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CHAPTER : 3. FORMULA. (TRIGONOMETRY)

$\sin\theta = \frac{opp}{Hyp}$, $\operatorname{cosec}\theta = \frac{Hyp}{opp}$

$\cos\theta = \frac{Adj}{Hyp}$ $\sec\theta = \frac{Hyp}{adj}$

$\tan\theta = \frac{opp}{adj}$ $\cot\theta = \frac{adj}{opp}$.

$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$, $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$, $\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$, $\cot\theta = \frac{1}{\tan\theta}$.

θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt2}$	$\sqrt3/2$	1
$\cos\theta$	1	$\frac{\sqrt3}{2}$	$\frac{1}{\sqrt2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$1/\sqrt3$	1	$\sqrt3$	∞
$\operatorname{cosec}\theta$	∞	2	$\sqrt2$	$2/\sqrt3$	1
$\sec\theta$	1	$2/\sqrt3$	$\sqrt2$	2	∞
$\cot\theta$	∞	$\sqrt3$	1	$1/\sqrt3$	0.

$\sin^2\theta + \cos^2\theta = 1$ | $\sec^2\theta - \tan^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$ | $\sec^2\theta = 1 + \tan^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$ | $\tan^2\theta = \sec^2\theta - 1$

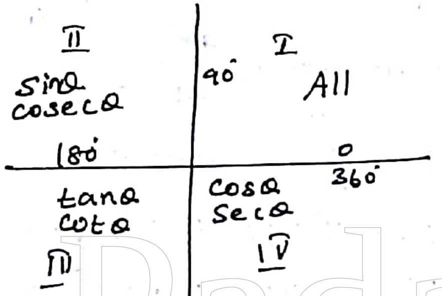
$\sin\theta = \sqrt{1-\cos^2\theta}$

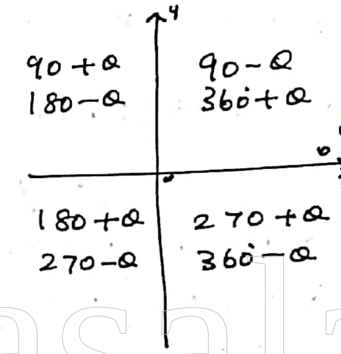
$\cos\theta = \sqrt{1-\sin^2\theta}$.

θ	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π
$n\pi$	30°	45°	60°	90°	180°	270°	360°

$Hyp = \sqrt{opp^2 + adj^2}$, $opp = \sqrt{Hyp^2 - adj^2}$

$adj = \sqrt{Hyp^2 - opp^2}$.





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தீர்வு

$\sin\theta = 0$, $\theta = n\pi,\ n\in Z$ | $\cos\theta = 0$, $\theta = (2n+1)\frac{\pi}{2}$ | $\tan\theta = 0$, $\theta = n\pi$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$.

$\sin 2A = 2\sin A\cos A$

$\cos 2A = \cos^2 A - \sin^2 A$
$= 1 - 2\sin^2 A$
$= 2\cos^2 A - 1$

$\sin 2A = \frac{2\tan A}{1+\tan^2 A}$

$\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$.

$\sin^2 A = \frac{1-\cos 2A}{2}$

$\cos^2 A = \frac{1+\cos 2A}{2}$

$\tan^2 A = \frac{1-\cos 2A}{1+\cos 2A}$.

$\sin 3A = 3\sin A - 4\sin^3 A$

$\cos 3A = 4\cos^3 A - 3\cos A$

$\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$.

$\sin\theta = 2\sin\theta/2\ \cos\theta/2$

$\cos\theta = \cos^2\theta/2 - \sin^2\theta/2$

$\cos\theta = 2\cos^2\theta/2 - 1$

$\cos\theta = 1 - 2\sin^2\theta/2$

$\sin A\cos B = \frac{1}{2}[\sin(A+B)+\sin(A-B)]$

$\cos A\sin B = \frac{1}{2}[\sin(A+B)-\sin(A-B)]$

$\cos A\cos B = \frac{1}{2}[\cos(A+B)+\cos(A-B)]$

$\sin A\sin B = \frac{1}{2}[\cos(A-B)-\cos(A+B)]$

$\sin C+\sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

$\sin C-\sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

$\cos C+\cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

$\cos C-\cos D = 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$

தீர்வு

(i) $\sin\theta = \sin\alpha$

$\theta = n\pi+(-1)^n\alpha,\ n\in Z$

(ii) $\cos\theta = \cos\alpha$

$\theta = 2n\pi\pm\alpha,\ n\in Z$

(iii) $\tan\theta = \tan\alpha$

$\theta = n\pi+\alpha,\ n\in Z.$

Inverse trigonometic functions

i) $\sin^{-1}x+\cos^{-1}x = \pi/2$

$\tan^{-1}x+\cot^{-1}x = \pi/2$

$\sec^{-1}x+\csc^{-1}x = \pi/2$

ii) $\sin^{-1}(-x) = -\sin^{-1}x$

$\cos^{-1}(-x) = \pi-\cos^{-1}x$

$\tan^{-1}(-x) = -\tan^{-1}x$

$\csc^{-1}(-x) = -\csc^{-1}x$

$\sec^{-1}(-x) = \pi-\sec^{-1}x$

$\cot^{-1}(-x) = -\cot^{-1}x.$

iii) $\sin^{-1}\left(\frac{1}{x}\right) = \csc^{-1}x$ | $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1}x$

$\tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1}x$ | $\csc^{-1}\left(\frac{1}{x}\right) = \sin^{-1}x$

$\sec^{-1}\left(\frac{1}{x}\right) = \cos^{-1}x$ | $\cot^{-1}\left(\frac{1}{x}\right) = \tan^{-1}(x)$

(iv) if $xy<1$, $\tan^{-1}x+\tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

(v) $\tan^{-1}x-\tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$

(vi) $\sin^{-1}x+\sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right)$

(vii) $\sin^2A-\sin^2B = \sin(A+B)\sin(A-B)$

1 radian $= \frac{180^\circ}{\pi} = 57^\circ 16'$

$1^\circ = \frac{\pi}{180}$ radian $= 0.01746$ radian

1) $\sin(-x) = -\sin x$ | $\cos(-x) = \cos x$

$\tan(-x) = -\tan x$ | $\sec(-x) = \sec x$

$\csc(-x) = -\csc x$ | $\cot(-x) = -\cot x$

Even function : $\cos x, \sec x$

odd function : $\sin x, \tan x, \csc x, \cot x$

	I	II	III	IV
$\sin x$	+	+	−	−
$\cos x$	+	−	−	+
$\tan x$	+	−	+	−
$\csc x$	+	+	−	−
$\sec x$	+	−	−	+
$\cot x$	+	−	+	−

Area formula

(i) $\Delta = \frac{1}{2}ab\sin C$ (iv) $\Delta = \frac{abc}{4R}$

(ii) $\Delta = \frac{1}{2}bc\sin A$

(iii) $\Delta = \frac{1}{2}ca\sin B.$

(v) $\Delta = 2R^2\sin A\sin B\sin C$

(vi) $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{a+b+c}{2}.$

sine formula

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R.$

Napier's formula

(i) $\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot C/2$

(ii) $\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot A/2$

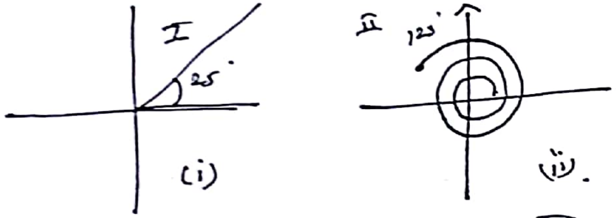
(iii) $\tan\left(\frac{C-A}{2}\right) = \frac{c-a}{c+a}\cot B/2$

Exercise 3.1 Chapter 3. TRIGONOMETRY.

1. Identify the quadrant in which an angle of each given measure lies.

(i) 25° which is lies in I quadrant.

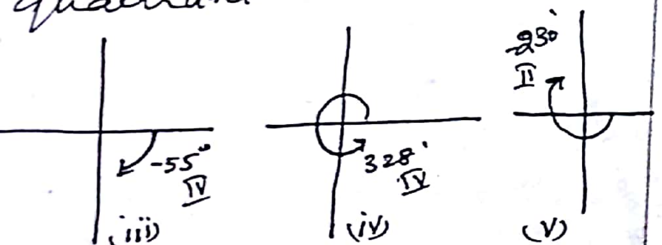
(ii) $825 = 2\times 360^\circ + 125^\circ$ which is lies in II quadrant



(iii) $-55^\circ \rightarrow$ which lies on IV quadrant. because angle is negative

(iv) 328° which lies on IV quadrant

(v) -230° which lies on II quadrant



2) For each given angle, find a coterminal angle with measure of θ such that $0 \le \theta \le 360^\circ$

Soln (i) $395^\circ = 360^\circ + 35^\circ$

$395 - 35 = 360^\circ$

$\therefore$ coterminal for 395° is 35

(ii) $525^\circ = 360^\circ + 165^\circ$

$\therefore$ Ans : 165

(iii) $1150^\circ = 360^\circ + 360^\circ + 360^\circ + 70^\circ$

$= 3(360^\circ) + 70^\circ$

coterminal angle $= 70^\circ$

(iv) $-270^\circ = -360^\circ + 90^\circ$

Coterminal is 90°

(v) $450^\circ = 360^\circ + 90^\circ$

Coterminal is 90°

(3) If $a\cos\theta - b\sin\theta = c$ show that $a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$

Soln $a\cos\theta - b\sin\theta = c$.

Squaring on both sides.

$(a\cos\theta - b\sin\theta)^2 = c^2$

$a^2\cos^2\theta + b^2\sin^2\theta \cancel{+} - 2ab\sin\theta\cos\theta = c^2$

$a^2(1-\sin^2\theta) + b^2(1-\cos^2\theta) \cancel{+} - 2ab\sin\theta\cos\theta = c^2.$

$\Rightarrow a^2 - a^2\sin^2\theta + b^2 - b^2\cos^2\theta - 2ab\sin\theta\cos\theta = c^2.$

$\Rightarrow -a^2\sin^2\theta - b^2\cos^2\theta - 2ab\sin\theta\cos\theta = c^2 - a^2 - b^2$

$a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta = a^2 + b^2 - c^2$

$(a\sin\theta + b\cos\theta)^2 = a^2+b^2-c^2$

$a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}.$

(4) If $\sin\theta + \cos\theta = m$ show that $\cos^6\theta + \sin^6\theta = \frac{4-3(m^2-1)^2}{4}$ where $m^2 \le 2$.

Soln LHS: $\cos^6\theta + \sin^6\theta$

$= (\cos^2\theta)^3 + (\sin^2\theta)^3$

$\because a^3+b^3 = (a+b)(a^2+b^2-ab)$

$= (\cos^2\theta + \sin^2\theta)\left[(\cos^2\theta)^2 + (\sin^2\theta)^2 - \cos^2\theta\sin^2\theta\right]$

$= 1\,(\cos^4\theta + \sin^4\theta - \cos^2\theta\sin^2\theta$

$= (\cos^2\theta)^2 + (\sin^2\theta)^2 - \cos^2\theta\sin^2\theta.$

$= (\cos^2\theta)^2 + (\sin^2\theta)^2 - 2\cos^2\theta\sin^2\theta + \cos^2\theta\sin^2\theta.$

$$\left[\because a^2+b^2+2ab=(a+b)^2\right]$$

$$= (\cos^2\alpha+\sin^2\alpha)^2-3\cos^2\alpha\sin^2\alpha$$

$$= 1-3\sin^2\alpha\cos^2\alpha.$$

RHS $\dfrac{4-3(m^2-1)^2}{4}$

$$=\frac{4-3\left[(\sin\alpha+\cos\alpha)^2-1\right]^2}{4}$$

$$=\frac{4-3(\sin^2\alpha+\cos^2\alpha+2\sin\alpha\cos\alpha-1)^2}{4}$$

$$=\frac{4-3(\cancel{1}+2\sin\alpha\cos\alpha-\cancel{1})^2}{4}$$

$$=\frac{4-3(2\sin\alpha\cos\alpha)^2}{4}$$

$$=\frac{4-3(4\sin^2\alpha\cos^2\alpha)}{4}$$

$$=\frac{\cancel{4}(1-3\sin^2\alpha\cos^2\alpha)}{\cancel{4}}$$

$$= 1-3\sin^2\alpha\cos^2\alpha$$

$\therefore$ RHS = LHS

Hence proved.

5) If $\dfrac{\cos^4\alpha}{\cos^2\beta}+\dfrac{\sin^4\alpha}{\sin^2\beta}=1$ prove that

(i) $\sin^4\alpha+\sin^4\beta=2\sin^2\alpha\sin^2\beta.$

Soln

$$\frac{\cos^4\alpha}{\cos^2\beta}+\frac{\sin^4\alpha}{\sin^2\beta}=1.$$

$$\frac{\sin^2\beta\cos^4\alpha+\cos^2\beta\sin^4\alpha}{\cos^2\beta\sin^2\beta}=1.$$

$$\cos^4\alpha(1-\cos^2\beta)+\cos^2\beta(1-\cos^2\alpha)^2 = \cos^2\beta(1-\cos^2\beta)$$

$$\cos^4\alpha-\cos^2\beta\cos^4\alpha+\cos^2\beta(1+\cos^4\alpha-2\cos^2\alpha]=\cos^2\beta(1)-\cos^4\beta$$

$$\cos^4\alpha-\cancel{\cos^2\beta\cos^4\alpha}+\cancel{\cos^2\beta}+\cancel{\cos^2\beta\cos^4\alpha}-2\cos^2\beta\cos^2\alpha=\cancel{\cos^2\beta}-\cos^4\beta$$

$$\cos^4\alpha+\cos^4\beta-2\cos^2\beta\cos^2\alpha$$

$$\Rightarrow (\cos^2\alpha-\cos^2\beta)^2=0$$

$$\cos^2\alpha-\cos^2\beta=0$$

$$\cos^2\alpha=\cos^2\beta$$

$$1-\sin^2\alpha=1-\sin^2\beta$$

$$\sin^2\alpha=\sin^2\beta \quad \text{—(1)}$$

(ii) $\sin^4\alpha+\sin^4\beta=2\sin^2\alpha\sin^2\beta$

LHS $\sin^4\alpha+\sin^4\beta$

$$=(\sin^2\alpha-\sin^2\beta)^2+2\sin^2\alpha\sin^2\beta$$

$$(a^2+b^2=(a-b)^2+2ab)$$

$$=0+2\sin^2\alpha\sin^2\beta \quad (\text{by } 1)$$

$$=2\sin^2\alpha\sin^2\beta.$$

(ii) $\dfrac{\cos^4\beta}{\cos^2\alpha}+\dfrac{\sin^4\beta}{\sin^2\alpha}=1$

LHS $\dfrac{\cos^4\beta}{\cos^2\alpha}+\dfrac{\sin^4\beta}{\sin^2\alpha}$

$$=\frac{\cos^2\beta\cos^2\beta}{\cos^2\alpha}+\frac{\sin^2\beta\sin^2\beta}{\sin^2\alpha}$$

$$=\frac{\cos^2\beta\cancel{\cos^2\alpha}}{\cancel{\cos^2\alpha}}+\frac{\sin^2\beta\cancel{\sin^2\alpha}}{\cancel{\sin^2\alpha}}$$

(by 1 & 2)

$$=\cos^2\beta+\sin^2\beta=1=\text{RHS}.$$

6) If $y=\dfrac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}$

Prove that $\dfrac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}=y.$

Soln $y = \frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}$

$= \frac{2\sin\alpha}{(1+\sin\alpha)+\cos\alpha} \cdot \frac{(1+\sin\alpha)-\cos\alpha}{(1+\sin\alpha)-\cos\alpha}$

$= \frac{2\sin\alpha[(1+\sin\alpha)-\cos\alpha]}{(1+\sin\alpha)^2-\cos^2\alpha}$

$= \frac{2\sin\alpha[1+\sin\alpha-\cos\alpha]}{(1+\sin\alpha)^2-(1-\sin\alpha)^2}$

$[\because (1-\sin\alpha)^2=(1+\sin\alpha)(1-\sin\alpha)]$

$= \frac{2\sin\alpha(1+\sin\alpha-\cos\alpha)}{(1+\sin\alpha)[1+\sin\alpha-1+\sin\alpha]}$

$= \frac{\cancel{2\sin\alpha}(1+\sin\alpha-\cos\alpha)}{(1+\sin\alpha)(\cancel{2\sin\alpha})}$

$y = \frac{1+\sin\alpha-\cos\alpha}{1+\sin\alpha}$

Hence proved.

7) If $x=\sum_{n=0}^{\infty}\cos^{2n}\theta$, $y=\sum_{n=0}^{\infty}\sin^{2n}\theta$

$z=\sum_{n=0}^{\infty}\cos^{2n}\theta\sin^{2n}\theta$, $0<\theta<\frac{\pi}{2}$

Show that $xyz = x+y+z$.

Soln $x=\sum_{n=0}^{\infty}\cos^{2n}\theta$

$=\cos^0\theta+\cos^2\theta+\cos^4\theta+\dots$

$=1+(\cos\theta)^2+(\cos^2\theta)^2+\dots$

$=\frac{1}{1-\cos^2\theta}$ $\left[\because 1+x+x^2\dots=\frac{1}{1-x}\right.$ if $|x|<1)$

$=\frac{1}{\sin^2\theta}$

$y=\sum_{n=0}^{\infty}\sin^{2n}\theta=\sin^0\theta+\sin^2\theta+\sin^4\theta+\dots$

$=1+\sin^2\theta+\sin^4\theta+\dots$

$=1+(\sin\theta)^2+(\sin^2\theta)^2+\dots$

$=\frac{1}{1-\sin^2\theta}=\frac{1}{\cos^2\theta}$.

Similary $z=\sum_{n=0}^{\infty}\cos^{2n}\theta\sin^{2n}\theta$

$=\cos^0\theta\sin^0\theta+\cos^2\theta\sin^2\theta+\cos^4\theta\sin^4\theta+\dots$

$=1+(\sin\theta\cos\theta)^2+(\sin^2\theta\cos^2\theta)^2+\dots$

$=\frac{1}{1-\sin^2\theta\cos^2\theta}$.

$x+y+z=\frac{1}{\sin^2\theta}+\frac{1}{\cos^2\theta}+\frac{1}{1-\sin^2\theta\cos^2\theta}$.

$=\frac{\cos^2\theta+\sin^2\theta}{\sin^2\theta\cos^2\theta}+\frac{1}{1-\sin^2\theta\cos^2\theta}$

$=\frac{1}{\sin^2\theta\cos^2\theta}+\frac{1}{1-\sin^2\theta\cos^2\theta}$

$=\frac{1-\sin^2\theta\cos^2\theta+\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta(1-\sin^2\theta\cos^2\theta)}$

$=\frac{1}{\sin^2\theta\cos^2\theta(1-\sin^2\theta\cos^2\theta)}$

$=\frac{1}{\sin^2\theta}\cdot\frac{1}{\cos^2\theta}\cdot\frac{1}{(1-\sin^2\theta\cos^2\theta)}$

$= x\,y\,z = RHS$.

Hence proved.

8) If $\tan^2\theta=1-k^2$ show that $\sec\theta+\tan^3\theta\,\mathrm{cosec}\,\theta=(2-k^2)^{3/2}$. Also find the values of k for which this result holds.

Soln $\tan^2\theta=1-k^2$

$1+\tan^2\theta=1+1-k^2$

$\sec^2\theta=2-k^2$

$(\sec^2\theta)^{3/2}=(2-k^2)^{3/2}$

$\sec^3\theta=(2-k^2)^{3/2}$.

$\sec\theta \sec^2\theta = (2-k^2)^{3/2}$

$\sec\theta(1+\tan^2\theta) = (2-k^2)^{3/2}$

$\sec\theta + \sec\theta\tan^2\theta = (2-k^2)^{3/2}$

$\sec\theta + \frac{1}{\cos\theta}\cdot\frac{\sin^2\theta}{\cos^2\theta} = (2-k^2)^{3/2}$

$\sec\theta + \frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\sin\theta}\cdot\frac{\sin^2\theta}{\cos^2\theta} = (2-k^2)^{3/2}$

$\sec\theta + \tan\theta\cdot\text{cosec}\,\theta\tan^2\theta = (2-k^2)^{3/2}$

$\sec\theta + \text{cosec}\,\theta\tan^3\theta = (2-k^2)^{3/2}$

Hence proved.

9) If $\sec\theta+\tan\theta = p$ obtain the values of $\sec\theta$, $\tan\theta$ and $\sin\theta$ in terms of p.

Soln $\sec\theta+\tan\theta = p$ — ①

$\sec^2\theta - \tan^2\theta = 1$ (∵ formula)

$(\sec\theta+\tan\theta)(\sec\theta-\tan\theta) = 1$

$p(\sec\theta-\tan\theta) = 1$

$\sec\theta-\tan\theta = \frac{1}{p}$ — ②

① + ② $2\sec\theta = p+\frac{1}{p}$

$2\sec\theta = \frac{p^2+1}{p}$ — ③

① − ② $2\tan\theta = p-\frac{1}{p}$

$2\tan\theta = \frac{p^2-1}{p}$ — ④

④ ÷ ③ $\frac{2\tan\theta}{2\sec\theta} = \frac{p^2+1/\cancel{p}}{p^2-1/\cancel{p}}$

$\frac{\frac{\sin\theta}{\cos\theta}}{1/\cos\theta} = \frac{p^2+1}{p^2-1}$

$\frac{\sin\theta}{\cancel{\cos\theta}}\times\cancel{\cos\theta} = \frac{p^2+1}{p^2-1}$

$\sin\theta = \frac{p^2+1}{p^2-1}$.

10) If $\cot\theta(1+\sin\theta) = 4m$ and $\cot\theta(1-\sin\theta) = 4n$ prove that $(m^2-n^2)^2 = mn$.

Soln (i) $\cot\theta(1+\sin\theta) = 4m$

$\cot\theta + \cot\theta\sin\theta = 4m$

$\cot\theta + \frac{\cos\theta}{\cancel{\sin\theta}}\cdot\cancel{\sin\theta} = 4m$

$\cot\theta+\cos\theta = 4m$. — ①

$\cot\theta(1-\sin\theta) = 4n$.

$\cot\theta - \cot\theta\sin\theta = 4n$

$\cot\theta - \frac{\cos\theta}{\cancel{\sin\theta}}\cdot\cancel{\sin\theta} = 4n$

$\cot\theta-\cos\theta = 4n$ — ②

①^2 + ②^2 ~~$2\cot\theta = 4$~~

$(\cot\theta+\cos\theta)^2 + (\cot\theta-\cos\theta)^2$
$= (4m)^2 - (4n)^2$

$\cot^2\theta + \cos^2\theta + 2\cot\theta\cos\theta$
$+\cot^2\theta + \cos^2\theta + 2\cot\theta\cos\theta$
$= 16m^2-16n^2$

$4\cot\theta\cos\theta = \cancel{16}^{4}(m^2-n^2)$

$\frac{\cos\theta}{\sin\theta}\cdot\cos\theta = 4(m^2-n^2)$.

$\frac{\cos^2\theta}{\sin\theta} = 4(m^2-n^2)$

Squaring both sides, we get

$\frac{\cos^4\theta}{\sin^2\theta} = 16(m^2-n^2)$ — ③

multiplying ① and ② we get

$[\cot\theta+\cos\theta)(\cot\theta-\cos\theta) = 16mn$.

$\cot^2\theta - \cos^2\theta = 16mn$.

$\frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta = 16mn$.

$\frac{\cos^2\theta - \cos^2\theta\sin^2\theta}{\sin^2\theta} = 16mn$.

$\frac{\cos^2\theta(1-\sin^2\theta)}{\sin^2\theta} = 16mn$.

$$\frac{\cos^2\theta \cos^2\theta}{\sin^2\theta} = 16mn.$$

$$\frac{\cos^4\theta}{\sin^2\theta} = 16mn.$$

From (3) and (4)

$$16(m^2-n^2)^2 = 16mn.$$

$$(m^2-n^2)^2 = mn.$$

(11) If $\csc\theta - \sin\theta = a^3$ then prove $\sec\theta - \cos\theta = b^3$ then prove that $a^2b^2(a^2+b^2) = 1$

Soln $a^3 = \csc\theta - \sin\theta$

$$= \frac{1}{\sin\theta} - \sin\theta = \frac{1-\sin^2\theta}{\sin\theta}$$

$$a = \frac{\cos^2\theta}{\sin\theta}$$

$$a = \left(\frac{\cos^2\theta}{\sin\theta}\right)^{1/3}$$

$$b^3 = \sec\theta - \cos\theta = \frac{1}{\cos\theta} - \cos\theta$$

$$= \frac{1-\cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta}$$

$$b = \left(\frac{\sin^2\theta}{\cos\theta}\right)^{1/3}$$

LHS: $a^2b^2(a^2+b^2)$

$$= \left(\frac{\cos^2\theta}{\sin\theta}\right)^{2/3}\cdot\left(\frac{\sin^2\theta}{\cos\theta}\right)^{2/3}\left[\left(\frac{\cos^2\theta}{\sin\theta}\right)^{2/3}+\left(\frac{\sin^2\theta}{\cos\theta}\right)^{2/3}\right]$$

$$= \frac{\cos^{4/3}\theta}{\sin^{2/3}\theta}\cdot\frac{\sin^{4/3}\theta}{\cos^{2/3}\theta}\left[\frac{\cos^{4/3}\theta}{\sin^{2/3}\theta}+\frac{\sin^{4/3}\theta}{\cos^{2/3}\theta}\right]$$

$$= \cos^{4/3-2/3}\theta\cdot\sin^{4/3-2/3}\theta\left[\frac{\cos^{6/3}\theta+\sin^{6/3}\theta}{\sin^{2/3}\theta\cos^{2/3}\theta}\right]$$

$$= \left(\cancel{\cos^{2/3}\theta}\cdot\cancel{\sin^{2/3}\theta}\right)\left[\frac{\cos^2\theta+\sin^2\theta}{\cancel{\sin^{2/3}\theta}\,\cancel{\cos^{2/3}\theta}}\right]$$

$$\cos^2\theta+\sin^2\theta = 1.$$

12) Eliminate θ from the equations $a\sec\theta - c\tan\theta = b$, $b\sec\theta + d\tan\theta = c$

Soln

$$a\sec\theta - c\tan\theta = b \quad —(1)$$

$$b\sec\theta + d\tan\theta = c \quad —(2)$$

(1) × b $\quad ab\sec\theta - bc\tan\theta = b^2$

(2) × a $\quad ab\sec\theta + ad\tan\theta = ac$

$\quad (-) \quad (-) \quad (-)$

$$-\tan\theta\,(bc+ad) = b^2-ac$$

$$\tan\theta = \frac{b^2-ac}{-(bc+ad)} = \frac{ac-b^2}{bc+ad}.$$

(1) × d ⇒ $ad\sec\theta - cd\tan\theta = bd$

(2) × c ⇒ $bc\sec\theta + cd\tan\theta = c^2$

$$(ad+bc)\sec\theta = bd+c^2$$

$$\sec\theta = \frac{bd+c^2}{ad+bc}.$$

We know that

$$\sec^2\theta - \tan^2\theta = 1$$

$$\left(\frac{bd+c^2}{ad+bc}\right)^2 - \left(\frac{ac-b^2}{bc+ad}\right)^2 = 1$$

$$\Rightarrow \frac{(bd+c^2)^2-(ac-b^2)^2}{(ad+bc)^2} = 1.$$

$$(bd+c^2)-(ac-b^2)^2 = (ad+bc)^2$$

$$(bd+c^2) = (ad+bc)^2+(ac-b^2)^2$$

பயிற்சி 3.2

1. Express each of the following angles in radian measure.

(i) $30° = \cancel{30}\times\frac{\pi}{\cancel{180}_6} = \frac{\pi}{6}$

2) $135° = \cancel{135}^{3}\times\frac{\pi}{\cancel{180}_4} = \frac{3\pi}{4}$

(iii) $-205° = -205 \times \frac{\pi}{180} = \frac{-41\pi}{36}$

(iv) $150° = 150° \times \frac{\pi}{180} = \frac{5\pi}{6}$.

(v) $330° = 330° \times \frac{\pi}{180} = \frac{11\pi}{6}$.

2) Find the degree measure corresponding to the following radian measures.

(i) $\frac{\pi}{3} = \frac{\pi}{3} \times \frac{180}{\pi} = 60°$

(ii) $\frac{\pi}{9} = \frac{\pi}{9} \times \frac{180}{\pi} = 20°$

(iii) $\frac{2\pi}{5} = \frac{2\pi}{5} \times \frac{180}{\pi} = 72°$

(iv) $\frac{7\pi}{3} = \frac{7\pi}{3} \times \frac{180}{\pi} = 420°$

(v) $\frac{10\pi}{9} = \frac{10\pi}{9} \times \frac{180}{\pi} = 200°$

③ What must be the radius of a circular running path around which an athlete must run 5 times in order to describe 1 km?

Soln 5 (circle around) = 1 km

$5(2\pi r) = 1000$ m.

$10 \times \frac{22}{7} \times r = 1000$

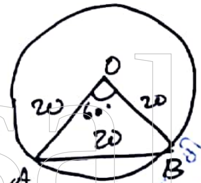
$r = \frac{1000 \times 7}{10 \times 22} = \frac{350}{11}$

$r = 31.818$ m

$r = 31.82$ m.

$11 \overline{)350}$ 31.81; 33; 20; 11; 90; 88; 20

4) In a circle of diameter 40 cm, a chord is of length 20 cm. Find the length of the minor arc of the chord?



Soln diameter $d = 40$ cm

$r = \frac{d}{2} = \frac{40}{2} = 20$ cm.

$AB = 20$ cm [chord]

$OA = OB = AB = 20$ cm

$\therefore \Delta AOB$ is equilateral Δ.

$\angle AOB = 60° = 60° \times \frac{\pi}{180} = \frac{\pi}{3}$ radians.

l be the length of the minor arc.

$\theta = \frac{l}{r}$

$l = r\theta = 20\left(\frac{\pi}{3}\right) = \frac{20\pi}{3}$

$20 \times \frac{22}{7} \times \frac{1}{3} = 20.95$ cm.

5) Find the degree measure of the angle subtended at the centre of radius 100 cm by an arc of length 22 cm.

Soln $r = 100$ cm.

length of arc = 22 cm.

$\theta = \frac{l}{r} = \frac{22}{100}$.

1 radian $= \frac{180°}{\pi}$

$\theta = 180° \times \frac{7}{22} \times \frac{22}{100}$.

$= \frac{18 \times 7}{10} = \frac{63}{5} = 12° \frac{3}{5}$

$= 12° 36'$ $\left[\frac{3}{5} = \frac{3}{5} \times 60 = 36'\right]$

6) What is the length of the arc intercepted by a central angle of measure 41° in a circle of radius 10 ft?

Soln $\theta = 41°$

$\theta = 41 \times \frac{\pi}{180} = \frac{41\pi}{180}$ radians

$r = 10$ ft

$\theta = \frac{l}{r}$

$l = r\theta$

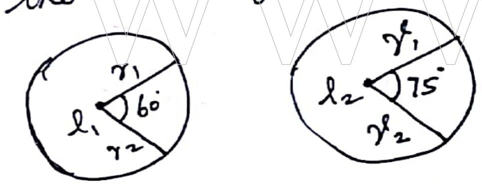
$l = 1\cancel{0} \times \frac{41\pi}{18\cancel{0}} = \frac{41}{189} \times \frac{\cancel{22}^{11}}{7}$

$l = \frac{41 \times 11}{9 \times 7} = \frac{451}{63}$

$l = 7.158 = 7.16$ ft.

7) If in two circles, arcs of same length subtend angles 60° and 75° at the center, find the ratio of their radii?

Soln



$\theta_1 = 60^\circ = 6\cancel{0} \times \frac{\pi}{18\cancel{0}} = \frac{\pi}{3}$ radians

$\theta_1 = \frac{l}{r_1} \Rightarrow l = \theta_1 r_1$

$l = \pi/3 \, r_1 \quad -(1)$

$\theta_2 = 75^\circ = 75^\circ \times \frac{\pi}{180} = \frac{15\pi}{36}$

$= \frac{5\pi}{12}$ radian.

$\theta_2 = \frac{l}{r_2} \Rightarrow l = \theta_2 r_2$

$l = \frac{5\pi}{12} \times r_2 \quad -(2)$

From (1) & (2)

$\frac{\pi}{3} r_1 = \frac{5\pi}{12} r_2$.

$\frac{r_1}{r_2} = \frac{5\cancel{\pi}}{\cancel{12}_4} \times \frac{\cancel{3}}{\cancel{\pi}}$

$\frac{r_1}{r_2} = 5/4$

$r_1 : r_2 = 5:4$.

8) The perimeter of a certain sector of a circle is equal to the length of the arc of a semi-circle having the same radius. Express the angle of the sector in degrees, min and sec.

Soln Perimeter of a sector $= l + 2r$

$= r\theta + 2r$

Length of a arc of a semi circle $= \pi r$

$r\theta + 2r = \pi r$

$\cancel{r}(\theta + 2) = \pi \cancel{r}$

$\theta + 2 = \pi$

$\theta = (\pi - 2)$ radians

$= (\pi - 2) \times \frac{180}{\pi} = \frac{180}{\pi} \times \pi - \frac{2 \times 180}{\pi}$

$= 180^\circ - \frac{360^\circ}{\pi}$

$= 180^\circ - 114^\circ 32' 44''$

$= 65^\circ 27' 16''$

$\frac{360}{\pi} = \frac{360}{22} \times 7$

$= 114^\circ \frac{12}{22}$

$= 114(\frac{12}{22} \times 60)$

$= 114^\circ 32' 44'$

$180^\circ . \cancel{60}\ \cancel{60}$

$114 . 32\ 44$

$65^\circ 27' 16''$

9) An airplane propeller rotates 1000 times per minute. Find the number of degrees that a point on the edge of the propeller will rotate in 1 second.

Soln One complete rotation $= 360^\circ$

No of degrees taken in $1' = 1000 \times 360$

No of degrees taken in $60' = 1000 \times 360$

No of degrees taken in 1'

$= \frac{1000 \times 36\cancel{0}}{6\cancel{0}}$

$= 6000$.

(c) A train is moving on a circular track of 1500m radius at the rate of 66 km/hr. what angle will it turn in 20 seconds?

Solution:-

length covered in 1hr by the train $= 66$ km.

length covered in (60×60) sec $= 66\times1000$ m.

Length covered in 1 sec

$= \frac{66\times100\cancel{0}}{6\cancel{0}\times6\cancel{0}}$ m. $= \frac{1100}{3}$

$l = \frac{1100}{3}$ m.

Angle made in 20 sec $= \frac{l}{r}$

$= \frac{1100}{3\times1500}$ radians

$\theta = \left(\frac{11}{45}\times\frac{180}{22}\times7\right)$ radians

$\theta = 14^\circ$

(d) A circular metallic plate of radius 8cm and thickness 6mm is melted and molded into a pie of radius 16cm and thickness 4mm. Find the angle of the sector.

soln:- $r = 8$ cm $h = 6\text{mm} = \frac{6}{10}$ cm.

volume of cyclinder = volume of a sector

$\cancel{\pi}r^2h = \frac{\theta}{360}\times\cancel{\pi}\times l^2\times h$

$8\times8\times\frac{6}{\cancel{10}} = \frac{\theta}{360}\times16\times16\times\frac{4}{\cancel{10}}$

$\theta = \frac{\cancel{8}\times\cancel{8}\times\cancel{6}^3\times\cancel{360}^{\cancel{180}\,45}}{\cancel{16}_2\times\cancel{16}_2\times\cancel{4}} = 3\times45$

$\theta = 135\,\text{cm}^2 = 13500^\circ$

$\theta = 13500\times\frac{\pi}{180} = 75\pi$ radians.

Exercise 3.3

1. Find the values of $\sin(480^\circ)$

(i) $\sin(480^\circ) = \sin(360^\circ+120^\circ)$

$= \sin120^\circ = \sin(90^\circ+30^\circ)$

$= \cos30^\circ = \sqrt{3}/2.$

(ii) $\sin(-1110^\circ) = -\sin(1110^\circ)$

$= -\sin(1080^\circ+30^\circ)$

$= -\sin[3\times360^\circ+30^\circ]$ $\quad 360\overline{)1110}^{\,3}$, $\frac{1080}{30}$

$= -\sin30^\circ = -\frac{1}{2}$

(3) $\cos(300^\circ) = \cos(360^\circ-60^\circ)$

$= \cos60^\circ = \frac{1}{2}.$

(4) $\tan1050^\circ = \tan(1080^\circ-30^\circ)$

$= \tan(3\times360^\circ-30^\circ)$

$= -\tan30^\circ = -\frac{1}{\sqrt{3}}$

(V) $\cot660^\circ = \cot(2\times360^\circ-60^\circ)$

$= -\cot60^\circ = -\frac{1}{\sqrt{3}}$

(VI) $\tan\left(\frac{19\pi}{3}\right) = \tan\left(\frac{19\times180^\circ}{3}\right)$

$= \tan1140^\circ$

$= \tan(3\times360^\circ+60^\circ)$

$= \tan60^\circ = \sqrt{3}.$

(Vii) $\sin\left(\frac{-11\pi}{3}\right) = \sin(-11\times60^\circ)$

$= \sin(-660^\circ)$

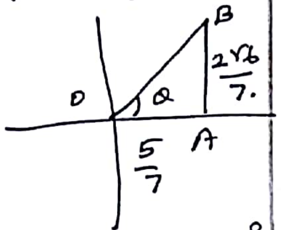
$= -\sin660^\circ$

$= -\sin(2\times360^\circ-60^\circ)$

$= +\sin60^\circ = \sqrt{3}/2.$

2) $\left(\frac{5}{7}, \frac{2\sqrt{6}}{7}\right)$ is a point on the terminal side of an angle θ in standard position. Determine the trigonometric function values of angle θ.

soln



$OB^2 = OA^2 + AB^2$

$= \left(\frac{5}{7}\right)^2 + \left(\frac{2\sqrt{6}}{7}\right)^2$

$= \frac{25}{49} + \frac{24}{49} = \frac{49}{49}$ $\quad \left[\because (2\sqrt{6})^2 = 4 \times 6 = 24\right]$

$OB^2 = 1$

$OB = 1.$

$\sin\theta = \frac{opp}{hyp} = \frac{AB}{OB} = \frac{2\sqrt{6}}{7}.$

$\cos\theta = \frac{adj}{hyp} = \frac{OA}{OB} = \frac{5}{7}.$

$\tan\theta = \frac{opp}{adj} = \frac{2\sqrt{6}/\cancel{7}}{5/\cancel{7}} = \frac{2\sqrt{6}}{5}$

$\cot\theta = \frac{5}{2\sqrt{6}},\quad \operatorname{cosec}\theta = \frac{7}{2\sqrt{6}}$

$\sec\theta = \frac{7}{5}.$

3) Find the values of the 5 trignometric functions for the following.

(i) $\cos\theta = -\frac{1}{2}$, θ lies in III quadrant

$AB^2 = AC^2 - BC^2$ $\qquad Hyp = 2$

$= 4 - 1 = 3$ $\qquad adj = 1$

$AB = \sqrt{3}. = opp.$

θ is in III quadrant.

$\tan\theta$, $\cot\theta$ are $+ve$.

$\therefore \sin\theta = \frac{opp}{hyp} = \frac{-\sqrt{3}}{2}$ $\qquad \tan\theta = \frac{opp}{adj} = \frac{+\sqrt{3}}{1} = \sqrt{3}.$

$\operatorname{cosec}\theta = \frac{-2}{\sqrt{3}}$ $\qquad \sec\theta = \frac{-2}{1} = -2$

$\cot\theta = \frac{1}{\sqrt{3}}$

(ii) Given $\cos\theta = \frac{2}{3}$, θ lies in the I quadrant.

$\cos\theta = \frac{2}{3} = \frac{adj}{hyp}.$

~~adj~~ $AB = \sqrt{3^2 - 2^2} = \sqrt{9-4} = \sqrt{5}$

$\theta \to$ I quadrant

All ratios are $+ve$.

$\sin\theta = \frac{opp}{hyp} = \frac{\sqrt{5}}{2}$ $\qquad \operatorname{cosec}\theta = \frac{2}{\sqrt{5}}$

$\cos\theta = \frac{2}{3}$ $\qquad \sec\theta = \frac{3}{2}$

$\tan\theta = \frac{opp}{hyp} = \frac{\sqrt{5}}{2}$ $\qquad \cot\theta = \frac{2}{\sqrt{5}}$

(iii) $\sin\theta = -\frac{2}{3}$, θ lies in the IV quadrant.

$\sin\theta = -\frac{2}{3} = \frac{opp}{hyp}.$

$Adj = \sqrt{3^2 - 2^2} = \sqrt{9-4} = \sqrt{5}.$

$\theta \to$ IV quadrant

$\cos\theta$, $\sec\theta$ are $+ve$

$\therefore \cos\theta = \frac{adj}{hyp} = +\frac{\sqrt{5}}{2}.$

$\sec\theta = \frac{2}{\sqrt{5}}$

$\operatorname{cosec}\theta = \frac{3}{-2}.$

$\tan\theta = \frac{opp}{adj} = \frac{-2}{\sqrt{5}}$

$\cot\theta = \frac{\sqrt{5}}{-2}.$

(iv) $\tan\theta = -2$, θ lies in the II quadrant.

$\tan\theta = -2 = \frac{-2}{1} = \frac{opp}{Adj}$

$opp = 2 \quad Adj = 1$

$Hyp = \sqrt{2^2+1^2} = \sqrt{4+1} = \sqrt{5}$.

$\theta \Rightarrow$ II quadrant

$\sin\theta$, $\csc\theta$ are +ve.

$\therefore \sin\theta = \frac{opp}{Hyp} = \frac{+2}{1} = 2$

$\csc\theta = \frac{1}{2}$

$\cot\theta = \frac{1}{-2}$

$\cos\theta = \frac{adj}{Hyp} = -\frac{1}{\sqrt{5}}$

$\sec\theta = -\sqrt{5}$

(v) $\sec\theta = \frac{13}{15}$, θ lies in the IV quadrant.

$\sec\theta = \frac{13}{5} = \frac{Hyp}{adj}$

$opp = \sqrt{13^2-5^2} = \sqrt{169-25}$

$= \sqrt{144} = 12$.

$\theta \Rightarrow$ IV quadrant

$\cos\theta$, $\sec\theta$ are +ve

$\cos\theta = \frac{5}{13}$, $\sin\theta = \frac{opp}{hyp} = -\frac{12}{13}$.

$\csc\theta = \frac{-13}{12}$ $\tan\theta = \frac{opp}{Adj} = -\frac{12}{5}$

$\cot\theta = \frac{-5}{12}$.

4) Prove that

$$\frac{\cot(180+\theta)\sin(90-\theta)\cos(-\theta)}{\sin(270+\theta)\tan(-\theta)\csc(360+\theta)} = \cos^2\theta\cot\theta.$$

Soln:- LHS:-

$\cot(180+\theta) = \cot\theta$

$\sin(90-\theta) = \cos\theta$

$\cos(-\theta) = \cos\theta$

$\sin(270+\theta) = -\cos\theta$

$\tan(-\theta) = -\tan\theta$

$\csc(360+\theta) = \csc\theta$.

$LHS = \frac{\cot\theta\ \cos\theta\ \cos\theta}{(-\cos\theta)(-\tan\theta)(\csc\theta)}$

$= \frac{\frac{\cos\theta}{\sin\theta}\cdot\cos^2\theta}{\cos\theta\cdot\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\sin\theta}}$

$= \cot\theta\cos^2\theta = RHS$.

5) Find all the angles between $0°$ and $360°$ which satisfy the equation $\sin^2\theta = 3/4$.

Soln $\sin^2\theta = 3/4$

$\sin\theta = \pm\sqrt{3}/2$.

$\sin\theta = \frac{\sqrt{3}}{2}$ (or) $\sin\theta = -\frac{\sqrt{3}}{2}$.

$\sin\theta = \sin 60°$ (or) $\sin\theta = -\sin 60°$

$\theta = 60°$ (or) $\theta = 180° - 60°$

$\theta = 120°$

6) Show that

$$\sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18} + \sin^2\frac{4\pi}{9} = 2.$$

LHS $\sin^2\left(\frac{\pi}{8}\times\frac{180}{\pi}\right) + \sin^2\left(\frac{\pi}{9}\times\frac{180}{\pi}\right) + \sin^2\left(\frac{7\pi}{18}\times\frac{180}{\pi}\right) + \sin^2\left(\frac{4\pi}{9}\times\frac{180}{\pi}\right)$

$= \sin^2 10 + \sin^2 20 + \sin^2 70 + \sin^2 80$

$= [\sin(90-80^\circ)]^2 + [\sin(90^\circ-70^\circ)]^2 + \sin^2 70^\circ + \sin^2 80^\circ$

$= \cos^2 80^\circ + \cos^2 70^\circ + \sin^2 70^\circ + \sin^2 80^\circ$

$= (\cos^2 80^\circ + \sin^2 80^\circ) + (\cos^2 70^\circ + \sin^2 70^\circ)$

$= 1 + 1 = 2 =$ RHS.

$[\because \sin^2\alpha + \cos^2\alpha = 1]$

Exercise 3·4

1· If $\sin x = \frac{15}{17}$ and $\cos y = \frac{12}{13}$, $0 < x < \pi/2$, $0 < y < \pi/2$ find the value of (i) $\sin(x+y)$.

Soln $0 < x < \pi/2$, $0 < y < \pi/2$

$\therefore x, y$ is in I quadrant.

All ratios are +ve.

$\sin x = \frac{15}{17} = \frac{opp}{hyp}$

adj $= \sqrt{17^2 - 15^2}$

$= \sqrt{289 - 225}$

$= \sqrt{64}$

$= 8$

$\cos y = \frac{12}{13}$

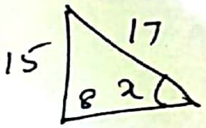
$\frac{12}{13} = \frac{adj}{hyp}$

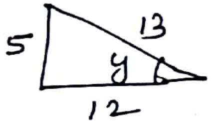
opp $= \sqrt{13^2 - 12^2}$

$= \sqrt{169 - 144}$

$= \sqrt{25}$

$= 5$.





$\sin x = \frac{15}{17}$.

$\cos x = \frac{8}{17}$

$\tan x = \frac{15}{8}$

$\sin y = 5/13$

$\cos y = 12/13$

$\tan y = \frac{5}{12}$.

(i) $\sin(x+y) = \sin x \cos y + \cos x \sin y$

$= \frac{15}{17} \cdot \frac{12}{13} + \frac{8}{17} \cdot \frac{5}{13}$.

$= \frac{180}{221} + \frac{40}{221} = \frac{220}{221}$.

(ii) $\cos(x-y) = \cos x \cos y + \sin x \sin y$

$= \frac{8}{17} \cdot \frac{12}{13} + \frac{15}{17} \cdot \frac{5}{13}$

$= \frac{96}{221} + \frac{75}{221} = \frac{171}{221}$.

(iii) $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$

$= \frac{\frac{15}{8} + \frac{5}{12}}{1 - \frac{15}{8} \cdot \frac{5}{12}} = \frac{\frac{180+40}{\cancel{96}}}{\frac{96-75}{\cancel{96}}}$

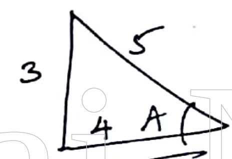
$= \frac{220}{21}$.

2) If $\sin A = \frac{3}{5}$ and $\cos B = \frac{9}{41}$, $0 < A < \pi/2$, $0 < B < \pi/2$ find the value of (i) $\sin(A+B)$.

Soln $0 < A < \pi/2$, $0 < B < \pi/2$ –

A, B lies in I quadrant

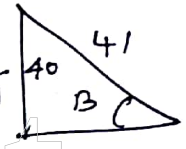
All ratios are +ve



$\sqrt{5^2 - 3^2}$

$= \sqrt{25 - 9}$

$= \sqrt{16} = 4$



$\sqrt{41^2 - 9^2}$

$= \sqrt{1681 - 81}$

$= \sqrt{1600}$

$= 40$.

$\sin A = \frac{3}{5}$

$\cos A = \frac{4}{5}$

$\sin B = \frac{40}{41}$

$\cos B = \frac{9}{41}$

(i) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$= \frac{3}{5} \cdot \frac{9}{41} + \frac{4}{5} \cdot \frac{40}{41}$

$= \frac{27}{205} + \frac{160}{205} = \frac{187}{205}$.

(ii) $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$= \frac{4}{5}\cdot\frac{9}{41} + \frac{3}{5}\cdot\frac{40}{41}$

$= \frac{36}{205} + \frac{120}{205} = \frac{156}{205}$.

3) Find $\cos(x-y)$ given that $\cos x = -\frac{4}{5}$ with $\pi < x < 3\pi/2$. $\sin y = -\frac{24}{25}$ with $\pi < y < \frac{3\pi}{2}$.

Soln Since $\pi < x < \frac{3\pi}{2}$

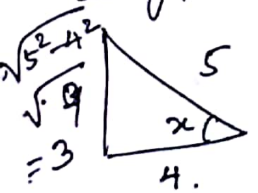
x lies in III quadrant.

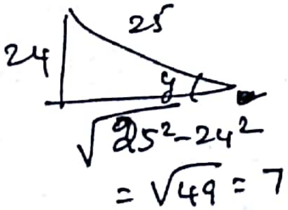
$\therefore \cos x, \tan x$ are +ve.

$\pi < y < 3\pi/2$

$\therefore y$ lies in III quadrant

$\cos y, \tan y$ are +ve





$\sin x = -\frac{3}{5}$ $\sin y = -\frac{24}{25}$

$\cos x = -\frac{4}{5}$ $\cos y = -\frac{7}{25}$

$\cos(x-y) = \cos x\cos y + \sin x\sin y$

$= \left(-\frac{4}{5}\right)\left(-\frac{7}{25}\right) + \left(-\frac{3}{5}\right)\left(-\frac{24}{25}\right)$

$= \frac{28}{125} + \frac{72}{125} = \frac{100}{125} = \frac{4}{5}$.

4) Find $\sin(x-y)$ given that $\sin x = \frac{8}{17}$ with $0 < x < \pi/2$ and $\cos y = -\frac{24}{25}$ with $\pi < y < \frac{3\pi}{2}$

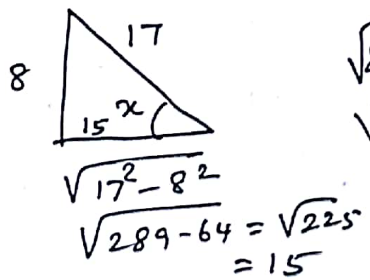
soln $0 < x < \pi/2$

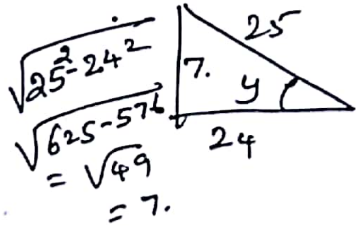
$\Rightarrow x$ lies in I quadrant

All ratios are +ve

$\pi < y < \frac{3\pi}{2} \Rightarrow y$ lies in III quadrant

$\therefore \tan y, \cot y$ are +ve.





$\sin x = \frac{8}{17}$ $\sin y = -\frac{7}{25}$

$\cos x = \frac{15}{17}$ $\cos y = -\frac{24}{25}$

$\sin(x-y) = \sin x\cos y - \cos x\sin y$

$= \left(\frac{8}{17}\right)\left(-\frac{24}{25}\right) - \left(\frac{15}{17}\right)\left(-\frac{7}{25}\right)$

$= -\frac{192}{425} + \frac{105}{425} = -\frac{87}{425}$.

5) Find the value of

(i) $\cos 105^\circ = \cos(60^\circ + 45^\circ)$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$

$= \frac{1}{2}\cdot\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}$

$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} = \frac{1-\sqrt{3}}{2\sqrt{2}}$

(ii) $\sin 105^\circ = \sin(60^\circ + 45^\circ)$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin 105^\circ = \sin 60^\circ \cos 45^\circ + \cos 45^\circ \sin 60^\circ$

$= \frac{\sqrt3}{2}\cdot\frac{1}{\sqrt2} + \frac{1}{2}\cdot\frac{1}{\sqrt2}$

$= \frac{\sqrt3}{2\sqrt2} + \frac{1}{2\sqrt2} = \frac{\sqrt3+1}{2\sqrt2}$

(iii) $\tan\left(\frac{7\pi}{12}\right) = \tan\left(\frac{7\times 180^\circ}{12}\right)$

$= \tan 105^\circ = \frac{\sin 105^\circ}{\cos 105^\circ}$

$= \frac{\sqrt3+1/\cancel{2\sqrt2}}{1-\sqrt3/\cancel{2\sqrt2}}$ (by (i) & (ii))

$= \frac{\sqrt3+1}{1-\sqrt3}\cdot\frac{(1+\sqrt3)}{(1+\sqrt3)}$

$= \frac{(\sqrt3+1)^2}{1^2-\sqrt3^2} = \frac{\sqrt3^2+1^2+2\sqrt3}{1-3}$

$= \frac{3+1+2\sqrt3}{-2} = \frac{4+2\sqrt3}{-2}$

$= \frac{\cancel{2}(2+\sqrt3)}{-\cancel{2}}$

$= -(2+\sqrt3)$.

6) Prove that

(i) $\cos(30^\circ+x) = \frac{\sqrt3\cos x - \sin x}{2}$.

Soln LHS:

$\cos(30^\circ+x) =$

$= \cos 30^\circ \cos x - \sin 30^\circ \sin x$

$= \frac{\sqrt3}{2}\cos x - \frac{1}{2}\sin x$

$= \frac{\sqrt3\cos x - \sin x}{2}$. Hence proved

(ii) $\cos(\pi+\alpha) = -\cos\alpha$

LHS $= \cos(\pi+\alpha) = -\cos\alpha$

Since $\pi+\alpha$ is III quadrant

(iii) $\sin(\pi+\alpha) = -\sin\alpha$

Since $\pi+\alpha$ lies in III quadrant.

7) Find a quadrant equation whose roots are $\sin 15^\circ$ and $\cos 15^\circ$.

Solution:

Sum of the roots $= \sin 15^\circ + \cos 15^\circ$

$= \sin(45^\circ-30^\circ) + \cos(45^\circ-30^\circ)$

$= (\sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ) + (\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ)$

$= \left(\frac{1}{\sqrt2}\cdot\frac{\sqrt3}{2} - \frac{1}{\sqrt2}\cdot\frac{1}{2}\right) + \left(\frac{1}{\sqrt2}\cdot\frac{\sqrt3}{2} + \frac{1}{\sqrt2}\cdot\frac{1}{2}\right)$

$= \frac{\sqrt3}{2\sqrt2} - \cancel{\frac{1}{2\sqrt2}} + \frac{\sqrt3}{2\sqrt2} + \cancel{\frac{1}{2\sqrt2}}$

$= \frac{\sqrt3+\sqrt3}{2\sqrt2} = \frac{2\sqrt3}{2\sqrt2}\cdot\frac{\sqrt2}{\sqrt2}$

$= \frac{\sqrt6}{2}$.

Product of the roots

$= \sin 15^\circ \cos 15^\circ = \frac{2\sin 15^\circ \cos 15^\circ}{2}$

$= \frac{\sin 2(15^\circ)}{2} = \frac{\sin 30^\circ}{2}$

$= \frac{1/2}{2} = \frac{1}{4}$. ($\sin 2A = 2\sin A\cos A$)

Formula

$x^2 - (\text{sum of roots})x + (\text{Pdt of root}) = 0$

$x^2 - \frac{\sqrt6}{2}x + \frac{1}{4} = 0$

$\frac{4x^2 - 2\sqrt6 x + 1}{4} = 0$

$4x^2 - 2\sqrt6 x + 1 = 0$

8) Expand $\cos(A+B+C)$. Hence prove that $\cos A\cos B\cos C = \sin A\sin B\cos C + \sin B\sin C\cos A + \sin C\sin A\cos B$. If $A+B+C=\pi/2$.

Soln Given $A+B+C=\frac{\pi}{2}$

$\cos(A+B+C) = \cos\pi/2 = 0$

$\Rightarrow \cos[(A+B)+C] = 0$

$\Rightarrow \cos(A+B)\cos C - \sin(A+B)\sin C = 0$

$\Rightarrow \cos C[\cos A\cos B - \sin A\sin B] - \sin C[\sin A\cos B + \cos A\sin B] = 0$

$\Rightarrow \cos A\cos B\cos C - \sin A\sin B\cos C - \sin A\cos B\sin C - \cos A\sin B\sin C = 0$

$\cos A\cos B\cos C = \sin A\sin B\cos C + \sin A\cos B\sin C + \cos A\sin B\sin C$

Hence proved.

9) Prove that

(i) $\sin(45^\circ+\alpha) - \sin(45^\circ-\alpha) = \sqrt{2}\sin\alpha$

Solution:- LHS:-

$\sin(45+\alpha) - \sin(45^\circ-\alpha)$

$= (\sin 45^\circ\cos\alpha + \cos 45^\circ\sin\alpha) - (\sin 45^\circ\cos\alpha - \cos 45^\circ\sin\alpha)$

$= \cancel{\frac{1}{\sqrt{2}}\cos\alpha} + \frac{1}{\sqrt{2}}\sin\alpha - \cancel{\frac{1}{\sqrt{2}}\cos\alpha} + \frac{1}{\sqrt{2}}\sin\alpha$

$= \frac{2\sin\alpha}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\cancel{2}\sqrt{2}\sin\alpha}{\cancel{2}}$

$= \sqrt{2}\sin\alpha$

(ii) $\sin(30^\circ+\alpha) + \cos(60^\circ+\alpha)$

$= [\sin 30^\circ\cos\alpha + \cos 30^\circ\sin\alpha] + \cos 60^\circ\cos\alpha - \sin 60^\circ\sin\alpha$

$= \frac{1}{2}\cos\alpha + \cancel{\frac{\sqrt{3}}{2}\sin\alpha} + \frac{1}{2}\cos\alpha - \cancel{\frac{\sqrt{3}}{2}\sin\alpha}$

$= \frac{2\cos\alpha}{2} = \cos\alpha =$ RHS.

10) If $a\cos(x+y) = b\cos(x-y)$ show that $(a+b)\tan x = (a-b)\cot y$.

Soln $a\cos(x+y) = b\cos(x-y)$

$a[\cos x\cos y - \sin x\sin y] = b[\cos x\cos y + \sin x\sin y]$

$a\cos x\cos y - a\sin x\sin y = b\cos x\cos y + b\sin x\sin y$

$a\cos x\cos y - b\cos x\cos y = b\sin x\sin y + a\sin x\sin y$

$(a-b)\cos x\cos y = (a+b)\sin x\sin y$

$(a-b)\frac{\cos y}{\sin y} = (a+b)\frac{\sin x}{\cos x}$

$(a-b)\cot y = (a+b)\tan x$

Hence proved.

11. Prove that $\sin 105^\circ + \cos 105^\circ = \cos 45^\circ$

LHS $\sin 105^\circ + \cos 105^\circ$

$= \sin(60^\circ + 45^\circ) + \cos(60^\circ + 45^\circ)$

$= \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ + \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$

$= \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$

$= \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{2}{2\sqrt{2}}$

$= \frac{1}{\sqrt{2}} = \cos 45^\circ$ H.P.

(12) Prove that $\sin 75^\circ - \sin 15^\circ = \cos 105^\circ + \cos 15^\circ$

LHS $\sin 75^\circ - \sin 15^\circ$

$= \sin(90^\circ - 15^\circ) - \sin 15^\circ$

$= \cos 15^\circ - \sin 15^\circ$ — (1)

$(\cos\theta = \sin(90 - \theta))$

RHS $\cos 105^\circ + \cos 15^\circ$

$= \cos(90^\circ + 15^\circ) + \cos 15^\circ$

$= -\sin 15^\circ + \cos 15^\circ$

$= \cos 15^\circ - \sin 15^\circ$ — (2)

by (1) & (2) LHS = RHS.

13) Show that $\tan 75^\circ + \cot 75^\circ = 4$

$\tan 75^\circ = \tan(45^\circ + 30^\circ)$

$= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$

$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1 / \cancel{\sqrt{3}}}{\frac{\sqrt{3}-1}{\cancel{\sqrt{3}}}}$

$= \frac{\sqrt{3}+1}{\sqrt{3}-1}.$

$\cot 75^\circ = \frac{1}{\tan 75^\circ} = \frac{\sqrt{3}-1}{\sqrt{3}+1}.$

LHS $= \tan 75^\circ + \cot 75^\circ$

$= \frac{\sqrt{3}+1}{\sqrt{3}-1} + \frac{\sqrt{3}-1}{\sqrt{3}+1}.$

$= \frac{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2}{\sqrt{3}^2 - 1^2}$

$= \frac{\sqrt{3}^2 + 1 + \cancel{2\sqrt{3}} + \sqrt{3}^2 + 1 - \cancel{2\sqrt{3}}}{3 - 1}$

$\frac{3+1+3+1}{2} = \frac{8}{2} = 4 = $ RHS.

14) Prove that $\cos(A+B)\cos C - \cos(B+C)\cos A = \sin B \sin(C-A)$

LHS $\cos(A+B)\cos C - \cos(B+C)\cos A$

$= (\cos A \cos B - \sin A \sin B)\cos C - [\cos B \cos C - \sin B \sin C)\cos A$

$= \cancel{\cos A \cos B \cos C} - \sin A \sin B \cos C - \cancel{\cos B \cos C \cos A} + \sin B \sin C \cos A$

$= \cos A \sin B \sin C - \sin A \sin B \cos C$

$= \sin B(\cos A \sin C - \sin A \cos C)$

$= \sin B(\sin C \cos A - \cos C \sin A)$

$= \sin B [\sin(C - A)]$

$= \sin B \sin(C - A)$

= RHS

H.P.

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15) Prove that $\sin(n+1)\theta \sin(n-1)\theta + \cos(n+1)\theta \cos(n-1)\theta = \cos 2\theta,\ n \in Z$.

Soln LHS.

$\sin(n+1)\theta \sin(n-1)\theta + \cos(n+1)\theta \cos(n-1)\theta$.

$(\because \cos(A-B) = \cos A \cos B + \sin A \sin B)$

$= \cos[(n+1)\theta - (n-1)\theta]$

$= \cos[\cancel{n\theta} + \theta - \cancel{n\theta} + \theta]$

$= \cos 2\theta = RHS.$

16) If $x\cos\theta = y\cos(\theta + \frac{2\pi}{3}) = z\cos(\theta + \frac{4\pi}{3})$ find the value $xy + yz + zx$.

Soln $x\cos\theta = \lambda$

$\cos\theta = \lambda/x$ — (1)

$y\cos(\theta + \frac{2\pi}{3}) = \lambda$

$\cos(\theta + \frac{2\pi}{3}) = \lambda/y$ — (2)

$z\cos(\theta + \frac{4\pi}{3}) = \lambda$

$\cos(\theta + \frac{4\pi}{3}) = \lambda/z$ — (3)

$\frac{\lambda}{x} + \frac{\lambda}{y} + \frac{\lambda}{z}$

$= \cos\theta + \cos(\theta + \frac{2\pi}{3}) + \cos(\theta + \frac{4\pi}{3})$

$= \cos\theta + \cos(120 + \theta) + \cos(240 + \theta)$

$= \cos\theta + \cos 120^\circ \cos\theta - \sin 120^\circ \sin\theta + \cos 240^\circ \cos\theta - \sin 240 \sin\theta$

$= \cos\theta - \frac{1}{2}\cos\theta - \cancel{\frac{\sqrt{3}}{2}\sin\theta} - \frac{1}{2}\cos\theta + \cancel{\sqrt{3}/2 \sin\theta}$.

$[\because \cos 120^\circ = \cos(90^\circ + 30^\circ) = -\sin 30^\circ = -\frac{1}{2}$

$\sin 120^\circ = \sin(90^\circ + 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$\cos 240^\circ = \cos(180^\circ + 60^\circ) = -\cos 60^\circ = -\frac{1}{2}$

$\sin 240^\circ = \sin(180^\circ + 60^\circ) = -\sin 60^\circ = -\frac{\sqrt{3}}{2}]$

$= \cos\theta - \frac{1}{2}\cos\theta - \frac{1}{2}\cos\theta$

$= \cos\theta - \cos\theta = 0$

$\lambda(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}) = 0.$

$\lambda\left(\frac{yz + xz + xy}{xyz}\right) = 0.$

$xy + xz + yz = 0.$

(17) Prove that.

(i) $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$

LHS $\sin(A+B)\sin(A-B)$

$= [\sin A \cos B + \cos A \sin B][\sin A \cos B - \cos A \sin B]$

$= \sin^2 A \cos^2 B - \cos^2 A \sin^2 B$

$[\because (a+b)(a-b) = a^2 - b^2]$

$= \sin^2 A(1 - \sin^2 B) - (1 - \sin^2 A)\sin^2 B$

$= \sin^2 A - \cancel{\sin^2 A \sin^2 B} - \sin^2 B + \cancel{\sin^2 A \sin^2 B}$

$= \sin^2 A - \sin^2 B.$

(ii) $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A.$

LHS $= \cos(A+B)\cos(A-B)$

$= [\cos A\cos B - \sin A\sin B]\cdot[\cos A\cos B + \sin A\sin B]$

$= \cos^2 A\cos^2 B - \sin^2 A\sin^2 B$ — (I)

$= \cos^2 A(1-\sin^2 B) - (1-\cos^2 A)\sin^2 B$

$= \cos^2 A - \sin^2 B\cos^2 A - \sin^2 B + \cos^2 A\sin^2 B$

$= \cos^2 A - \sin^2 B$ — (2)

LHS $= \cos^2 A\cos^2 B - \sin^2 A\sin^2 B$ by (I)

$= (1-\sin^2 A)\cos^2 B - \sin^2 A(1-\cos^2 B)$

$= \cos^2 B - \sin^2 A\cos^2 B - \sin^2 A + \sin^2 A\cos^2 B$

$= \cos^2 B - \sin^2 A$ — (3)

by (2) & (3)

(ii) is proved.

(iii) $\sin^2(A+B) - \sin^2(A-B) = \sin 2A\sin 2B$

LHS $\sin^2(A+B) - \sin^2(A-B)$

$= \sin[A+B+A-B]\sin[(A+B)-(A-B)]$

$= \sin[2A]\sin[A+B-A+B]$

$= \sin 2A\sin 2B$ = RHS.

iv) $\cos 8\theta\cos 2\theta = \cos^2 5\theta \sin^2 3\theta$

RHS $\cos^2 5\theta - \sin^2 3\theta$

$= \cos(5\theta+3\theta)\cos(5\theta-3\theta)$

$(\because \cos^2 B - \sin^2 A = \cos(A+B)\cos(A-B))$

$= \cos 8\theta\cos 2\theta$ = LHS.

(18) Show that $\cos^2 A + \cos^2 B - 2\cos A\cos B\cos(A+B) = \sin^2(A+B)$

Soln LHS.

$\cos^2 A + \cos^2 B - 2\cos A\cos B\cos(A+B)$

$= \cos^2 A + (1-\sin^2 B) - 2\cos A\cos B\cos(A+B)$

$= \cos^2 A + 1 - \sin^2 B - 2\cos A\cos B\cos(A+B)$

$= \cos^2 A - \sin^2 B + 1 - 2\cos A\cos B\cos(A+B)$

$= \cos(A+B)\cos(A-B) + 1 - 2\cos A\cos B\cos(A+B)$

$= \cos(A+B)[\cos(A-B) - 2\cos A\cos B] + 1$

$= \cos(A+B)[\cos A\cos B + \sin A\sin B - 2\cos A\cos B] + 1$

$= \cos(A+B)[\sin A\sin B - \cos A\cos B] + 1$

$= -\cos(A+B)[\cos A\cos B - \sin A\sin B] + 1$

$= -\cos(A+B)\cos(A+B) + 1$

$= -\cos^2(A+B) + 1$

$= 1 - \cos^2(A+B)$

$= \sin^2(A+B)$ $(\because \sin^2\theta = 1-\cos^2\theta)$

19) If $\cos(\alpha-\beta) + \cos(\beta-\gamma) + \cos(\gamma-\alpha) = -\frac{3}{2}$.

Prove that $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$

Soln

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos(\beta-\gamma) = \cos\beta\cos\gamma + \sin\alpha\sin\gamma$

$\cos(\gamma-\alpha) = \cos\gamma\cos\alpha + \sin\gamma\sin\alpha$

LHS: Add these.

$\Rightarrow 2[\cos\alpha\cos\beta+\sin\alpha\sin\beta + \cos\beta\cos\gamma+\sin\beta\sin\gamma) + \cos\gamma\cos\alpha+\sin\alpha\sin\gamma) = \frac{-3}{2}$

$\Rightarrow 2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta + 2\cos\beta\cos\gamma + 2\sin\beta\sin\gamma + 2\cos\gamma\cos\alpha + 2\sin\alpha\sin\gamma + 3 = 0.$

$\Rightarrow 2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta + 2\cos\beta\cos\gamma + 2\sin\beta\sin\gamma + 2\cos\gamma\cos\alpha + 2\sin\alpha\sin\gamma + (\cos^2\alpha+\sin^2\alpha) + (\cos^2\beta+\sin^2\beta) + (\sin^2\gamma+\cos^2\gamma) = 0$

$(\because 3 = 1+1+1)$

$\Rightarrow (\cos^2\alpha+\cos^2\beta+\cos^2\gamma + 2\cos\alpha\cos\beta + 2\cos\beta\cos\gamma + 2\cos\gamma\cos\alpha) + (\sin^2\alpha+\sin^2\beta + \sin^2\gamma + 2\sin\alpha\sin\beta + 2\sin\beta\sin\gamma + 2\sin\gamma\sin\alpha) = 0$

$(\cos\alpha+\cos\beta+\cos\gamma)^2 + (\sin\alpha+\sin\beta+\sin\gamma)^2 = 0$

$\Rightarrow \cos\alpha+\cos\beta+\cos\gamma = 0$

$\sin\alpha+\sin\beta+\sin\gamma = 0$

$[\because (a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca]$

20) Show that:

i) $\tan(45^\circ+A) = \frac{1+\tan A}{1-\tan A}$

LHS $= \tan(45^\circ+A)$

$= \frac{\tan 45^\circ + \tan A}{1-\tan 45^\circ \tan A}$

$= \frac{1+\tan A}{1-\tan A}$ $(\tan 45^\circ = 1)$

$=$ RHS.

ii) $\tan(45^\circ-A) = \frac{1-\tan A}{1+\tan A}$

LHS $= \tan(45^\circ-A) = \frac{\tan 45^\circ - \tan A}{1+\tan 45^\circ \tan A}$

$= \frac{1-\tan A}{1+\tan A}$ $(\tan 45^\circ = 1)$

21) Prove that $\cot(A+B) = \frac{\cot A \cos B - 1}{\cot A \cos B}$.

LHS $\cot(A+B) = \frac{1}{\tan(A+B)}$

$= \frac{1}{\frac{\tan A+\tan B}{1-\tan A\tan B}} = \frac{1-\tan A\tan B}{\tan A+\tan B}$

$= \frac{1-\frac{1}{\cot A}\cdot\frac{1}{\cot B}}{\frac{1}{\cot A}+\frac{1}{\cot B}}$

$= \frac{\frac{\cot A\cot B - 1}{\cot A\cot B}}{\frac{\cot B+\cot A}{\cot A\cot B}}$

$= \frac{\cot A\cot B - 1}{\cot A+\cot B} =$ RHS.

22) If $\tan x = \frac{n}{n+1}$ and $\tan y = \frac{1}{2n+1}$ find $\tan(x+y)$

Sol: $\tan x = \frac{n}{n+1}$; $\tan y = \frac{1}{2n+1}$

$\tan(x+y) = \frac{\tan x+\tan y}{1-\tan x\tan y}$

$= \frac{\frac{n}{n+1}+\frac{1}{2n+1}}{1-\frac{n}{n+1}\cdot\frac{1}{2n+1}}$

$= \frac{\frac{n(2n+1)+n+1}{\cancel{(n+1)(2n+1)}}}{\frac{(n+1)(2n+1)-n}{\cancel{(n+1)(2n+1)}}}$

$$= \frac{2n^2+n+n+1}{2n^2+\cancel{n}+2n+1-\cancel{n}}$$

$$= \frac{2n^2+2n+1}{2n^2+2n+1} = 1 = RHS.$$

23) Prove that $\tan\left(\frac{\pi}{4}+\theta\right)\tan\left(\frac{3\pi}{4}+\theta\right) = -1$

Soln LHS: $\tan\left(\frac{\pi}{4}+\theta\right)\tan\left(\frac{3\pi}{4}+\theta\right)$

$= \tan(45°+\theta)\tan(135°+\theta)$

$= \tan(45°+\theta)\tan(180°-45°+\theta)$

$= \tan(45°+\theta)[\tan(180°-(45°-\theta))]$

$= \tan(45°+\theta)[-\tan(45°-\theta)]$

$= -\tan(45°+\theta)\tan(45°-\theta)$

$= -\left[\frac{\tan 45°+\tan\theta}{1-\tan 45°\tan\theta}\right]\left[\frac{\tan 45°-\tan\theta}{1+\tan 45°\tan\theta}\right]$

$= -\left[\frac{1+\tan\theta}{1-\tan\theta}\right]\left[\frac{1-\tan\theta}{1+\tan\theta}\right]$

$= -1 \quad [\because \tan 45°=1]$

$= RHS$

Hence proved.

24) Find the values of $\tan(\alpha+\beta)$ given that $\cot\alpha = 1/2$, $\alpha \in (\pi, 3\pi/2)$ and $\sec\beta = -5/3$, $\beta \in (\pi/2, \pi)$.

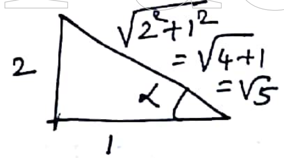
Soln Since $\alpha \in (\pi, 3\pi/2)$

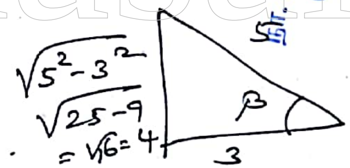
α lies in III quadrant

$\therefore \tan\alpha, \cot\alpha$ are $+ve$.

$\beta \in (\pi/2, \pi)$, β lies in II quadrant

$\Rightarrow \sin\beta, \operatorname{cosec}\beta$ are $+ve$.





$\cot\alpha = \frac{1}{2}$ $\qquad$ $\sec\beta = -\frac{5}{3}$

$\tan\alpha = 2$ $\qquad$ $\tan\beta = -\frac{4}{3}$.

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$= \frac{2-4/3}{1-2\left(-\frac{4}{3}\right)} = \frac{2-4/3}{1+8/3}$$

$$= \frac{6-4/\cancel{3}}{3+8/\cancel{3}} = \frac{2}{11}.$$

25) If $\theta+\varphi = \alpha$ then $\tan\theta = k \times \tan\varphi$ prove that $\sin(\theta-\varphi) = \frac{k-1}{k+1}\sin\alpha$

Soln Given $\theta+\varphi=\alpha$

$\tan\theta = k\tan\varphi$

$\frac{\tan\theta}{\tan\varphi} = k$

$\frac{\sin\theta}{\cos\theta} \Big/ \frac{\sin\varphi}{\cos\varphi} = k.$

$\frac{\sin\theta\cos\varphi}{\cos\theta\sin\varphi} = \frac{k}{1}.$

$\frac{\sin\theta\cos\varphi - \cos\theta\sin\varphi}{\sin\theta\cos\varphi + \cos\theta\sin\varphi} = \frac{k-1}{k+1}$

$\frac{\sin(\theta-\varphi)}{\sin(\theta+\varphi)} = \frac{k-1}{k+1}$

$\frac{\sin(\theta-\varphi)}{\sin\alpha} = \frac{k-1}{k+1}$

$\sin(\theta-\varphi) = \frac{k-1}{k+1}\sin\alpha$

Hence proved.

Exercise 3.5

1. Find the values of cos2A, A lies in the first quadrant when

Soln (i) $\cos A = \frac{15}{17}$.

$\cos 2A = 2\cos^2 A - 1$

$= 2\left(\frac{15}{17}\right)^2 - 1 = 2\left(\frac{225}{289}\right) - 1$

$= \frac{450-289}{289} = \frac{161}{289}$.

(ii) $\sin A = \frac{4}{5}$

$\cos 2A = 1 - 2\sin^2 A = 1 - 2\left(\frac{4}{5}\right)^2$

$= 1 - 2\left(\frac{16}{25}\right) = 1 - \frac{32}{25}$

$= \frac{25-32}{25} = \frac{-7}{25}$.

(iii) $\tan A = \frac{16}{63}$.

$\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{16}{63}\right)^2}{1+\left(\frac{16}{63}\right)^2}$

$= \frac{1 - \frac{256}{3969}}{1 + \frac{256}{3969}}$

$= \frac{\frac{3969-256}{\cancel{3969}}}{\frac{3969+256}{\cancel{3969}}} = \frac{3713}{4225}$.

2) If θ is an acute angle then find.

(i) $\sin\left(\pi/4 - \theta/2\right)$ when $\sin\theta = \frac{1}{25}$.

$\sin\theta = \frac{1}{25}$

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{1}{625}} = \sqrt{\frac{625-1}{625}}$

$= \sqrt{\frac{624}{25}} = \frac{\sqrt{4\times4\times39}}{25} = \frac{4\sqrt{39}}{25}$

$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-\frac{4\sqrt{39}}{25}}{2}}$

$= \sqrt{\frac{25-4\sqrt{39}}{2\times25}} = \sqrt{\frac{25-4\sqrt{39}}{50}}$.

$\cos\theta/2 = \sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\frac{1+\frac{4\sqrt{39}}{25}}{2}}$

$= \sqrt{\frac{25+4\sqrt{39}}{2\times25}} = \sqrt{\frac{25+4\sqrt{39}}{50}}$.

$\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right) = \sin\frac{\pi}{4}\cos\frac{\theta}{2} - \cos\frac{\pi}{4}\sin\frac{\theta}{2}$

$= \frac{1}{\sqrt2}\cos\theta/2 - \frac{1}{\sqrt2}\sin\theta/2$

$= \frac{1}{\sqrt2}\left[\cos\frac{\theta}{2} - \sin\theta/2\right]$

$= \frac{1}{\sqrt2}\left[\sqrt{\frac{25+4\sqrt{39}}{50}} - \sqrt{\frac{25-4\sqrt{39}}{50}}\right]$

$= \frac{1}{\sqrt2\sqrt{50}}\left(\sqrt{25+4\sqrt{39}} - \sqrt{25-4\sqrt{39}}\right)$

$= \frac{1}{\sqrt2\times5\sqrt2}\left(\sqrt{25+4\sqrt{39}} - \sqrt{25-4\sqrt{39}}\right)$

$= \frac{1}{10\sqrt2}\left(\sqrt{25+4\sqrt{39}} - \sqrt{25-4\sqrt{39}}\right)$

(ii) $\cos\left(\pi/4 + \theta/2\right)$ when

$\sin\theta = 8/9$

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{64}{81}}$

$= \sqrt{\frac{81-64}{81}} = \frac{\sqrt{17}}{9}$.

$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-\sqrt{17}/9}{2}}$

$= \sqrt{\frac{9-\sqrt{17}}{9\times2}} = \sqrt{\frac{9-\sqrt{17}}{18}}$.

$\cos\theta/2 = \sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\frac{1+\sqrt{17}/9}{2}}$

$$= \sqrt{\frac{9+\sqrt{17}}{9\times 2}} = \sqrt{\frac{9+\sqrt{17}}{18}}$$

$$\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right) = \cos\pi/4 \cos\theta/2 - \sin\pi/4 \sin\frac{\theta}{2}$$

$$= \frac{1}{\sqrt2}\cos\theta/2 - \frac{1}{\sqrt2}\sin\theta/2$$

$$= \frac{1}{\sqrt2}\left[\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right]$$

$$= \frac{1}{\sqrt2}\left[\sqrt{\frac{9+\sqrt{17}}{18}} - \sqrt{\frac{9-\sqrt{17}}{18}}\right]$$

3) If $\cos\theta = \frac{1}{2}\left(a+\frac{1}{a}\right)$, show that $\cos 3\theta = \frac{1}{2}\left(a^3+\frac{1}{a^3}\right)$.

solution $\cos\theta = \frac{1}{2}\left(a+\frac{1}{a}\right)$

$$\text{LHS} = \cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$= 4\left[\left(\frac{1}{2}\left(a+\frac{1}{a}\right)\right)^3\right] - 3\left[\frac{1}{2}\left(a+\frac{1}{a}\right)\right]$$

$$= \cancel{4}\left[\frac{1}{\cancel{8}_2}\left(a^3+\frac{1}{a^3}+3a^2\cdot\frac{1}{a}+3a\cdot\frac{1}{a^2}\right)\right] - \frac{3}{2}\left(a+\frac{1}{a}\right)$$

$$= \frac{1}{2}a^3 + \frac{1}{2a^3} + \cancel{\frac{3}{2}a} + \cancel{\frac{3}{2a}} - \cancel{\frac{3}{2}a} - \cancel{\frac{3}{2a}}$$

$$= \frac{1}{2}\left(a^3+\frac{1}{a^3}\right) = \text{RHS}.$$

4. Prove that $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$.

Soln. $\cos 5\theta = \cos(3\theta+2\theta)$

$$= \cos 3\theta\cos 2\theta - \sin 3\theta \sin 2\theta$$

$$= (4\cos^3\theta - 3\cos\theta)(2\cos^2\theta - 1)$$

$$- [3\sin\theta - 4\sin^3\theta][2\sin\theta\cos\theta]$$

$$= 8\cos^5\theta - 6\cos^3\theta - 4\cos^3\theta + 3\cos\theta$$

$$- 6\sin^2\theta\cos\theta + 8\sin^4\theta\cos\theta$$

$$= 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta + 8(1-\cos^2\theta)^2\cos\theta$$

$$- 6(1-\cos^2\theta)\cos\theta$$

$$= 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta - 6\cos\theta + 8(1+\cos^4\theta - 2\cos^2\theta)\cos\theta$$

$$+ 6\cos^3\theta$$

$$= 8\cos^5\theta - 10\cos^3\theta - 3\cos\theta + 6\cos^3\theta$$

$$= 8\cos^5\theta - 10\cos^3\theta - 16\cos^3\theta$$

$$+ 8\cos\theta + 8\cos^5\theta$$

$$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

$$= \text{RHS}.$$

5) Prove that $\sin 4\alpha = 4\tan\alpha \cdot \frac{(1-\tan^2\alpha)}{(1+\tan^2\alpha)^2}$

soln LHS: $\sin 4\alpha = \sin 2(2\alpha)$

$$= 2\sin 2\alpha\cos 2\alpha$$

$$= 2\left(\frac{2\tan\alpha}{1+\tan^2\alpha}\right)\left(\frac{1-\tan^2\alpha}{1+\tan^2\alpha}\right)$$

$$= 4\tan\alpha\,\frac{(1-\tan^2\alpha)}{(1+\tan^2\alpha)^2} = \text{RHS}.$$

6) If $A+B=45^\circ$ show that $(1+\tan A)(1+\tan B) = 2$.

Soln $A+B=45^\circ \Rightarrow B = 45^\circ - A$

LHS $(1+\tan A)(1+\tan B)$

$$= (1+\tan A)(1+\tan(45^\circ - A))$$

$$= (1+\tan A)\left(1+\frac{\tan 45^\circ - \tan A}{1+\tan 45^\circ\tan A}\right)$$

$$= (1+\tan A)\left(1+\frac{1-\tan A}{1+\tan A}\right)$$

$(\because \tan 45^\circ = 1)$

$$= \cancel{(1+\tan A)}\left(\frac{1+\cancel{\tan A} + 1 - \cancel{\tan A}}{\cancel{(1+\tan A)}}\right)$$

$$= 2 = \text{RHS}.$$

7) Prove that $(1+\tan 1^\circ)(1+\tan 2^\circ)(1+\tan 3^\circ)\ldots(1+\tan 44^\circ)$ is a multiple of 4

soln LHS $=$

$$(1+\tan 1^\circ)(1+\tan 44^\circ)(1+\tan 2^\circ)(1+\tan 43^\circ)(1+\tan 3^\circ)(1+\tan 42^\circ)$$

$$= (2)(2)(2)\cdots(2) \quad (22\text{ times})$$

$$= 2^{22} = (2^2)^{11} = 4^{11}$$

= which is a multiple of 4

$\left[\because 1+44 = 45^\circ \right.$
$\left.(1+\tan 1^\circ)(1+\tan 44^\circ) = 2 \quad \text{Qn No}: 6\right]$.

8). Prove that $\tan\left(\frac{\pi}{4}+\theta\right)-\tan\left(\frac{\pi}{4}-\theta\right) = 2\tan 2\theta$.

Soln. LHS $= \tan\left(\frac{\pi}{4}+\theta\right)-\tan\left(\frac{\pi}{4}-\theta\right)$

$$= \left[\frac{\tan \pi/4 + \tan\theta}{1-\tan \pi/4 \cdot \tan\theta}\right] - \left[\frac{\tan \pi/4 - \tan\theta}{1+\tan \pi/4 \tan\theta}\right]$$

$$= \left[\frac{1+\tan\theta}{1-\tan\theta}\right] - \left[\frac{1-\tan\theta}{1+\tan\theta}\right]$$

$$= \frac{(1+\tan\theta)^2-(1-\tan\theta)^2}{(1+\tan\theta)(1-\tan\theta)}$$

$$= \frac{(1+\tan^2\theta+2\tan\theta)-(1+\tan^2\theta-2\tan\theta)}{1-\tan^2\theta}$$

$$= \frac{2\tan\theta+2\tan\theta}{1-\tan^2\theta}$$

$$= \frac{4\tan\theta}{1-\tan^2\theta} = 2\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$= 2\tan 2\theta$.

9) Show that $\cot\left(7\frac{1}{2}\right)^\circ = \sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$

Soln LHS $= \cot\left(7\frac{1}{2}\right)^\circ = \dfrac{\cos\left(7\frac{1}{2}\right)^\circ}{\sin\left(7\frac{1}{2}\right)}$.

$$= \frac{2\sin\left(7\frac{1}{2}\right)\cos\left(7\frac{1}{2}\right)}{2\sin^2\left(7\frac{1}{2}\right)}$$

$$= \frac{\sin 2\left(7\frac{1}{2}\right)}{1-\cos 2\left(7\frac{1}{2}\right)} \qquad \left[\because 2\sin A\cos A = \sin 2A,\ 2\sin^2 A = 1-\cos 2A\right]$$

$$= \frac{\sin 15^\circ}{1-\cos 15^\circ} = \frac{\sin(45^\circ-30^\circ)}{1-\cos(45^\circ-30^\circ)}$$

$$= \frac{\sin 45^\circ\cos 30^\circ - \cos 45^\circ \sin 30^\circ}{1-[\cos 45^\circ\cos 30^\circ + \sin 45^\circ \sin 30^\circ]}$$

$$= \frac{\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\cdot\frac{1}{2}}{\frac{1}{1} - \left(\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2}\right)} = \frac{\frac{\sqrt{3}-1}{\cancel{2\sqrt{2}}}}{\frac{2\sqrt{2}-\sqrt{3}-1}{\cancel{2\sqrt{2}}}}$$

$$= \frac{(\sqrt{3}-1)}{(2\sqrt{2}-\sqrt{3}-1)} \cdot \frac{2\sqrt{2}+\sqrt{3}+1}{2\sqrt{2}+\sqrt{3}+1}$$

$$= \frac{(\sqrt{3}-1)(2\sqrt{2}+\sqrt{3}+1)}{(2\sqrt{2})^2-(\sqrt{3}+1)^2}$$

$$= \frac{2\sqrt{6}+3+\cancel{\sqrt{3}}-2\sqrt{2}-\cancel{\sqrt{3}}-1}{8-(3+1+2\sqrt{3})}$$

$$= \frac{2\sqrt{6}-2\sqrt{2}+2}{8-3-1-2\sqrt{3}}$$

$$= \frac{2(\sqrt{6}-\sqrt{2}+1)}{4-2\sqrt{3}} = \frac{\cancel{2}(\sqrt{6}-\sqrt{2}+1)}{\cancel{2}(2-\sqrt{3})}$$

$$= \left(\frac{\sqrt{6}-\sqrt{2}+1}{2-\sqrt{3}}\right)\cdot\frac{(2+\sqrt{3})}{(2+\sqrt{3})}$$

$$= \frac{2\sqrt{6}-2\sqrt{2}+2+\sqrt{18}-\sqrt{6}+\sqrt{3}}{2^2-\sqrt{3}^2}$$

$$= \frac{\sqrt{6}+\sqrt{3}-2\sqrt{2}+2+3\sqrt{2}}{4-3}$$

$$= \frac{\sqrt{6}+\sqrt{3}+\sqrt{2}+2}{1}$$

$= \sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6} \qquad (2=\sqrt{4})$

$=$ RHS.

10) Prove that $(1+\sec 2\theta)(1+\sec 4\theta)\cdots(1+\sec 2^n\theta) = \tan 2^n\theta \cot\theta$.

Soln LHS: $(1+\sec 2\theta)(1+\sec 4\theta)\cdots\cdots(1+\sec 2^n\theta)$

$$= \left(1+\frac{1}{\cos 2\theta}\right)\left(1+\frac{1}{\cos 4\theta}\right)\cdots\left(1+\frac{1}{\cos 2^n\theta}\right)$$

$$= \frac{(\cos 2\theta + 1)}{\cos 2\theta}\,\frac{(\cos 4\theta + 1)}{\cos 4\theta} \cdots \left(\frac{\cos 2^n\theta + 1}{\cos 2^n\theta}\right)$$

$$= \frac{(\cos 2\theta + 1)(\cos 4\theta + 1) \cdots (\cos 2^n\theta + 1)}{\cos 2\theta \cdot \cos 4\theta \cdots \cos 2^n\theta}$$

$$= \frac{(2\cos^2\theta)(2\cos^2 2\theta)\cdot(2\cos^2 2^{n-1}\cdot x)}{\cos 2\theta \cdot \cos 2^2\theta \cdots \cos 2^n\theta}$$

$$= \frac{2^n \cos\theta}{\cos^{2n}\theta}\left[\cos x \cdot \cos 2x \cdot \cos 2^2 x \cdots \cos 2^{n-1}x\right]$$

$$= \frac{2^n \cos\theta}{\cos 2^n\theta} \cdot \frac{\cos 2^n\theta}{2^n \cdot \sin x}$$

$$\left[\because \cos A \cos 2A \cdot \cos 2^2 A \cdots \cos 2^{n-1}A = \frac{\sin 2^n A}{2^n \sin A}\right]$$

by eg 3.32.

$$= \tan 2^n\theta \cdot \cot\theta = RHS.$$

11) Prove that

$$32\sqrt{3}\sin\frac{\pi}{48}\cdot\cos\frac{\pi}{48}\cdot\cos\frac{\pi}{24}\cos\frac{\pi}{12}\cos\pi/6 = 3.$$

Soln LHS $= 32\sqrt{3}\sin\frac{\pi}{48}\cos\frac{\pi}{48}\cos\frac{\pi}{24}\cos\frac{\pi}{12}\cos\pi/6$.

$$= \frac{32\sqrt{3}}{2}\left(2\sin\frac{\pi}{48}\cos\frac{\pi}{48}\right)\cos\frac{\pi}{24}\cdot\cos\frac{\pi}{12}\cos\frac{\pi}{6}$$

(Multiply divide by 2)

$$= \frac{32\sqrt{3}}{2}\left(\sin 2\frac{\pi}{48}\right)\cos\frac{\pi}{24}\cdot\cos\frac{\pi}{12}\cdot\cos\frac{\pi}{6}$$

$(\because \sin 2A = 2\sin A\cos A)$

$$= 16\sqrt{3}\left(\frac{2}{2}\sin\frac{\pi}{24}\cos\frac{\pi}{24}\right)\cos\frac{\pi}{12}\cdot\cos\frac{\pi}{6}.$$

$$= 8\sqrt{3}\left[\sin 2\frac{\pi}{24}\right]\cos\frac{\pi}{12}\cdot\cos\frac{\pi}{6}.$$

$$= 8\sqrt{3}\left[\frac{2}{2}\sin\frac{\pi}{12}\cos\frac{\pi}{12}\right]\cos\frac{\pi}{6}$$

$$= 4\sqrt{3}\left(\sin 2\left(\frac{\pi}{12}\right)\right)\cos\pi/6$$

$$= 4\sqrt{3}\left(\frac{2}{2}\sin\pi/6\right)\cos\pi/6)$$

$$= 2\sqrt{3}\sin 2(\pi/6) = 2\sqrt{3}\sin\frac{\pi}{3}$$

$$= 2\sqrt{3}\sin 60^\circ = 2\sqrt{3}\cdot\frac{\sqrt{3}}{2}$$

$$= 3 = RHS.$$

Exercise 3.6

1. Express each of the following as a sum or difference.

(i) $\sin 35^\circ \cos 28^\circ$

$$= \frac{1}{2}(2\sin 35^\circ \cos 28^\circ)$$

$$= \frac{1}{2}\left[\sin(35^\circ + 28^\circ) + \sin(35^\circ - 28^\circ)\right]$$

$$\left(\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]\right)$$

$$= \frac{1}{2}\left[\sin 63^\circ + \sin 7^\circ\right)$$

(ii) $\sin 4x \cos 2x = \frac{1}{2}(2\sin 4x\cos 2x)$

$$= \frac{1}{2}\left[\sin(4x + 2x) + \sin(4x - 2x)\right]$$

$$= \frac{1}{2}\left[\sin 6x + \sin 2x\right]$$

(iii) $2\sin 10\theta\cos 2\theta$

$$= \frac{1}{2}\left[\sin(10\theta + 2\theta) + \sin(10\theta - 2\theta)\right]$$

$$= \frac{1}{2}\left[\sin 12\theta + \sin 8\theta\right]$$

(iv) $\cos 5\theta \cos 2\theta$

$$= \frac{1}{2}(2\cos 5\theta\cos 2\theta]$$

$$= \frac{1}{2}\left[\cos(5\theta + 2\theta) + \cos(5\theta - 2\theta)\right]$$

$$= \frac{1}{2}\left[\cos 7\theta + \cos 3\theta\right]$$

(v) $\sin 5\theta \sin 4\theta = \frac{1}{2}(2\sin 5\theta \sin 4\theta)$

$= \frac{1}{2}[\cos(5\theta - 4\theta) - \cos(5\theta + 4\theta)]$

$= \frac{1}{2}[\cos\theta - \cos 9\theta]$

2) Express each of the following as a product.

(i) $\sin 75^\circ - \sin 35$

$= 2\cos\left(\frac{75^\circ + 35}{2}\right)\sin\left(\frac{75^\circ - 35^\circ}{2}\right)$

$= 2\cos 55^\circ \sin 20^\circ$

$\left(\because \sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}\right.$

(ii) $\cos 65^\circ + \sin 15^\circ$

$= 2\cos\left(\frac{65^\circ + 15}{2}\right)\cos\left(\frac{65^\circ - 15}{2}\right)$

$= 2\cos 40^\circ \cos 25$

(iii) $\sin 5\theta + \sin 4\theta$

$= 2\sin\left(\frac{5\theta + 4\theta}{2}\right)\cos\left(\frac{50 - 40}{2}\right)$

$= 2\sin 45^\circ \cos 5^\circ = 2 \cdot \frac{1}{\sqrt{2}}\cos 5^\circ$

$= \sqrt{2}\cos 5^\circ$

(iv) $\cos 35^\circ - \cos 75^\circ$

$= 2\sin\left(\frac{35^\circ + 75^\circ}{2}\right)\sin\left(\frac{75^\circ - 35^\circ}{2}\right)$

$= 2\sin 55^\circ \sin 20^\circ$

3) Show that $\sin 12^\circ \sin 48^\circ \sin 54^\circ = \frac{1}{8}$

Soln LHS $= \sin 12^\circ \sin 48^\circ \sin 54^\circ$

$= \frac{1}{2}[\cos(12^\circ - 48^\circ) - \cos(12^\circ + 48^\circ)]\sin 54^\circ$

$= \frac{1}{2}[\cos(-36^\circ) - \cos 60^\circ]\sin 54^\circ$

$= \frac{1}{2}\left[\cos 36^\circ - \frac{1}{2}\right]\sin 54^\circ$

$= \frac{1}{2}\left[\frac{\sqrt{5}+1}{4} - \frac{1}{2}\right]\left[\frac{\sqrt{5}+1}{4}\right]$

$= \frac{1}{2}\left[\frac{\sqrt{5}+1-2}{4}\right]\left[\frac{\sqrt{5}+1}{4}\right]$

$= \frac{1}{2}\left[\frac{\sqrt{5}-1}{4}\right]\left[\frac{\sqrt{5}+1}{4}\right]$

$= \frac{1}{2}\left(\frac{\sqrt{5}^2 - 1^2}{16}\right) = \frac{1}{32}(5-1) = \frac{4}{32}$

$= \frac{1}{8} = $ RHS.

4) Show that $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15} = \frac{1}{128}$

LHS $\left\{\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{7\pi}{15}\right\} \cdot \left\{\cos\frac{3\pi}{15} \cdot \cos\frac{7\pi}{15}\right\}\cos\frac{5\pi}{15}$

$= -\left\{\cos\frac{\pi}{15} \cdot \cos\frac{2\pi}{15} \cdot \cos\frac{4\pi}{15} \cdot \cos\frac{8\pi}{15}\right\} \cdot \left\{\cos\frac{3\pi}{15} \cdot \cos\frac{6\pi}{15}\right\}\cos\frac{\pi}{3}$

$\left[\because \cos\frac{5\pi}{15} = \cos\pi/3 = \frac{1}{2}\right.$

$\cos\frac{7\pi}{15} = \cos\left(\pi - \frac{8\pi}{15}\right) = -\cos\frac{8\pi}{15}$

$= -\frac{1}{2}\left(\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right)\left(\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}\right)$

$= -\frac{1}{2}\left(\frac{\sin 2^4\frac{\pi}{15}}{2^4\sin\frac{\pi}{15}}\right) \times \left[\frac{\sin 2^2 \times \frac{3\pi}{15}}{2^2\sin\frac{3\pi}{15}}\right]$

$\left(\because \cos A \cos 2A \cos 2^{n-1}A = \frac{\sin 2^n A}{2^n \sin A}\right]$

$= -\frac{1}{2}\left[\frac{\sin\frac{16\pi}{15}}{16\sin\frac{\pi}{15}}\right]\left[\frac{\sin 12\pi/15}{4\sin\frac{3\pi}{15}}\right]$

$= -\frac{1}{2}\left[\frac{\sin\left(\pi + \frac{\pi}{15}\right)}{16\sin\pi/15}\right]\left[\frac{\sin\left(\pi - \frac{3\pi}{15}\right)}{4\sin\frac{3\pi}{15}}\right]$

$$= -\frac{1}{2}\left[\frac{-\sin\frac{\pi}{15}}{16\sin\frac{\pi}{15}}\right]\left[\frac{\sin\frac{3\pi}{15}}{4\sin\frac{3\pi}{15}}\right]$$

$$= -\frac{1}{2}\left(\frac{-1}{16}\right)\left(\frac{1}{4}\right) = \frac{1}{128}$$

$= RHS$.

5) Show that

$$\frac{\sin 8x \cos x - \sin 6x \cos 3x}{\cos 2x \cos x - \sin 3x \sin 4x} = \tan 2x.$$

Soln LHS:

$$= \frac{\frac{1}{2}[\sin 9x + \sin 7x] - \frac{1}{2}[\sin 9x + \sin 3x]}{\frac{1}{2}[\cos 3x + \cos x] - \frac{1}{2}[\cos x - \cos 7x]}$$

$(\because \sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)])$

$$= \frac{\cancel{\tfrac{1}{2}}[\cancel{\sin 9x} + \sin 7x - \cancel{\sin 9x} - \sin 3x]}{\cancel{\tfrac{1}{2}}[\cos 3x + \cancel{\cos x} - \cancel{\cos x} + \cos 7x]}$$

$$= \frac{(\sin 7x - \sin 3x)}{(\cos 3x + \cos 7x)}$$

$$= \frac{2\cos\left(\frac{7x+3x}{2}\right)\sin\left(\frac{7x-3x}{2}\right)}{2\cos\left(\frac{7x+3x}{2}\right)\cos\left(\frac{7x-3x}{2}\right)}$$

$$= \frac{\cancel{2}\,\cancel{\cos 5x}\,\sin 2x}{\cancel{2}\,\cancel{\cos 5x}\,\cos 2x} = \tan 2x = RHS.$$

$[\because \sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$.

$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}]$

6) Show that $\frac{(\cos\theta - \cos 3\theta)(\sin 8\theta + \sin 2\theta)}{(\sin 5\theta - \sin\theta)(\cos 4\theta - \cos 6\theta)} = 1$

Soln

$$LHS = \frac{2\sin\left(\frac{\theta+3\theta}{2}\right)\sin\left(\frac{3\theta-\theta}{2}\right)\cdot 2\sin\left(\frac{8\theta+2\theta}{2}\right)\cos\left(\frac{8\theta-2\theta}{2}\right)}{2\cos\left(\frac{5\theta+\theta}{2}\right)\sin\left(\frac{5\theta-\theta}{2}\right)\cdot 2\sin\left(\frac{4\theta+6\theta}{2}\right)\sin\left(\frac{6\theta-4\theta}{2}\right)}$$

$$= \frac{\sin 2\theta \sin\theta \sin 5\theta \cos 3\theta}{\cos 3\theta \sin 2\theta \sin 5\theta \sin\theta} = 1 = RHS.$$

7) Prove that $\sin x + \sin 2x + \sin 3x = \sin 2x(1 + 2\cos x)$.

Soln LHS: $(\sin x + \sin 3x) + \sin 2x$

$= 2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right) + \sin 2x$

$= 2\sin 2x \cos(-x) + \sin 2x$

$= 2\sin 2x \cos x + \sin 2x$

$= \sin 2x\,(2\cos x + 1)$

$= \sin 2x\,(1 + 2\cos x)$

$= RHS$.

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8) Prove that $\frac{\sin 4x + \sin 2x}{\cos 4x + \cos 2x} = \tan 3x$.

Soln

$$LHS = \frac{\sin 4x + \sin 2x}{\cos 4x + \cos 2x}$$

$$= \frac{\cancel{2}\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)}{\cancel{2}\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)}$$

$$= \frac{\sin 3x \cancel{\cos x}}{\cos 3x \cancel{\cos x}} = \tan 3x = RHS.$$

9) P.T $1 + \cos 2x + \cos 4x + \cos 6x = 4\cos x \cos 2x \cos 3x$.

Soln LHS: $(1 + \cos 2x) + \cos 4x + \cos 6x$

$= 2\cos^2 x + \cos 4x + \cos 6x$.

$= 2\cos^2 x + 2\cos\frac{4x+6x}{2}\cos\frac{4x-6x}{2}$

$= 2\cos^2 x + 2\cos 5x \cos(-x)$

$= 2\cos^2 x + 2\cos 5x \cos x$

$= 2\cos x\,(\cos x + \cos 5x)$

$= 2\cos x\left(2\cos\frac{x+5x}{2}\cos\frac{x-5x}{2}\right)$

$= 2\cos x\,[2\cos 3x \cos(-2x)]$

$= 4\cos x \cos 2x \cos 3x$

$= RHS$.

10) P.T. $\sin\frac{\theta}{2}\sin\frac{7\theta}{2}+\sin\frac{3\theta}{2}\sin\frac{11\theta}{2} = \sin 2\theta \sin 5\theta$.

Soln

LHS :. $\sin\theta/_2 \sin\frac{7\theta}{2} + \sin\frac{3\theta}{2}\sin\frac{11\theta}{2}$.

$$= \frac{1}{2}\left[\cos\left(\frac{\theta}{2}-\frac{7\theta}{2}\right)-\cos\left(\frac{\theta}{2}+\frac{7\theta}{2}\right)\right]$$

$$+\frac{1}{2}\left[\cos\left(\frac{3\theta}{2}-\frac{11\theta}{2}\right)-\cos\left(\frac{3\theta}{2}+\frac{11\theta}{2}\right)\right]$$

$$= \frac{1}{2}\left[\cos(-3\theta)-\cos 4\theta+\cos(-4\theta)-\cos 7\theta\right]$$

$$= \frac{1}{2}\left[\cos 3\theta-\cancel{\cos 4\theta}+\cancel{\cos 4\theta}-\cos 7\theta\right]$$

$$= \frac{1}{2}\left[\cos 3\theta-\cos 7\theta\right]$$

$$= \frac{1}{2}\left[2\sin\left(\frac{3\theta+7\theta}{2}\right)\sin\left(\frac{7\theta-3\theta}{2}\right)\right]$$

$$= \frac{\cancel{2}}{\cancel{2}}\sin(5\theta)\sin 2\theta = \text{RHS}.$$

11) P.T $\cos(30-A)\cos(30+A)+\cos(45-A)\cos(45^\circ+A) = \cos 2A+\frac{1}{4}$

LHS :- $\cos^2 30^\circ-\sin^2 A+\cos^2 45^\circ-\sin^2 A$

$[\because \cos(A+B)\cos(A-B) = \cos^2 A-\sin^2 B]$

$$= \left(\frac{\sqrt{3}}{2}\right)^2-2\sin^2 A+\left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 3/4+\frac{1}{2}-(1-\cos 2A)$$

$$= \frac{3}{4}+\frac{1}{2}-1+\cos 2A$$

$$= \frac{1}{4}+\cos 2A = \text{RHS}.$$

12) P.T $\dfrac{\sin x+\sin 3x+\sin 5x+\sin 7x}{\cos x+\cos 3x+\cos 5x+\cos 7x} = \tan 4x$.

Soln LHS :

$$= \frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)+2\sin\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)}{2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)+2\cos\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)}$$

$$= \frac{\sin 2x\cos x+\sin 6x\cos x}{\cos 2x\cos x+\cos 6x\cos x}$$

$$= \frac{\cancel{\cos x}(\sin 2x+\sin 6x)}{\cancel{\cos x}(\cos 2x+\cos 6x)}$$

$$= \frac{\cancel{2}\sin\left(\frac{6x+2x}{2}\right)\cos\left(\frac{6x-2x}{2}\right)}{\cancel{2}\cos\left(\frac{6x+2x}{2}\right)\cos\left(\frac{6x-2x}{2}\right)}$$

$$= \frac{\sin 4x\cancel{\cos 2x}}{\cos 4x\cancel{\cos 2x}}$$

$$= \tan 4x$$

$$= \text{RHS}.$$

13) P.T $\dfrac{\sin(4A-2B)+\sin(4B-2A)}{\cos(4A-2B)+\cos(4B-2A)} = \tan(A+B)$

Soln LHS:

$$= \frac{2\sin\left(\frac{4A-2B+4B-2A}{2}\right)\cos\left(\frac{4A-2B-4B+2A}{2}\right)}{2\cos\left(\frac{4A-2B+4B-2A}{2}\right)\cos\left(\frac{4A-2B-4B+2A}{2}\right)}$$

$$= \frac{\sin\left(\frac{2A+2B}{2}\right)\cancel{\cos\left(\frac{6A-6B}{2}\right)}}{\cos\left(\frac{2A+2B}{2}\right)\cancel{\cos\left(\frac{6A-6B}{2}\right)}}$$

$$= \frac{\sin(A+B)}{\cos(A+B)} = \tan(A+B).$$

$= \text{RHS}$.

14) S.T $\cot(A+15^\circ)-\tan(A-15^\circ) = \dfrac{4\cos 2A}{1+2\sin 2A}$.

Soln LHS: $\cot(A+15^\circ)-\tan(A-15^\circ)$

$$= \frac{\cos(A+15^\circ)}{\sin(A+15^\circ)}-\frac{\sin(A-15^\circ)}{\cos(A-15^\circ)}$$

$$= \frac{\cos(A+15)\cdot\cos(A-15)-\sin(A+15)\sin(A-15)}{\sin(A+15^\circ)\cos(A-15^\circ)}$$

$$=\frac{(\cos^2 A-\sin^2 15°)-(\sin^2 A-\sin^2 15°)}{\frac{1}{2}\left[\sin(A+15°+A-15°)+\sin(A+15°-A+15°)\right]}$$

$$=\frac{2\left[\cos^2 A-\cancel{\sin^2 15°}-\sin^2 A+\cancel{\sin^2 15°}\right]}{\sin 2A+\sin 30°}$$

$$=\frac{2(\cos^2 A-\sin^2 A)}{\sin 2A+\frac{1}{2}}$$

$$=\frac{2(\cos 2A)}{2\sin 2A+1}\cdot 2$$

$$=\frac{4\cos 2A}{2\sin 2A+1}=RHS.$$

Exercise 3.7

1) If $A+B+C=180°$ P.T

(i) $\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$

Soln (i) $\sin 2A+\sin 2B+\sin 2C$

$$=2\sin\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right)+2\sin C\cos C$$

$$=2\sin(A+B)\cos(A-B)+2\sin C\cos C$$

$$=2\sin(180°-C)\cos(A-B)+2\sin C\cos C$$

$$=2\sin C\cos(A-B)+2\sin C\cos C$$

$$=2\sin C[\cos(A-B)+\cos C]$$

$$=2\sin C[\cos(A-B)-\cos(A+B)]$$

$$=2\sin C(2\sin A\sin B)$$

$$=4\sin A\sin B\sin C$$

$[\because A+B+C=180°$

$C=\pi-(A+B)$

$\cos C=\cos(\pi-(A+B))$

$\cos C=-\cos(A+B)]$

(ii) $\cos A+\cos B-\cos C=-1+4\cos\frac{A}{2}\cos B/2\sin C/2$.

Soln

LHS: $\cos A+\cos B-\cos C$

$$=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)-\left(1-2\sin^2\frac{C}{2}\right)$$

$$=2\sin C/2\cos\left(\frac{A-B}{2}\right)-1+2\sin^2 C/2$$

$[\because A+B+C=\pi \Rightarrow A/2+B/2+C/2=\frac{\pi}{2}$

$\frac{A+B}{2}=90-C/2$

$\cos\left(\frac{A+B}{2}\right)=\cos(90-C/2)=\sin C/2]$

$$=2\sin C/2\left[\cos\frac{A-B}{2}+\sin\frac{C}{2}\right]-1$$

$$=2\sin\frac{C}{2}\left[\cos\frac{A-B}{2}+\cos\left(\frac{A+B}{2}\right)\right]-1$$

$$=-1+2\sin\frac{C}{2}\left[2\cos\frac{A}{2}\cos\frac{B}{2}\right]$$

$$=-1+4\cos A/2\cos B/2\sin C/2$$

$=RHS.$

(iii) $\sin^2 A+\sin^2 B+\sin^2 C=2+2\cos A\cos B\cos C$

Soln LHS: $\sin^2 A+\sin^2 B+\sin^2 C$

$$=\frac{1}{2}\left[2\sin^2 A+2\sin^2 B+2\sin^2 C\right]$$

$$=\frac{1}{2}\left[(1-\cos 2A)+(1-\cos 2B)+(1-\cos 2C)\right]$$

$$=\frac{1}{2}\left[3-(\underbrace{\cos 2A+\cos 2B}+\cos 2C)\right]$$

$$=\frac{1}{2}\left[3-\left(2\cos\frac{2A+2B}{2}\cdot\cos\frac{2A-2B}{2}+\cos 2C\right)\right]$$

$$=\frac{3}{2}-\cos(A+B)\cos(A-B)-\frac{1}{2}\cos 2C$$

$$=\frac{3}{2}-\cos(\pi-C)\cos(A-B)-\frac{1}{2}(2\cos^2 C-1)$$

$$=\frac{3}{2}+\cos C\cos(A-B)-\cos^2 C+\frac{1}{2}$$

$$=3/2+\frac{1}{2}+\cos C[\cos(A-B)-\cos C]$$

$$=2+\cos C[\cos(A-B)-\cos(\pi-(A+B))]$$

$$=2+\cos C[\cos(A-B)+\cos(A+B)]$$

$= \frac{3}{2}\ 2 + \cos C(2\cos A\cos B)$

$= 2 + 2\cos A\cos B\cos C$

$= RHS.$

(iv) $\sin^2 A + \sin^2 B - \sin^2 C = 2\sin A\sin B\cos C.$

Soln

LHS: $\sin^2 A + \sin^2 B - \sin^2 C.$

$= \frac{1}{2}[2\sin^2 A + 2\sin^2 B - 2\sin^2 C]$

$= \frac{1}{2}[1 - \cos 2A + \cancel{1} - \cos 2B \cancel{-1} + \cos 2C]$

$= \frac{1}{2}[1 - \cos 2A + \cos 2B - \cos 2C)]$

$= \frac{1}{2}[1 - (\cos 2A + \cos 2B - \cos 2C)]$

$= \frac{1}{2} - \frac{1}{2}(\cos 2A + \cos 2B - \cos 2C)$

$= \frac{1}{2} - \frac{1}{2}[2\cos\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right) - \cos 2C]$

$= \frac{1}{2} - \frac{1}{2}[2\cos(A+B)\cos(A-B) - 2\cos^2 C + 1]$

$= \frac{1}{2} - \cos(A+B)\cos(A-B) + \cos^2 C - \frac{1}{2}$

$= -[-\cos C\cos(A-B)] + \cos^2 C$

$(\because \cos(A+B) = \cos(180° - C) = -\cos C)$

$= \cos C\cos(A-B) + \cos^2 C$

$= \cos C[\cos(A-B) + \cos C)$

$= \cos C(\cos(A-B) + \cos(\pi - (A+B))]$

$= \cos C[\cos(A-B) - \cos(A+B)]$

$= \cos C(2\sin A\sin B)$

$\therefore = 2\sin A\sin B\cos C = RHS.$

(v) $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan A/2 = 1$

Soln $A + B + C = 180°$

$A + B = 180° - C$

$A/2 + B/2 = 90 - C/2$

$\tan(A/2 + B/2) = \tan(90 - C/2) = \cot\frac{C}{2}$

$$\frac{\tan A/2 + \tan B/2}{1 - \tan A/2\tan B/2} = \frac{1}{\tan\frac{C}{2}}$$

$\tan A/2\tan C/2 + \tan B/2\tan C/2 = 1 - \tan A/2\tan B/2$

$\tan A/2\tan B/2 + \tan B/2\tan C/2 + \tan A/2\tan C/2 = 1$

Hence proved

(vi) $\sin A + \sin B + \sin C = 4\cos A/2\cos B/2\cos C/2.$

Soln LHS

$\sin A + \sin B + \sin C$

$= 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) + 2\sin C/2\cos\frac{C}{2}$

$2\sin\left(90 - \frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right) + 2\sin\frac{C}{2}\cdot\cos\frac{C}{2}.$

$= 2\cos C/2\cos\left(\frac{A-B}{2}\right) + 2\sin\frac{C}{2}\cos\frac{C}{2}$

$= 2\cos\frac{C}{2}[\cos\frac{A-B}{2} + \sin\frac{C}{2}]$

$= 2\cos\frac{C}{2}[\cos\left(\frac{A-B}{2}\right) + \sin\left(90 - \left(\frac{A+B}{2}\right)\right)$

$= 2\cos\frac{C}{2}[\cos\left(\frac{A-B}{2}\right) + \cos\frac{A+B}{2})$

$= 2\cos\frac{C}{2}(2\cos A/2\cos B/2)$

$= 4\cos A/2\cos B/2\cos\frac{C}{2})$

$= RHS.$

(vii) $\sin(B+C-A)+\sin(C+A-B)+\sin(A+B-C) = 4\sin A\sin B\sin C$.

Soln LHS: $\sin(180^\circ-A-A)+\sin(180^\circ-B-B)+\sin(180^\circ-C-C)$

$= \sin(180^\circ-2A)+\sin(180^\circ-2B)+\sin(180^\circ-2C)$

$= \sin 2A+\sin 2B+\sin 2C$. $[\because A+B+C=180^\circ$, $B+C=180^\circ-A$, $A+B=180^\circ-C$, $C+A=180^\circ-B]$

$= 2\sin\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right)+2\sin C\cos C$

$= 2\sin(A+B)\cos(A-B)+2\sin C\cos C$

$= 2\sin(180^\circ-C)\cos(A-B)+2\sin C\cos C$

$= 2\sin C[\cos(A-B)+\cos C]$

$= 2\sin C[\cos(A-B)+\cos(\pi-(A+B))]$

$= 2\sin C[\cos(A-B)-\cos(A+B)]$

$= 2\sin C\,(2\sin A\sin B) = 4\sin A\sin B\sin C = RHS$.

2) If $A+B+C=2S$. Prove that $\sin(S-A)\sin(S-B)+\sin S\sin(S-C)=\sin A\sin B$

Soln $A+B+C=2S$.

LHS: $\sin(S-A)\sin(S-B)+\sin S\sin(S-C)$

$= \frac{1}{2}[\cos(S-A-S+B)-\cos(S-A+S-B)] + \frac{1}{2}[\cos(S-S+C)-\cos(S+S-C)]$

$= \frac{1}{2}[\cos(B-A)-\cos(2S-(A+B))] + \cos C-\cos(2S-C)$

$= \frac{1}{2}[\cos(A-B)-\cos C]+\frac{1}{2}[\cos C-\cos(2S-C)]$

$= \frac{1}{2}[\cos(A-B)-\cancel{\cos C}+\cancel{\cos C}-\cos(A+B)]$

$= \frac{1}{2}[\cos(A-B)-\cos(A+B)]$ $(A+B+C=2S$, $\because B+C=2S-A$, $\because 2S-C=A+B)$

$= \frac{1}{2}(2\sin A\sin B)$

$= \sin A\sin B = RHS$.

3) If $x+y+z=xyz$, Prove that

$$\frac{2x}{1-x^2}+\frac{2y}{1-y^2}+\frac{2z}{1-z^2}=\frac{2x}{1-x^2}\cdot\frac{2y}{1-y^2}\cdot\frac{2z}{1-z^2}.$$

Soln. Let $x=\tan A$, $y=\tan B$, $z=\tan C$.

$x+y+z-xyz=0$.

$\tan A+\tan B+\tan C-\tan A\tan B\tan C=0$.

$\Rightarrow \tan(A+B)+\tan C=0$ $\left[\because \tan(A+B+C)=\frac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\ldots}=0\right]$

$\Rightarrow \tan(A+B+C)=0$

$\Rightarrow \tan 2(A+B+C)=0$

$\Rightarrow \tan(2A+2B+2C)=0$

$\Rightarrow \tan 2A+\tan 2B+\tan 2C=\tan 2A\tan 2B\tan 2C$ — (1)

$[\because \tan A+\tan B+\tan C=\tan A\tan B\tan C$ if $\tan(A+B+C)=0]$

Since $x=\tan A$ $\therefore \tan 2A=\frac{2\tan A}{1-\tan^2 A}=\frac{2x}{1-x^2}$

lly $\tan 2B=\frac{2y}{1-y^2}$, $\tan 2C=\frac{2z}{1-z^2}$.

Sub in ① we get

$$\frac{2x}{1-x^2}+\frac{2y}{1-y^2}+\frac{2z}{1-z^2}=\frac{2x}{1-x^2}\cdot\frac{2y}{1-y^2}\cdot\frac{2z}{1-z^2}.$$

4) If $A+B+C=\pi/2$ prove the following

(i) $\sin 2A+\sin 2B+\sin 2C = 4\cos A\cos B\cos C$.

LHS $= \sin 2A+\sin 2B+\sin 2C$

$= 2\sin\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right)+2\sin C\cos C$

$= 2\sin(A+B)\cos(A-B)+2\sin C\cos C$

$= 2\sin(\pi/2-C)\cos(A-B)+2\sin C\cos C$ $\quad (\because A+B+C=\pi/2,\ A+B=\pi/2-C)$

$= 2\cos C\cos(A-B)+2\sin C\cos C$

$= 2\cos C[\cos(A-B)+\sin C]$

$= 2\cos C[\cos(A-B)+\sin(\pi/2-(A+B))]$

$= 2\cos C(\cos(A-B)+\cos(A+B))$ $\quad (\because \sin(\pi/2-\theta)=\cos\theta)$

$= 2\cos C(2\cos A\cos B)$

$= 4\cos A\cos B\cos C$ = RHS

(ii) $\cos 2A+\cos 2B+\cos 2C = 1+4\sin A\sin B\sin C$

LHS $= \cos 2A+\cos 2B+\cos 2C$

$= 2\cos\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right)+(1-2\sin^2 C)$

$= 2\cos(A+B)\cos(A-B)+1-2\sin^2 C$

$= 2\cos(\pi/2-C)\cos(A-B)-2\sin^2 C+1$

$= 1+2\sin C\cos(A-B)-2\sin^2 C$

$= 1+2\sin C[\cos(A-B)-\sin C)$

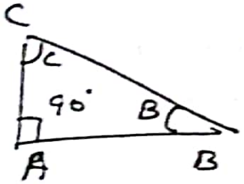
$= 1+2\sin C[\cos(A-B)-\sin(\pi/2-(A+B))]$

$= 1+2\sin C(\cos(A-B)-\cos(A+B)]$

$= 1+2\sin C(2\sin A\sin B)$

$= 1+4\sin A\sin B\sin C$.

5) If ΔABC is a right ~~triangle~~ and if $\angle A=\pi/2$ then prove that.



(i) $\cos^2 B+\cos^2 C=1$

Sol: (i) $\cos^2 B+\cos^2 C=1$

$\angle A=\pi/2 \Rightarrow B+C=\pi/2$

LHS: $\cos^2 B+\cos^2 C$ $\quad (\because C=\pi/2-B)$

$= \cos^2 B+[\cos(\pi/2-B)]^2$

$= \cos^2 B+(\sin B)^2 = \cos^2 B+\sin^2 B=1$.

(ii) $\sin^2 B+\sin^2 C=1$

LHS: $\sin^2 B+\sin^2 C = \sin^2 B+[\sin(\pi/2-B)]^2$ $\quad (\because C=\pi/2-B)$

$= \sin^2 B+\cos^2 B=1$ = RHS

(iii) $\cos B-\cos C = -1+2\sqrt{2}\cos\frac{B}{2}\sin\frac{C}{2}$.

LHS $\cos B-\cos C = \cos(\pi/2-C)-\cos C$

$= \sin C-\cos C = \sin C-(1-2\sin^2\frac{C}{2})$

$= 2\sin\frac{C}{2}\cos\frac{C}{2}-1+2\sin^2\frac{C}{2}$.

$= -1 + 2\sin C/2 \, [\cos C/2 + \sin C/2]$

$= -1 + 2\sin C/2 \, [\cos(45^\circ - \frac{B}{2}) + \sin(45^\circ - B/2)]$

$= -1 + 2\sin C/2 \, [\cos 45^\circ \cos B/2 + \sin 45^\circ \sin B/2$
$+ \sin 45^\circ \cos B/2 - \cos 45^\circ \sin B/2]$

$(\because B + C = 90^\circ \Rightarrow B/2 + C/2 = 45^\circ$
$C/2 = 45^\circ - B/2\}$

$[\because \cos(A - B) = \cos A \cos B + \sin A \sin B$
$\sin(A - B) = \sin A \cos B - \cos A \sin B]$

$= -1 + 2\sin C/2 \left[\frac{1}{\sqrt{2}} \cos B/2 + \cancel{\frac{1}{\sqrt{2}} \sin B/2} + \frac{1}{\sqrt{2}} \cos B/2 - \cancel{\frac{1}{\sqrt{2}} \sin B/2}\right]$

$= -1 + 2\sin C/2 \left(\frac{\sqrt{2}\,\cancel{2} \cos B/2}{\cancel{\sqrt{2}}}\right)$

$= -1 + 2\sqrt{2} \sin C/2 \cos B/2 = R.H.S.$

Exercise 3.8

1. Find Principal solution (முதன்மை மதிப்பு).

(i) $\sin\theta = -\frac{1}{\sqrt{2}} < 0$

$\sin\theta = -\sin \pi/4 = \sin(-\pi/4)$

$\theta = -\pi/4.$

(ii) $\cot\theta = \sqrt{3} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} > 0.$

$\tan\theta = \frac{1}{\sqrt{3}} = \tan \pi/6 \Rightarrow \theta = \pi/6.$

(iii) $\tan\theta = -\frac{1}{\sqrt{3}} = -\tan \pi/6 = \tan(-\pi/6)$

$\theta = -\pi/6.$

2) Solve: (i) $\sin^4 x - \sin^2 x.$ $(0 \le \theta \le 360^\circ)$

$\sin^4 x - \sin^2 x = 0$

$\sin^2 x(\sin^2 x - 1) = 0$

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$\sin^2 x = 0$ | $\sin^2 x - 1 = 0$

$\sin x = 0$ | $\sin^2 x = +1$

$x = 0, \pi$ | $\sin x = \pm 1.$

$\sin x = 1$ | $\sin x = -1$

$x = 90^\circ$ | $x = \pi + \pi/2 = 3\pi/2.$

$\therefore$ Value of x are $0, \pi/2, 3\pi/2, \pi.$

(ii) $2\cos^2 x + 1 = -3\cos x$

$2\cos^2 x + 3\cos x + 1 = 0$

$(\cos x + 1)(2\cos x + 1) = 0$

$\cos x + 1 = 0$ (or) $2\cos x + 1 = 0$

$\cos x = -1$ | $2\cos x = -1$

$\cos x = -\cos 0^\circ$ | $\cos x = -\frac{1}{2} = -\cos \pi/3$

$\cos x = \cos(\pi - 0)$ | $\cos x = \cos(\pi - \pi/3)$

$x = \pi$ | $x = \pi - \pi/3 = 2\pi/3$

(or) $\cos x = \cos(\pi + \pi/3)$

$x = \pi + \pi/3 = 4\pi/3.$

$\therefore x = \pi, 2\pi/3, 4\pi/3.$

(iii) $2\sin^2 x + 1 = 3\sin x$

$2\sin^2 x - 3\sin x + 1 = 0$

$(\sin x - 1)(2\sin x - 1) = 0$

$\sin x - 1 = 0$ (or) $2\sin x - 1 = 0$

$\sin x = 1$ | $2\sin x = 1$

$\sin x = \sin \pi/2$ | $\sin x = \frac{1}{2} = \sin \pi/6$

$x = \pi/2$ | $x = \pi/6$.

$2 \to -3$: $\frac{-2}{2}, \frac{-1}{2}$.

Ans x values are $\pi/2, \pi/6$.

(iv) $\cos 2x = 1 - 3\sin x$.

$\cos 2x = 1 - 3\sin x$

$\cancel{1} - 2\sin^2 x = \cancel{1} - 3\sin x$

$\cancel{-}2\sin^2 x = \cancel{-}3\sin x$

$2\sin^2 x - 3\sin x = 0$

$\sin x(2\sin x - 3) = 0$

$\sin x = 0$ (or) $2\sin x - 3 = 0$

$\sin x = \sin 0$ or $\sin \pi$ | $2\sin x = 3$

$x = 0, \pi$. | $\sin x = 3/2$. not possible

$\therefore$ Ans: $x = 0, \pi$.

3) Solve: தீர்க்க.

(i) $\sin 5x - \sin x = \cos 3x$.

LHS $= 2\cos\left(\frac{5x+x}{2}\right)\sin\left(\frac{5x-x}{2}\right) = \cos 3x$.

$2\cos 3x \sin 2x = \cos 3x$

$2\cos 3x \sin 2x - \cos 3x = 0$

$\cos 3x(2\sin 2x - 1) = 0$

$\cos 3x = 0$ (or) $2\sin 2x - 1 = 0$

$3x = (2n+1)\pi/2$ | $2\sin 2x = 1$

$x = (2n+1)\pi/6$ | $\sin 2x = 1/2 = \sin \pi/6$

$2x = n\pi + (-1)^n \cdot \pi/6$

$x = n\pi/2 + (-1)^n \cdot \pi/12$.

$\therefore$ தீர்வு $x = (2n+1)\pi/6$ (or) $\frac{n\pi}{2} + (-1)^n \cdot \pi/12$.

(ii) $2\cos^2\theta + 3\sin\theta - 3 = 0$.

$2(1-\sin^2\theta) + 3\sin\theta - 3 = 0$

$2 - 2\sin^2\theta + 3\sin\theta - 3 = 0$

$-2\sin^2\theta + 3\sin\theta - 1 = 0$.

$2\sin^2\theta - 3\sin\theta + 1 = 0$

$(\sin\theta - 1)(2\sin\theta - 1) = 0$

$2 \to -3$: $\frac{-2}{2}, \frac{-1}{2}$; $-1, -\frac{1}{2}$

$\sin\theta = 1$ (or) $2\sin\theta - 1 = 0$

$\sin\theta = \sin \pi/2$ | $2\sin\theta = 1$

$\theta = n\pi + (-1)^n \cdot \pi/2$ | $\sin\theta = 1/2 = \sin \pi/6$

$\theta = n\pi + (-1)^n \cdot \pi/6$.

Ans: $\theta = n\pi + (-1)^n \pi/2$)

(or) $\theta = n\pi + (-1)^n \cdot \pi/6$.

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(iii) $\cos\theta + \cos 3\theta = 2\cos 2\theta$.

$\Rightarrow 2\cos\left(\frac{\theta+3\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right) - 2\cos 2\theta = 0$

$2\cos 2\theta \cos\theta - 2\cos 2\theta = 0$

$2\cos 2\theta(\cos\theta - 1) = 0$

$2\cos 2\theta = 0$ (or) $\cos\theta - 1 = 0$

$\cos 2\theta = 0$ | $\cos\theta = 1$

$2\theta = (2n+1)\frac{\pi}{2}$ | $\cos\theta = \cos 0$

$\theta = (2n+1)\pi/4$ | $\theta = 2n\pi \pm 0 = 2n\pi$

Ans: $\theta = (2n+1)\pi/4$ (or) $2n\pi$.

(iv) $\sin\theta + \sin 3\theta + \sin 5\theta = 0$

$2\sin\left(\frac{\theta+5\theta}{2}\right)\cos\left(\frac{5\theta-\theta}{2}\right) + \sin 3\theta = 0$

$2\sin 3\theta \cos 2\theta + \sin 3\theta = 0$

$\sin 3\theta(2\cos 2\theta + 1) = 0$

$\sin 3\theta = 0$ (or) $2\cos 2\theta + 1 = 0$

$\sin 3\theta = 0$ | $2\cos 2\theta = -1$

$3\theta = n\pi$ | $\cos 2\theta = -\frac{1}{2}$

$\theta = \frac{n\pi}{3}$ | $\cos 2\theta = -\cos\pi/3$

$\cos 2\theta = \cos(\pi - \pi/3)$

$\cos 2\theta = \cos 2\pi/3$

$2\theta = 2n\pi \pm 2\pi/3$

$\theta = n\pi \pm \pi/3$.

Ans: $\theta = \frac{n\pi}{3}$ (or) $n\pi \pm \pi/3$, $n \in Z$

(v) $\sin 2\theta - \cos 2\theta - \sin\theta + \cos\theta = 0$

$\sin 2\theta - \sin\theta + \cos\theta - \cos 2\theta = 0$

$\left(2\cos\frac{2\theta+\theta}{2}\sin\frac{2\theta-\theta}{2}\right)\left(2\sin\left(\frac{\theta+2\theta}{2}\right)\cdot\sin\left(\frac{2\theta-\theta}{2}\right)\right) = 0$

$\Rightarrow 2\cos\frac{3\theta}{2}\sin\frac{\theta}{2} + 2\sin\frac{3\theta}{2}\sin\frac{\theta}{2} = 0$

$2\sin\frac{\theta}{2}\left[\cos\frac{3\theta}{2} + \sin\frac{3\theta}{2}\right] = 0$

$2\sin\theta/2 = 0$ | or $\cos\frac{3\theta}{2} + \sin\frac{3\theta}{2} = 0$

$\sin\theta/2 = 0$ | $\cos\frac{3\theta}{2} = -\sin\frac{3\theta}{2}$

$\theta/2 = n\pi$ | $\frac{\sin\frac{3\theta}{2}}{\cos 3\theta/2} = -1$

$\theta = 2n\pi$ | $\tan\frac{3\theta}{2} = \tan\left(\frac{-\pi}{4}\right)$

$\frac{3\theta}{2} = n\pi - \pi/4$

$\theta = \frac{2}{3}n\pi - \frac{2}{3}\pi/4$

$\theta = \frac{2}{3}n\pi - \frac{\pi}{6}$.

Ans $\theta = 2n\pi$ (or) $\frac{2}{3}n\pi - \frac{\pi}{6}$

$n \in Z$.

(vi) $\sin\theta + \cos\theta = \sqrt{2}$

Dividing by $\sqrt{2}$;

$\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = 1$

$\sin\pi/4\,\sin\theta + \cos\pi/4\,\cos\theta = 1$

$\cos\left(\theta - \frac{\pi}{4}\right) = 1$.

$\cos(\theta - \pi/4) = \cos 0$

$\theta - \pi/4 = 2n\pi \pm 0$

$\theta = 2n\pi + \pi/4$

$\theta = \frac{8n\pi + \pi}{4} = \frac{(8n+1)\pi}{4}$

$n \in Z$.

(vii) $\sin\theta + \sqrt{3}\cos\theta = 1$

$(\div 2)$ $\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta = 1/2$

$\sin\pi/6\,\sin\theta + \cos\pi/6\,\cos\theta = \frac{1}{2}$

$\cos(\theta - \pi/6) = \cos\pi/3$

$\theta - \pi/6 = 2n\pi \pm \pi/3$

$\theta = 2n\pi \pm \pi/3 + \pi/6$.

$\theta = 2n\pi + \pi/3 + \pi/6$ (or) $\theta = 2n\pi - \frac{\pi}{3} + \frac{\pi}{6}$

$= 2n\pi + \pi/2 \qquad \theta = 2n\pi - \pi/6.$

(vii) $\cot\theta + \operatorname{cosec}\theta = \sqrt{3}$

$\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta} = \sqrt{3}$

$\frac{\cos\theta + 1}{\sin\theta} = \sqrt{3}$

$\cos\theta + 1 = \sqrt{3}\sin\theta$

$\sqrt{3}\sin\theta - \cos\theta = 1.$

$(\div 2)$ $\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta = \frac{1}{2}$

$\sin\theta - \pi/3 \cos\theta = \frac{1}{2}$

$-\cos(\theta + \pi/3) = \frac{1}{2}$

$\cos(\theta + \pi/3) = -\frac{1}{2} = \cos\left(\frac{2\pi}{3}\right)$

$\theta + \pi/3 = 2n\pi \pm \frac{2\pi}{3}, \; n \in N.$

$\theta = 2n\pi \pm \frac{2\pi}{3} - \pi/3 = 2n\pi \pm 2\pi/3 - \frac{\pi}{3}$

$\theta = 2n\pi + \frac{2\pi}{3} - \pi/3$ (or) $\theta = 2n\pi - \frac{2\pi}{3} - \frac{\pi}{3}$

$\theta = 2n\pi + \pi/3$ (or) $\theta = 2n\pi - \pi$

(ix) $\tan\theta + \tan(\theta + \pi/3) + \tan\left(\theta + \frac{2\pi}{3}\right) = \sqrt{3}$

LHS $\tan\theta + \frac{\tan\theta + \tan\frac{\pi}{3}}{1 - \tan\theta\tan\frac{\pi}{3}} + \frac{\tan\theta + \tan\frac{2\pi}{3}}{1 - \tan\theta\tan\frac{2\pi}{3}} = \sqrt{3}$

$\tan\theta + \frac{\tan\theta + \sqrt{3}}{1 - \sqrt{3}\tan\theta} + \frac{\tan\theta - \sqrt{3}}{1 + \sqrt{3}\tan\theta} = \sqrt{3}$

$(\tan 2\pi/3 = -\sqrt{3})$

$\tan\theta + \frac{(1 + \sqrt{3}\tan\theta)(\tan\theta + \sqrt{3}) + (\tan\theta - \sqrt{3})(1 - \sqrt{3}\tan\theta)}{1 - 3\tan^2\theta} = \sqrt{3}.$

$\frac{\tan\theta + \tan\theta + \cancel{\sqrt{3}} + \cancel{\sqrt{3}\tan^2\theta} + 3\tan\theta + \tan\theta - \cancel{\sqrt{3}\tan^2\theta} - \cancel{\sqrt{3}} + 3\tan\theta}{1 - 3\tan^2\theta} = \sqrt{3}$

$\tan\theta + \frac{8\tan\theta}{1 - 3\tan^2\theta} = \sqrt{3}$

$\frac{\tan\theta - 3\tan^3\theta + 8\tan\theta}{1 - 3\tan^2\theta} = \sqrt{3}$

$\frac{9\tan\theta - 3\tan^3\theta}{1 - 3\tan^2\theta} = \sqrt{3}$

$\frac{3(3\tan\theta - \tan^3\theta)}{1 - 3\tan^2\theta} = \sqrt{3}$

$3\tan 3\theta = \sqrt{3}$

$\tan 3\theta = \sqrt{3}/3 = \frac{1}{\sqrt{3}} = \tan\frac{\pi}{6}$

$3\theta = n\pi + \frac{\pi}{6}$

$\theta = \frac{n\pi}{3} + \frac{\pi}{18}, \; n \in z.$

(x) $\cos 2\theta = \frac{\sqrt{5}+1}{4} = \cos 36^\circ$

$\cos 2\theta = \cos\pi/5$

$2\theta = 2n\pi \pm \pi/5$

$\theta = n\pi \pm \pi/10, \; n \in z.$

(xi) $2\cos^2 x - 7\cos x + 3 = 0$

$(\cos x - 3)(2\cos x - 1) = 0$

$\cos x = 3$ or $2\cos x - 1 = 0$

$\cos x = 3$ not possible | $2\cos x = 1$

$\cos x = \frac{1}{2}$

$\cos x = \frac{1}{2} = \cos\pi/3$

$x = 2n\pi \pm \pi/3.$

$\therefore x = 2n\pi \pm \frac{\pi}{3}, \; n \in z.$

6: $-\frac{6}{2}, -\frac{1}{2}$; $-3, -\frac{1}{2}$

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Exercise : 3.9.

1. In ΔABC, if $\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$ prove that a^2, b^2, c^2 are in arithmetic progression.

Soln using Sine formula.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k.$$

$\sin A = ak$, $\sin B = bk$, $\sin C = ck$.

$$\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)} \quad (\because \sin A = \sin(B+C),\ \sin C = \sin(A+B))$$

$$\frac{\sin(B+C)}{\sin(A+B)} = \frac{\sin(A-B)}{\sin(B-C)}$$

$$\sin(B+C)\sin(B-C) = \sin(A-B)\sin(A+B)$$

$$\sin^2 B - \sin^2 C = \sin^2 A - \sin^2 B.$$

$$k^2b^2 - k^2c^2 = k^2a^2 - k^2b^2.$$

$$b^2 - c^2 = a^2 - b^2.$$

$$b^2 + b^2 = a^2 + c^2.$$

$$2b^2 = a^2 + c^2.$$

$\therefore a^2, b^2, c^2$ are in A.P.

2) The angles of a triangle ABC, are in arithmetic progression and if $b:c = \sqrt{3}:\sqrt{2}$ find $\angle A$.

Soln $\angle A, \angle B, \angle C$ are in A.P

$$2\angle B = \angle A + \angle C$$

$$3\angle B = \angle A + \angle B + \angle C$$

$$3\angle B = 180^\circ$$

$$\angle B = \frac{180^\circ}{3} = 60^\circ$$

Now $\frac{b}{c} = \frac{\sin B}{\sin C}$

$$\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sin 60^\circ}{\sin C} = \frac{\sqrt{3}/2}{\sin C} \quad (\because b:c = \sqrt{3}:\sqrt{2})$$

$$\sin C = \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}.$$

$\angle C = 45^\circ$ $\therefore \angle A = 180^\circ - (\angle B + \angle C) = 180^\circ - (60^\circ + 45^\circ)$

$$= 180^\circ - 105 = 75^\circ$$

$\angle A = 75^\circ$, $\angle B = 60^\circ$, $\angle C = 45^\circ$.

3) In a ΔABC if $\cos C = \frac{\sin A}{2\sin B}$ show that the triangle is isoceles.

Soln $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k.$

$\sin A = ka$, $\sin B = bk$, $\sin C = ck$.

$$\cos C = \frac{\sin A}{2\sin B}$$

$$2\cos C \sin B = \sin A$$

$$2\left(\frac{a^2+b^2-c^2}{2ab}\right)(kb) = ka$$

$$a^2 + b^2 - c^2 = a^2 \Rightarrow b = c.$$

$$b^2 = c^2$$

$\therefore \Delta ABC$ is isoceles (இருசமபக்க Δ).

4) In a ΔABC, P.T $\frac{\sin B}{\sin C} = \frac{c - a\cos B}{b - a\cos C}.$

Soln $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k.$

$\Rightarrow a = k\sin A,\ b = k\sin B,\ c = k\sin C.$

LHS: $a\cos A + b\cos B + c\cos C$

$= k\sin A\cos A + k\sin B\cos B + k\sin C\cos C$

$= \frac{k}{2}(2\sin A\cos A + 2\sin B\cos B + 2\sin C\cos C]$

$= \frac{k}{2}[\sin 2A + \sin 2B + \sin 2C].$

$= \frac{k}{2}[4\sin A\sin B\sin C]$ (Qn : 1(i) Ex 3.7)

$= \frac{k}{\cancel{2}}[\overset{2}{\cancel{4}}\sin A\sin B\sin C]$

$= (2k\sin A\sin B\sin C)$ $=$ RHS. ($\because k\sin A = a$)

$= 2a\sin B\sin C$

6) In ΔABC, $\angle A = 60^\circ$ P.T $b+c = 2a\cos\left(\frac{B-C}{2}\right)$

Soln

$A+B+C = 180^\circ$

$B+C = 180^\circ - A = 180^\circ - 60^\circ = 120^\circ$

$\frac{B+C}{2} = \frac{120^\circ}{2} = 60^\circ$

Using sine formula $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$

$a = k\sin A,\ b = k\sin B,\ c = k\sin C$

RHS $= 2a\cos\left(\frac{B-C}{2}\right) = 2(k\sin A)\cos\left(\frac{B-C}{2}\right)$

$= 2k\sin 60^\circ\cos\left(\frac{B-C}{2}\right) = 2k\sin\frac{B+C}{2}\cos\frac{B-C}{2}$

$= k\left[\sin\left(\frac{B+C+B-C}{2}\right) + \sin\left(\frac{B+C-B+C}{2}\right)\right]$

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$= k(\sin B + \sin C) = k\sin B + k\sin C$

$= b+c =$ LHS.

7) In ΔABC. Prove (i) $a\sin\left(\frac{A}{2}+B\right) = (b+c)\sin\frac{A}{2}$

Soln (i) $\frac{b+c}{a} = \frac{\cancel{k}\sin B + \cancel{k}\sin C}{\cancel{k}\sin A} = \frac{\cancel{k}(\sin B + \sin C)}{\cancel{k}\sin A}$

$\frac{\sin B + \sin C}{\sin A} = \frac{\cancel{2}\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)}{\cancel{2}\sin A/2\cos A/2}.$

$\frac{b+c}{a} = \frac{\sin(90 - A/2)\cos\left(\frac{B-C}{2}\right)}{\sin A/2\cos A/2} = \frac{\cancel{\cos A/2}\cos\frac{B-C}{2}}{\sin A/2\cancel{\cos A/2}}$

$= \frac{\cos\frac{B-C}{2}}{\sin A/2} = \frac{\cos\left(\frac{B-(\pi-A-B)}{2}\right)}{\sin A/2}$

$= \frac{\cos(-90^\circ + A/2 + B)}{\sin A/2} = \frac{\cos(-(90^\circ - (A/2+B)))}{\sin A/2}$

$\frac{b+c}{a} = \frac{\cos(90^\circ - (A/2+B))}{\sin A/2}$

$\frac{b+c}{a} = \frac{\sin(A/2+B)}{\sin A/2}.$

$\frac{b+c}{a}\sin A/2 = \sin(A/2+B)$

$(b+c)\sin A/2 = a\sin(A/2+B).$

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(iii) $a(\cos B + \cos C) = 2(b+c)\sin^2 A/2$.

LHS: $a(\cos B + \cos C) = a\left[\frac{c^2+a^2-b^2}{2ac} + \frac{a^2+b^2-c^2}{2ab}\right]$

$= \frac{a}{2a}\left[\frac{c^2+a^2-b^2}{c} + \frac{a^2+b^2-c^2}{b}\right]$

$= \frac{1}{2bc}\left[bc^2+a^2b-b^3+a^2c+b^2c-c^3\right]$

$= \frac{1}{2bc}\left[-(b^3+c^3)+(a^2b+a^2c)+bc^2+b^2c\right]$

$= \frac{1}{2bc}\left[-(b+c)(b^2-bc+c^2)+a^2(b+c)+bc(b+c)\right]$

$= \frac{1}{2bc}\left[-(b+c)(b^2-bc+c^2)+a^2+bc\right]$

$= \frac{1}{2bc}(b+c)\left[-b^2+bc-c^2+a^2+bc\right]$

$= \frac{b+c}{2bc}\left[a^2-b^2-c^2+2bc\right]$

$= \frac{b+c}{2bc}\left[a^2-b^2-c^2+2bc\right]$

$= \frac{b+c}{2bc}\left[(a+b-c)(a-b+c)\right]$

$= \frac{b+c}{2bc}\left[(a+b-c)(a+b+c-2b)\right]$

$= \frac{b+c}{2bc}\left((a+b+c-2c)(a+b+c-2b)\right]$

$= \frac{b+c}{2bc}\left((2s-2c)(2s-2b)\right]$

$= \frac{b+c}{2bc}(\cancel{4})^{2}(s-c)(s-b)$

$= \frac{2(b+c)(s-b)(s-c)}{bc} = 2(b+c)\sin^2\frac{A}{2}$

(iii) $\frac{a^2-c^2}{b^2} = \frac{\sin(A-C)}{\sin(A+C)}$.

Soln $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$.

$a = k\sin A$, $b = k\sin B$, $c = k\sin C$

LHS: $\frac{a^2-c^2}{b^2} = \frac{k^2\sin^2 A - k^2\sin^2 C}{k^2\sin^2 B}$

$= \frac{\sin^2 A - \sin^2 C}{\sin^2 B} = \frac{\sin(A+C)\sin(A-C)}{\sin^2[180^\circ-(A+C)]}$

$= \frac{\cancel{\sin(A+C)}\sin(A-C)}{\sin^{\cancel{2}}(A+C)} = \frac{\sin(A-C)}{\sin(A+C)} = RHS$.

(iv) $\frac{a\sin(B-C)}{b^2-c^2} = \frac{b\sin(C-A)}{c^2-a^2} = \frac{c\sin(A-B)}{a^2-b^2}$.

Soln Let $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$.

$a = k\sin A$, $b = k\sin B$, $c = k\sin C$.

LHS: $\frac{a\sin(B-C)}{b^2-c^2} = \frac{k\sin A\sin(B-C)}{k^2\sin^2 B - k^2\sin^2 C}$

$= \frac{k\sin(B+C)\sin(B-C)}{k^2(\sin^2 B - \sin^2 C)}$

$= \frac{1}{k}\frac{(\sin^2 B - \cancel{\sin^2 C})}{(\sin^2 B - \cancel{\sin^2 C})} = \frac{1}{k}$.

இதேபோன்று, $\frac{b\sin(C-A)}{c^2-a^2} = \frac{1}{K}$ & $\frac{c\sin(A-B)}{a^2-b^2} = \frac{1}{K}$

$\therefore \frac{a\sin(B-C)}{a^2-b^2} = \frac{b\sin(C-A)}{c^2-a^2} = \frac{c\sin(A-B)}{a^2-b^2}$

(v) $\frac{a+b}{a-b} = \tan\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right)$

Soln $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$.

$a = k\sin A$, $b = k\sin B$, $c = k\sin C$.

LHS: $\frac{a+b}{a-b} = \frac{2k\sin A + 2k\sin B}{2k\sin A - 2k\sin B} = \frac{\sin A + \sin B}{\sin A - \sin B}$

$= \frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}$

$= \tan\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right) =$ RHS.

8) In $\triangle ABC$, P.T $(a^2-b^2+c^2)\tan B = (a^2+b^2-c^2)\tan C$

Soln $\frac{a^2-b^2+c^2}{a^2+b^2-c^2} = \frac{\tan C}{\tan B} = \tan C \cdot \cot B$.

LHS: $\frac{a^2-b^2+c^2}{a^2+b^2-c^2} = \frac{k^2\sin^2 A - k^2\sin^2 B + k^2\sin^2 C}{k^2\sin^2 A + k^2\sin^2 B - k^2\sin^2 C}$

$= \frac{\sin^2 A - \sin^2 B + \sin^2 C}{\sin^2 A - \sin^2 C + \sin^2 B} = \frac{\sin(A+B)\sin(A-B) + \sin^2 C}{\sin(A+C)\sin(A-C) + \sin^2 B}$

$= \frac{\sin C\sin(A-B) + \sin^2 C}{\sin B\sin(A-C) + \sin^2 B}$

$= \frac{\sin C\,[\sin(A-B) + \sin C]}{\sin B\,[\sin(A-C) + \sin B]}$

$= \frac{\sin C\,(\sin(A-B) + \sin(A+B))}{\sin B\,(\sin(A-C) + \sin(A+C))}$

$= \frac{\sin C\,(2\sin A\cos B)}{\sin B\,(2\sin A\cos C)}$

$= \frac{\sin C}{\cos C} \cdot \frac{\cos B}{\sin B} = \tan C \cot B =$ RHS.

9) Refer book.

Soln. Maximum area means equilateral $\triangle$

(அதிகபட்ச பரப்பு அனைத்து பக்கங்கள் சமமாகும்)

$a + a + a = 120\text{m}$

$3a = 120$

$a = \frac{120}{3} = 40\text{m}$.

Ans. 40m, 40m, 40m.

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10). Refer book.

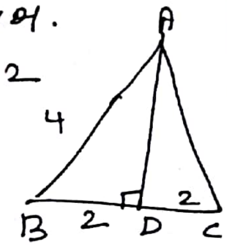
Soln Length = 12 m = நீளம்

Perimeter = 12 m = சுற்றளவு.

by Problem No. 9 $a+a+a = 12$

$3a = 12$

$a = 4$ m.



$AD = \sqrt{AB^2 - BD^2} = \sqrt{4^2 - 2^2} = \sqrt{16-4}$

$= \sqrt{12} = 2\sqrt{3}$.

∴ Area of largest triangle (பெரிய முக்கோணத்தின் பரப்பு)

$ABC = \frac{1}{2} \times BC \times AD = \frac{1}{\cancel{2}} \times 4 \times \cancel{2}\sqrt{3}$

$= 4\sqrt{3}$ sq.m.

11) Derive Projection formula (வீழல் சூத்திரம்).

(i) Law of Sines: சைன் விதி.

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$

$a = k\sin A$, $b = k\sin B$, $c = k\sin C$.

$$\boxed{a = b\cos C + c\cos B}$$

RHS $b\cos C + c\cos B = k\sin B\cos C + k\sin C\cos B$

$= k[\sin B\cos C + \cos B\sin C]$

$= k\sin(B+C) = k\sin(180° - A)$

$= k\sin A = a$.

இதுபோல, $b = c\cos A + a\cos C$

$c = a\cos B + b\cos A$.

(ii) Law of cosines: கொசைன் விதி

$$\boxed{a = b\cos C + c\cos B}$$

RHS:- $b\cos C + c\cos B = \cancel{b}\left(\frac{a^2+b^2-c^2}{2a\cancel{b}}\right) + \cancel{c}\left(\frac{c^2+a^2-b^2}{2\cancel{c}a}\right)$

$= \frac{a^2+b^2-c^2}{2a} + \frac{c^2+a^2-b^2}{2a} = \frac{a^2+\cancel{b^2}-\cancel{c^2}+\cancel{c^2}+a^2-\cancel{b^2}}{2a}$

$= \frac{2a^2}{2a} = a = LHS$.

Exercise 3.11

1. Find principal value (முதன்மை மதிப்பு)

(i) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = y$

$\frac{1}{\sqrt{2}} = \sin y$

$\sin \pi/4 = \sin y$

$y = \pi/4$

(ii) $\cos^{-1}(\sqrt{3}/2) = y$

$\sqrt{3}/2 = \cos y$

$\cos y = \cos \pi/6$

$y = \pi/6$.

(iii) $\operatorname{cosec}^{-1}(-1) = y$

$-1 = \operatorname{cosec} y$.

$-\operatorname{cosec} \pi/2 = \operatorname{cosec} y$

$\operatorname{cosec}(-\pi/2) = \operatorname{cosec} y$

$y = -\pi/2$

(iv) $\sec^{-1}(-\sqrt{2}) = y$

$-\sqrt{2} = \sec y$

$-\sec \pi/4 = \sec y$

$\sec(\pi - \pi/4) = \sec y$

$y = \pi - \pi/4 = 3\pi/4$.

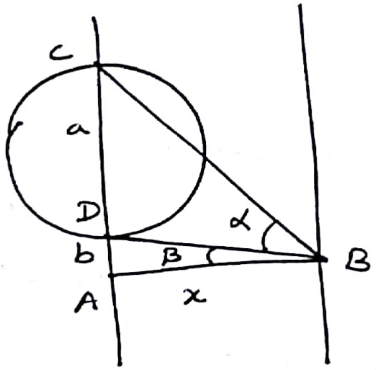
(v) $y = \tan^{-1}\sqrt{3}$

$\sqrt{3} = \tan y$

$\tan \pi/3 = \tan y$

$y = \pi/3$ $\quad\therefore$ Ans : $\pi/3$.

2) Refer book.



Soln $DC = a$.

$\angle CBD = \alpha$

$DA = b$

$AB = x$. $\angle DBA = \beta$

In ΔABD.

$\tan\beta = \frac{opp}{adj} = \frac{DA}{AB} = \frac{b}{x}$ — (1)

$\beta = \tan^{-1}(b/x)$

In ΔABC, $\tan(\alpha+\beta) = \frac{AC}{AB} = \frac{a+b}{x}$

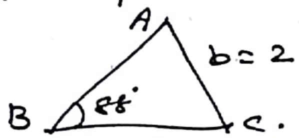
$\alpha+\beta = \tan^{-1}\left(\frac{a+b}{x}\right)$

$\alpha = \tan^{-1}\left(\frac{a+b}{x}\right) - \beta$

$\alpha = \tan^{-1}\left(\frac{a+b}{x}\right) - \tan^{-1}\left(\frac{b}{x}\right)$ (by (1))

Exercise 3.10

1) Refer book. $\angle B = 88°$, $a = 23$, $b = 2$.



Soln $\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{\sin A}{a} = \frac{\sin B}{b}$

$\Rightarrow \frac{\sin A}{a} = \frac{\sin 88°}{2} \Rightarrow \sin A = 23\frac{\sin 88°}{2}$

$\Rightarrow \sin A = 23 \times 0.999 = 22.99$ which is not possible.

solution of Δ does not exist.

(முக்கோணத்திற்கு தீர்வு இல்லை)

2) In ΔABC, $a = 4$, $b = 6$, $c = 8$.

S.T $4\cos B + 3\cos C = 2$

Soln LHS: $4\cos B + 3\cos C$

$= 4\left(\frac{a^2+c^2-b^2}{2ac}\right) + 3\left(\frac{a^2+b^2-c^2}{2ab}\right)$

$= 4\left(\frac{16+64-36}{2\cdot4\cdot8}\right) + 3\left(\frac{16+36-64}{2\cdot4\cdot6}\right)$

$= 4\left(\frac{80-36}{64}\right) + 3\left(\frac{52-64}{48}\right) = \frac{11}{4} - \frac{3}{4}$

$= 8/4 = 2 = RHS.$

3) In ΔABC if $a = \sqrt{3}-1$, $b = \sqrt{3}+1$ and $\angle C = 60°$.

Soln $\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot C/2$.

$= \dfrac{(\sqrt{3}+1)-(\sqrt{3}-1)}{(\sqrt{3}+1)+(\sqrt{3}-1)} \cot\dfrac{60^\circ}{2}$

$= \dfrac{\cancel{\sqrt{3}}+1-\cancel{\sqrt{3}}+1}{2\sqrt{3}+1-1} \cot 30^\circ = \dfrac{\cancel{2}}{\cancel{2}\sqrt{3}} \cdot \sqrt{3} = 1$

$\tan\dfrac{A-B}{2} = 1 = \tan 45^\circ$

$\dfrac{A-B}{2} = 45^\circ \Rightarrow A-B = 90^\circ$ — ①

$A+B = 180^\circ - C = 180^\circ - 60^\circ = 120^\circ$ — ②

① + ② ⇒ $2A = 210^\circ \Rightarrow \angle A = 105^\circ$

① − ② ⇒ $2B = 30^\circ \Rightarrow \angle B = 15^\circ$

Sine formula (சைன் விதி)

$\dfrac{c}{\sin C} = \dfrac{a}{\sin A} \Rightarrow c = a\dfrac{\sin C}{\sin A}$

$c = \dfrac{(\sqrt{3}+1)\sin 60^\circ}{\sin 105^\circ} = \dfrac{(\sqrt{3}+1)\sqrt{3}/2}{\dfrac{(\sqrt{3}+1)}{2\sqrt{2}}}$

$= \dfrac{(\sqrt{3}+1)}{2} \times \sqrt{3} \times \dfrac{2\sqrt{2}}{(\sqrt{3}+1)} = \sqrt{6}$.

$\angle A = 105^\circ$ $\angle B = 15^\circ$ and $c = \sqrt{6}$.

4) In ΔABC, P.T area $\Delta = \dfrac{b^2+c^2-a^2}{4\cot A}$.

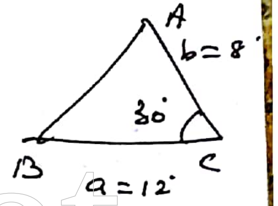
Soln RHS $= \dfrac{b^2+c^2-a^2}{4\cot A} = \dfrac{b^2+c^2-a^2}{4\dfrac{\cos A}{\sin A}}$

$= \dfrac{b^2+c^2-a^2}{4\cos A} \times \sin A = \dfrac{b^2+c^2-a^2}{4\left(\dfrac{b^2+c^2-a^2}{2bc}\right)} \cdot \sin A$

$= \dfrac{2bc}{4}\sin A = \dfrac{1}{2}bc\sin A =$ area of ΔABC

$\Delta = \dfrac{b^2+c^2-a^2}{4\cot A}$.

5) In ΔABC, $a = 12$ cm, $b = 8$ cm, $\angle C = 30^\circ$



Soln Area of $\Delta ABC = \dfrac{1}{2}ab\sin C$

$= \dfrac{1}{2} \times 12 \times 8 \times \sin 30^\circ$

$= 48\left(\dfrac{1}{2}\right) = 24$ sq.m.

6) In ΔABC, if $a = 18$ cm, $b = 24$ cm $c = 30$ cm S.T area = 216 sq cm.

Soln area of ΔABC $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

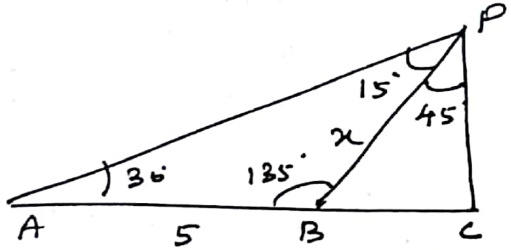
$s = \dfrac{a+b+c}{2} = \dfrac{18+24+30}{2} = \dfrac{72}{2} = 36$

$\Delta = \sqrt{36(36-18)(36-24)(36-30)}$

$= \sqrt{36 \times 18 \times 12 \times 6} = \sqrt{6 \times 6 \times 6 \times 3 \times 6 \times 2 \times 2 \times 3}$

$= 6 \times 6 \times 2 \times 3 = 216$ sq.cm.

7) Refer book.



In ΔABP

$\angle PAB = 30°$

$\angle PBC = 45°$

$\angle APB = 15°$

Sine formula (சைன் விதி)

$$\frac{5}{\sin 15°} = \frac{x}{\sin 30°} \Rightarrow x = \frac{5}{\sin 15°} \cdot \sin 30°$$

$$x = 5 \times \frac{2\cancel{\sin 15°}\cos 15°}{\cancel{\sin 15°}} \qquad \begin{pmatrix}\sin 30° = \sin 2(15°) \\ = 2\sin A\cos A \\ \sin 2A = 2\sin A\cos A\end{pmatrix}$$

$$x = 10 \cos 15° \quad — ①$$

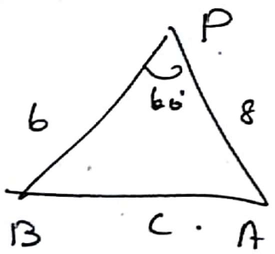
$$\cos 15 = \cos(45° - 30°) = \cos 45° \cos 30° + \sin 45° \sin 30°$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}.$$

$$\therefore ① \Rightarrow x = \cancel{10}^{5}\left(\frac{\sqrt{3}+1}{\cancel{2}\sqrt{2}}\right) = \frac{5(\sqrt{3}+1)}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{5\sqrt{2}(\sqrt{3}+1)}{2}.$$

8) Refer book.



$PA = 8$, $PB = 6$

$a = 6$ $b = 8$, $\angle C = 60°$

Cosine formula (கோசைன் சூத்திரம்)

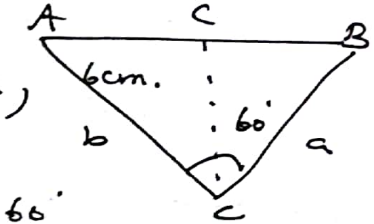
$$c^2 = a^2 + b^2 - 2ab\cos C = 36 + 64 - 2 \cdot 6 \cdot 8 \cos 60°$$

$$= 100 - 12 \times 8 \times \frac{1}{2} = 100 - 48 = 52.$$

$$c^2 = 52 \Rightarrow c = \sqrt{52} = \sqrt{2 \times 2 \times 13} = 2\sqrt{13}$$

$c = 2\sqrt{13}$ km.

9) Refer book.



Cosine formula (கோசைன்)

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$10^2 = a^2 + 6^2 - 2a(6)\cos 60°$$

$$100 = a^2 + 36 - \cancel{12}^{6} a \cdot \frac{1}{\cancel{2}}$$

$$100 - 36 = a^2 - 6a$$

$$64 = a^2 - 6a \Rightarrow a^2 - 6a - 64 = 0$$

$$a = \frac{+6 \pm \sqrt{36 - 4 \cdot 1(-64)}}{2 \cdot 1} = \frac{6 \pm \sqrt{36 + 256}}{2}$$

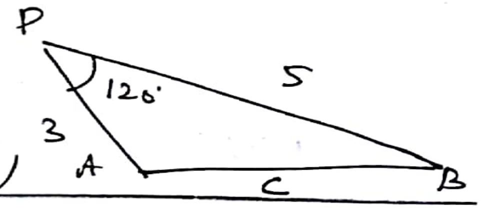
$$= \frac{6 \pm \sqrt{292}}{2} = \frac{6 \pm 3\sqrt{73}}{2}$$

$$= \frac{\cancel{2}(3 \pm \sqrt{73})}{\cancel{2}} \qquad \left(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.\right)$$

$a = (3 + \sqrt{73})$ km $\qquad$ $(3 - \sqrt{73}$ is $-$ve$)$.

10) Refer book.

$a = 5$, $b = 3$ $\angle C = 120°$



Cosine formula (கோசைன் சூத்திரம்)

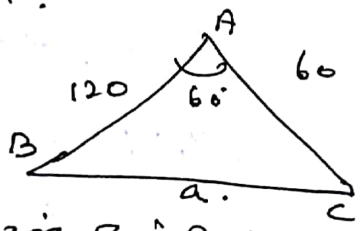
$c^2 = a^2+b^2-2ab\cos C = 25+9-2(5)(3)\cos 120^\circ$

$= 34-30\cos(180^\circ-60^\circ) = 34-30(-\cos 60^\circ)$

$= 34-30\left(-\frac{1}{2}\right) = 34+15 = 49$

$c^2 = 49.$ $c = \sqrt{49} = 7$ km.

11) Refer book.



Soln $b = 60$ $c = 120$ $\angle A = 60^\circ$

Using cosine formula (கொசைன் சூத்திரம்)

$a^2 = b^2+c^2-2bc\cos A$

$= 120^2+60^2-2(120)(60)\cos 60^\circ$

$= 14400+3600-14400\left(\frac{1}{2}\right)$

$a^2 = 14400+3600-7200 = 10800.$

$a = \sqrt{100\times4\times9\times3} = 60\sqrt{3}.$

Perimeter (சுற்றளவு) $s = a+b+c = 120+60+60\sqrt{3}$

$= 180+60\sqrt{3} = 60(3+\sqrt{3})$ அலகு.

Area $\Delta ABC = \frac{1}{2}bc\sin A = \frac{1}{\cancel{2}}\times\overset{30}{\cancel{60}}\times120\times\sin 60^\circ$

$= 30\times120\times\frac{\sqrt{3}}{2} = 30\times60\times\sqrt{3}$

$= 1800\sqrt{3} = 1800\times1.732$

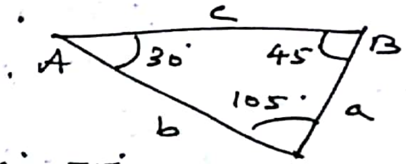
$= 3117.6.$

1 sq ft cost (செலவு) 500

3117.6 sq ft cost (செலவு) $= 3117.6\times500$

$= ₹\ 1,55,8800.$

12) Refer book



Soln $\angle BAC = 30^\circ$ $\angle ABC = 45^\circ$

$\angle C = 180^\circ-(30^\circ+45^\circ) = 180^\circ-75^\circ$

$\angle C = 105^\circ$

$AB = 100$ km.

Using sine formula (சைன் விதி)

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{a}{\sin 30^\circ} = \frac{100}{\sin 105^\circ}$

$\frac{a}{1/2} = \frac{100}{\sin 105^\circ} \Rightarrow \frac{\overset{50}{\cancel{100}}}{\sin 105^\circ} = \cancel{2}a$

$a = \frac{50}{\sin 105^\circ}$ —(1)

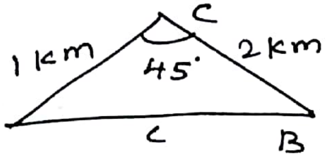
$\sin(105^\circ) = \sin(60^\circ+45^\circ) = \sin 60^\circ\cos 45^\circ + \cos 60^\circ\sin 45^\circ$

$= \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} + \frac{1}{2}\cdot\frac{1}{\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}.$

(1) $\Rightarrow$ $a = \frac{50}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{50\times2\sqrt{2}}{\sqrt{3}+1} = \frac{100\sqrt{2}}{(\sqrt{3}+1)}\cdot\frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}$

$= \frac{100\sqrt{2}(\sqrt{3}-1)}{\sqrt{3}^2-1^2} = \frac{100\sqrt{2}(\sqrt{3}-1)}{3-1}.$

$a = \frac{\cancel{100}^{50}(\sqrt{6}-\sqrt{2})}{\cancel{2}} = 50(\sqrt{6}-\sqrt{2})$ km.



13) Refer book

soln $\angle ACB = 45°$

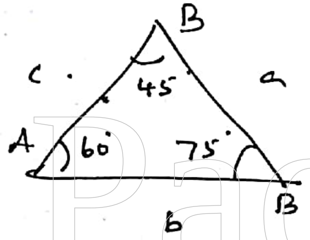
Cosine formula (கோசைன் சூத்திரம்)

$c^2 = a^2+b^2-2ab\cos C = 2^2+1^2-2(2)(1)\cos 45$

$= 4+1-4\left(\frac{1}{\sqrt{2}}\right) = 5 - 2\times\sqrt{2}\times\cancel{\sqrt{2}}\cdot\frac{1}{\cancel{\sqrt{2}}}$

$c^2 = (5-2\sqrt{2})$ km

$c = \sqrt{5-2\sqrt{2}}$ km.



14) Refer book.

$AC = 4$ km $\angle A = 60°$

$\angle B = 45°$ $\angle C = 180° - (A+B)$

$\angle C = 180° - (60°+45°)$

$\angle C = 180° - 105° = 75°$

Using Sine formula (சைன் விதி)

$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{a}{\sin 60°} = \frac{b}{\sin 45°}$

$\frac{a}{\sqrt{3}/2} = 4/\frac{1}{\sqrt{2}} \Rightarrow a = \cancel{4}^{2}\times\sqrt{2}\times\frac{\sqrt{3}}{\cancel{2}}$

$a = 2\sqrt{6}$ km.

Using sine formula (சைன் விதி)

$\frac{c}{\sin C} = \frac{a}{\sin A}$. — (1)

$\sin 75° = \sin(45°+30°) = \sin 45°\cos 30° + \cos 45°\sin 30°$

$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$.

(1) $\Rightarrow \frac{c}{\sin 75°} = \frac{2\sqrt{6}}{\sin 60°}$

$\frac{c}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{2\sqrt{6}}{\sqrt{3}/2} = \frac{2\sqrt{6}\times 2}{\sqrt{3}} = \frac{4\cancel{\sqrt{3}}\sqrt{2}}{\cancel{\sqrt{3}}}$

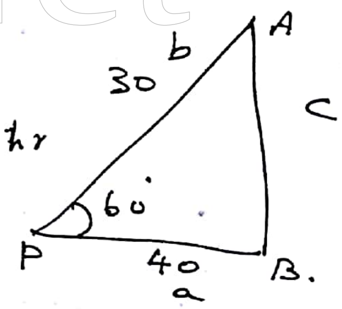
$c\left(\frac{2\sqrt{2}}{\sqrt{3}+1}\right) = 4\sqrt{2} \Rightarrow c = \frac{(\sqrt{3}+1)\cancel{4}^{2}\sqrt{2}}{\cancel{2}\cancel{\sqrt{2}}}$

$c = 2(\sqrt{3}+1)$.

Total $= a+b+c = 2\sqrt{6}+4+2(\sqrt{3}+1)$

$= 2\sqrt{6}+4+2\sqrt{3}+2$

$= (2\sqrt{6}+6+2\sqrt{3})$ km.



15) Refer book

soln வேகம் (speed) = 60 km hr

time (நேரம்) = 1/2 hr.

Distance = speed × time

(தூரம்) = வேகம் × நேரம்

$= 60\times\frac{1}{2} = 30$ km

PA = 30 km

speed by II vehicle
2வது வாகனம் வேகம் } = 80 km hr.

(Time) நேரம் = 1/2 hr.

தூரம் (Distance) $= 80 \times \frac{1}{2} = 40$

$PB = 40$

$\angle APB = 60^\circ$

Cosine formula (கொசைன் சூத்திரம்)

$$c^2 = a^2 + b^2 - 2ab\cos C$$

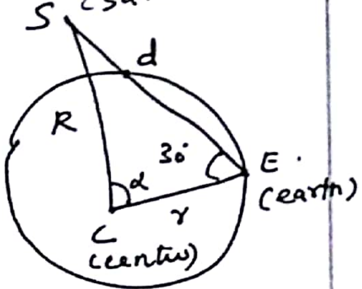
$$c^2 = 40^2 + 30^2 - 2(40)(30)\cos 60^\circ$$

$$= 1600 + 900 - 2(40)(30)\left(\frac{1}{2}\right)$$

$$= 2500 - 40(30) = 1300$$

$$c = \sqrt{1300} = 10\sqrt{3}\ \text{km.}$$

16) Refer book.



Soln

$CE = r$, $CS = R$, $SE = d$

$\angle SCE = \alpha$.

Cosine rule.

$$d^2 = r^2 + R^2 - 2rR\cos\alpha$$

$$\frac{d^2}{R^2} = \frac{r^2}{R^2} + \frac{R^2}{R^2} - \frac{2rR\cos\alpha}{R^2} \quad (\div R^2)$$

$$\frac{d^2}{R^2} = \frac{r^2}{R^2} + 1 - \frac{2r}{R}\cos\alpha$$

$$d^2 = R^2\left[\frac{r^2}{R^2} + 1 - \frac{2r}{R}\cos\alpha\right]$$

$$d = R\sqrt{1 + (r/R)^2 - \frac{2r}{R}\cos\alpha}$$

— x —.