RAVI MATHS TUITION CENTER, PH - 8056206308

Important 2 marks for quarterly

Date : 30-Aug-19

12th Standard

Business Maths

Reg.No. :

Time : 02:30:00 Hrs

Total Marks : 100

MY YOUTUBE CHANNEL NAME
SR MATHS TEST PAPERS
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41 x 2 = 82

1) Solve the following equations by using Cramer's rule

2x + 3y = 7; 3x + 5y = 9

$$\Delta = \begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix} = 10 - 9 = 1 \neq 0$$

Since $\Delta \neq 0$ we can apply Cramer's rule and the system is consistent with unique solution.

$$\Delta x = \begin{vmatrix} 7 & 3 \\ 9 & 5 \end{vmatrix} = 7(5) - 9(3)$$

= 35 - 27 = 8

$$\Delta y = \begin{vmatrix} 2 & 7 \\ 3 & 9 \end{vmatrix} = 2(9) - 3(7)$$

= 18 - 21 =-3

$$\therefore \ x = \frac{\Delta x}{\Delta} = \frac{8}{1} = 8$$

$$y = \frac{\Delta y}{\Delta} = \frac{-3}{1} = 3$$

$\therefore$ Solution set is {8, -3)

2) Find the rank of the matrix $A = \begin{pmatrix} 1 & -3 & 4 & 7 \\ 9 & 1 & 2 & 0 \end{pmatrix}$

Given $A = \begin{pmatrix} 1 & -3 & 4 & 7 \\ 9 & 1 & 2 & 0 \end{pmatrix}$

$$\begin{pmatrix} 1 & -3 & 4 & 0 \\ 0 & 28 & -34 & -63 \end{pmatrix} R_2 \to R_2 - 9R_1$$

$$-\begin{pmatrix} 1 & -3 & 4 & 0 \\ 0 & 0 & \frac{10}{3} & -63 \end{pmatrix} R_2 \to R_2 + \frac{28}{3}. R_1$$

The last equivalent matrix is in echelon form and there are 2 non-zero rows

$\therefore \rho(A) = 2$

3) Find k if the equations 2x+3y−z=5,3x−y+4z=2,x+7y−6z=k are consistent.

Given non-homogeneous equations are

2x + 3y - z = 5, 3x - y + 4z = 2, x + 7y - 6z = k

Augmented matrix	Elementary Transformation
$\begin{pmatrix} 2 & 3 & -1 & 5 \\ 3 & -1 & 4 & 2 \\ 1 & 7 & -6 & k \end{pmatrix}$	
$\begin{pmatrix} 1 & 7 & -6 & k \\ 3 & -1 & 4 & 2 \\ 2 & 3 & -1 & 5 \end{pmatrix}$	$R_1 \leftrightarrow R_3$
$\begin{pmatrix} 1 & 7 & -6 & k \\ 0 & -22 & 22 & 2-3k \\ 0 & -11 & 11 & 5-2k \end{pmatrix}$	$R_2 \to R_2 - 3R_1$
$\begin{pmatrix} 1 & 7 & -6 & k \\ 0 & -22 & 22 & 2-3k \\ 0 & 0 & 0 & 2(5-2k)-(2-3k) \end{pmatrix}$	$R_2 \to R_2 - 3R_1$ $R_3 \to R_3 - 2R_1$
$\begin{pmatrix} 1 & 7 & -6 & k \\ 0 & -22 & 22 & 2-3k \\ 0 & 0 & 0 & 10-4k-2+3k \end{pmatrix}$	$R_3 \to 2R_3 - R_2$
$\begin{pmatrix} -1 & 7 & -6 & k \\ 0 & -22 & 22 & 2-3k \\ 0 & 0 & 0 & 8-k \end{pmatrix}$	

Here $\rho(A) = 2$

Since the given system is consistent, ρ (A, B) must be equal to 2.

This can happen only when

8-k=0 ⇒ k=8

4) Find k if the equations x+y+z=1,3x−y−z=4,x+5y+5z=k are inconsistent.

x +y + z = I, 3x - Y - z = 4, x + 5y + 5z = k

Augmented matrix [A,B]	Elementary Transformation
$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & -1 & -1 & 4 \\ 1 & 5 & 5 & k \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -4 & -4 & 1 \\ 0 & 4 & 4 & k-1 \end{pmatrix}$	$R_2 \to R_3 - 3R_1$ $R_3 \to R_3 - R_1$
$\sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -4 & -4 & 1 \\ 0 & 0 & 0 & k \end{pmatrix}$	$R_3 \to R_3 + R_2$

Here clearly $\rho(A) = 2$

Since the given system is inconsistent $\rho(A) \neq \rho(A, B)$

This can hak can take any value other than zero.

∴ k can take any value other than zero.

5) Find the rank of the following matrices

$$\begin{pmatrix} 1 & -1 \\ 3 & -6 \end{pmatrix}$$

Let $A = \begin{pmatrix} i & -1 \\ 3 & -6 \end{pmatrix}$

Order of A is 2 x 2

$\therefore \rho(A) \le 2$ [Since minimum of (2,2) is 2]

Consider the second order minor

$$\begin{vmatrix} 1 & -1 \\ 3 & -6 \end{vmatrix} = -6-(-3)$$

= -+6+3

= -3

$\neq 0$

There is a minor of order 2, which is not zero

$\therefore \rho(A) = 2$

6) Find the rank of the following matrices

$$\begin{pmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{pmatrix}$$

Let $A = \begin{pmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{pmatrix}$

The order of A is 3 x 3

$\therefore \rho(A) \le 3$ [Since of (3,3) is 3]

Let us transform the matrix A to an echelon form

Matrix A	**Elementary Transformation**
$A = \begin{pmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 1 & 1 \\ 3 & 1 & -5 \\ 2 & -1 & 1 \end{pmatrix}$	$R_1 \leftrightarrow R_3$
$\sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & -2 & -8 \\ 0 & -3 & -1 \end{pmatrix}$	$R_2 \rightarrow R_2 3R_1$ $R_3 - R_3 - 2R_1$
$\sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & -4 \\ 0 & -3 & -1 \end{pmatrix}$	$R_2 \rightarrow R_2 \div 2$
$\sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & -4 \\ 0 & 0 & 11 \end{pmatrix}$	$R_3 \rightarrow R_3 - 3R_2$

This matrix is in echelon from and number of nonzero rows is 3.

$\therefore \rho(A) = 3$

7) Find the rank of the following matrices

$$\begin{pmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{pmatrix}$$

$$A=\begin{pmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{pmatrix}$$

The order of A is 3 x 4

$\therefore \rho(A) \leq 3$ LSiuke_Ulihimum of (3,4) is 3]

Let us transform the matrix A to an echelon fo

Matrix A	Elementary Transformation
$A = \begin{pmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 5 & -7 & 2 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & -5 & -1 \end{pmatrix}$	$R_1 \leftrightarrow R_3$
$\sim \begin{pmatrix} 1 & 5 & -7 & 2 \\ 0 & -7 & 8 & -7 \\ 3 & 1 & -5 & -1 \end{pmatrix}$	$R_2 \rightarrow R_2 + R_1$
$\sim \begin{pmatrix} 1 & 5 & -7 & 2 \\ 0 & -7 & 8 & -7 \\ 0 & -14 & 16 & -7 \end{pmatrix}$	$R_3 \rightarrow R_3 + 3R_1$
$\sim \begin{pmatrix} 1 & 5 & -7 & 2 \\ 0 & -7 & 8 & -7 \\ 0 & 0 & 0 & 7 \end{pmatrix}$	$R_3 \rightarrow R_3 - 2R^2$

The matrix is in echelon form and the number of non-zero matrix is 3

$\therefore \rho(A) = 3$

8) Solve the following equation by using Cramer's rule

5x + 3y = 17; 3x + 7y = 31

$$\Delta = \begin{vmatrix} 5 & 3 \\ 3 & 7 \end{vmatrix} = 5(7) - 3(3)$$

= 119 - 93 = 26

$$\Delta x = \begin{vmatrix} 17 & 3 \\ 31 & 7 \end{vmatrix} = 17(7) - 31(3)$$

= 119 - 93 = 2

$$\Delta y = \begin{vmatrix} 5 & 17 \\ 3 & 31 \end{vmatrix} = 5(31) - 17(3)$$

$= 155 - 51 = 104$

$$x = \frac{\Delta x}{\Delta} = \frac{26}{26} = 1$$

$$y = \frac{\Delta y}{\Delta} = \frac{104}{26} = 4$$

$\therefore$ Solution set is {I, 4}

9) Integrate the following with respect to x. $\left(9x^2 - \frac{4}{x^2}\right)^2$

$$\int \left(9x^2 - \frac{4}{x^2}\right)^2 dx$$

$$= \int \left[(9x^2)^2 - 2(9x^2)\left(\frac{4}{x^2}\right) + \left(\frac{4}{x^2}\right)^2\right] dx$$

$\left[\because (a-b)^2 = a^2 - 2ab + b^2\right]$

$$= \int \left(81x^4 - 72 + \frac{16}{x^4}\right) dx$$

$$= 81\frac{x^{4+1}}{4+1} - 72x + 16\frac{x^{-4+1}}{-4+1} + c$$

$\left[\because \frac{16}{x^4} = 16x^{-4}\right]$

$$= 81\frac{x^5}{5} - 72x + 16\frac{x^{-3}}{-3} + c$$

$$= \frac{81}{5}x^5 - 72x - \frac{16}{3x^3} + c$$

10) Integrate the following with respect to x. $\sqrt{x}$ (x^3 − 2x + 3)

$$\int \sqrt{x}\left(x^2 - 2x + 3\right) dx$$

$$= \int x^{\frac{1}{2}}\left(x^3 - 2x + 3\right) dx$$

$$= \int \left(x^{3+\frac{1}{2}} - 2x^{1+\frac{1}{2}} + 3x^{\frac{1}{2}}\right) dx$$

$$= \frac{x^{\frac{7}{2}+1}}{\frac{7}{2}+1} - \frac{2x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{3x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

$$= \frac{x^{\frac{9}{2}}}{\frac{9}{2}} - 2\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 3\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{2}{9}x^{\frac{9}{2}} - 2\times\frac{2}{5}x^{\frac{5}{2}} + 3\times\frac{2}{3}x^{\frac{3}{2}} + c$$

$$= \frac{2}{9}x^{\frac{9}{2}} - \frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + c$$

11) Integrate the following with respect to x. $\frac{8x+13}{\sqrt{4x+7}}$

$\int \frac{8x+13}{\sqrt{4x+7}}dx$

$= \int \frac{8x+14-1}{\sqrt{4x+7}}dx$

$= \int \frac{2(4x+7)-1}{\sqrt{4x+7}}dx$

$= 2\int \frac{(4x+7)}{\sqrt{4x+7}}dx - \int \frac{1}{\sqrt{4x+7}}dx$

$= 2\int \sqrt{4x+7}dx - \int \frac{1}{\sqrt{4x+7}}dx$

$= 2\int (4x+7)^{\frac{1}{2}}dx - \int (4x+7)^{-\frac{1}{2}}dx$

$= 2\frac{(4x+7)^{\frac{1}{2}+1}}{4\left(\frac{1}{2}+1\right)} - \frac{(4x+7)^{-\frac{1}{2}+1}}{4\left(\frac{-1}{2}+1\right)} + c$

$= 2\frac{(4x+7)^{\frac{3}{2}}}{4\left(\frac{3}{2}\right)} - \frac{(4x+7)^{\frac{1}{2}}}{4\left(\frac{1}{2}\right)} + c$

$= \cancel{2}\frac{(4x+7)^{\frac{3}{2}}}{\cancel{6}\,3} - \frac{(4x+7)^{\frac{1}{2}}}{2} + c$

$= \frac{(4x+7)^{\frac{3}{2}}}{3} - \frac{(4x+7)^{\frac{1}{2}}}{2} + c$

12) Integrate the following with respect to x. $\frac{1}{\sqrt{x+1}+\sqrt{x-1}}$

$\int \frac{1}{\sqrt{x+1}+\sqrt{x-1}}dx$

=Multiplying and dividing the conjugate of the denominator we get

$= \int \frac{\sqrt{x+1}-\sqrt{x-1}dx}{\left(\sqrt{x+1}+\sqrt{x-1}\right)\left(\sqrt{x+1}-\sqrt{x-1}\right)}$

$= \int \frac{\sqrt{x+1}-\sqrt{x-1}}{(x+1)-(x-1)}dx$

$\left[\because (a+b)(a-b) = a^2 - b^2\right]$

$= \int \frac{\sqrt{x+1}-\sqrt{x-1}}{\cancel{x}+1-\cancel{x}+1}dx$

$= \int \frac{\sqrt{x+1}-\sqrt{x-1}}{2}dx$

$= \frac{1}{2}\int \left((x+1)^{\frac{1}{2}} - (x-1)^{\frac{1}{2}}\right)dx$

$= \frac{1}{2}\left[\frac{(x+1)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x-1)^{\frac{3}{2}}}{\frac{3}{2}}\right] + c$

$= \frac{1}{\cancel{2}\times\frac{3}{\cancel{2}}}\left[(x+1)^{\frac{3}{2}} - (x-1)^{\frac{3}{2}}\right] + c$

$= \frac{1}{3}\left[(x+1)^{\frac{3}{2}} - (x-1)^{\frac{3}{2}}\right] + c$

13) If f(x) = x + b, f (1)= 5 and f (2) = 13, then find f (x)

Given f'(x)=x+b, f(1)=5 and f(2)=13

f'(x)=x+b

$\Rightarrow \int f'(x)dx = \int (x+b)dx$

[∴ Integration is the reverse process of differentiation]

$\Rightarrow f(0) = \frac{x^2}{2} + bx + c \quad ---(1)$

Given f(1)=5

$\Rightarrow 5 = \frac{1^2}{2} + b(1) + c$

$\Rightarrow 5 = \frac{1}{2} + b(1) + c \Rightarrow 5 - \frac{1}{2} = b + c$

$\Rightarrow \frac{10-2}{2} = b + c \Rightarrow b + c = \frac{9}{2}$

$\Rightarrow 2b + 2c = 9 \quad ---(2)$

$Also \quad f(2) = 13 \Rightarrow 13 = \frac{2^2}{2} + b(2) + c$

⇒ 13 = 2+2b+c

⇒ 13-2=2b+c

⇒ 2b+c = 11 ---(3)

(2) - (3)⟶2b+2c = 9

-2b + -c=-11

c = -2

Substituting c = -2 in (3) we get

2b - 2 = 11⇒2b = 11+2⇒2b=13

$\Rightarrow b = \frac{13}{2}$

Substituting $b = \frac{13}{2}$,c = -2 in(1) we get,

$f(x) = \frac{x^2}{2} + \frac{13}{2}x - 2$

14) If f ' (x) = $8x^3 - 2x$ and f (2)= 8, then find f (x)

Given f'(x)= $8x^3$-2x,f(2)=8

f'(x)= $8x^3$-2x

$\Rightarrow \int f'(x)dx = \int \left(8x^3 - 2x\right)dx$

$\Rightarrow f(x) = \frac{\overset{2}{\cancel{8}}x^4}{\cancel{4}} - \frac{\cancel{2}x^2}{\cancel{2}} + c$

⇒ f(x)= $2x^4$-x^2+c ---(1)

Given f(2) = 8

⇒ 8 = $2(2^4)$-2^2+c

⇒ 8 = 32 - 4+c

⇒ 8 = 32 - 4+c

⇒ 8 - 28 = c

⇒ c = -20

Substituting c = -20 in (1) we get,

f(x) = $2x^4$-x^2-20

15)

Integrate the following with respect to x.
$e^{x \log a} + e^{a \log a} - e^{n\log x}$

$\int \left(e^{x \quad loga} + e^{a \quad loga} - e^{nlogx}\right)dx$

$= \int \left(e^{loga^{x}} + e^{loga^{a}} - e^{logx^{n}}\right)dx$

$\left[\because m \quad log \quad n = logn^{m}\right]$

$\left[\because e^{logx} = x\right]$

$= \int \left(a^{x} + a^{a} - x^{n}\right)dx$

$= \left[\frac{a^{x}}{log \quad a}\right] + a^{a}(x) - \frac{x^{n+1}}{n+1} + c$

$\left[\because \quad \int a^{x} = \frac{a^{x}}{log \quad a}\right]$

16) If f ' (x) = ex and f (0)= 2, then find f (x)

Given f'(x) = e^x

⇒ ∫ f'(x) dx = ∫ e^x dx [Taking integration both sides]

⇒ f(x) = e^x+c (1)

Also, f(0) = 2

⇒ 2 = e^0 + c

⇒ 2 = 1+ c

⇒ 2 - 1 = c

⇒ c = 1

Substituting c= 1 in (1) we get,

f(x) = e^x+1

17) Integrate the following with respect to x.

$2\cos x - 3\sin x + 4\sec^2 x - 5\operatorname{cosec}^2 x$

$\int (2 \cos x - 3 \sin x + 4 \sec^2 x - 5\operatorname{cosec}^2 x)dx$

$= 2 \int\cos x\, dx - 3\int\sin x\, dx + 4\int\sec^2 x\, dx - 5 \int\operatorname{cosec}^2 x\, dx$

$= 2 (\sin x) - 3 (-\cos x) + 4 \tan x - 5 (-\cot x) + c$

$= 2\sin x + 3\cos x + 4 \tan x + 5\cot x + c$

18) Integrate the following with respect to x.

$\sin^3 x$

$\int \sin^3 x\, dx$

We know that $\sin 3x = 3\sin x - 4 \sin^3 x$

⇒ $4 \sin^3 x = 3\sin x - \sin 3x$

⇒ $\sin^3 x = \frac{1}{4}(3sinx - sin3x)$

$\therefore \int \sin^3 x\, dx = \frac{1}{4} \int (3 \sin x - 3 \sin 3x)\, dx$

$= \frac{3}{4}\int sin \quad x \quad dx - \frac{1}{4}\int sin \quad 3x \quad dx$

$= \frac{3}{4}(-\cos\ x) - \frac{1}{4}\left(-\frac{\cos\ 3x}{3}\right) + c \qquad \left[\because \int \sin\ ax\ dx = \frac{-1}{a}\cos\ ax + c\right]$

$= -\frac{3}{4}\cos\ x + \left(\frac{\cos\ 3x}{12}\right) + c$

19) Integrate the following with respect to x.

$\frac{1}{x\ \log\ x}$

$Let\ \ I = \int \frac{1}{x\ \ logx}dx$

$put\ \ log\ \ x = t$

$\Rightarrow \frac{1}{x}dx = dt$

$\therefore I = \int \frac{1}{t}dt = \log|t| + c$

$= \log|\log x| + c \qquad [\because t = logx]$

20) Integrate the following with respect to x.

$e^x\left[\frac{1}{x^2} - \frac{2}{x^3}\right]$

$Let\ \ I = \int e^x\left[\frac{1}{x^2} - \frac{2}{x^3}\right]dx$

$Letf(x) = \frac{1}{x^2} = x^{-2}$

$\Rightarrow f'(x) = -2x^{-2-1}$

$= -2x^{-3} = \frac{-2}{x^3}$

$\Rightarrow f'(x) = \frac{-2}{x^3}$

$\therefore I = \int e^x\left[f(x) + f'(x)\right]dx$

$= e^x(x) + c$

$= e^x\frac{1}{x^2} + c$

$= \frac{e^x}{x^2} + c$

21) Integrate the following with respect to x

$\frac{1}{\sqrt{9x^2-7}}$

$\int \frac{dx}{\sqrt{9x^2-7}}$

$= \int \frac{dx}{\sqrt{9\left(x^2-\frac{7}{9}\right)}}$

$= \frac{1}{3}\int \frac{dx}{\sqrt{x^2\left(\frac{\sqrt{7}}{3}\right)^2}}$

$\left[\because \int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x + \sqrt{x^2+a^2}\right| + c\right]$

$= \frac{1}{3}\log\left|x + \sqrt{x^2 - \frac{7}{9}}\right| + c$

$= \frac{1}{3}\log\left|x+\sqrt{\frac{9x^2-7}{3}}\right|+c$

$= \frac{1}{3}\log\left|3x+\sqrt{9x^2-7}\right|+c$

22) Integrate the following with respect to x

$\sqrt{4x^2-5}$

$\int\sqrt{4x^2-5}dx$

$= \int\sqrt{4\left(x^2-\frac{5}{4}\right)}dx$

$= 2\int\sqrt{x^2-\left(\frac{\sqrt{5}}{2}\right)^2}dx$

$= 2\left[\frac{x}{2}\sqrt{x^2-\frac{5}{4}}-\frac{5}{4(2)}\log\left|x+\sqrt{x^2-\frac{5}{4}}\right|\right]+c$

$= \frac{x}{2}\sqrt{4x^2-5}-\frac{5}{4}\log\left|2x+\sqrt{4x^2+5}\right|+c$

$= \frac{1}{4}\left[2x\sqrt{4x^2-5}-5log\left|2x+\sqrt{4x^2-5}\right|\right]+c$

23) Using second fundamental theorem, evaluate the following:

$\int_0^3\frac{e^xdx}{1+e^x}$

Let I = $\int_0^3\frac{e^x}{1+e^x}dx$

put t = 1+ e^x

⇒ dt = e^x dx

When x = 0, t = 1+e^0 = 1+1=2

When x = 3, t = 1+e^3

$\therefore I = \int_2^{1+e^3}\frac{dt}{t} = [log \quad t]_2^{1+e^3}$

= log (1+e^3) - log2

$= \log\left(\frac{1+e^3}{2}\right)$

24) Evaluate the following

$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

$\Gamma\left(\frac{9}{2}\right) = \frac{7}{2}\Gamma\left(\frac{7}{2}\right)$

$= \frac{7}{2}\Gamma\left(\frac{5}{2}\right)$

$= \frac{7}{2}\times\frac{5}{2}\times\Gamma\left(\frac{3}{2}\right)$

$= \frac{7}{2}\times\frac{5}{2}\times\left(\frac{3}{2}\right)\times\Gamma\left(\frac{1}{2}\right)$

$= \frac{7}{2} \times \frac{5}{2} \times \frac{3}{2} \times \frac{1}{2}\sqrt{\pi}$

$= \frac{105}{16}\sqrt{\pi}$

25) Evaluate the following

$\int_0^\infty e^{-4x}x^4 dx$

Let I $= \int_0^\infty e^{-4x}x^4 dx$

Here n = 4 and a = 4

$\therefore I = \frac{4!}{4^{4+1}} = \frac{4!}{4^5}$

$\left[\because \int_0^\infty x^n e^{-ax}dx = \frac{n!}{a^{n+1}}\right]$

$= \frac{4 \times 3 \times 2 \times 1}{4 \times 4 \times 4 \times 4 \times 4^2}$

$= \frac{3}{128}$

26) If f'(x) = $8x^3 - 2x^2$, f(2) = 1, find f(x)

Given f'(x) = $8x^3 - 2x^2$

$\therefore \int f'(x)\,dx = \int (8x^3 - 2x^2)\,dx$

$\Rightarrow f(x) = \frac{8x^4}{4} - \frac{2x^3}{3} + c$

$\Rightarrow f(x) = 2x^4 - \frac{2x^3}{3} + c$ --------(1)

Also, f(2) = 1

$\Rightarrow 1 = 2(2^4) - \frac{2(2^3)}{3} + c$

$\Rightarrow 1 = 32 - \frac{16}{3} + c$

$\Rightarrow 1 - 32 + \frac{16}{3} = c \Rightarrow -31 + \frac{16}{3} = c$

$\Rightarrow \frac{-93+16}{3} = c$

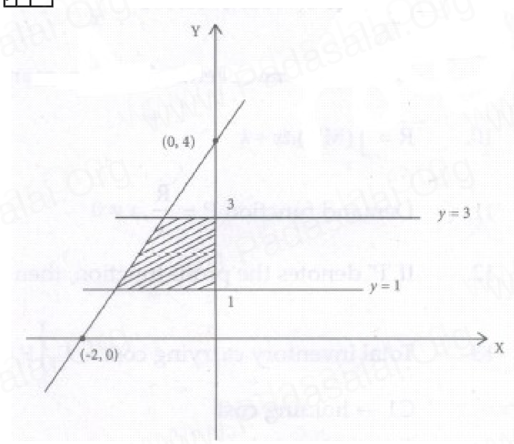
$\Rightarrow c = \frac{-77}{3}$

$\therefore (1) \longrightarrow f(x) = 2x^4 - \frac{2x^3}{3} - \frac{-77}{3}$

27) Find the area bounded by the lines y − 2x − 4 = 0, y = 1, y = 3 and the y-axis

y-2x-4=0

x	0	-2
y	4	0



Given y - 2.x - 4 =0

$\Rightarrow$ y-4=2x

$\Rightarrow \quad x = \frac{1}{\not 2} \times \overset{2}{\not 4}\ (y-4)$

Since the area lies to the left of Y-axis, with the limitsy = 1 &y = 3.

Area $= \int_1^3 -xdy$

$= \int_1^3 -\left(\frac{1}{2}\right)(y-4)dy$

$= \frac{1}{2}\int_1^3 (4-y)dy = \frac{1}{2}\left[4y - \frac{y^2}{2}\right]_1^3$

$= \frac{1}{2}\left[\left(4(3) - \frac{3^2}{2}\right) - \left(4(1) - \frac{1^2}{2}\right)\right]$

$= \frac{1}{2}\left[\left(12 - \frac{9}{2}\right) - \left(4 - \frac{1}{2}\right)\right]$

$= \frac{1}{2}\left[\left(\frac{24-9}{2}\right) - \left(\frac{8-1}{2}\right)\right]$

$= \frac{1}{2}\left[\frac{15}{2} - \frac{7}{2}\right] = \frac{1}{2}\left[\frac{8}{2}\right]$

$= \frac{1}{\not 2} \times \overset{2}{\not 4} \quad A = 2$ sq. units.

28) Find the area of the region lying in the first quadrant bounded by the region $y = 4x^2$, x = 0, y = 0 and y = 4

Given curve $y = 4x^2$ is an open upward parabola

$\Rightarrow \frac{y}{4} = x^2$

The limits are from y = 0 to y = 4

Since the shaded region lies to the right of Y-axis,

required area $= \int_0^4 xdy$

$= \int_0^4 \sqrt{\frac{y}{4}}dy$

$= \frac{1}{2}\int_0^4 \sqrt{y}dy = \frac{1}{2}\int_0^4 y^{\frac{1}{2}}dy$

$= \frac{1}{2}\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 = \frac{1}{\not 2} \times \frac{\not 2}{3}\left[y^{\frac{3}{2}}\right]_0^4$

$= \frac{1}{3}\left[4^{\frac{3}{2}} - 0\right] = \frac{1}{3}\left[4^{\frac{3}{2}}\right]$

$= \frac{1}{3}4^1\sqrt{4} = \frac{1}{3}.4(2)$

$A = \frac{8}{2}$ sq.units

29) The marginal cost function is $MC = 300\,x^{\frac{2}{5}}$ and fixed cost is zero. Find out the total cost and average cost functions.

Given $MC = 300x^{\frac{2}{5}}$

$\Rightarrow \frac{dC}{dx} = 300x^{\frac{2}{5}}$

$\int dC = 300\int x^{\frac{2}{5}}dx$

$\Rightarrow C = 300\frac{x^{\frac{2}{5}+1}}{\frac{2}{5}+1} + k$

$\Rightarrow C = 300\frac{x^{\frac{7}{5}}}{\frac{7}{5}} + k$

$\Rightarrow C = 300 \times \frac{5}{7}x^{\frac{7}{5}} + k \quad \ldots (1)$

Given fixed cost is zero ⇒ k = 0

∴(1) becomes,

$C = \frac{1500}{7}x^{\frac{7}{5}} + 0 \Rightarrow C = \frac{1500}{7}x^{\frac{7}{5}}$

Average cost function $(AC) = \frac{C}{x}$

$\Rightarrow AC = \frac{1500}{7}\frac{x^{\frac{7}{5}}}{x}$

$\Rightarrow AC = \frac{1500}{7}x^{\frac{7}{5}-1}$

$\Rightarrow AC = \frac{1500}{7}x^{\frac{2}{5}}$

30) If the marginal revenue function is R'(x)= 1500 − 4x − 3x^2. Find the revenue function and average revenue function.

Given

$MR = R'(x) = 1500 - 4x - 3x^2$

$\Rightarrow \int R'(x) = \int(1500 - 4x - 3x^2)dx$

$\Rightarrow R(x) = 1500x - \frac{4x^2}{2} - \frac{3x^3}{3} + k$

$\Rightarrow R(x) = 1500x - 2x^2 - x^3 + k$

When x=0, R=0⇒k=0

$\Rightarrow R(x) = 1500x - 2x^2 - x^3$

Average revenue function $= \frac{R(x)}{x}$

$= \frac{1500x-2x^2-x^3}{x}$

AR=1500-2x-x^2

31) Calculate consumer’s surplus if the demand function p = 122 − 5x − 2x^2 and x = 6

Given demand functionp = 122 - 5x - 2x^2

and x=6

When x_0=6, p_0=122-5(6)-2(6)^2

=122-30-72

=122-102

P_0=20

p_0x_0=20x6=120

Consumer's Surplus

CS= $= \int_0^x f(x)dx - p_0x_0$

$= \int_0^6(122 - 5x - 2x^2)dx - 120$

$= \left[122x - \frac{5x^2}{2} - \frac{2x^3}{3}\right]_0^6 - 120$

$= 122(6) - 5\frac{(6^2)}{2} - 2\frac{(6^3)}{3} - 120$

$= 732 - \frac{180}{2} - \frac{432}{3} - 120$

=732-90-144-120

=732-354

C.S = 378 units

32) If the supply function for a product is p = 3x + 5x^2. Find the producer’s surplus when x = 4.

Given supply function $P = 3x + 5x^2$ and x = 4.

Whenx_0=4,p_0 = $3(4)+5(4^2)$

=12+80

=92

$\therefore p_0x_0$=92x4=368

Producer's Surplus

$PS = p_0x_0 - \int_0^x g(x)dx$

$= 368 - \int_0^4 (3x + 5x^2)dx$

$$= 368 - \left(\frac{3x^2}{2} + \frac{5x^3}{3}\right)_0^4$$

$$= 368 - \left(\frac{3(4^2)}{2} + 5\frac{(4^3)}{3}\right)$$

$$= 368 - \left(24 + \frac{320}{3}\right)$$

=368-(24+106.66)

=368-(130.66)

P.S = 237.3 units

33) Find the differential equation of the following

$y = cx + c - c^3$

Given equation is y = cx + c - c3 ....(1)

Differentiating w.r.t 'x' we get,

$\frac{dy}{dx}$=c(1) + 0-0 ⇒ $\frac{dy}{dx}$=c ....(2)

Substituting (2) in (1) we get,

$y=x\left(\frac{dy}{dx}\right) + \left(\frac{dy}{dx}\right) - \left(\frac{dy}{dx}\right)^3$ which is the required differential equation.

34) Solve: ydx − xdy = 0

Given ydx - xdy =0

⇒ ydx=xdy

Separating the variables we get,

$\frac{dx}{x} = \frac{dy}{y}$

Integrating both sides we get,

$\int \frac{dx}{x} = \int \frac{dy}{y}$

logx=logy+logc

logx=logy

[∵ log m+log n =log mn]

x=cy

35) Find the curve whose gradient at any point P(x, y) on it is $\frac{x-a}{y-b}$ and which passes through the origin.

Given Gradient =$\frac{x-a}{y-b}$

$\Rightarrow \frac{dy}{dx} = \frac{x-a}{y-b}$

Separating the variables we get,

(y - b) dy = (x - a) dx

Integrating both sides we get,

$\int(y-b)dy = \int(x-a)$ dx

$\frac{y^2}{2} - by = \frac{x^2}{2} - ax$ +c ......(1)

Since the curve passes through the origin (0, 0), we get

0-0 = 0-0+c ⇒ c=0

∴ (1) be comes,

$\frac{y^2}{2} - by = \frac{x^2}{2} - ax$

$\Rightarrow \frac{y^2-2by}{2} = \frac{x^2-2ax}{2}$

y^2-2by =x^2-2ax

Adding and subtracting b^2 in the L.H.S and a^2 in the R.H.S we get

$\underbrace{y^2-2by+b^2}-b^2 = x^2-2ax+a^2-a^2$

$(y-b)^2-b^2 = (x-a)^2-a^2$

$\Rightarrow (y-b)^2 = (x-a)^2+b^2-a^2$

36) Solve the following:

$x\frac{dy}{dx} + 2y = x^4$

$\frac{dy}{dx} + \frac{2}{y}y = x^3$

The given differential equation is of this form [Divided by x]

$\frac{dy}{dx}$ +Py=Q where

P=$\int\frac{2}{x}$ and Q=x^3

∴ $\int Pdx = \int\frac{2}{x}dx$ =2 logx=logx^2

∴ Integrating factor (I. F) =$e^{\int pdx} = e^{logx^2}$ =x^2

Hence the solution is

$ye^{\int pdx} = \int Q.\, e^{\int p.dx}dx + c$

$\Rightarrow y.x^2=\int x^3.\, x^2dx + c$

$\Rightarrow x^2y=\int x^5dx + c$

$\Rightarrow x^2y=\frac{x^6}{6}$ +c

37) Solve the following differential equations

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$

The auxiliary equation is

m^2 - 6m + 8 = 0

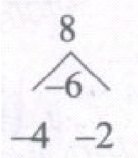
⇒ (m-4)(m-2)=0

⇒ m=2,4

The roots are real and different

∴ Complementary function CF is $Ae^{2x} + Be^{4x}$

∴ The general solution is $y = Ae^{2x} + Be^{4x}$



38) Solve the following differential equations: $\frac{d^2y}{dx^2} + 16y = 0$

The auxiliary equation is $m^2 + 16 = 0$

$m_2 = -16$

$\Rightarrow m_2 = \pm\sqrt{-16} = \pm 4i$

Hence α=0 and β=4

∴ Complementary function CF is

$e^{ax} = [A \cos \beta x + B \sin \beta x]$

$CF = e^{0x}[A \cos 4x + B \sin 4x]$

$= A \cos 4x + B \sin 4x$

$[\because e^{o} = 1]$

∴ The general solution is $y = A \cos 4x + B \sin 4x$

39) Find the differential equation of the following

$xy = c^2$

Differentiating w.r.t 'x' we get,

$x.\frac{dy}{dx} + y(1) = 0$ [Product rule]

$\Rightarrow x\frac{dy}{dx} + y = 0$ which is the required differentiated equation.

40) Evaluate Δ(log ax).

$\Delta(\log ax) = \log (ax + h) - \log (ax)$

$= \log\left(\frac{ax+h}{ax}\right)$

$= \log\left(\frac{ax}{ax} + \frac{h}{ax}\right)$

$= \log\left(1 + \frac{h}{ax}\right)$

$\therefore \Delta (\log ax) = \log\left(1 + \frac{h}{ax}\right)$

41) If $f(x) = x^2 + 3x$ than show that $\Delta f(x) = 2x + 4$

Given $f(x) = x^2 + 3h$

$LHS = \Delta f(x)$

$= f(x + h) - f(x)$

$= [(x + h)^2 + 3 (x + h)] - [x^2 + 3x]$

$= \cancel{x^2} + h^2 + 2xh + \cancel{3x} + 3h - \cancel{x^2} - \cancel{3x}$

$= h^2 + 2xh + 3h$

when $h = 1$, $LHS = 1^2 + 2x(1) + 3(1)$

$= 1+2x+3$

$= 2x+4 =$ RHS

Hence proved.

9 x 3 = 27

42) Consider the matrix of transition probabilities of a product available in the market in two brands A and B.

$$\begin{matrix} & A & B \\ A & 0.9 & 0.1 \\ B & 0.3 & 0.7 \end{matrix}$$

Determine the market share of each brand in equilibrium position.

Transition probability matrix

$$T = \begin{matrix} & \begin{matrix} A & B \end{matrix} \\ \begin{matrix} A \\ B \end{matrix} & \begin{pmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{pmatrix} \end{matrix}$$

At equilibrium, (A B) T=(AB) where A+B=1

$$(A\ B)\begin{pmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{pmatrix} = (A\ B)$$

$0.9A + 0.3B = A$

$0.9A+0.3(1-A) = A$

$0.9A-0.3A+0.3 = A$

$0.6A + 0.3 = A$

$0.4A = 0.3$

$A=\frac{0.3}{0.4} = \frac{3}{4}$

$B=1-\frac{3}{4} = \frac{1}{4}$

Hence the market share of brand A is 75% and the market share of brand B is 25%

43) Evaluate $\int 3^{2x+3}dx$

$\int 3^{2x+3}dx = \int 3^{2x}.\,3^3dx$

$= 3^3\int 3^{2x}dx$

$= 27\frac{3^{2x}}{2log3} + c$

$[\int ma^{mx+n}dx = \int ma^{mx+n}d(mx+n)\frac{1}{loga}a^{mx+n} + c, a > o \quad and \quad a \neq 1\,]$

44) Evaluate $\int \frac{dx}{1-25x^2}$

$$\int \frac{dx}{1-25x^2} = \frac{1}{25}\int \frac{dx}{\left(\frac{1}{5}\right)^2 - x^2}$$

$$= \frac{1}{25}\left[\frac{1}{2\left(\frac{1}{5}\right)} log\left|\frac{\frac{1}{5}+x}{\frac{1}{5}-x}\right|\right] + c$$

$$= \frac{1}{10}log\left|\frac{1+5x}{1-5x}\right| + c$$

45) Evaluate $\int \frac{dx}{4x^2-1}$

$$\int \frac{dx}{4x^2-1} = \int \frac{dx}{4\left(x^2-\frac{1}{4}\right)}$$

$$= \frac{1}{4}\int \frac{dx}{x^2-\left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{4}\left[\frac{1}{2\left(\frac{1}{2}\right)} log\left|\frac{x-\frac{1}{2}}{x-\frac{1}{2}}\right|\right] + c$$

$$= \frac{1}{4}log\left|\frac{2x-1}{2x+1}\right| + c$$

46) Evaluate $\int \sqrt{x^2 - 16}\, dx$

$\int \sqrt{x^2 - 16}\ dx = \int \sqrt{x^2 - 4^2}\ dx$

$= \frac{x}{2}\sqrt{x^2 - 4^2} - \frac{4^2}{2} log\left|x + \sqrt{x^2 - 4^2}\right| + c$

$= \frac{x}{2}\sqrt{x^2 - 16} - 8log\left|x + \sqrt{x^2 - 16}\right| + c$

47) Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} cosx\ dx$

Let f (x) = cos x

f (−x) = cos(−x) = cos x

⇒ f (x) = − f (x)

∴ f (x) is an even function

∴ $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} cosx\ dx = 2\int_{0}^{\frac{\pi}{2}} cosx$

$= 2\ [sinx]_{0}^{\frac{\pi}{2}}$

$= \left[sin\frac{\pi}{2} - sin0\right]$

= 2

48) Find the order and degree of the following differential equations

$(i)\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 + 4y = 0$

$(ii)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3y = 0$

$(iii)\frac{d^2y}{dx^3} - 3\left(\frac{dy}{dx}\right)^6 + 2y = x^2$

$(iv)\left[1 + \frac{d^2y}{dx^2}\right]^{\frac{3}{2}} = a\frac{d^2y}{dx^2}$

(v) $y' + (y'')^2 = (x+y)^2$

$(vi)\frac{d^3y}{dx^3} - \left(\frac{dy}{dx}\right)^{\frac{1}{2}} = 0$

$(vi)y = 2\left(\frac{dy}{dx}\right)^2 + 4x\frac{dy}{dx}$

$(i)\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 + 4y = 0$

Highest order derivative is $\frac{d^2y}{dx^2}$

∴ order = 2

Power of the highest order derivative $\frac{d^2y}{dx^2}$ is 1.

∴ Degree = 1

$(ii)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3y = 0$

Highest order derivative is $\frac{d^2y}{dx^2}$

∴ order = 2

Power of the highest order derivative $\frac{d^2y}{dx^2}$ is 1

∴ Degree = 1

$(iii)\frac{d^2y}{dx^3} - 3\left(\frac{dy}{dx}\right)^6 + 2y = x^2$

∴ order = 3, ∴ Degree = 1

$(iv)\left[1 + \frac{d^2y}{dx^2}\right]^{\frac{3}{2}} = a\frac{d^2y}{dx^2}$

Here we eliminate the radical sign.

Squaring both sides, we get

$\left[1 + \frac{d^2y}{dx^2}\right]^{\frac{3}{2}} = a^2(\frac{d^2y}{dx^2})^2$

∴ order = 2, ∴ Degree = 3

(v) y' +(y")^2 =(x + y)"^{2}

y' +(y") =x^2 + 2xy" +(y")^2

y' = x^2 + 2xy ⇒ $\frac{dy}{dx} = x^2 + 2x\frac{d^2y}{dx^2}$

∴ order = 2, ∴ Degree = 1

$(vi)\frac{d^3y}{dx^3} - \left(\frac{dy}{dx}\right)^{\frac{1}{2}} = 0$

Here we eliminate the radical sign.

For this write the equation as

$\frac{d^3y}{dx^3} = \left(\frac{dy}{dx}\right)^{\frac{1}{2}}$

Squaring both sides, we get

$\frac{d^3y}{dx^3} = \left(\frac{dy}{dx}\right)$

∴ order = 3, ∴ Degree = 2

$(vi)y = 2\left(\frac{dy}{dx}\right)^2 + 4\text{x}\frac{dy}{dx}$

$y = 2\left(\frac{dy}{dx}\right)^2 + 4\text{x}\frac{1}{\left(\frac{dy}{dx}\right)}$

$y\left(\frac{dy}{dx}\right) = \left(\frac{dy}{dx}\right)^3 + 4\text{x}$

∴ order = 1, ∴ Degree = 3

49) Solve 9y" − 12y' + 4y = 0

Given (9D^2−12D + 4)y = 0

The auxiliary equation is

(3m - 2)^2 = 0

(3m−2)(3m−2) = 0 ⇒ m = $\frac{2}{3}, \frac{2}{3}$

Roots are real and equal

The C.F. is (Ax + B)$e^{\frac{2}{3}x}$

50) Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0$

Given $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0$

$(D^2-4D+5)y = 0$

The auxiliary equation is

$m^2-4m + 5 = 0$

$\Rightarrow (m-2)^2-4 + 5 = 0$

$(m-2)^2 = -1$

$m - 2 = \pm\sqrt{-1}$

$m = 2 \pm i$, it is if the form $\alpha \pm i\beta$

$\therefore$ C.F $= e^{2x}[A \cos x + B \sin x]$

The general solution is $y = e^{2x}[A \cos x + B \sin x]$

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QUARTERLY IMPORTANT 3 MARKS WITH ANSWERS

12th Standard

Business Maths

MY YOUTUBE CHANNEL NAME
SR MATHS TEST PAPERS
CHECK FOR MATHS VIDEOS AND MATERIALS

Total Marks : 150

50 x 3= 150

1) If A= $\begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$ and B= $\begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$, then find the rank of AB and the rank of BA.

Given

A= $\begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$ and B= $\begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$

$$AB = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}\begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-2+5 & -2+4-1 & 3-6+1 \\ 2+6+20 & -4-12+45 & 6+18-4 \\ 3+4+15 & -6-8+3 & 9+12-3 \end{pmatrix} = \begin{pmatrix} -6 & 1 & -2 \\ 28 & -12 & 20 \\ 22 & -11 & 18 \end{pmatrix} = \begin{pmatrix} -6 & 1 & -2 \\ 28 & -12 & 20 \\ 22 & -11 & 18 \end{pmatrix}$$

Matrix (AB)	**Elementary Transformation**
$AB = \begin{pmatrix} -6 & 1 & -2 \\ 28 & -12 & 20 \\ 22 & -11 & 18 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & -6 & -2 \\ -12 & 28 & 20 \\ -11 & 22 & 18 \end{pmatrix}$	$C_1 \leftrightarrow C_2$
$\sim \begin{pmatrix} 1 & -6 & -2 \\ -12 & 28 & 20 \\ -11 & 22 & 18 \end{pmatrix}$	$R_2 \to R_2 + 12R_1$
$\sim \begin{pmatrix} 1 & -6 & -2 \\ 0 & -44 & -4 \\ 0 & -44 & -4 \end{pmatrix}$	$R_3 \to R_3 + 11R_1$

Matrix (AB)	Elementary Transformation
$\sim \begin{pmatrix} 1 & -6 & -2 \\ 0 & -44 & -4 \\ 0 & 0 & 0 \end{pmatrix}$	$R_3 \to R_3 - R_2$

The matrix is in echelon form and the number of non-zero rows is 2. $\therefore \rho(AB) = 2$

Now $BA = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$

$$= \begin{pmatrix} 1-4+9 & 1+6-6 & -1-8+9 \\ -2+8-18 & -2-12+12 & 2+16-18 \\ 5+2-3 & 5-3+2 & -5+4-3 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 1 & 0 \\ -12 & -2 & 0 \\ 4 & 4 & -4 \end{pmatrix}$$

Matrix (BA)	Elementary Transformation
$BA = \begin{pmatrix} 6 & 1 & 0 \\ -12 & -2 & 0 \\ 4 & 4 & -4 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 6 & 0 \\ -2 & -12 & 0 \\ 4 & 4 & -4 \end{pmatrix}$	$C_1 \leftrightarrow C_2$
$\sim \begin{pmatrix} 1 & 6 & 0 \\ 0 & 0 & 0 \\ 4 & 4 & -4 \end{pmatrix}$	$R_2 \to R_2 + 2R_1$
$\sim \begin{pmatrix} 1 & 6 & 0 \\ 0 & 0 & 0 \\ 0 & -20 & -4 \end{pmatrix}$	$R_3 \to R_3 - 4R_1$

The number of non-zero rows is 2.

$\therefore \rho(BA) = 2$

2) Show that the equations5x+3y+7z=4,3x+26y+2z=9,7x+2y+10z =5 are consistent and solve them by rank method.

Given non-homogeneous equations are

$5x + 3y + 7z = 4$

$3x + 26y + 2z = 9$

$7x + 2y + 10z = 5$

The matrix equation corresponding to the given system is

$$\begin{pmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \\ 5 \end{pmatrix}$$

Augmented matrix [A, B]	Elementary Transformation

Augmented matrix [A, B]	Elementary Transformation
$\begin{pmatrix} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{pmatrix}$	
$\sim \begin{pmatrix} 3 & 26 & 2 & 9 \\ 5 & 3 & 7 & 4 \\ 7 & 2 & 10 & 5 \end{pmatrix}$	$R_1 \leftrightarrow R_2$
$\begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} & 3 \\ 5 & 3 & 7 & 4 \\ 7 & 2 & 10 & 5 \end{pmatrix}$	$R_1 \to R_1 \div 3$
$\begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} & 3 \\ 0 & \frac{-121}{3} & \frac{11}{3} & -11 \\ 7 & 2 & 5 & 5 \end{pmatrix}$	$R_2 \to R_2 - 5R_1$
$\begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} & 3 \\ 0 & \frac{-121}{3} & \frac{11}{3} & -11 \\ 0 & \frac{-176}{3} & \frac{16}{3} & -16 \end{pmatrix}$	$R_3 \to R_3 - 7R_1$
$\sim \begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} & 3 \\ 0 & \frac{-11}{3} & \frac{1}{3} & -1 \\ 0 & \frac{-11}{3} & \frac{1}{3} & -1 \end{pmatrix}$	$R_2 \to R_2 \div 11$ $R_3 \to R_3 \div 16$
$\begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} & 3 \\ 0 & \frac{-11}{3} & \frac{1}{3} & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$R_3 \to R_3 - R_2$

Here $\rho(A) = \rho(A, B) = 2 < Number of unknowns.$

$\therefore$ The system is consistent with infinitely many solutions let us rewrite the above echelon form into matrix form

$$\begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} \\ 0 & \frac{-11}{3} & \frac{1}{3} \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$$

$$x + \frac{26}{3}y + \frac{2}{3}z = 3$$

$$x + \frac{26}{3}y + \frac{2}{3}z = 3$$

|letz = k where kE R

$$(2) \Rightarrow \frac{-11}{3}y + \frac{k}{3} = -1 \quad \Rightarrow \quad \frac{-11}{3}y = -1 - \frac{k}{3} = \frac{-3-k}{3}$$

$$\Rightarrow \quad -11y = -3 - k$$ 11y=3+k

$$\Rightarrow \quad y = \frac{1}{11}(3 + k)$$

Substituting $y = \frac{1}{11}(3 + k)$ and z = kin (1) we get,

$$x + \frac{26}{3}\left(\frac{3+k}{11}\right) + \frac{2}{3}k = 3$$

$$= \frac{26}{3}\left(\frac{3+k}{11}\right) - \frac{2k}{3} + 3$$

$$\frac{78 - 26k}{33} - \frac{2k}{3} + 3$$

$$\frac{78 - 26k - 22k + 99}{33}$$

$$\frac{78 - 26k - 22k + 99}{33}$$

$$\frac{21 - 48k}{33} = \frac{3(7 - 16k)}{33}$$

$$= \frac{1}{11}(7 - 6k)$$

$$\therefore \text{ Solution set is } \left\{\frac{1}{11}(7 - 16k). \frac{1}{11}(3 + k), k\right\} \text{ K} \in \text{R}$$

Hence, for different values of k, we get infinitely many solutions.

3) The price of three commodities X,Y and Z are x,y and z respectively Mr.Anand purchases 6 units of Z and sells 2 units of X and 3 units of Y. Mr.Amar purchases a unit of Y and sells 3 units of X and 2units of Z. Mr.Amit purchases a unit of X and sells 3 units of Y and a unit of Z. In the process they earn `5,000/-, `2,000/- and `5,500/- respectively Find the prices per unit of three commodities by rank method.

Given that the price of commodities X, Y and Z are x, y and z respectively

By the given data

Transaction	x	y	z	Earning
Mr. Anand	+2	+3	-6	Rs.5000
Mr. Amar	+3	-1	+2	Rs.2000
Mr. Amit	-1	+3	+1	Rs.5500

Here, purchasing is taken as negative symbol and selling is taken as positive symbol

Thus, the non-homogeneous equations are

2x + 3y - 6z = 5000

3x - y + 2z = 2000

-x + 3y + z = 550

The matrix equation corresponding to the given system is

$$\begin{pmatrix} 2 & 3 & -6 \\ 3 & -1 & 2 \\ -1 & 3 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5000 \\ 2000 \\ 5500 \end{pmatrix}$$

Augmented matrix	Elementary Transformation
$\begin{pmatrix} 2 & 3 & -6 & 5000 \\ 3 & -1 & 2 & 2000 \\ -1 & 3 & 1 & 5500 \end{pmatrix}$	
$\begin{pmatrix} -1 & 3 & 1 & 5500 \\ 3 & -1 & 2 & 2000 \\ 2 & 3 & -6 & 5000 \end{pmatrix}$	R_1 ⊠ R_3
$\begin{pmatrix} 1 & -3 & -1 & -5000 \\ 3 & -1 & 2000 & 2000 \\ 2 & 3 & -6 & 5500 \end{pmatrix}$	$R_1 \rightarrow R_1(-1)$
$\begin{pmatrix} 1 & -3 & -1 & -5500 \\ 0 & 8 & 5 & 18500 \\ 0 & 9 & -4 & 16000 \end{pmatrix}$	$R_2 \rightarrow r_2 - 3R_1$ $R_3 \rightarrow R_3 - 2R_1$
$\begin{pmatrix} 1 & -3 & -1 & -5500 \\ 0 & 1 & \frac{65}{8} & \frac{18500}{8} \\ 0 & 1 & \frac{-4}{9} & \frac{16000}{9} \end{pmatrix}$	$R_2 \rightarrow R_2 \div 8$ $R_3 \rightarrow R_3 \div 9$
$\begin{pmatrix} 1 & -3 & -1 & -5500 \\ 0 & 1 & \frac{5}{8} & \frac{18500}{8} \\ 0 & 0 & \frac{-4}{9} - \frac{5}{8} & \frac{16000}{9} - \frac{18500}{8} \end{pmatrix}$	$R_3 \rightarrow R_3 - R_2$

Augmented matrix	Elementary Transformation
$\begin{pmatrix} 1 & -3 & -1 & -5500 \\ 0 & 1 & \frac{5}{8} & \frac{18500}{8} \\ 0 & 0 & \frac{-77}{72} & \frac{-38500}{72} \end{pmatrix}$	

.Clearly the last equivalent matrix is in echelon form and it has three non-zero rows . . :. $\therefore \rho(A) = \rho([A, B]) = 3$ Number of unknowns. $\therefore$ The given system is consistent and has unique solution. To find the solution, let us rewrite the above : echelon form into the matrix form.

$$\begin{pmatrix} 1 & -3 & -1 \\ 0 & 1 & \frac{5}{8} \\ 0 & 0 & \frac{-77}{72} \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}\begin{pmatrix} -5000 \\ \frac{18500}{8} \\ \frac{-38500}{72} \end{pmatrix}$$

$\Rightarrow x - 3y - z = -5500$

$y + \frac{5}{8}z = \frac{18500}{8}$

$\frac{-77}{72}z = \frac{38500}{72}$

$(3) \Rightarrow \quad \frac{-77z}{72} = \frac{-38500}{72}$

$\Rightarrow z = \frac{-38500}{-77}$

$\Rightarrow z = 500$

$(2) \Rightarrow y + \frac{5}{8}(500) = \frac{18500}{8} \quad y = \frac{18500}{8} - \frac{2500}{8}$

$= \frac{16000}{8}$ (2000) $\quad \Rightarrow y = 2000$

$(1) \Rightarrow x - 3(2000) - 500 = -5500$

$\Rightarrow x - 6000 - 500 = -5500$

$\Rightarrow x - 6000 - 500 = -5500$

$\Rightarrow x = -5500 + 6500$

$\Rightarrow x = 10000$

Hence, the prices per unit of three commodities are Rs1000, Rs 2000 and Rs500 respectively

4) An amount of `5,000/- is to be deposited in three different bonds bearing 6%, 7% and 8% per year respectively. Total annual income is `358/-. If the income from first two investments is `70/- more than the income from the third, then find the amount of investment in each bond by rank method.

Let the amount of investment in each bond be

Rs.x Rs.Y,Rs z respectively.

Given x +y +z = 5000 ..(1)

Also $\frac{6x}{100} + \frac{7y}{100} + \frac{8z}{100} = 358$

$\therefore Interest = \frac{PNR}{100} = \frac{x \times 1 \times 6}{100} = \frac{6x}{100}$

$\Rightarrow \frac{6x + 7y + 8z}{100} = 358$

$\Rightarrow 6x + 7y + \ + 8z = 35800$

Given that $\frac{6x}{100} + \frac{7y}{100} = 70 + \frac{8z}{100}$

$\Rightarrow 6x + 7y = 7008z$

$\Rightarrow 6x + 7y - 8z = 7000$

The matrix equation corresponding to the given system is

$$\begin{pmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5000 \\ 35800 \\ 7000 \end{pmatrix}$$

Augmented matrix [A,B]	**Elementary Transformation**
$\begin{pmatrix} 1 & 1 & 1 & 5000 \\ 6 & 7 & 8 & 35800 \\ 6 & 7 & -8 & 7000 \end{pmatrix}$	
$\begin{pmatrix} 1 & 1 & 1 & 5000 \\ 0 & 1 & 2 & 5800 \\ 0 & 1 & -14 & -2300 \end{pmatrix}$	$R_2 \to R_2 - 6R_1$ $R_3 \to R_3 - 6R_1$
$\begin{pmatrix} 1 & 1 & 1 & 5000 \\ 0 & 1 & 2 & 5800 \\ 0 & 0 & -16 & -28800 \end{pmatrix}$	$R_3 \to R_3 - R_2$

The last equivalent matrix is in echelon form and $\rho(A) = \rho([A, B]) = 3 = Numberofunknowns$

Thus, the given system is consistent with unique solution. To find the solution, let us rewrite the above echelon form into the matrix form

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -16 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5000 \\ 5800 \\ -28800 \end{pmatrix}$$

$\Rightarrow x + y + z = 5000$ ...(1)

$\Rightarrow y + 2z = 5800$ ..(2)

$\Rightarrow -16z = -28800$ ...(3)

(3) $\Rightarrow -16z = -28800$

$\Rightarrow z = \frac{28800}{-16} = 1800$

Substituting z = 1800 in (2) we get,

y + 2(1800) = 5800

⇒ y + 3600 = 5800

⇒ y=5800-3600

⇒ y = 2200

Substituting y = 2200 and z = 1800 in (1) we get

⇒ x + 2200 + 1800 = 5000

⇒ x + 4000 = 5000

⇒ x = 5000 - 4000

⇒ x = 1000

Hence, the amount of investment in each bond is Rs. 1000, Rs.2200 and Rs. 1800 respectively

5) A total of Rs 8,600 was invested in two accounts. One account earned $4\frac{3}{4}$ % annual interest and the other earned $6\frac{1}{2}$ % annual interest. If the total interest for one year was Rs 431.25, how much was invested in each account? (Use determinant method).

Let the amount invested in the two accounts be Rs x and Rs. y respectively

By the given data, x + y = 8600 ..(1)

$$4\frac{3}{4} \times \frac{x}{100} + 6\frac{1}{2} \times \frac{y}{100} = 431.25 \quad \left[\therefore interest = \frac{PNR}{100} \right]$$

$$\Rightarrow \frac{19x}{400} + \frac{13y}{3200} = 431.25$$

$$\Rightarrow \frac{19x + 26y}{400} = 431.25$$

19x + 26y = 172500 ...(2)

$$\Delta = \begin{vmatrix} 1 & 1 \\ 19 & 26 \end{vmatrix} = 1(26) - 1(19)$$

= 26-19 =7

$$\Delta x = \begin{vmatrix} 8600 & 1 \\ 172500 & 26 \end{vmatrix} = 8600(26) - 1(172500)$$

= 223600 - 172500= 51100

$$\Delta y = \begin{vmatrix} 1 & 8600 \\ 19 & 172500 \end{vmatrix} = 1(172500) - 19(8600)$$

= 172500 - 163400 = 9100

$$x = \frac{\Delta x}{\Delta} - \frac{51100}{7} = 7300$$

$$y = \frac{\Delta y}{\Delta} = \frac{9100}{7} = 1300$$

∴ Investment in the interest of $4\frac{3}{4}$ % account is Rs. 7300 and investment in the rate of $6\frac{1}{2}$ account is Rs. 1300.

6)

In a market survey three commodities A, B and C were considered. In finding out the index number some fixed weights were assigned to the three varieties in each of the commodities. The table below provides the information regarding the consumption of three commodities according to the three varieties and also the total weight received by the commodity

Commodity Variety	Variety			Total weight
	I	II	III	
A	1	2	3	11
B	2	4	5	21
C	3	5	6	27

Find the weights assigned to the three varieties by using Cramer's Rule.

Let the weight assigned to the three varieties be Rs. x, Rs. y and Rs. z respectively

By the given data,

x + 2y + 3z = 11

2x + 4y + 5z = 21

3x + 5y + 6z = 27

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{vmatrix} = 1\begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} - 2\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} + 3\begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix}$$

= 1(24 - 25) -2(12 - 15) + 3(10 - 12)

= 1(-1) -2 (-3) + 3(-2)

= =-1+6-6=-1 $\neq$ 0.

Since $\Delta \neq 0$ the system is consistent with unique solution and Cramer's rule can be applied.

$$\Delta x = \begin{vmatrix} 11 & 2 & 3 \\ 21 & 4 & 5 \\ 27 & 5 & 6 \end{vmatrix}$$

$$= \ = 11\begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} - 2\begin{vmatrix} 21 & 5 \\ 27 & 6 \end{vmatrix} + 3\begin{vmatrix} 21 & 4 \\ 27 & 5 \end{vmatrix}$$

= 11(24 - 25) - 2(126 - 135) + 3(105 - 108)

= 11(-1) - 2(-9) + 3 (-3)

= 11+18-9

= -2

$$\Delta y = \begin{vmatrix} 1 & 11 & 3 \\ 2 & 21 & 5 \\ 3 & 27 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} 21 & 5 \\ 27 & 6 \end{vmatrix} - 11\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} + 3\begin{vmatrix} 2 & 21 \\ 3 & 27 \end{vmatrix}$$

= 1(126 - 135) - 11(12 -15) + 3(54 - 63)

= - 9 - 11(-3) + 3(-9)

= - 9 + 33 - 27

=3

$$\Delta z = \begin{vmatrix} 1 & 2 & 11 \\ 2 & 4 & 21 \\ 3 & 5 & 27 \end{vmatrix} = 1\begin{vmatrix} 4 & 21 \\ 5 & 27 \end{vmatrix} - 2\begin{vmatrix} 2 & 21 \\ 3 & 27 \end{vmatrix} + 11\begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix}$$

= 1(108 - 105) - 2(54 - 63) + 11(10 - 12)

= 1(3) - 2(-9) + 11(-2)

= 3 + 18 - 22

= -1

$$x = \frac{\Delta x}{\Delta} = \frac{-2}{-1} = 2$$

$$y = \frac{\Delta y}{\Delta} = \frac{-3}{1} = 3$$

and $z = \frac{\Delta z}{\Delta} = \frac{-1}{-1} = 1$

Hence, the weights assigned to the three varieties are 2, 3 and 1 respectively

7) The cost of 2kg. of wheat and 1kg. of sugar is Rs 100. The cost of 1kg. of wheat and 1kg. of rice is Rs 80. The cost of 3kg. of wheat, 2kg. of sugar and 1kg of rice is Rs 220. Find the cost of each per kg., using Cramer's rule.

Let the cost of lkg of wheat be Rs. x, lkg of sugar be Rs. y and lkg of rice be Rs. z.

By the given data,

2x+ Y = 100

x +z = 80

3x+ 2y+z = 220

$$\Delta = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 2\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} + 0$$

= 2 (0 - 2) -1 (1 - 3) + 0

= 2(-2) - 1(- 2)

= - 4 + 2 =-2

$$\Delta x = \begin{vmatrix} 100 & 1 & 0 \\ 80 & 0 & 1 \\ 220 & 2 & 1 \end{vmatrix} = 100\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 80 & 1 \\ 220 & 1 \end{vmatrix} + 0$$

= 100(0 - 2) - 1 (80 - 220)

= 100(- 2) - 1(- 140)

= - 200 + 140 = - 60.

$$\Delta y = \begin{vmatrix} 2 & 100 & 0 \\ 1 & 80 & 1 \\ 3 & 220 & 1 \end{vmatrix} = 2\begin{vmatrix} 80 & 1 \\ 220 & 1 \end{vmatrix} - 100\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} + 0$$

= 2 (80 - 220) - 100 (1 - 3)

= 2 (- 140) - 100 (-2)

= - 280 + 200 = - 80.

$$\Delta z = \begin{vmatrix} 2 & 1 & 100 \\ 1 & 0 & 80 \\ 3 & 2 & 220 \end{vmatrix}$$

$2\begin{vmatrix}0 & 80\\2 & 220\end{vmatrix} - 1\begin{vmatrix}1 & 80\\3 & 220\end{vmatrix} + 100\begin{vmatrix}1 & 0\\3 & 2\end{vmatrix}$

= 2(0 - 160) - 1(220 - 240) + 100(2 - 0)

= 2(- 160) - 1(- 20) + 100(2)

= - 320 + 20 + 200

= -100

$x = \frac{\Delta x}{\Delta} = \frac{-60}{-2} = 30$

$y = \frac{\Delta y}{\Delta} = \frac{-80}{-2} = 40$

$z = \frac{\Delta z}{\Delta} = \frac{-100}{-2} = 50$

$\therefore$ The cost of lkg of wheat is Rs.30

The cost of lkg sugar is Rs.40 and

The cost of 1 kg of rice is Rs.50

8) Solve the following equation by using Cramer's rule

x + 4y + 3z =2,2x−6y + 6z=−3, 5x− 2y + 3z =−5

$\Delta = \begin{vmatrix}1 & 4 & 3\\2 & -6 & 6\\5 & -2 & 3\end{vmatrix}$

$= 1\begin{vmatrix}-6 & 6\\-2 & 3\end{vmatrix} - 4\begin{vmatrix}2 & 6\\5 & 3\end{vmatrix} + 3\begin{vmatrix}2 & -6\\5 & -2\end{vmatrix}$

= 1(-18 + 12) - 4(6 - 30) +3 (- 4 +30)

= 1(- 6) - 4(- 24) + 3(26)

= - 6 + 96 + 78 = 168 $\neq$ 0

Since $\Delta \neq 0$ the system is consistent with unique solution and Cramer's rule can be applied

$\Delta x = \begin{vmatrix}2 & 4 & 3\\-3 & -6 & 6\\-5 & -2 & 3\end{vmatrix}$

$= 2\begin{vmatrix}-6 & 6\\-2 & 3\end{vmatrix} - 4\begin{vmatrix}-3 & 6\\-5 & 3\end{vmatrix} + 3\begin{vmatrix}-3 & -6\\-5 & -2\end{vmatrix}$

= 2 (- 18 + 12) - 4(- 9 +30) + 3(6 -30)

= 2(- 6) - 4(21) + 3(- 24)

= -12-84-72=-168

$\Delta y = \begin{vmatrix}1 & 2 & 3\\2 & - & 6\\5 & -5 & 3\end{vmatrix} = 1\begin{vmatrix}-3 & 6\\-5 & 3\end{vmatrix} - 2\begin{vmatrix}2 & 6\\5 & 3\end{vmatrix} + 3\begin{vmatrix}2 & -3\\5 & -5\end{vmatrix}$

= 1 (-9+30)-2(6-30)+3(- 10+ 15)

= 1(21) - 2(- 24) + 3(5)

= 21 + 48 + 15 = 84

$$\Delta z = \begin{vmatrix} 1 & 4 & 2 \\ 2 & -6 & -3 \\ 5 & -2 & -5 \end{vmatrix}$$

$$= 1\begin{vmatrix} -6 & -3 \\ -2 & -5 \end{vmatrix} - 4\begin{vmatrix} 2 & -3 \\ 5 & -5 \end{vmatrix} + 2\begin{vmatrix} 2 & -6 \\ 5 & -2 \end{vmatrix}$$

= 1(30-6)-4(-10+ 15)+2(-4+30)

= 24 - 4(5) + 2(26)

= 24 - 20 + 52 = 56

$$x = \frac{\Delta x}{\Delta} = \frac{-168}{168} = -1$$

$$y = \frac{\Delta y}{\Delta} = \frac{84}{168} = \frac{1}{2}$$

$$z = \frac{\Delta z}{\Delta} = \frac{56}{168} = \frac{1}{3}$$

Solution set is $\left\{-1, \frac{1}{2}, \frac{1}{3}\right\}$

9) Integrate the following with respect to x. $\frac{1}{\sqrt{x+1}+\sqrt{x-1}}$

$$\int \frac{1}{\sqrt{x+1}+\sqrt{x-1}}dx$$

=Multiplying and dividing the conjugate of the denominator we get

$$= \int \frac{\sqrt{x+1}-\sqrt{x-1}dx}{\left(\sqrt{x+1}+\sqrt{x-1}\right)\left(\sqrt{x+1}-\sqrt{x-1}\right)}$$

$$= \int \frac{\sqrt{x+1}-\sqrt{x-1}}{(x+1)-(x-1)}dx$$

$$\left[\because (a+b)(a-b) = a^2 - b^2\right]$$

$$= \int \frac{\sqrt{x+1}-\sqrt{x-1}}{x+1-x+1}dx$$

$$= \int \frac{\sqrt{x+1}-\sqrt{x-1}}{2}dx$$

$$= \frac{1}{2}\int\left((x+1)^{\frac{1}{2}} - (x-1)^{\frac{1}{2}}\right)dx$$

$$= \frac{1}{2}\left[\frac{(x+1)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x-1)^{\frac{3}{2}}}{\frac{3}{2}}\right] + c$$

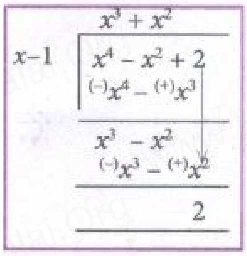
$$= \frac{1}{2\times\frac{3}{2}}\left[(x+1)^{\frac{3}{2}} - (x-1)^{\frac{3}{2}}\right] + c$$

$= \frac{1}{3}\left[(x+1)^{\frac{3}{2}} - (x-1)^{\frac{3}{2}}\right] + c$

10) Integrate the following with respect to x.

$\frac{x^4-x^2+2}{x-1}$

$\int \frac{x^4-x^2+2}{x-1}dx$



$= \int \left(x^3 + x^2 + \frac{2}{x-1}\right)dx$

$= \frac{x^4}{3} - \frac{x^3}{3} + 2log|x-1| + c$

11) Integrate the following with respect x

$\frac{3x+2}{(x-2)(x-3)}$

$\int \frac{(3x+2)dx}{(x-2)(x-3)}$

$= \int \left(\frac{-8}{x-2} + \frac{11}{x-3}\right)dx$

$= -8\log|x-2| + 11\log|x-3| + c$

$= -11\log|x-3| - 8\log|x-2| + c$

$\frac{3x+2}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$

⇒ 3x+2 = A (x-3)+B(x-2)

Put x = 3

9+2 = B(1) ⇒ B = 11

Put x = 2

8 = A(-1) ⇒ A = -8

$\frac{3x+2}{(x-2)(x-3)} = \frac{-8}{x-2} + \frac{11}{x-3}$

12) Integrate the following with rexpect to x

$\frac{a^x - e^{xlogb}}{e^{xloga}b^x}$

$\int \frac{a^x - e^{x \ log \ b}}{e^{x \ log \ a}b^x}dx$

$= \int \frac{a^x - e^{logb^x}}{e^{loga^x}b^x}dx \quad [\because m \ log \ n = logn^m]$

$= \int \frac{a^x - b^x}{a^x . b^x}dx \quad [\because e^{logx} = x]$

$= \int \frac{a^x}{a^x b^x}dx - \int \frac{a^x}{a^x b^x}dx$

$= \int \frac{1}{b^x}dx - \int \frac{1}{a^x}dx$

$= \int b^{-x}dx - \int a^{-x}dx \quad \left[\because \int a^{-x}dx = \frac{a^{-x}}{-log \ a} + c\right]$

$$= \frac{b^{-x}}{-\log b} - \frac{a^{-x}}{-\log a} + c$$

$$= -\frac{b^{-x}}{\log b} + \frac{a^{-x}}{\log a} + c$$

$$= \frac{1}{b^x \log b} + \frac{1}{\log a . a^x} + c$$

$$= \frac{1}{a^x \log a} - \frac{-1}{b^x \log b} + c$$

13) Integrate the following with respect to x

$$\frac{6x+7}{\sqrt{3x^2+7x-1}}$$

$$Let \quad I = \int \frac{6x+7}{\sqrt{3x^2+7x-1}} dx$$

$$Let \quad t = 3x^2 + 7x - 1$$

$$\Rightarrow dt = (6x+7)dx$$

$$\therefore I = \int \frac{dt}{\sqrt{t}} = \int t^{\frac{1}{2}} dt$$

$$= \frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2\sqrt{t} + c$$

$$= 2\sqrt{3x^2 + 7x - 1} + c$$

$$\left[\because t = 3x^2 + 7x + c \right]$$

14) Integrate the following with respect to x.

$$(4x + 2)\sqrt{x^2 + x + 1}$$

$$Let \quad I = \int (4x+2)\sqrt{x^2 + x + 1} dx$$

$$put \quad t = x^2 + x + 1$$

$$dt = (2x+1)dx$$

$$\therefore I = 2\int (2x+1)\sqrt{x^2 + x + 1} dx$$

$$= 2\int \sqrt{t} dt$$

$$= 2\int t^{\frac{1}{2}} dt = 2\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

$$= 2\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{4}{3} t^{\frac{3}{2}} + c$$

$$= \frac{4}{3}\left(x^2 + x + 1\right)^{\frac{3}{2}} + c \qquad \left[\because t = x^2 + x + 1 \right]$$

15) Integrate the following with respect to x.

$$e^{3x}\left[\frac{3x-1}{9x^2}\right]$$

$$Let \quad I = \int e^{3x}\left[\frac{3x-1}{9x^2}\right] dx$$

$$= \int e^{3x}\left(\frac{3x}{9x^2} - \frac{1}{9x^2}\right) dx$$

$= \int e^{3x}\left[\frac{3}{9x} - \frac{1}{9x^2}\right]dx$

$Let f(x) = \frac{1}{9x} and \quad a = 3$

$\Rightarrow f(x) = \frac{1}{9x}.x^{-1}$

$f'(x) = \frac{1}{9}.(-1)x^{-1-1}$

$\Rightarrow f'(x) = \frac{-1}{9}x^{-2} = \frac{-1}{9x^2}$

$\therefore I = \int e^{3x}\left(3.f(x) + f'(x)\right)dx$

$= e^{3x}f(x) + c$

$\left[\because \int e^{ax}\left[af(x) + f'(x)\right]dx = e^{ax}f(x) + c\right]$

$= e^{3x}\frac{1}{9x} + c = \frac{e^{3x}}{9x} + c$

16) Using second fundamental theorem, evaluate the following:

$\int_1^e \frac{dx}{x(1+logx)^3}$

Let I = $\int_1^e \frac{dx}{x(1+\log x)^3}$

Let t = 1+log x

$\Rightarrow dt = \frac{1}{x}dx$

When x=1, t=1+log 1=1+0 = 1

When x=e, t=1+log e=1+1 = 2

$\therefore I = \int_1^2 \frac{dt}{t^3} = \int_1^2 t^{-3}dt$

$= \left[\frac{t^{-3+1}}{-3+1}\right]_1^2$

$= \left[\frac{t^{-2}}{-2}\right]_1^2 = \left[-\frac{1}{2t^2}\right]_1^2$

$= -\frac{1}{2}\left[\frac{1}{2^2} - \frac{1}{1^2}\right]$

$= -\frac{1}{2}\left[\frac{1}{4} - 1\right] = -\frac{1}{2}\left(\frac{-3}{4}\right) = \frac{3}{8}$

17) Evaluate the following using properties of definite integrals:

$\int_0^{\frac{\pi}{2}} \frac{sin^7x}{sin^7x+cos^7x}dx$

Let I = $\int_0^{\frac{\pi}{2}} \frac{sin^7x}{sin^7x+cos^7x}dx$ ---(1)

By the property, $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \int_0^{\frac{\pi}{2}} \frac{sin^7\left(\frac{\pi}{2}-x\right)}{sin^7\left(\frac{\pi}{2}-x\right)+cos^7\left(\frac{\pi}{2}-x\right)}dx$

$I = \int_0^{\frac{\pi}{2}} \frac{cos^7x \quad dx}{cos^7x+sin^7x}$ ---(2)

$\left[\because \sin\left(\frac{\pi}{2}-x\right) = \cos\ x \ \text{ and } \ \cos\ \left(\frac{\pi}{2}-x\right) = x\right]$

Adding (1) and (2) we get,

$2I = \int_0^{\frac{\pi}{2}} \left(\frac{\sin^7 x}{\sin^7 x+\cos^7 x} + \frac{\cos^7 x}{\cos^7 x+\sin^7 x}\right) dx$

$= \int_0^{\frac{\pi}{2}} \frac{\sin^7 x+\cos^7 x}{\sin^7 x+\cos^7 x} dx$

$= \int_0^{\frac{\pi}{2}} dx = [x]_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

$\therefore 2I = \frac{\pi}{2}$

$\Rightarrow I = \frac{\pi}{4}$

18) Evaluate the following using properties of definite integrals:

$\int_0^1 \log\left(\frac{1}{x} - 1\right) dx$

Let $I = \int_0^1 \log\left(\frac{1}{x} - 1\right) dx$

$I = \int_0^1 \log\left(\frac{1-x}{x}\right) dx \quad ---(1)$

By the property, $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \int_0^1 \log\left(\frac{1-(1-x)}{1-x}\right) dx$

$I = \int_0^1 \log\left(\frac{1-1+x}{1-x}\right) dx$

$= \int_0^1 \log\left(\frac{x}{1-x}\right) dx \quad ---(2)$

Adding (1) and (2) we get,

$2I = \int_0^1 \left[\log\left(\frac{1-x}{x}\right) + \log\left(\frac{x}{1-x}\right)\right] dx$

$= \int_0^1 \left[\log\left(\frac{1-x}{x} \times \frac{x}{1-x}\right)\right] dx$

[$\because$ log m + log n= log mn]

$= \int_0^1 \log 1 dx = 0 \quad [\because \log 1 = 0]$

2I = 0

$\Rightarrow$ I = 0

19) Evaluate the following using properties of definite integrals:

$\int_0^1 \frac{x}{(1-x)^{\frac{3}{4}}} dx$

Let $I = \int_0^1 \frac{x}{(1-x)^{\frac{3}{4}}} dx$

$= -\int_0^1 \frac{-x}{(1-x)^{\frac{3}{4}}} dx$

[Multiply and divide by -1]

$= -\int_0^1 \frac{1-x-1}{(1-x)^{\frac{3}{4}}} dx$

[Adding and subtracting 1 in the numberator]

$= \int_1^0 \frac{1-x-1}{(1-x)^{\frac{3}{4}}} dx$

$\left[\because \int_a^b f(x)dx = -\int_b^a f(x) \quad dx \right]$

$= \int_1^0 \left(\frac{1-x}{(1-x)^{\frac{3}{4}}} - \frac{1}{(1-x)^{\frac{3}{4}}} \right) dx$

$= \int_1^0 \left((1-x)^{1-\frac{3}{4}} - (1-x)^{-\frac{3}{4}} \right) dx$

$= \int_1^0 (1-x)^{\frac{1}{4}} dx - \int_1^0 (1-x)^{-\frac{3}{4}} dx$

$= \left[\frac{(1-x)^{\frac{1}{4}+1}}{-1\left(\frac{1}{4}+1\right)} - \frac{(1-x)^{-\frac{3}{4}+1}}{-1\left(-\frac{3}{4}+1\right)} \right]_1^0$

$= \left[-\frac{(1-x)^{\frac{5}{4}}}{\frac{5}{4}} + \frac{(1-x)^{\frac{1}{4}}}{\frac{1}{4}} \right]_1^0$

$= \left[-\frac{4}{5}(1-x)^{\frac{5}{4}} + 4(1-x)^{\frac{1}{4}} \right]_1^0$

$= -\frac{4}{5}\left(1^{\frac{5}{4}}\right) + 4\left(1^{\frac{1}{4}}\right) - 0$

$= -\frac{4}{5}(1) + 4(1) = -\frac{4}{5} + 4$

$= \frac{-4+20}{5} = \frac{16}{5}$

20) Evaluate the following integrals:

$\int_{-1}^1 x^2 e^{-2x} dx$

Let $I = \int_{-1}^1 x^2 e^{-2x} dx$

$u = x^2 \quad dv = e^{-2x}$

$u^1 = 2x \quad v = \frac{e^{-2x}}{-2} = \frac{-e^{-2x}}{2}$

$u^4 = 2 \quad v_1 = +\frac{e^{-2x}}{4}$

$v_2 = -\frac{e^{-2x}}{8}$

Using Bernoulli's formula

$I = uv = u^1 v_1 + u^4 v_2$

$= \left[x^2\left(\frac{-e^{-2x}}{2}\right) - 2x\left(\frac{e^{-2x}}{4}\right) + 2\left(\frac{-e^{-2x}}{8}\right) \right]_{-1}^1$

$= \left(e^{-2x}\left[\frac{-x^2}{2} - \frac{x}{2} - \frac{1}{4}\right]\right)$

$= \left[e^{-2}\left(-\frac{1}{2} - \frac{1}{2} - \frac{1}{4}\right)\right] - \left[e^{2}\left(-\frac{1}{2} + \frac{1}{2} - \frac{1}{4}\right)\right]$

$\left(e^{-2x}\left[\frac{-x^2}{2} - \frac{x}{2} - \frac{1}{4}\right]\right)$

$= e^{-2}\left(-\frac{5}{4}\right) - e^{2}\left(-\frac{1}{4}\right)$

$= \frac{-5}{4}e^{-2} + \frac{1}{4}e^{2}$

$= \frac{1}{4}\left(e^2 - \frac{5}{e^2}\right) = \frac{1}{4}\left(\frac{e^4 - 5}{e^2}\right)$

21) Evaluate the following integrals:

$\int_0^3 \frac{xdx}{\sqrt{x+1} + \sqrt{5x+1}}$

Let $I = \int_0^3 \frac{xdx}{\sqrt{x+1} + \sqrt{5x+1}}$

$I = \int_0^3 \frac{x\left(\sqrt{x+1} - \sqrt{5x+1}\right)}{\left(\sqrt{x+1} + \sqrt{5x+1}\right)\left(\sqrt{x+1} - \sqrt{5x+1}\right)}$

$= \int_0^3 \frac{x\left(\sqrt{x+1} - \sqrt{5x+1}\right)dx}{(x+1) - (5x+1)}$

$\left[\because (a+b)(a-b) = a^2 - b^2\right]$

$= \int_0^3 \frac{x\left(\sqrt{x+1} - \sqrt{5x+1}\right)dx}{x + \not{1} - 5x - \not{1}}$

$= \int_0^3 \frac{\not{x}\left(\sqrt{x+1} - \sqrt{5x+1}\right)dx}{-4\not{x}}$

$= -\frac{1}{4}\int_0^3 \left(\sqrt{x+1} - \sqrt{5x+1}\right)dx$

$= -\frac{1}{4}\left[\frac{(x+1)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(5x+1)^{\frac{3}{2}}}{5\left(\frac{3}{2}\right)}\right]_0^3$

$= -\frac{1}{4}\left[\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{1}{15}(5x+1)^{\frac{3}{2}}\right]_0^3$

$= -\frac{1}{2}\left[\frac{1}{3}(x+1)^{\frac{3}{2}} - \frac{1}{15}(5x+1)^{\frac{3}{2}}\right]_0^3$

$$= -\frac{1}{2}\left[\frac{1}{3}(4)^{\frac{3}{2}} - \frac{1}{15}(16)^{\frac{3}{2}}\right] - \left[\frac{1}{3}(1)^{\frac{3}{2}} - \frac{1}{15}(1)^{\frac{3}{2}}\right]$$

$$= -\frac{1}{2}\left[\left(\frac{1}{3}(4)\sqrt{4} - \frac{1}{15}16\sqrt{16}\right) - \left(\frac{1}{3} - \frac{1}{15}\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{8}{3} - \frac{64}{15}\right) - \frac{1}{3} + \frac{1}{5}\right]$$

$$= -\frac{1}{2}\left[\frac{8}{3} - \frac{64}{15} - \frac{1}{3} + \frac{1}{15}\right]$$

$$= -\frac{1}{2}\left[\frac{8}{3} - \frac{1}{3} - \frac{64}{15} + \frac{1}{15}\right]$$

$$= -\frac{1}{2}\left[\frac{7}{3} - \frac{63}{15}\right] = -\frac{1}{2}\left[\frac{105-189}{45}\right]$$

$$= -\frac{1}{2}\left[\frac{-84}{45}\right] = \frac{42}{45} = \frac{14}{15}$$

$$\therefore\ I = \frac{14}{15}$$

22) Elasticity of a function $\frac{Ey}{Ex}$ is given by $\frac{Ey}{Ex} = \frac{-7x}{(1-2x)(2+3x)}$. Find the function when x = 2, y = $\frac{3}{8}$

$$\frac{EY}{Ex} = \frac{-7x}{(1-2x)(2+3x)}$$

Also, it is given that x = 2, when y $= \frac{3}{8}$

$$\Rightarrow \frac{x}{y}\frac{dy}{dx} = \frac{-7x}{(1-2x)(2+3x)}$$

$$\Rightarrow \frac{dy}{y} = \frac{-7\not{x}\,dx}{\not{x}(1-2x)(2+3x)}$$

$$= \frac{-7dx}{(1-2x)(2+3x)}$$

$$\Rightarrow \frac{dy}{y} = \frac{7dx}{(2x+1)(3x+2)}$$

$$\int \frac{dy}{y} = \int \frac{7dx}{(2x-)(3x+2)}$$

$$\int \frac{dy}{y} = \int \left(\frac{2}{2x-1} - \frac{3}{3x+2}\right)dx$$

$$log\quad y = 2\int \frac{1}{2x-1}dx - 3\int \frac{dx}{3x+2}$$

$$= 2\frac{log|2x-1|}{2} - 3\frac{log|3x+2|}{3} + logc$$

$$= log\quad y - log\quad c = log\left|\frac{2x-1}{3x+2}\right|$$

$$\Rightarrow log\left|\left(\frac{y}{c}\right)\right| = log\left|\frac{2x-1}{3x+2}\right|$$

$$\Rightarrow \frac{y}{c} = \frac{2x-1}{3x+2}y = c\left(\frac{2x-1}{3x+2}\right) \qquad ....(1)$$

When $x = 2, y = \frac{3}{8}$

$$\Rightarrow \frac{3}{8} = c\left(\frac{4-1}{8}\right)$$

$\frac{3}{8} = c\left(\frac{3}{8}\right) = c = 1$

$y = \left(\frac{2x-1}{3x+2}\right)$

$\Rightarrow y = \frac{2x-1}{3x+2}$

$\frac{7}{(2x-1)(3x+2)} = \frac{A}{2x-1} + \frac{B}{3x+2}$

7=A(3x+2)+B(2x-1)

$Put \quad x = \frac{-2}{3} 7 = B\left(\frac{-4}{3} - 1\right) 7 = B\left(\frac{-7}{3}\right)$

$Put \quad x = \frac{1}{2} 7 = A\left(\frac{3}{2} + 2\right) \Rightarrow 7 = A\left(\frac{7}{2}\right)$

$\therefore \frac{7}{(2x-1)(3x+2)} = \frac{2}{2x-1} - \frac{3}{3x+2}$

23) If the marginal cost function of x units of output is $\frac{a}{\sqrt{ax+b}}$ and if the cost of output is zero. Find the total cost as a function of x.

Given marginal cost function = $\frac{a}{\sqrt{ax+b}}$

$\Rightarrow MC = \frac{a}{\sqrt{ax+b}} \Rightarrow \frac{dC}{dx} = \frac{a}{\sqrt{ax+b}}$

$\Rightarrow dC = \frac{a}{\sqrt{ax+b}} dx$

$\Rightarrow \int dC = a \int \frac{dx}{\sqrt{ax+b}}$

$\Rightarrow C = a \int (ax+b)^{\frac{-1}{2}} dx$

$\Rightarrow \quad C = \frac{a(ax+b)^{\frac{-1}{2}+1}}{\left(-\frac{1}{2}+1\right)a} + k$

$\Rightarrow C = \frac{(ax+b)^{\frac{1}{2}}}{1} + k$

$\Rightarrow C = 2\sqrt{ax+b} + k \quad \ldots (1)$

Since the cost of output is zero.

C = 0, when x = 0

$\therefore (1) \rightarrow 0 = 2\sqrt{0+b} + k$

$\Rightarrow 0 = 2\sqrt{b} + k$

$\Rightarrow k = -2\sqrt{b}$

∴(1)becomes

$C = 2\sqrt{ax+b} - 2\sqrt{b}$

24) Given the marginal revenue function $\frac{4}{(2x+3)^2}$ -1, show that the average revenue function is P = $\frac{4}{6x+9}$ -1

Given marginal revenue function $= \frac{4}{(2x+3)^2} - 1$

$\Rightarrow MR = \frac{dR}{dx} = 4(2x+3)^{-2} - 1$

$\Rightarrow dR = (4(2x+3)^{-2} - 1)dx$

$\Rightarrow \int dR = 4\int (2x+3)^{-2} dx - \int dx$

$\Rightarrow R = 4\frac{(2x+3)^{-2+1}}{(-2+1)^2} - x$

$\Rightarrow R = 4\frac{(2x+3)^{-1}}{-2} - x$

$\Rightarrow R = \frac{-2}{(2x+3)} - x + k \quad \ldots(1)$

When x=0, R=0

$\Rightarrow 0 = \frac{-2}{3} - 0 + k \Rightarrow = \frac{2}{3}$

∴ (1) becomes

$R = \frac{-2}{2x+3} - x + \frac{2}{3}$

We know that R=Px⇒P=$\frac{R}{x}$

∴ Average revenue function

$P = \frac{R}{x} = \frac{R}{x} = \frac{-2}{x(2x+3)} - \frac{x}{x} + \frac{2}{3x}$

$= \frac{-2}{x(2x+3)} - 1 + \frac{2}{3x}$

$= \frac{-2}{x(2x+3)} + \frac{2}{3x} - 1$

$= \frac{-6+2(2x+3)}{3x(2x+3)} - 1$

$= \frac{-6+4x+6}{3x(2x+3)} - 1$

$= \frac{4x}{3x(2x+3)} - 1$

$= \frac{4}{3(2x+3)} - 1$

$P = \frac{4}{6x+9} - 1$

Hence Proved.

25)

A firm's marginal revenue function is MR = $20e^{-x/10}\left(1 - \frac{x}{10}\right)$. Find the corresponding demand function.

Given marginal revenue function.

$MR = \frac{DR}{dx} = 20e^{\frac{-x}{10}}\left(1 - \frac{x}{10}\right)$

$dR = 20e^{\frac{-x}{10}}\left(1 - \frac{x}{10}\right)dx$

$R = \int 20e^{\frac{-x}{10}}\left(1 - \frac{x}{10}\right)dx$

We know that $\int e^{ax}[af(x) + f'(x)]dx = e^{ax}f(x) + c$

Here $a = \frac{-1}{10}, f(x) = x, \quad f'(x) = 1$

$R = 20\int e^{\frac{-x}{10}}\left[-\frac{1}{10}x + 1\right]dx = 20e^{\frac{-x}{10}}x + k$

$\Rightarrow R = 20e^{\frac{-x}{10}}x + k \quad \ldots(1)$

When x=0, R=0

0=0+k⇒k=0

(1)becomes

$R = 20xe^{\frac{-x}{10}}$

Demand function P

$= \frac{R}{x} = \frac{20xe^{\frac{-x}{10}}}{x}$

$\Rightarrow \quad P = 20e^{\frac{-x}{10}}$

26) The marginal cost function of a commodity is given by MC = $\frac{14000}{\sqrt{7x+4}}$ and the fixed cost is Rs.18,000.Find the total cost and average cost.

Given $MC = \frac{14000}{\sqrt{7x+4}}$

$\Rightarrow \frac{dc}{dx} = \frac{14000}{\sqrt{7x+4}}$

$\Rightarrow dC = \frac{14000}{\sqrt{7x+4}}dx$

$\Rightarrow \int dC = 14000\int \frac{dx}{\sqrt{7x+4}}$

$\Rightarrow = 14000\int (7x+4)^{-\frac{1}{2}}dx$

$\Rightarrow C = 14000\frac{(7x+4)^{-\frac{1}{2}+1}}{\left(-\frac{1}{2}+1\right)7} + k$

$\Rightarrow C = 14000\frac{(7x+4)^{\frac{1}{2}}}{\frac{7}{2}} + k$

$\Rightarrow \quad C = \overset{2000}{14000} \times \frac{2}{7}\sqrt{7x+4} + k$

$\Rightarrow C = 4000\sqrt{7x+4} + k \quad \ldots(1)$

Given fixed cost is 18,000

When x=0, C=18000

⇒18000=4000√4+k

⇒18000=4000(2)+k

⇒k=18000-8000

⇒k=10000

∴(1) becomes,

C = $4000\sqrt{7x+4} + 10000$,

Average Cost (AC)

$\frac{C}{x}$

$= \frac{4000}{x}\sqrt{7x+4} + \frac{10000}{x}$

27) The price elasticity of demand for a commodity is $\frac{p}{x^3}$. Find the demand function if the quantity of demand is 3, when the price is Rs. 2

Given elasticity of demand = $\frac{p}{x^3}$

$\Rightarrow \quad \frac{-p}{x} \cdot \frac{dx}{dp} = \frac{p}{x^3}$

$\Rightarrow \quad \frac{-x^3 dx}{x} = p. \frac{dp}{p}$

$\Rightarrow \quad -x^2dx = dp$

$\Rightarrow \quad -\int x^2dx = \int dp$

$\Rightarrow -\frac{x^3}{3} + k = p \quad \ldots(1)$

When p=2, x=3

$\Rightarrow -\frac{3^3}{3} + k = 2$

k=2+9⇒ k=11

∴ (1) becomes

$P = -\frac{x^3}{3} + 11$

$= 11 - \frac{x^3}{3}$

28) Solve: $y(1 - x) - x\frac{dy}{dx} = 0$

Given $y(1-x) - x\frac{dy}{dx} = 0$

$\Rightarrow y(1-x) = x\frac{dy}{dx}$

Separating the variables we get,

$\Rightarrow \frac{(1-x)}{x}dx = \frac{dy}{y}$

$\Rightarrow \left(\frac{1}{x} - 1\right) = \frac{dy}{y}$

Integrating both sides we get,

$\int \left(\frac{1}{x} - 1\right)dx = \int \frac{dy}{y}$

⇒ logx - x =logy+c

29) Solve: cosx(1 + cos y)dx − sin y(1 + sin x)dy = 0

cosx(1 + cos y)dx − sin y(1 + sin x)dy

Separating the variables we get

$\frac{cosx}{1+sinx}dx = \frac{siny}{16cosy}dy$

Integrating both sides we get

$\int \frac{cosx}{1+sinx}dx = \int \frac{siny}{16cosy}dy$

put 1+sinx = t ⇒ cosx dx =dt

Also 1+cosy =s ⇒ -siny dy =ds

⇒ siny dy =-ds

$\Rightarrow \int \frac{dt}{t} = -\int \frac{ds}{s}$

⇒ log t=log s + log c

$\Rightarrow \log t = \log\left(\frac{c}{s}\right)$

[∵ log m- logn=log$\frac{m}{n}$]

$\Rightarrow t = \frac{c}{s}$

$\Rightarrow 1+sinx = \frac{c}{1+cosy}$

[∵ t=1+sinx & s=1+cosy]

⇒ (1+sinx)(1+cosy)=c

30) Solve: $\frac{dy}{dx} = y \sin 2x$

Separating the variables. ,we get,

$\frac{dy}{x}$ =sin2x dx

Integrating both sides we get,

$\int \frac{dy}{y} = \int sin2x$

$\Rightarrow$ log y=$\frac{-cos2x}{1}$ +c

31) Solve the following homogeneous differential equations.

$x\frac{dy}{dx} - y = \sqrt{x^2+y^2}$

$x\frac{dy}{dx} = y + \sqrt{x^2+y^2}$

$\frac{dy}{dx} = \frac{\left[y+\sqrt{x^2+y^2}\right]}{x}$

Since the numerator and denominator is a homogeneous function of degree 1,

put y=vx and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ v+x$\frac{dv}{dx} = \frac{vx+\sqrt{x^2+v^2x^2}}{x} = \frac{vx+x\sqrt{1+v^2}}{x}$

$= x\left[\frac{v+\sqrt{1+v^2}}{x}\right] = v + \sqrt{1+v^2}$

$\Rightarrow$ x$\frac{dv}{dx} = v + \sqrt{1+v^2} - v = \sqrt{1+v^2}$

Separating the variables we get,

$\frac{dx}{\sqrt{1+v^2}} = \frac{dx}{x}$

Integrating both sides we get,

$\int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x}$

$\left[\therefore \int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x + \sqrt{x^2+a^2} + c\right| \right]$

$\Rightarrow log\left|v + \sqrt{v^2+1}\right|$ =log x + log c

$\Rightarrow log(v + \sqrt{v^2+1})$ =log xc

$\Rightarrow$ v+$\sqrt{v^2+1}$ =xc

Replace v by $\frac{y}{x}$ we get,

$\frac{y}{x} + \sqrt{\frac{y^2}{x^2} + 1}$ =xc

$\Rightarrow \frac{y}{x} + \sqrt{\frac{x^2+y^2}{x}}$ =xc

$\Rightarrow$ y+$\frac{y+\sqrt{\frac{x^2+y^2}{x}}}{x}$ =xc

$\Rightarrow$y+$\sqrt{x^2+y^2}$ =x^2c

32) Solve the following:

$\frac{dy}{dx} + ycosx = sinxcosx$.

The given differential equation is of the follows

$\frac{dy}{dx}$ +Py=Q where

P=cosx, Q=sinx cosx

$\therefore \int Pdx = \int cosxdx$

$\therefore$ Integrating factor (I.F) $= e^{\int p.dx} = e^{sinx}$

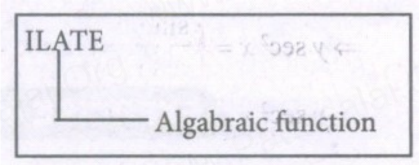
Hence, the solution is

$ye^{\int p.dx} = \int Qe^{\int p.dx}dx + c$

$\Rightarrow y.\, e^{sinx} = \int sinxcosx.\, e^{sinx}dx + c$

put t=sinx $\Rightarrow$ dt =cosx dx

$\Rightarrow ye^{sinx} = \int te^{t}dt$ .....(1)



put u=t; dv=et dt

du=dt; v=et

Using integration by parts

$\int udv = uv - \int vdu$

$\Rightarrow \int te^{t}dt = te^{t} - \int e^{t}dt$ $=te^{t}-e^{t}$ ....(2)

Substituting (2) in (1) we get,

$ye^{sinx} = t\,e^{t} - e^{t} + c$

$\Rightarrow y\,e^{sinx} = e^{t}(t-1)+c$

$\Rightarrow y\,e^{sinx} = e^{sinx}(sinx-1)+c$ [$\because$ t =sinx]

33) Solve the following:

$\frac{dy}{dx} + \frac{3x^3}{1+x^2}y = \frac{1+x^2}{1+x^3}$

The given differential equation is of this form

$\frac{dy}{dx}$ +Py=Q Where

P=$\frac{3x^2}{1+x^3}$; Q=$\frac{1+x^2}{1+x^3}$

$\therefore \int pdx = \int \frac{3x^2}{1+x^3}$ dx

put t=1+x^3 $\Rightarrow$ dt=3x^2 dx

=$\int \frac{dt}{t}$ =logt=log(1+x^3) [$\because$ t=1+x^3]

$\therefore$ Integrating factor (I. F) $= e^{\int pdx} = e^{log(1+x^3)}$

=1+x^3

Hence the solution is

$ye^{\int pdx} = \int Q.\, e^{\int pdx}dx + c$

$$\Rightarrow \quad y(1+x^3) = \int\left(\frac{1+x^2}{1+x^3}\cdot 1+x^3\right)dx + c$$

$=\int(1+x^2)dx$ +c

$\Rightarrow$ y(1+x^3)=x+$\frac{x^3}{3}$ +c

34) If $\frac{dy}{dx}$ + y tan x = sin x and if y = 0 when x = $\frac{\pi}{3}$ express y in terms of x.

Given differential equation is of the form

$\frac{dy}{dx}$ +Py=Q where

P=2 tanx; Q=sinx

$\int pdx = \int 2tanxdx$ =2(log secx)=log sec^{2}x

$\therefore$ Integrating factor (I. F) $= e^{\int pdx} = e^{log \quad sec^{2x}}$

$=\sec^2 x$

Hence the solution is

$y e^{\int p dx} = \int Q . e^{\int p dx} dx + c$

$\Rightarrow y \sec^2 x = \int sinx sec^2 x dx + c$

$\Rightarrow y \sec^2 x = \int sinx . \frac{1}{cos^2 x} dx + c$

$\Rightarrow y \sec^2 x = \int \frac{sinx}{cosx} . \frac{1}{cosx} dx + c$

$\Rightarrow y \sec^2 x = \int tanx \ secx dx + c$

$\Rightarrow y \sec^2 x = secx + c$ ....(1)

Also its givenwhen $x=\frac{\pi}{3}$, y=0

$\therefore 0(sec^2 \frac{\pi}{3}) = sec\frac{\pi}{3} + c$

$\Rightarrow 0=2+c \Rightarrow c=-2$

$\therefore$ (1) becomes

y sec2x =secx-2

35) Solve the following differential equations $(D^2-10D+25)y=4e^{5x} + 5$

The auxiliary equation is

$m^2 - 10m + 25 = 0$

(m - 5) = 0

$\Rightarrow$ m = 5, 5

The roots are real and equal.

$\therefore$ Complementary function CF is $(Ax + B) e_{5x}$

Particular Integral PI1 $=\frac{1}{\phi(D)} f_1(x)$

$=\frac{1}{(D-5)^2} .4 x e^{5x} = 4.e^{5x}.\frac{x^2}{2}$

$=2x^2 e^{5x}$ [$\because (D-5)^2 = 0$ when D=5]

$PI_2=\frac{1}{\phi(D)} f_2(x)$

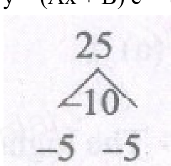
$=\frac{1}{(D-5)^2}5 = \frac{5.e^{0x}}{(D-5)^2} = \frac{5.e^{0x}}{(0-5)^2}$

$=\frac{5}{25} = \frac{1}{5}$

$\therefore y = CF + PI_1 + PI2$

$\therefore$ The general solution is

$y = (Ax + B) e^{5x} + 2x^2 e^{5x} + 5$



36) Solve the following differential equations $(3D^2 + D - 14)y=13e^{2x}$

The auxiliary equation is

$3m^2 + m - 14 = 0$

$(m-2)\frac{7}{3} = 0$

$\Rightarrow m=2, -\frac{7}{3}$

The roots are real and different

$\therefore$ CF is $Ae^{2x} + Be^{-\frac{7}{3}x}$

Particular Integral PI$=\frac{1}{\phi(D)} .f(x)$

PI=$\frac{1}{(3D^2+D-14)}$ .13 e2x

$=\frac{13.e^{2x}}{3D^2+D-14}$

$=\frac{13e^{2x}}{(D-2)(3D+7)} = \frac{13e^{2x}}{3(D-2)\left(D+\frac{7}{3}\right)}$

$=\frac{13xe^{2x}}{3\left(2+\frac{7}{3}\right)}$

$=\frac{13x\cdot e^{2x}}{3\left(\frac{13}{3}\right)} = x\,e^{2x}$

∴ y= CF + PI

∴ The general solution is

y=Ae^{2x}+$Be^{-\frac{7}{3}x}$ +xe^{2x}

−42
1
−6 7
$-\frac{6}{3}$ $\frac{7}{3}$
−2 $\frac{7}{3}$
(x−2) (3x + 7)

37) Solve: $\frac{dy}{dx}$ + e^x+ye^x = 0

⇒ $\frac{dy}{dx}$ =-e^x(1+y)

Separating the variables we get,

⇒ $\frac{dy}{1+y}$ =-e^x dx

⇒ log(1+y) =-e^x+c

38) Solve: $\log\left(\frac{dy}{dx}\right)$ = ax + by

Given $\log\left(\frac{dy}{dx}\right)$ = ax + by

⇒ $\frac{dy}{dx}$ =e^{ax+by}

[Since logarithmic & exponential are reversible functions]

⇒ $\frac{dy}{dx}$ =e^{ax} x e^{by} [∵ a^m x a^n =a^{m+n}]

Separating the variables we get,

$\frac{dy}{e^{-by}}$ =eax dx

⇒ e-by dy =eax dx

Integrating both sides we get,

$\int e^{-by} = \int e^{ax}$ dx

⇒ $\frac{e^{-by}}{-b} = \frac{e^{ax}}{a}$ +c

$\Rightarrow \frac{-e^{-by}}{b} = \frac{e^{ax}}{a}$ +c

$\Rightarrow \frac{e^{ax}}{a} = \frac{e^{-by}}{b}$ +c

39) If $y = x^3 - x^2 + x - 1$ calculate the values of y for x = 0,1,2,3,4,5 and form the forward differences table.

when x = 0, y = 0 + 0 + 0 - 1 ⇒ Y = -1

when x = 1, $y = 1^3 - 1^2 + 1 - 1$ ⇒ y=0.

when x = 2, $y = 2^3 - 2^2 + 2 - 1$ ⇒ y = 8 - 4 + 1 ⇒ y=5

when x = 3, $y = 3^3 - 3^2 + 3 - 1$ ⇒ y = 27 - 9 + 2 ⇒ y=20

when x = 4, $y = 4^3 - 4^2 + 4 - 1$ ⇒ y= 64 -16 + 3 ⇒ y= 51

when x = 5, $y = 5^3 - 5^2 + 5 - 1$ ⇒ y = 125 - 25 + 4 ⇒ y=104.

Hence, the forward difference table is

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
0	-1					
1	0	1	4	6		
2	5	5	10	6	0	
3	20	15	16	6	0	0
4	51	31	22			
5	104	53				

40) Evaluate $\Delta\left[\frac{1}{(x+1)(x+2)}\right]$ by taking '1' as the interval of differencing

By partial fraction method

$$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$\Rightarrow \frac{1}{(x+1)(x+2)} = \frac{A(x+2)+B(x+1)}{(x+1)(x+2)}$$

⇒ 1 = A (x + 2) + B (x + 1)

when x = -1, 1 = A [-1 + 2] ⇒ 1 = A

when x = -2, 1 =B [-2+ 1] ⇒1 = - B

⇒ B = -1.

$$\therefore \frac{1}{(x+1)(x+2)} = \frac{1}{x+1} + \frac{1}{x+2}$$

$$\therefore \triangle\left[\frac{1}{(x+1)(x+2)}\right] = \triangle\left[\frac{1}{x+1} - \frac{1}{x+2}\right]$$

$$\therefore \left(\frac{1}{x+1+1} - \frac{1}{x+1}\right) - \left(\frac{1}{x+1+2} - \frac{1}{x+2}\right)$$ [∵ Δ f(x) = f(x+1) - f(x)]

$$= \left(\frac{1}{x+2} - \frac{1}{x+1}\right) - \left(\frac{1}{x+3} - \frac{1}{x+2}\right)$$ where h = 1

$$= \frac{1}{x+2} - \frac{1}{x+1} - \frac{1}{x+3} + \frac{1}{x+2}$$

$$= \frac{2}{x+2} - \frac{1}{x+1} - \frac{1}{1+3}$$

$$= \frac{2(x+1)(x+3) - 1(x+2)(x+3) - 1(x+1)(x+2)}{(x+1)(x+2)(x+3)}$$

$$= \frac{2(x^2+4x+3) - (x^2+5x+6) - (x^2+3x-2)}{(x+1)(x+2)(x+3)}$$

$$= \frac{-2}{(x+1)(x+2)(x+3)}$$

$$\therefore = \triangle\left[\frac{1}{(x+1)(x+2)}\right] = \frac{-2}{(x+1)(x+2)(x+3)}$$

41)

Show that the equations x−4y+7z=14,3x+8y−2z=13,7x−8y+26z =5 are inconsistent.

The matrix equation corresponding to the given system is

$$\begin{pmatrix}1 & -4 & 7\\ 3 & 8 & -2\\ 7 & -8 & 26\end{pmatrix}\begin{pmatrix}X\\ Y\\ Z\end{pmatrix}=\begin{pmatrix}14\\ 13\\ 5\end{pmatrix}$$

AX=B

Augmented matrix [A,B]	Elementary Transformation
$\begin{pmatrix}1 & -4 & 7 & 14\\ 3 & 8 & -2 & 13\\ 7 & -8 & 26 & 5\end{pmatrix}$	
$\sim\begin{pmatrix}1 & -4 & 7 & 14\\ 0 & 20 & -23 & -29\\ 0 & 20 & -23 & -93\end{pmatrix}$	$R_2 \to R_2 - 3R_1$ $R_3 \to R_3 - 7R_1$
$\sim\begin{pmatrix}1 & -4 & 7 & 14\\ 0 & 20 & -23 & -29\\ 0 & 0 & 0 & 64\end{pmatrix}$	$R_3 \to R_3 - R_2$
$\rho(A) = 2, \rho([A, B]) = 3$	

The last equivalent matrix is in the echelon form. [A, B] has 3 non-zero rows and [A] has 2 non-zero rows.

$\therefore \rho([A, B]) = 3$,$\rho(A) = 2$,

$\rho(A) \neq ([A, B])$

The system is inconsistent and has no solution.

42) Evaluate $\int \frac{5+5e^{2x}}{e^x+e^{-x}}dx$

$\int \frac{5+5e^{2x}}{e^x+e^{-x}}dx = \int \frac{e^x\left(e^{-x}+e^x\right)}{e^x+e^{-x}}dx$

$= 5\int e^x dx$

$= 5e^x + c$

43) Evaluate $\int (logx)^2 dx$

$\int (logx)^2 dx \quad = \quad \int udv$

= uv − ∫ vdu

= x (log x)^2 − 2∫ logxdx...(*)

$= x(logx)^2 - 2\int udv$

$= x(logx)^2 - 2[uv - \int udv]$

$= x(logx)^2 - 2[xlogx - \int dx]$

$= x(logx)^2 - 2xlogx + x + c$

$= x[(log)^2 - logx^2 + 2] + c$

For ∫ log x dx in (*)	
Take u = (log x) Differentiate $du = \frac{1}{x}dx$	and dv = dx Integrate v = x

44) Sketch the graph $y = |x+3|$ and evaluate $\int_{-6}^{0} |x+3|$ dx.

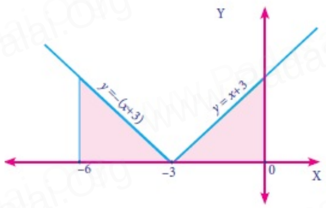
$$y = |x+3| = \begin{cases} x+3 & if \quad x \geq -3 \\ -(x+3) & if \quad x < -3 \end{cases}$$

Required area = $\int_b^a ydx = \int_{-6}^0 ydx$

$= \int_b^a ydx = \int_{-6}^0 ydx$

$= \int_{-6}^{-3}(x+3)dx + \int_{-3}^0 (x+3)dx$

$$= -\left[\frac{(x+3)^2}{2}\right]_{-6}^{-3} + \left[\frac{(x+3)^2}{2}\right]_{-3}^{0} = \left[0 - \frac{9}{2}\right] + \left[\frac{9}{2} - 0\right] = 9 \text{ sq. units}$$



45) Given $y_3 = 2$, $y_4 = -6$, $y_5 = 8$, $y_6 = 9$ and $y_7 = 17$ Calculate $\Delta^4 y_3$

Given $y_3 = 2$, $y_4 = -6$, $y_5 = 8$, $y_6 = 9$ and $y_7 = 17$

$\Delta^4 y_3 = (E-1)^4 y_3$

$= (E^4 - 4E^3 + 6E^2 - 4E+1)y_3$

$= E^4y_3 - 4E^3y_3 + 6E^2y_3 - 4Ey_3 + y_3$

$= y_7 - 4y_6 + 6y_5 - 4y_4 + y_3$

$= 17 - 4(9) + 6(8) - 4(-6) + 2$

$= 17 - 36 + 48 + 24 + 2 = 55$

5 x 5 = 25

46) The total cost of 11 pencils and 3 erasers is Rs 64 and the total cost of 8 pencils and 3 erasers is Rs 49. Find the cost of each pencil and each eraser by Cramer's rule.

Let 'x' be the cost of a pencil

Let 'y' be the cost of an eraser

∴ By given data, we get the following equations

11x+3y=64

8x+3y=49

$$\Delta = \begin{vmatrix} 11 & 3 \\ 8 & 3 \end{vmatrix} = 9 \neq 0, \quad \text{It has unique solution.}$$

$$\Delta_x \begin{vmatrix} 64 & 3 \\ 49 & 3 \end{vmatrix} = 45 \qquad \Delta_y \begin{vmatrix} 11 & 64 \\ 8 & 49 \end{vmatrix} = 27$$

∴ By Cramer's rule

$$x = \frac{\Delta x}{\Delta} = \frac{45}{9} = 5$$

$$y = \frac{\Delta y}{\Delta} = \frac{27}{9} = 3$$

∴ The cost of a pencil is Rs 5 and the cost of an eraser is Rs 3.

47) Evaluate $\int \frac{cos2x}{sin^2x cos^2x}dx$

$$\int \frac{cos2x}{sin^2x cos^2x}dx \quad = \int \left(cosec^2x - sec^2x\right)dx$$

$= -\cot x - \tan x + c$

[Change into simple integrands

$\frac{cos2x}{sin^2xcos^2x} = \frac{cos^2x - sin^2x}{sin^2xcos^2x} = \frac{1}{sin^2x} - \frac{1}{cos^2x} = \quad = cosec^2x - sec^2x$

48) Evaluate $\int \left[\frac{1}{log \quad x} - \frac{1}{(log \quad x)^2}\right]$ dx

$\int \left[\frac{1}{log \quad x} - \frac{1}{(log \quad x)^2}\right]dx = \int \left[\frac{1}{z} - \frac{1}{z^2}\right]e^z dx$

$= \int e^x$ [f(z) +f'(z)] dx

$= e^z$ f(z) + c

$= e^z \left[\frac{1}{z}\right] + c$

$= \frac{x}{log \quad x} + c$

Take z = log x
∴dz = $\frac{1}{x}$ dx
⇒dx e^x dz [∵ x = e^x]
and f(z) = $\frac{1}{z}$
∴ f'(z) = $-\frac{1}{z^2}$

49) Evaluate $\int_a^b \frac{\sqrt{logx}}{x} dx$ a,b > 0

$\int_a^b \frac{\sqrt{logx}}{x} dx = \int_a^b (logx)^{\frac{1}{2}} \frac{dx}{x}$ $\left[\because [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} \right] + c$

$= \left[2\frac{(logx)^{\frac{3}{2}}}{3}\right]_a^b$

$= \frac{2}{3}\left[(logb)^{\frac{3}{2}} - (loga)^{\frac{3}{2}}\right]$

50) Solve sec^2x tan y dx + sec^2y tan x dy = 0

Separating the variables, we get

$\frac{sec^x x}{tanx}dx + \frac{sec^2}{tany}dy = 0$

Integrating, we get

$\int \frac{sec^x x}{tanx}$ dx + $\int \frac{sec^2}{tany}dy$ = c

log tan x + log tan y = log c

log(tan x tan y) = log c

tan x tan y = c

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QUARTERLY IMPORTANT 5 MARKS WITH ANSWERS Date : 30-Aug-19

12th Standard

Business Maths

MY YOUTUBE CHANNEL NAME
SR MATHS TEST PAPERS
CHECK FOR MATHS VIDEOS AND MATERIALS

Total Marks : 125

4 x 2 = 8

1) The price of three commodities X,Y and Z are x,y and z respectively Mr.Anand purchases 6 units of Z and sells 2 units of X and 3 units of Y. Mr.Amar purchases a unit of Y and sells 3 units of X and 2units of Z. Mr.Amit purchases a unit of X and sells 3 units of Y and a unit of Z. In the process they earn `5,000/-, `2,000/- and `5,500/- respectively Find the prices per unit of three commodities by rank method.

Given that the price of commodities X, Y and Z are x, y and z respectively

By the given data

Transaction	x	y	z	Earning
Mr. Anand	+2	+3	-6	Rs.5000
Mr. Amar	+3	-1	+2	Rs.2000
Mr. Amit	-1	+3	+1	Rs.5500

Here, purchasing is taken as negative symbol and selling is taken as positive symbol

Thus, the non-homogeneous equations are

2x + 3y - 6z = 5000

3x - y + 2z = 2000

-x + 3y + z = 550

The matrix equation corresponding to the given system is

$$\begin{pmatrix} 2 & 3 & -6 \\ 3 & -1 & 2 \\ -1 & 3 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5000 \\ 2000 \\ 5500 \end{pmatrix}$$

Augmented matrix	Elementary Transformation
$\begin{pmatrix} 2 & 3 & -6 & 5000 \\ 3 & -1 & 2 & 2000 \\ -1 & 3 & 1 & 5500 \end{pmatrix}$	
$\begin{pmatrix} -1 & 3 & 1 & 5500 \\ 3 & -1 & 2 & 2000 \\ 2 & 3 & -6 & 5000 \end{pmatrix}$	R_1 ⊠ R_3
$\begin{pmatrix} 1 & -3 & -1 & -5000 \\ 3 & -1 & 2000 & 2000 \\ 2 & 3 & -6 & 5500 \end{pmatrix}$	$R_1 \rightarrow R_1\,(-1)$
$\begin{pmatrix} 1 & -3 & -1 & -5500 \\ 0 & 8 & 5 & 18500 \\ 0 & 9 & -4 & 16000 \end{pmatrix}$	$R_2 \rightarrow r_2 - 3R_1$ $R_3 \rightarrow R_3 - 2R_1$
$\begin{pmatrix} 1 & -3 & -1 & -5500 \\ 0 & 1 & \frac{65}{8} & \frac{18500}{8} \\ 0 & 1 & \frac{-4}{9} & \frac{16000}{9} \end{pmatrix}$	$R_2 \rightarrow R_2 \div 8$ $R_3 \rightarrow R_3 \div 9$
$\begin{pmatrix} 1 & -3 & -1 & -5500 \\ 0 & 1 & \frac{5}{8} & \frac{18500}{8} \\ 0 & 0 & \frac{-4}{9}-\frac{5}{8} & \frac{16000}{9}-\frac{18500}{8} \end{pmatrix}$	$R_3 \rightarrow R_3 - R_2$
$\begin{pmatrix} 1 & -3 & -1 & -5500 \\ 0 & 1 & \frac{5}{8} & \frac{18500}{8} \\ 0 & 0 & \frac{-77}{72} & \frac{-38500}{72} \end{pmatrix}$	

.Clearly the last equivalent matrix is in echelon form and it has three non-zero rows . . :. $\therefore \rho(A) = \rho([A, B]) = 3$ Number of unknowns. $\therefore$ The given system is consistent and has unique solution. To find the solution, let us rewrite the above : echelon form into the matrix form.

$$\begin{pmatrix} 1 & -3 & -1 \\ 0 & 1 & \frac{5}{8} \\ 0 & 0 & \frac{-77}{72} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{pmatrix} -5000 \\ \frac{18500}{8} \\ \frac{-38500}{72} \end{pmatrix}$$

$\Rightarrow x - 3y - z = -5500$

$y + \frac{5}{8}z = \frac{18500}{8}$

$\frac{-77}{72}z = \frac{38500}{72}$

$$(3) \Rightarrow \quad \frac{-77z}{\cancel{72}} = \frac{-38500}{\cancel{72}}$$

$\Rightarrow z = \frac{-38500}{-77}$

$\Rightarrow z = 500$

$(2) \Rightarrow y + \frac{5}{8}(500) = \frac{18500}{8} \quad y = \frac{18500}{8} - \frac{2500}{8}$

$= \frac{\cancel{16000}^{2000}}{\cancel{8}}$ $\Rightarrow y = 2000$

$(1) \Rightarrow x - 3\,(2000) - 500 = -5500$

$\Rightarrow x - 6000 - 500 = -5500$

$\Rightarrow x - 6000 - 500 = -5500$

$\Rightarrow x = -5500 + 6500$

$\Rightarrow x = 10000$

Hence, the prices per unit of three commodities are Rs1000, Rs 2000 and Rs500 respectively

2) A new transit system has just gone into operation in Chennai. Of those who use the transit system this year, 30% will switch over to using metro train next year and 70% will continue to use the transit system. Of those who use metro train this year, 70% will continue to use metro train next year and 30% will switch over to the transit system. Suppose the population of Chennai city remains constant and that 60% of the commuters use the transit system and 40% of the commuters use metro train this year.

(i) What percent of commuters will be using the transit system after one year?

(ii) What percent of commuters will be using the transit system in the long run?

Transition probability matrix

$$T = \begin{matrix} & \begin{matrix} A & B \end{matrix} \\ \begin{matrix} A \\ B \end{matrix} & \begin{pmatrix} \cdot 70 & \cdot 30 \\ \cdot 30 & \cdot 70 \end{pmatrix} \end{matrix}$$

Where A represents the percentage of people using transit system and B represents the percentage of people using metro train.

By the.given data

A 60% = ·60

andB 40% =·4

= ((-6)(-7)+(-4)(-3) (-6)(-3)+(-4)(-7))

= (-42+·12 ·18+·28)

= (-54 -46)

$\therefore$ A = 54%and B = 46%

(i) The percent of Commuters using the transit system after one year is 54% and the percent of commuters using the metro train after one year is 46%

(ii) Equilibrium will be reached in the long run. At equilibrium we must have

(A B) T = (A B)

where A+B = 1

$\Rightarrow (A \quad B)\begin{pmatrix} \cdot 7 & \cdot 3 \\ \cdot 3 & \cdot 7 \end{pmatrix} = (A \quad B)$

(-7A + ·3B ·3A +.7B) = (A B)

Equaling the entries on both sides, we get

·7A+ ·3B = A

$\Rightarrow \cdot 7A + \cdot 3(1 - A) = A$

$[\because A + B = 1 \Rightarrow B = 1 - A] \;\; \Rightarrow \cdot 7A + \cdot 3(1 - A) = A$

$\Rightarrow \cdot 3 = A - \cdot 7A + \cdot 3A$

$\Rightarrow \cdot 3 = A(\cdot 3 + \cdot 3)$

$\Rightarrow 3 = A(\cdot 6)$

$\Rightarrow A = \frac{\cdot 3}{\cdot 6} = \frac{1}{2} = \cdot 50$

∴ The percent of commuters using the transit system in the long run is 50%

3) A salesman has the following record of sales during three months for three items A,B and C, which have different rates of commission.

Months	Sales of units			Total commission drawn (in Rs)
	A	B	C	
January	90	100	20	800
February	130	50	40	900
March	60	100	30	850

Let the rate of commission on the items A, B and C be x, y and z respectively.

By the given data, the non-homogeneous equations are

90x + 100y + 20z = 800

⇒ 9x + 10y +2z = 80

130x+50y+40z=900

⇒ 13x+5y+4z=90

60x + 100y + 30z = 850

⇒ 6x + 10y + 3z = 85

$\Delta = \begin{vmatrix} 9 & 10 & 2 \\ 13 & 5 & 4 \\ 6 & 10 & 3 \end{vmatrix}$

$9\begin{vmatrix} 5 & 4 \\ 10 & 3 \end{vmatrix} - 10\begin{vmatrix} 13 & 4 \\ 6 & 3 \end{vmatrix} + 2\begin{vmatrix} 13 & 5 \\ 6 & 10 \end{vmatrix}$

= 9 (15 - 40) - 10 (39 - 24) + 2(130 - 30)

= 9 (- 25) - 10(15) + 2(100)

= - 225 - 150 + 200

= -175

Since $\Delta \neq 0$ Cramer's rule can be applied and the system is consistent with unique solution.

$\Delta x = \begin{vmatrix} 80 & 10 & 2 \\ 90 & 5 & 4 \\ 85 & 10 & 3 \end{vmatrix}$

$= 80\begin{vmatrix} 5 & 4 \\ 10 & 3 \end{vmatrix} - 10\begin{vmatrix} 90 & 4 \\ 85 & 3 \end{vmatrix} + 2\begin{vmatrix} 90 & 5 \\ 85 & 10 \end{vmatrix}$

= 80(15 - 40) - 10(270 - 340) + 2(900 - 425)

= 80 (- 25) - 10 (- 70) + 2 (475)

= - 2000 + 700 + 950

= -350

$\Delta y = \begin{vmatrix} 9 & 80 & 2 \\ 13 & 90 & 4 \\ 6 & 85 & 3 \end{vmatrix}$

$= 9\begin{vmatrix} 90 & 4 \\ 85 & 3 \end{vmatrix} - 80\begin{vmatrix} 13 & 4 \\ 6 & 3 \end{vmatrix} + 2\begin{vmatrix} 13 & 90 \\ 6 & 85 \end{vmatrix}$

= 9(270 - 340) - 80(39 - 24) +2(1105 - 540)

= 9(- 70) - 80(15) + 2(565)

= - 630 - 1200 + 1130

= -700

$$\Delta z = \begin{vmatrix} 9 & 10 & 80 \\ 13 & 5 & 90 \\ 6 & 10 & 85 \end{vmatrix}$$

= $9\begin{vmatrix} 5 & 90 \\ 10 & 85 \end{vmatrix} - 10\begin{vmatrix} 13 & 90 \\ 6 & 85 \end{vmatrix} + 80\begin{vmatrix} 13 & 5 \\ 6 & 10 \end{vmatrix}$

= 9(425 - 900) - 10(1105 - 540) + 80(130 - 30)

= 9(- 475) - 10(565) + 80 (100)

= - 4275 - 5650 + 8000

= - 1925

$$x = \frac{\Delta x}{\Delta} = \frac{-350}{-175} = 2$$

$$y = \frac{\Delta y}{\Delta} = \frac{-700}{-175} = 4$$

$$z = \frac{\Delta z}{\Delta} = \frac{-1925}{-175} = 11$$

∴ The rate of commission on the items A, Band Care 2%, 4% and 11%

4) In an examination the number of candidates who secured marks between certain interval were as follows

Marks	0-19	20-39	40-59	60-79	80-99
No.of.candidates	41	62	65	50	17

Given

Marks	0-19	20-39	40-59	60-79	80-99
No.of.candidates	41	62	65	50	17

This can be rewritten as

Marks:	**Below 19**	**Below 39**	**Below 59**	**Below 79**	**Below 99**
No. of. candidates:	41	(41 + 62) =103	(41 + 62 + 65) = 168	(41 + 62 + 65 + 50) =218	(41 + 62 + 65 + 50 + 17) = 235

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
Below 19	41				
Below 39	103	62	3	-18	0
Below 59	168	65	-15	-18	
Below 79	218	50	-33		
Below 99	235	17			

Since we have to find below 70, use Newton's backward interpolation formula

∴ xn+nh = 70 ⇒ 99 + n(20) = 70

⇒ 20n = 70 - 99 = -29

⇒ n = $\frac{-29}{20}$ = -1.45

∴ $y_{70} = y_n + \frac{n}{1!}\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n$

$= 235 - 1.45(17) + \frac{(-1.45)(-1.45+1)}{2}(-33) + \frac{(-1.45)(-1.45+2)(-18)}{6}$

$= 235 - 24.65 + \frac{(-1.45)(-.45)}{2}(-33) +$ (-1.45)(-0.45)(0.55)(-3)

= 235 - 24.65 - 10.76625 - 1.076625

= 235 - 36.49

= 198.5

Hence, the number of students who have scored below 70 are 198 (app).

7 x 3 = 21

5) Show that the equations 2x+y+z=5,x+y+z=4,x−y+2z = 1 are consistent and hence solve them.

The matrix equation corresponding to the given system is

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{pmatrix}\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix}$$

A X=B

Augmented matrix [A,B]	Elementary Transformation
$\begin{pmatrix} 2 & 1 & 1\,5 \\ 1 & 1 & 1\,4 \\ 1 & -1 & 2\,1 \end{pmatrix}$ $\sim \begin{pmatrix} 1 & 1 & 1\,4 \\ 2 & 1 & 1\,5 \\ 1 & -1 & 2\,1 \end{pmatrix}$ $\begin{pmatrix} 1 & 1 & 1 & 4 \\ 0 & -1 & -1 & -3 \\ 1 & -2 & 1 & -3 \end{pmatrix}$ $\sim \begin{pmatrix} 1 & 1 & 1 & 4 \\ 2 & -1 & -1 & -3 \\ 0 & 0 & 3 & 3 \end{pmatrix}$	$R_1 \leftrightarrow R_2$ $R_2 \to R_2 - 2R_1$ $R_3 \to R_3 - R_1$ $R_3 \to R_3 - 2R_2$
$\rho(A) = 3,\ \rho([A, B]) = 3$	

Obviously the last equivalent matrix is in the echelon form. It has three non-zero rows.

$\rho(A) = 3,\ \rho([A, B]) = 3$ = Number of unknowns .

The given system is consistent and has unique solution.

To find the solution, let us rewrite the above echelon form into the matrix form.

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 3 \end{pmatrix}$$

x+y+z=4 (1)

y+z=3 (2)

3z=3 (3)

$(3) \Rightarrow z = 1$

$(2) \Rightarrow y = 3 - z = 2$

$(1) \Rightarrow x = 4 - y - z$

x=1

$\therefore$ x=1, y=2,z=1

6) Show that the equations x+y+z=6,x+2y+3z=14,x+4y+7z =30 are consistent and solve them.

The matrix equation corresponding to the given system is

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 6 \\ 14 \\ 30 \end{pmatrix}$$

A X=B

Augmented matrix [A,B]	Elementary Transformation
$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{pmatrix}$ $\sim \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 2 & 4 & 16 \end{pmatrix}$ $\sim \begin{pmatrix} 1 & 1 & 1\,6 \\ 0 & 1 & 2\,8 \\ 0 & 0 & 0\,0 \end{pmatrix}$	$R_2 \to R_2 - R_1$ $R_3 \to R_3 - R_2$ $R_3 \to R_3 - 2R_2$
$\rho(A) = 2, \rho([A, B]) = 2$	

Obviously the last equivalent matrix is in the echelon form. It has two non-zero rows.

$\rho(A) = 2, \rho([A, B]) = 2$

$\rho(A) = 2, \rho([A, B]) = 2$ < Number of unknowns.

The given system is consistent and has infinitely many solutions.

The given system is equivalent to the matrix equation,

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix}$$

x+y+z=6 (1)

y+2z=8 (2)

(2) $\Rightarrow$ Y=8-2Z,

(2) $\Rightarrow$ X=6-Y-Z=6-(8-2z)-Z=z-2

Let us take z=k,k$\in$ R, we get x=k−2,y=8−2k, Thus by giving different values for k we get different solutions. Hence the given system has infinitely many solutions.

7) Investigate for what values of 'a' and 'b' the following system of equations x+y+z=6,x+2y+3z=10, x+2y+az = b have

(i) no solution

(ii) a unique solution

(iii) an infinite number of solutions.

The matrix equation corresponding to the given system is

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & a \end{pmatrix}\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ b \end{pmatrix}$$

AX=B

Augmented matrix [A,B]	Elementary Transformation
$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & a & b \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & a-1 & b-6 \end{pmatrix}$	$R_2 \rightarrow R_2 - R_1$ $R_3 \rightarrow R_3 - R_1$
$\sim \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & a-3 & b-10 \end{pmatrix}$	

Case (i) For no solution:

The system possesses no solution only when $\rho(A) \neq ([A,B])$ which is possible only whena−3=0 andb−10$\neq$ 0.

Hence for a=3,b$\neq$ 10 , the system possesses no solution.

Case (ii) For a unique solution:

The system possesses a unique solution only when $\rho(A) = ([A,B])$ =number of unknowns.

i.e when$\rho(A) = \rho([A,B])$ =3

Which is possible only when a−3$\neq$ 0 and b may be any real number as we can observe .

Hence for a$\neq$ and b $\in$ R, the system possesses a unique solution.

Case (iii) For an infinite number of solutions:

The system possesses an infinite number of solutions only when

$\rho(A) = \rho([A,B])$ < number of unknowns

i,e when $\rho(A) = \rho([A,B])$ =2< 3(number of unknowns) which is possible only when a−3=0,b−10=0

Hence for a = 3, b =10, the system possesses infinite number of solutions.

8) The total number of units produced (P) is a linear function of amount of over times in labour (in hours) (l), amount of additional machine time (m) and fixed finishing time (a)

i.e, P = a + bl + cm

From the data given below, find the values of constants a, b and c

Day	Production (in Units P)	Labour (in Hrs l)	Additional Machine Time (in Hrs m)
Monday	6,950	40	10
Tuesday	6,725	35	9
Wednesday	7,100	40	12

Estimate the production when overtime in labour is 50 hrs and additional machine time is 15 hrs.

We have, P = a + bl + cm

Putting above values we have

6,950 = a + 40b + 10c

6,725 = a + 35b + 9c

7,100 = a + 40b + 12c

The Matrix equation corresponding to the given system is

$$\begin{pmatrix} 1 & 40 & 10 \\ 1 & 35 & 9 \\ 1 & 40 & 12 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 6950 \\ 6725 \\ 7100 \end{pmatrix}$$

Augmented matrix [A,B]	Elementary Transformation
$\begin{pmatrix} 1 & 40 & 10 & 6950 \\ 1 & 35 & 9 & 6725 \\ 1 & 40 & 12 & 7100 \end{pmatrix}$ $\begin{pmatrix} 1 & 40 & 10 & 6950 \\ 0 & -5 & -1 & -225 \\ 0 & 0 & 2 & 150 \end{pmatrix}$	$R_2 \to R_2 - R_1$ $R_3 \to R_3 - R_1$
$\rho(A) = 3, \rho([A,B]) = 3$	

$\therefore$ The given system is equivalent to the matrix equation

$$\begin{pmatrix} 1 & 40 & 10 \\ 0 & -5 & -1 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 6950 \\ -225 \\ 150 \end{pmatrix}$$

a+40b+10c=6950 (1)

-5b-c=-225 (2)

2c=150 (3)

c-=75

Now, (2)⇒ -5b-75=-225

b=30

and (1)⇒ a+1200+750=6950

a = 5000

a = 5000, b = 30, c = 75

$\therefore$ The production equation is P = 5000 + 30l + 75m

$\therefore$ $P_{at\ l = 50,\ m=15}$ = 5000 + 30(50) + 75(15)

= 7625 units.

$\therefore$ The production = 7,625 units.

9) 80% of students who do maths work during one study period, will do the maths work at the next study period. 30% of students who do english work during one study period, will do the english work at the next study period.

Initially there were 60 students do maths work and 40 students do english work.

Calculate,

(i) The transition probability matrix

(ii) The number of students who do maths work, english work for the next subsequent 2 study periods.

(i) Transition probability matrix T= $\begin{matrix} M \\ E \end{matrix} \overset{\begin{matrix} M & E \end{matrix}}{\begin{pmatrix} 0.8 & 0.2 \\ 0.7 & 0.3 \end{pmatrix}}$

After one study period, $\overset{\begin{matrix} M & E \end{matrix}}{\begin{pmatrix} 60 & 40 \end{pmatrix}}$ $\begin{matrix} M \\ E \end{matrix} \overset{\begin{matrix} M & E \end{matrix}}{\begin{pmatrix} 0.8 & 0.2 \\ 0.7 & 0.3 \end{pmatrix}}$ = $\overset{\begin{matrix} M & E \end{matrix}}{\begin{pmatrix} 76 & 24 \end{pmatrix}}$

So in the very next study period, there will be 76 students do maths work and 24 students do the English work.

After two study periods, $\begin{matrix} M \\ E \end{matrix} \overset{\begin{matrix} M & E \end{matrix}}{\begin{pmatrix} 0.8 & 0.2 \\ 0.7 & 0.3 \end{pmatrix}}$ =(60.8+16.8 15.2+7.2)

=(77.6 22.4)

After two study periods there will be 78 (approx) students do maths work and 22 (approx) students do English work.

10) The demand and supply function of a commodity are $p_d = 18 - 2x - x_2$ and $p_s = 2x - 3$. Find the consumer's surplus and producer's surplus at equilibrium price.

Given $P_d = 18 - 2x - x^2$; $P_s = 2x - 3$

We know that at equilibrium prices $p_d = p_s$

$18 - 2x - x^2 = 2x - 3$

$x^2 + 4x - 21 = 0$

$(x - 3)(x + 7) = 0$

$x = -7$ or 3

The value of x cannot be negative, $x = 3$

When $x_0 = 3$

ஃ $p_0 = 18 - 2(3) - (3)^2 = 3$

$CS = \int_0^{x_o} f(x)\ dx - x_0p_0$

$= \int_0^3 (18 - 2x - x^2)\ dx - 3 \times 3$

$= \left[18x - x^2 - \frac{x^3}{3}\right]_0^3 - 9$

$= 18(3) - (3)^2 - \left(\frac{3^3}{3}\right) - 9$

CS = 27 units

$PS = x_0P_0 - \int_0^{x_o} g(x)$

$= (3 \times 3) - \int_0^3 (2x - 3)$

$= 9 - (x^2 - 3x)_0^3$

= 9 units

Hence at equilibrium price,

(i) the consumer's surplus is 27 units

(ii) the producer's surplus is 9 units.

11) Estimate the production for 1964 and 1966 from the following data

Year	1961	1962	1963	1964	1965	1966	1967
Production	200	220	260	-	350	-	430

Since five values are given, the polynomial which fits the data is of degree four.

Hence $\Delta^5 y_k = 0$ (i.e) $(E-1)^5 y_k = 0$

i.e., $(E^5 - 5E^4 + 10E^3 - 10E^2 + 5E - 1)y_k = 0$

$E^5y_k - 5E^4y_k + 10E^3y_k - 10E^2y_k + 5Ey_k - y_k = 0$ (1)

Put k = 0 in (1)

$E^5y_0 - 5E^4y_0 + 10E^3y_0 - 10E^2y_0 + 5Ey_0 - y_0$

$y_5 - 5y_4 + 10y_3 - 10y_2 + 5y_1 - y_0 = 0$

$y_5 - 5(350) + 10y_3 - 10(260) + 5(220) - 200 = 0$

$y_5 + 10y_3 = 3450$ (2)

1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Put k = 1 in (1)

$E^5y_1 - 5E^4y_1 + 10E^3y_1 - 10E^2y_1 + 5Ey_1 - y_0 = 0$

$y_6 - 5y_5 + 10y_4 - 1\ 0y_3 - y_1 = 0$

$430 - 5y_5 + 10(350) - 10y_3 + 5(260) - 220 = 0$

$5y_5 + 10y_3 = 5010$

(3) – (2) ⇒ $4y_5 = 1560$

$y_5 = 390$

From $390 + 10y_3 = 3450$

$10y_3 = 3450 - 390$

$y_3 \cong 306$

14 x 5 = 70

12) Evaluate $\int \frac{3x+2}{(x-2)^2(x-3)} dx$

$\int \frac{3x+2}{(x-2)^2(x-3)} dx = \int \left[\frac{11}{(x-2)} - \frac{8}{(x-2)^2} + \frac{11}{(x-3)}\right] dx$

$= 11\int \frac{dx}{(x-2)} - 8\int \frac{dx}{(x-2)^2} + 11\int \frac{dx}{(x-3)}$

$= 11log|x-2| + \frac{8}{x-2} + 11log|x-3| + c \quad = 11log\left|\frac{x-3}{x-2}\right| + \frac{8}{x-2} + c$

[By partial fractions,
$\frac{3x+2}{(x-2)^2(x-3)} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{C}{(x-3)} \Rightarrow \frac{3x+2}{(x-2)^2(x-3)} = \frac{8}{(x-2)^2} + \frac{11}{(x-3)}$]

13) Evaluate $\int \frac{3x^2+6x+1}{(x+3)(x^2+1)}dx$

$\int \frac{3x^2+6x+1}{(x+3)(x^2+1)}dx \quad = \int \left[\frac{1}{(x+3)}\frac{2x}{(x^2+1)}\right]dx$

$\int \frac{dx}{(x+3)} + \int \frac{2x}{(x^2+1)}dx$

$= log|x+3| + log|x^2+1| + c$

$= log|(x+3)(x^2+1)| + c$

$= log|x^3+3x^2+x+3| + c$

[By partial fractions,
$\frac{3x^2+6x+1}{(x+3)(x^2+1)} = \frac{A}{(x+3)} + \frac{Bx+C}{(x^2+1)} \Rightarrow \frac{3x^2+6x+1}{(x+3)(x^2+1)} = \frac{1}{(x+3)} + \frac{2x}{(x^2+1)}$

14) If f(x) = $\begin{cases} x^2, & -2 \le x < 1 \\ x, & 1 \le x < 2 \\ x-4, & 2 \le x \le 4 \end{cases}$, then find the following

(i) $\int_{-2}^{1} f(x)dx$

(ii) $\int_{-2}^{1} f(x)dx$

(iii) $\int_{2}^{3} f(x)dx$

(iv) $\int_{-2}^{1.5} f(x)dx$

(v) $\int_{1}^{3} f(x)dx$

(i) $\int_{-2}^{1} f(x)dx = \int_{-2}^{1} x^2dx = \left[\frac{x^3}{3}\right]_{-2}^{1} = \frac{1}{3} - \left(\frac{-8}{3}\right) = 3$

(ii) $\int_{1}^{2} f(x)dx = \int_{1}^{2} xdx = \left[\frac{x^2}{2}\right]_{1}^{2} = \frac{4}{2} - \frac{1}{2} = \frac{3}{2}$

(iii) $\int_{2}^{3} f(x)dx = \int_{2}^{3} (x-4)dx\left[\frac{x^2}{2} - 4x\right]_{2}^{3} = \left(\frac{9}{2} - 12\right) - \left(\frac{4}{2} - 8\right) = \frac{15}{2} + 6 = \frac{-3}{2}$

(iv) $\int_{-2}^{1.5} f(x)dx = \int_{-2}^{1} f(x)dx + \int_{1}^{1.5} f(x)dx = 3 + \int_{1}^{1.5} xdx$ using (i)

$= 3 + \left[\frac{x^2}{2}\right]_{1}^{1.5} = 3 + \frac{2.25}{2} - \frac{1}{2} = 3 + \frac{1.25}{2} = 3.62$

(v) $\int_{1}^{3} f(x)dx = \int_{1}^{2} f(x)dx + \int_{2}^{3} f(x)dx$

$= \frac{3}{2} + \left(\frac{-3}{2}\right) = 0$ using (ii) and (iii)

15) A firm has the marginal revenue function given by MR = $\frac{a}{(x+b)^2}$ - c where x is the output and a, b, c are constants. Show that the demand function is given by $x = \frac{a}{b(p+c)} - b$.

MR = a $(x + b)^{-2}$ – c

$R = \int (x+b)^2$ dx - c $\int dx$

R = $\frac{a(x+b)^{-2}}{-1}$ - cx + k

R = $\frac{a}{(x+b)}$ - cx + k

When x = 0, R = 0

$0 = \frac{a}{b} - c(0) + k$

$k = \frac{a}{b}$

$R = -\frac{a}{x+b}$ - cx+$\frac{a}{b}$

$= \frac{-ab+a(x+b)}{b(x+b)} - cx \quad R = \frac{ax}{b(x+b)} - cx$

Demand function $P = \frac{R}{x}$

$P = \frac{a}{b(x+b)} - c$

$P + c = \frac{a}{b(x+b)}$

$b(x+b) = \frac{a}{P+c}$

$x = \frac{a}{b(P+c)} - b$

16) The marginal cost and marginal revenue with respect to commodity of a firm are given by C'(x) = 8 + 6x and R'(x)= 24. Find the total Profit given that the total cost at zero output is zero.

Given MC = 8 + 6x

$C(x) = \int (8+6x)dx$ + k_1

= 8x + $3x^2$ + k_1 (1)

But given when x = 0, C = 0 ⇒ k_1 = 0

∴ C(x) = 8x + $3x^2$ (2)

Given that MR = 24

R(x) = $\int MR$ dx + k_2

$= \int 24 + k_2$

Revenue = 0, when x = 0 ⇒ $k_2 = 0$

R(x) = 24x (3)

Total Profit functions P(x) = R(x) – C(x)

$P(x) = 24x - 8x - 3x^2$

$= 16x - 3x^2$

17) The elasticity of demand with respect to price p for a commodity is $\eta_d = \frac{p+2p^2}{100-p-p^2}$. Find demand function where price is Rs. 5 and the demand is 70.

$\eta_d = \frac{p+2p^2}{100-p-p^2}$

$\frac{-p}{x}\frac{dx}{dp} = \frac{p(2p+1)}{100-p-p^2}$

$\frac{-dx}{x} = \frac{-(2p+1)}{p^2+p-100}dp$

$\int \frac{dx}{x} = \int \frac{2p+1}{p^2+p-100}dp$

log x = log(p^2 + p = 100) + log k

ஃ x = k(p^2 + p −100)

When x = 70, p = 5,

70 = k(25 + 5 − 100)

⇒ k = –1

Hence x = 100 − p − p^2

R = px

Revenue = p(100 – p – p^2)

18) The demand and supply functions under pure competition are $P_d = 16 - x^2$ and $p_s = 2x^2 + 4$. Find the consumer's surplus and producer's surplus at the market equilibrium price.

For market equilibrium, $P_d = p_s$

⇒ $16-x^2=2x^2+4$

⇒ $16-4=2x^2+x^2$

⇒$3x^2=12$

⇒ $x^2=4$

⇒ $x=\pm 2$

Since x=2 is not possible $x_0=2$

$p_0=16-2^2=16-4=12$

∴ $p_0x_0=2x12=24$

Consumer's Surplus CS $\int_0^{x0} f(x)dx - p_0x_0$

$= \int_0^2 (16 - x^2)dx - 24$

$= \left[16x - \frac{x^3}{3}\right]_0^2 - 24$

$= 16(2) - \frac{2^3}{3} - 24$

$= 32 - \frac{8}{3} - 24$

$= 8 - \frac{8}{3} - 24$

$= \frac{16}{3}$ units

Producer's Surplus $PS = p_0x_0 - \int_0^{x0} g(x)dx$

$= 24 - \int_0^2 (2x^2 + 4)dx$

$= 24 - \left[\frac{2x^3}{3} + 4x\right]_0^2$

$= 24 - \left[\frac{16}{3} + 8\right]$

$= 24 - \frac{16}{3} - 8 = 16 - \frac{16}{3}$

$= \frac{48-16}{3} = \frac{32}{3}$ units

19) The demand and supply curves are given by $P_d = \frac{16}{x+4}$ and $P_s = \frac{x}{2}$. Find the Consumer's surplus and producer's surplus at the market equilibrium price.

For market equilibrium, $P_d = P_s$ $P_d = \frac{16}{x+4} = \frac{x}{2} \Rightarrow 32 = x(x+4)$

⇒ $32-x^2+4x$

⇒ $x^2+4x-32=0$

⇒ (x+8)(x-4)=0

⇒ x=-8, x=4

Since x = - 8 is not possible, $X_o = 4$

$\therefore p_0 = \frac{16}{4+4} = \frac{16}{8} = 2$

$p_0x_0 = 2(4) = 8$

Consumer's Surplus $CS = \int_0^{x0} f(x)dx - p_0x_0$

$Cs = \int_0^4 \frac{16}{x+4}dx - 8$

$= [16log(x+4)]_0^4 - 8$

$= 16\left[log(4+4) - log(0+4)\right] - 8$

$= 16[log \quad 8 - log4] - 8$

$= 16log\left(\frac{8}{4}\right) - 8$

CS=(16 log 2-8)units.

Producer's Surplus $PS = p_0x_0 - \int_0^{x0} g(x)dx$

$= 8 - \int_0^4 \frac{x}{2}dx = 8 - \left[\frac{x^2}{4}\right]_0^4$

$= 8 - \left[\frac{4^2}{4}\right] = 8 - \frac{16}{4} = 8 - 4$

PS=4units

20) Solve $ydx - xdy - 3x^2y^2e^{x3}dx = 0$

Given equation can be written as $\frac{ydx-xdy}{y^2} - 3x^2e^{x^2}dx = 0$

Integrating, $\int \frac{ydx-xdy}{y^2}$ - $\int 3x^2e^{x3}$ dx = c

$\int d\left(\frac{x}{y}\right)$ - $\int e^tdt$ = c (where $t=x^3$ and $dt = 3x^2dx$)

$\frac{x}{y}$ - e^t = c

$\frac{x}{y}$ - e^{x^2} = c

21) Solve $\frac{dy}{dx}$ −3ycotx=sin2x given that y = 2 when x = $\frac{\pi}{2}$

Given $\frac{dy}{dx}$ − (3 cot x).y= sin 2x

It is of the form $\frac{dy}{dx}$ + Py =Q

Here P= − 3 cot x,Q = sin 2x

$\int Pdx = \int -3 \cot xdx = -3 \log \sin x = - \log \sin^3x = \log \frac{1}{\sin^3x}$

I.F. = $e^{log\frac{1}{sin^3x}} = \frac{1}{sin^3x}$

The required solution is y (I.F) = $\int$Q(I.F)dx+c

$y\frac{1}{sin^3x} = \int sin2x\frac{1}{sin^3x}dx + c$

$\int \frac{1}{sin^3x} = \int 2sinxcosx \times \frac{1}{sin^3x}dx + c$

= $2\int \frac{1}{sinx} \times \frac{cosx}{sinx}dx + c$

= $\int$cos ecx cot xdx + c

$y\frac{1}{sin^3x} = -2cosecx + c$

Now y = 2 when x = $\frac{\pi}{2}$

(1) ⇒ $2\frac{1}{1}$ = −2×1+c ⇒ c= 4

∴ (1) ⇒ $y\frac{1}{sin^3x}$ = −2cosecx + 4

22) Using graphic method, find the value of y when x = 38 from the following data:

x	10	20	30	40	50	60
y	63	55	44	34	29	22

From the graph in Fig. 5.1 we find that for x = 38, the value of y is equal to 35

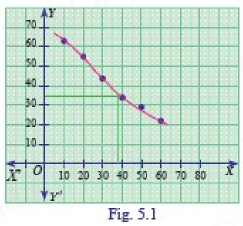

Fig. 5.1

(i) Take a suitable scale for the values of x and y, and plot the various points on the graph paper for given values of x and y.

(ii) Draw a suitable curve passing through the plotted points.

(iii) Find the point corresponding to the value x = 38 on the curve and then read the corresponding value of y on the y- axis, which will be the required interpolated valu

23) From the following table find the number of students who obtained marks less than 45.

Marks	30-40	40-50	50-60	60-70	70-80

No. of Students	31	42	51	35	31

Let x be the marks and y be the number of students

By converting the given series into cumulative frequency distribution, the difference table is as follows.

x	y	Δy	$\Delta^2 y$	$\Delta^2 y$	$\Delta^4 y$
Less than 40	31				
		42			
50	73		9		
		51		-25	
60	124		-16		
		35		12	
70	159		-4		
		31			
80	190				

$y_{(x=x_0+nh)} = y_0 + \frac{n}{n!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-)(n-2)}{3!}\Delta^3 y_0 \ldots$

To find y at x = 45 ∴ x_0+nh = 45 , x0=40,h=10⇒ $n = \frac{1}{2}$

$y_{(x=45)}$=$31 + \frac{1}{2} \times 42 + \frac{\frac{1}{2}\left(\frac{-1}{2}\right)}{2}(9) + \frac{\frac{1}{2}\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)}{6} \times (-25)$

$31 + \frac{1}{2} \times 42 + \frac{\frac{1}{2}\left(\frac{-1}{2}\right)}{2}(9) + \frac{\frac{1}{2}\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)}{6} \times (-25) + \frac{\frac{1}{2}\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)\left(\frac{-5}{2}\right)}{24} \times (-25)$

= $31 + 21 - \frac{9}{8} - \frac{25}{16} - \frac{37 \times 15}{128}$

= 47.867=48

24) The population of a certain town is as follows

Year : x	1941	1951	1961	1971	1981	1991
Population in lakhs:y	20	24	29	36	46	51

Using appropriate interpolation formula, estimate the population during the period 1946.

Solution:

x	1941	1951	1961	1971	1981	1991
y	20	24	29	36	46	51

$y_{(x=x_0+nh)} = y_0 + \frac{n}{n!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-)(n-2)}{3!}\Delta^3 y_0 + ..$

To find y at x = 1946 ∴ x_0+nh = 1946 , x_0=1941,h=10

1941+n(10) =1946⇒ n = 0.5

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1941	20					
		4				
1951	24		1			
		5		1		
1961	29		2		0	
		7		1		-9
1971	36		3		-9	
		10		-8		
1981	46		-5			
		5				
1991	51					

$y_{(x=1946)} = 20 + \frac{0.5}{1!}(4) + \frac{0.5(0.5-1)}{2!}(1) + \frac{0.5(0.5-1)(0.5-2)}{3!}(1) + \frac{0.5(0.5-1)(0.5-2)(0.5-3)}{4!}(0) + \frac{0.5(0.5-1)(0.5-2)}{5!}(-9)$

= 20+2-0.125+0.0625-0.24609

= 21.69 lakhs

25) From the following table of half- yearly premium for policies maturing at different ages. Estimate the premium for policies maturing at the age of 63.

Age	45	50	55	60	65
Premium	114.84	96.16	83.32	74.48	63.48

Let age = x and premium = y

To find y at x = 63. So apply Newton's backward interpolation formula

$$y_{(x=x_0+nh)} = y_0 + \frac{n}{n!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-)(n-2)}{3!}\Delta^3 y_0 + ..$$

x	y	Δy	$\Delta^2 y$	$\Delta^2 y$	$\Delta^4 y$
45	114.84				
		-18.68			
50	96.16		5.84		
		-12.84		-1.84	
55	83.32		4		0.68
		-8.84		-1.16	
60	74.48		2.84		
		-6			
65	68.48				

$$y_{(x=6.5)} = 68.48 + \frac{\frac{-2}{5}}{1!}(-6) + \frac{\frac{-2}{5}\left(\frac{-2}{5}+1\right)}{2!}2.84 + \frac{\frac{-2}{5}\left(\frac{-2}{5}+1\right)\left(\frac{-2}{5}+2\right)}{3!}(-1.16) + \frac{\frac{-2}{5}\left(\frac{-2}{5}+1\right)\left(\frac{-2}{5}+2\right)\left(\frac{-2}{5}+3\right)}{3!}(0.68)$$

= 68.48 + 2.4 – 0.3408 + 0.07424 – 0 – 0.028288

y(63) = 70.437