text{ s.}$$

and

**Calculation of  $t_2$ :**

Using  $v = \frac{s}{t_2}$ ,

$$t_2 = \frac{s}{v} = \frac{200}{340} = 0.588 \text{ s.}$$

$$\therefore \text{Total time } t = t_1 + t_2$$

$$= 6.3887 + 0.588$$

$$= 6.97 \text{ s}$$

The splash has heard at 6.97 s

3. For the travelling harmonic wave  $y(x, t) = 2.0 \cos 2\pi [8t - 0.0060x + 0.27]$ , where  $x$  and  $y$  are in cm and  $t$  in s. Calculate the phase difference between oscillatory motion of two points separated by a distance of,

(a) 300 cm (b) 0.75 m (c)  $\frac{\lambda}{4}$

**Solution:**

Here,

$$y = 2.0 \cos 2\pi (8t - 0.0060x + 0.27)$$

$$= 2.0 \cos [2\pi (8t - 0.0060x) + 2\pi (0.27)]$$

Standard equation for a travelling wave is,

$$y = r \cos \left[ \frac{2\pi}{\lambda} (vt - x) + \phi \right]$$

Here,

$$\phi = \frac{2\pi}{\lambda} x = 2\pi \times 0.006x$$

$$\frac{2\pi}{\lambda} = 0.006$$

(a) When  $x = 300 \text{ cm}$ ,

$$\phi = 2\pi \times 0.006 \times 300$$

$$= 3.6\pi \text{ rad.}$$

(b) When  $x = 0.75 \text{ m} = 75 \text{ cm}$ ,

$$\phi = 2\pi \times 0.006 \times 75$$

$$= 0.9\pi \text{ rad}$$

(c) When  $x = \frac{\lambda}{4}$ ,

$$\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\lambda}{2} \text{ rad}$$

4. A transverse harmonic wave on a string is described by  $y(x, t) = 5.0 \sin (48t + 0.0264x + \frac{\pi}{6})$ , where  $x$  and  $y$  are in cm and  $t$  in sec. The positive direction of  $x$  is from left to right.
- (a) What are its amplitude and frequency?
- (b) What is the least distance between two success in crests in the wave?

**Solution:**

Here,

$$y(x, t) = 5.0 \sin (48t + 0.0264x + \frac{\pi}{6})$$

The general equation of a plane progressive wave is,

$$y(x, t) = a \sin \left[ \frac{2\pi}{\lambda} (vt + x) + \phi \right]$$

It is observed that the given equation represent a travelling waveform right to left.

$$\text{Velocity, } V = \frac{48}{0.0264} = 1818.18 \text{ cm s}^{-1}, r = 5 \text{ cm.}$$

**(a) Amplitude and frequency:**

Amplitude,

$$\frac{2\pi}{\lambda} = 0.0264$$

or

$$\lambda = \frac{2\pi}{0.0264} \text{ cm} = \frac{2 \times 3.14}{0.0264} = \frac{6.28}{0.0264} = 237.8 \text{ cm}$$

frequency,

From the equation

$$v = \lambda \nu,$$

$$\begin{aligned} \nu &= \frac{v}{\lambda} = \frac{1818.18}{2\pi} \times 0.0264 = \frac{1818.18}{2 \times 3.14} \times 0.0264 \\ &= \frac{1818.18}{6.28} \times 0.0264 = 289.51 \times 0.0264 = 7.64 \text{ Hz} \end{aligned}$$

(b) To find least distance between two successive crests in the wave.

$$\begin{aligned} \lambda &= \frac{2\pi}{0.0264} = \frac{2 \times 3.14}{0.0264} = \frac{6.28}{0.0264} \\ &= 237.8 \text{ cm} = 2.38 \text{ m} \end{aligned}$$

(c) When  $x = \frac{\lambda}{4}$

$$\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2} \text{ rad.}$$

### Creative Questions (HOTS)

**1. How can we distinguish experimentally between longitudinal and transverse waves?**

**Ans.** We can distinguish between longitudinal and transverse ways by performing polarization experiments. Transverse waves can be polarised while longitudinal waves cannot be polarised.

**2. Two astronauts on the surface of the moon cannot talk to each other why?**

**Ans.** Sound waves require material medium for their propagation. As there is no atmosphere on the moon, hence the sound wave cannot propagate on the moon.

**3. How does the frequency of a tuning fork change, when the temperature is increased?**

**Ans.** As the temperature increases, the length of the prong of the tuning fork increases. This increases the wavelength of the stationary waves set up in the tuning fork. As frequency,  $\nu \propto \frac{1}{\lambda}$  so frequency of the tuning fork decreases.

**4. The beats are not heard if the difference in frequencies of the two sounding notes is more than 10. Why?**

**Ans.** If the difference in frequencies of the two waves is more than 10, we shall hear more than 10 beats for second. Due to persistence of hearing, our ear is not able to distinguish between two sounds as separate. If the time interval between

them is less than  $\left(\frac{1}{10}\right)^{\text{th}}$  of a second. Hence

beats heard will not be distinct if the number of beats produced per second is more than 10.

**5. Explain why we cannot here an echo in a small room?**

**Ans.** For an echo of a simple sound to be heard, the minimum distance between speaker and the walls should be 17m. As the length of a room is less than 17m, so we do not hear on echo.

**6. What is the difference between an echo and a reverberation?**

**Ans.** An echo is produced when sound reflected from a distant obstacle comes back after an interval of  $\frac{1}{10}$  second or more. In an echo, the original and reflected sounds are heard repetitively. Reverberation on the other hand connects of successive reflections which follow each other so quickly that they cannot produce separate echoes.

**7. Tube A has both ends open, while tube B has one end closed, otherwise they are identical the ratio of fundamental frequency of tubes A & B?**

**Ans.** The fundamental frequency for tube A with both into open is

$$\nu_B = \frac{V}{2L}$$

The fundamental frequency for tube B with one end closed is

$$\nu_B = \frac{V}{4L}; \quad \frac{\nu_A}{\nu_B} = \frac{V/2L}{V/4L} = 2$$

### VALUE BASED QUESTIONS

1. (i) What is the science (physics concept) behind a stethoscope? What is the principle?

- (ii) How will you make a child to understand the, Intensity of light or sound from Density without using formulae.

**Ans. (i)** The principle behind a stethoscope is multiple reflections of sound. It consists of a long tube made of rubber or metal. When sound pulses pass through one end of the tube, the pulses get concentrated to the other end due to several reflections on the inner surface of the tube. Using this the doctor hears the patient's heart beat as concentrated rays.

- (ii) Density means the closeness of the particles. As the particles are closely packed, Density is more. For example we say Dense forest, it means the trees are closely packed so that even light cannot pass into. Density can be used for solid, liquid and gas. But intensity means closeness of the waves or lines of force. Where the waves or lines of force are closer together. Intensity is more for example laser is a highly intensified beam we say. Because in single beam, it has more number of coherent waves, packed closer together. Intensity can be used for light or sound or heat or electric lines of force or magnetic lines of force.

2. Rakesh asked his grandpa, (who was once a scientist) that whether the planets orbiting around the sun are regular shaped i.e properly spherical in shape. His Grandpa answered No, they are not at all. All are irregularly shaped. Then Rakesh asked him again, how to find that they are irregular in shape. What is his grandpa's explanation?

- (i) How does a RADAR and SONAR Work?

**Ans.** Rakesh's grandpa explained, the science behind this question is Doppler effect. In Doppler effect, the velocity of light or sound is  $c = n\lambda$

Where  $n$  = frequency of sound (light)

$\lambda$  = Wavelength of sound (light)

Scientists would transmit some light pulses on different parts (points) on the surface of a planet. They would measure the returning time, after the reflection of light from the planet if there is an apparent change in frequencies of light from different parts of the planet's surface, it means the planets are irregularly shaped. For example if the light pulses hit the top of the mountain on the planet, the time of returning or shift in frequency will be noted. If the light pulse, incident on a valley or a river or a sea, the returning time or frequency will be more. So using this difference in frequencies or time, we can analyse the planets contain peaks and deep parts on them. Thus planets are irregularly shaped.

- (i) RADAR - It means Radio Detection and Ranging. To find the speed of an aeroplane it is used. The radar sends high frequency radio waves towards an aeroplane. The reflected waves are detected by the receiver of the radar station. The difference in frequencies is used to determine its speed.

SONAR- It means Sound Navigation and Ranging.

Sound waves generated from a ship fitted with SONAR are transmitted in water towards an approaching submarine. The frequency  $W$  of the reflected waves is measured and hence the speed of the submarine is calculated.





11<sup>th</sup>  
STD.

## Govt. MODEL QUESTION PAPER

TIME ALLOWED : 2.30 HOURS

## PHYSICS

MAXIMUM MARKS : 70

- Instructions :** (1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.  
(2) Use Blue or Black ink to write and underline and pencil to draw diagrams.

**PART - I****Note :** (i) Answer all the questions: (15 × 1 = 15)

(ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer.

- A substance whose mass is 4.27 g occupies 1.3 cm<sup>3</sup>. The number of significant figure in density is  
(a) 1 (b) 2 (c) 3 (d) 4
- Which of the following physical quantities have same dimensional formula?  
(a) Torque and Work done  
(b) Energy and Angular momentum  
(c) Force and Torque  
(d) Angular momentum and Linear momentum
- The maximum value of fractional error in division of two quantities i.e.,  $x = \frac{A}{B}$  is  
(a)  $\frac{\Delta x}{x} = \mp \left( \frac{\Delta A}{A} - \frac{\Delta B}{B} \right)$  (b)  $\frac{\Delta x}{x} = \left( -\frac{\Delta A}{A} + \frac{\Delta B}{B} \right)$   
(c)  $\frac{\Delta x}{x} = \left( -\frac{\Delta A}{A} + \frac{\Delta B}{B} \right)$  (d)  $\frac{\Delta x}{x} = \left( \frac{\Delta A}{A} + \frac{\Delta B}{B} \right)$
- The unit vector in the direction of  $\vec{A} = \hat{i} + \hat{j} + \hat{k}$  is  
(a)  $\hat{i} + \hat{j} + \hat{k}$  (b)  $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{2}}$   
(c)  $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$  (d)  $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{6}}$
- The position vector of the particle is  $\vec{r} = 3t^2 \hat{i} + 5t \hat{j} + 9 \hat{k}$ . What is the acceleration of the particle?  
(a) 6 m s<sup>-2</sup> along  $\hat{i}$  (b) 5 m s<sup>-2</sup> along  $\hat{j}$   
(c) 9 m s<sup>-2</sup> along  $\hat{k}$  (d) zero
- A body is whirled in a horizontal circle of radius vector  $\vec{r}$ . It has an angular velocity of  $\vec{\omega}$ . The velocity at any point on circular path is  
(a)  $v = r\omega$  (b)  $v = \frac{\omega}{r}$   
(c)  $v = \frac{r}{\omega}$  (d)  $v = m \frac{\omega}{r}$
- When a fast moving bus suddenly stops, the passenger is thrown forward because of  
(a) inertia of rest (b) inertia of direction  
(c) moment of inertia (d) inertia of motion
- In studying motion of a body, the starting of motion is more difficult than maintaining it because, the coefficient of static friction and kinetic friction satisfy the relation  
(a)  $\mu_s > \mu_k$  (b)  $\mu_s < \mu_k$  (c)  $\mu_s = \mu_k$  (d)  $\mu_s = \frac{1}{2} \mu_k$
- If two masses  $m_1$  and  $m_2$  are experiencing the same force, then the ratio of respective acceleration is  
(a)  $\frac{a_1}{a_2} = \frac{m_1}{m_2}$  (b)  $\frac{a_1}{a_2} = 1$   
(c)  $\frac{a_1}{a_2} = \frac{m_2}{m_1}$  (d)  $\frac{a_1}{a_2} = \sqrt{\frac{m_1}{m_2}}$
- What is the work done by the gravity when an object of mass  $m$  is taken from ground to some height  $h$  with constant velocity?  
(a)  $W = mgh$  (b)  $W = -mgh$   
(c)  $W = 0$  (d)  $W = 2 mgh$
- If the work done is independent of path, then the force is  
(a) Non-conservative force  
(b) conservative force  
(c) Newton's force (d) Centrifugal force
- One horse power is  
(a) 707 W (b) 786 W (c) 746 W (d) 647 W
- Four round objects namely a ring, a disc, a hollow sphere and a solid sphere with same radius  $R$  and made of same material start to roll down an inclined plane at the same time. The object that will reach the bottom third is  
(a) Solid sphere (b) disc  
(c) hollow sphere (d) ring
- The power delivered by the torque is  
(a)  $P = \vec{\tau} \cdot \vec{\theta}$  (b)  $P = \vec{\tau} \times \vec{\theta}$   
(c)  $P = \tau \theta \sin \theta$  (d)  $P = 0$  (zero always)
- The center of mass for a uniform rod of mass  $M$  and length  $\frac{1}{2}$  i.e.,  $0.5 l$  lies at the  
(a)  $l$  (b)  $0.75 l$  (c)  $0.5 l$  (d)  $0.25 l$

**PART - II****Answer any six questions in which question No. 20 is compulsory:** (6 × 2 = 12)

- Write down the number of significant figures in the following: (i) 0.007 (ii) 400.
- Write any two limitations of dimensional analysis. Give relevant examples.

11<sup>th</sup>  
STD.

## QUARTERLY COMMON EXAMINATION - 2019

TIME : 2.30 Hours

## PHYSICS

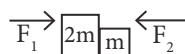
MAXIMUM MARKS : 70

- Instructions :** (1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.
- (2) Use only Blue or Black ink to write and underline and pencil to draw diagrams.

## PART - I

- Note :** (i) Answer all the questions: (15 × 1 = 15)
- (ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.

- The significant figure of the number 0.003401 is:  
(a) 6 (b) 3 (c) 5 (d) 4
- If the force is proportional to square of velocity, then the dimension of proportionality constant is:  
(a) [MLT<sup>0</sup>] (b) [MLT<sup>-1</sup>] (c) [ML<sup>-2</sup>T] (d) [ML<sup>-1</sup>T<sup>0</sup>]
- If a particle has negative velocity and negative acceleration, its speed  
(a) increases (b) decreases  
(c) remains same (d) zero
- A physical quantity is given by  $X = \frac{a^2 \sqrt{b}}{c^3}$ . If the percentage errors of measurement in a, b and c are 3%, 2% and 1% respectively, then the percentage error in X is  
(a) 5% (b) 10% (c) 8% (d) 6%
- If the object dropped vertically from the top of the building takes 2 second to reach the ground then the height of the building is ( $g = 10 \text{ ms}^{-2}$ )  
(a) 10m (b) 16m (c) 20m (d) 25m
- Two blocks of masses m and 2 m are placed on a smooth horizontal surface as shown. In the first case only a force  $F_1$  is applied from the left. Later only a force  $F_2$  is applied from the right. If the force acting at the interface of the two blocks in the two cases is same then  $F_1 : F_2$



- (a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 1 : 3

- Consider a circular leveled road of radius 10 m having coefficient of static friction 0.81. Three cars (A, B and C) are travelling with speed  $7 \text{ ms}^{-1}$ ,  $8 \text{ ms}^{-1}$  and  $10 \text{ ms}^{-1}$  respectively, which car will skid when it moves in the circular level road? ( $g = 10 \text{ ms}^{-2}$ ) :  
(a) A (b) B  
(c) C (d) Both B and C
- The centrifugal force appears to exist  
(a) only in any inertial frames  
(b) only in rotation frames  
(c) in any accelerated frames  
(d) both in inertial and non inertial frames
- A ball of mass 1 kg and another of mass 2 kg are dropped from a tall building whose height is 80 m. After, a fall of 40 m each towards Earth, their respective kinetic energies will be in the ratio of  
(a)  $\sqrt{2} : 1$  (b)  $1 : \sqrt{2}$  (c) 2 : 1 (d) 1 : 2
- If the linear momentum of the object is increased by 0.3% then the kinetic energy is increased by :  
(a) 0.1 % (b) 0.2 % (c) 0.4 % (d) 0.6 %
- What is the minimum velocity with which a body of mass m must enter a vertical loop of radius R so that it can complete the loop  
(a)  $\sqrt{2gR}$  (b)  $\sqrt{3gR}$  (c)  $\sqrt{5gR}$  (d)  $\sqrt{gR}$
- A close cylindrical container is partially filled with water. As the container rotates in a horizontal plane about a perpendicular bisector, its moment of inertia  
(a) increases (b) decreases  
(c) remains constant (d) depends on direction of rotation
- A rigid body rotates with an angular momentum L. If its kinetic energy is reduced to one fourth ( $1/4$ ) their angular momentum becomes:  
(a) L (b)  $L/2$  (c) 2L (d)  $L/\sqrt{2}$
- The speed of the center of a wheel rolling on a horizontal surface is  $V_0$ . A point on the rim in level with the center will be moving at a speed of:  
(a) 0 (b)  $V_0$  (c)  $\sqrt{2} V_0$  (d)  $2 V_0$
- Which of the following is scalar quantity?  
(a) momentum (b) work  
(c) force (d) Displacement

11<sup>th</sup>

STD.

## PUBLIC EXAM QUESTION PAPER MARCH - 2020

With Answer

## PHYSICS

TIME ALLOWED : 3.00 HOURS

MAXIMUM MARKS : 70

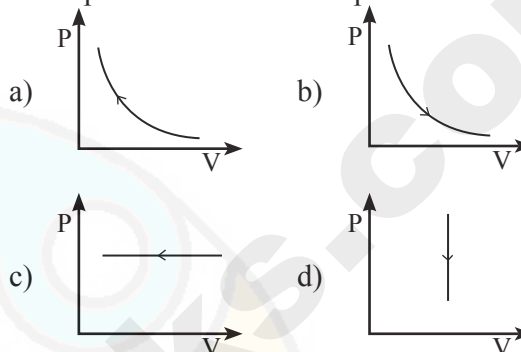
- Instructions :** (1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.
- (2) Use Blue or Black ink to write and underline and pencil to draw diagrams.

- Note :** (1) Answer all the questions. **(15 × 1 = 15)**
- (ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.

## PART - I

- When a car takes a sudden left turn on a curved road, passengers are pushed towards the right due to :  
a) absence of inertia   b) inertia of direction  
c) inertia of motion   d) inertia of rest
- A spring of force constant  $k$  is cut into two pieces such that the length of one piece is double the length of the other. Then the longer piece will have a force constant of :  
a)  $6k$    b)  $\frac{2}{3}k$    c)  $\frac{3}{2}k$    d)  $3k$
- The length of a body is measured as 3.51 m. If the accuracy is 0.01 m, then the percentage error in the measurement is :  
a) 0.035 %   b) 351%  
c) 1 %   d) 0.28 %
- Which one of the following is a scalar quantity?  
a) Speed  
b) Velocity  
c) Displacement  
d) Linear momentum
- If the distance between the Earth and Sun is twice its present value, the number of days in a year will be :  
a) 730   b) 1032  
c) 64.5   d) 182.5

6. Which one of the following P-V diagrams corresponds to isobaric compression?



- An air column in pipe which is closed at one end, is in resonance with the vibrating body of frequency 83 Hz. Then the length of the air column is : (velocity of sound in air = 332 ms<sup>-1</sup>)  
a) 1.5 m  
b) 0.5 m  
c) 2.0 m  
d) 1.0 m
- Identify the unit vector in the following :  
a)  $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$    b)  $\hat{i} + \hat{j}$   
c)  $\frac{\hat{i}}{\sqrt{2}}$    d)  $\hat{k} - \frac{\hat{i}}{\sqrt{2}}$
- The dimensional formula for Moment of Inertia :  
a)  $ML^{-1}T^{-1}$   
b)  $ML^2T^{-2}$   
c)  $MLT^2$   
d)  $ML^2$
- A body of mass 20 kg moving with a speed of 10 ms<sup>-1</sup> on a horizontal smooth surface collides with a massless spring of spring constant 5 N/m. If the mass stops after collision, distance of compression of the spring will be :  
a) 10 m   b) 50 m  
c) 5 m   d) 20 m