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11 th Mathematics study matrial 2018-19

Unit : 6 Two dimentional Analytical Geometry

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Topics:

1. Locus of a point
2. Stright Lines
3. Angle between two lines
4. Pair of stright lines

15- Aug 2018

Chapter - 6 Two dimensional Analytical Geometry.

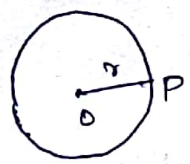
6.2. locus of a point:

Point:
A point is an exact position or location on a plance surface.

Locus: A path traced out by a moving point under a certain condition is called the locus of that point.

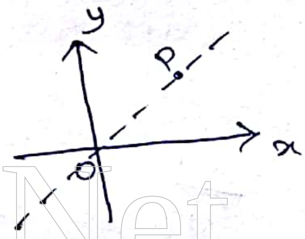
Example:

1. **Circle:**



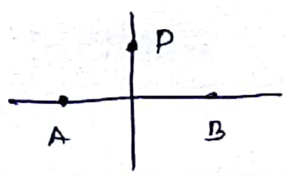
A point moves equal distance from a fixed point 'O'

2. **Angle bisector of the angle $\angle xoy$:**



A point P moves such that it is equidistance from two fixed lines ox & oy

3. **perpendicular bisector of the line segment AB:**



A point P moves such that Pt is equidistance from two fixed points A & B

Procedure for finding the equation of the locus of a point:

1. If we are finding the equation of the locus of a point 'P' assign coordinates say (h,k) by P.
2. Express the given condition as equation in terms of the known quantites and unknown parameters.
3. Eliminate the parameters.
4. Replace h by x, k by y in the resulting equation

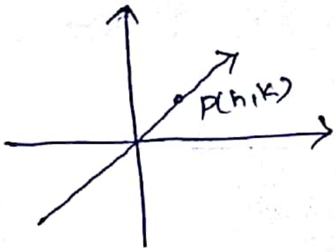
We get the locus of the point P.

Problems:

1. Find the locus of a point of a which moves such that the distance from the x axis is equal to the distance from the y axis.

Soln

Let $P(h,k)$ be the point on the locus



from the given data

$h = k \rightarrow (1)$

Replace h by x, k by y in (1)

$x = y$

it is the locus of the point 'P'

it is a line passing through origin.

Ex-2

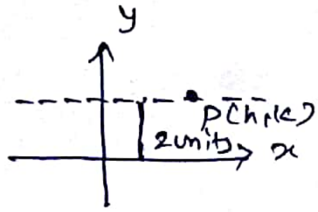
2. Find the locus of a point 'P' that moves at constance distance of

(i) two units from the x axis

(ii) 3 units from the y axis.

Soln

(i) Let $P(h,k)$ be a point on the locus



from the given data

$k = 2 \rightarrow (1)$

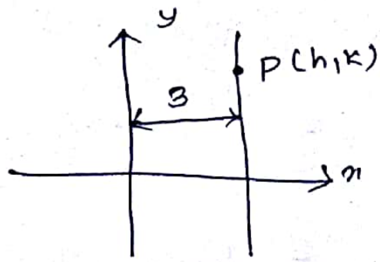
Replace k by y in (1)

$y = 2$

it is required locus of given pt 'P'

it is a line parallel to x axis.

(ii) Let $P(h,k)$ be a point on the locus



from the given data

$h = 3 \rightarrow (2)$

Put $h = x$ in (2)

$x = 3$

it is required equ. of locus of given pt

it is a line parallel to y axis.

3. Find the path traced out by the point $(ct, c/t)$ $t \neq 0$ is the parameter & c is a constant.

Soln

Let $p(h,k)$ be a point on the locus

from the given data

$h = ct$ | $k = c/t \rightarrow (1)$

$\frac{h}{c} = t$

Using this in (1)

$$k = \frac{c^2}{h}$$

$$hk = c^2$$

put $h = x$, $k = y$

$$\therefore xy = c^2$$

it is the required locus of the point 'P'

Ex-1

4 Find the locus of P, if all values of α the coordinates of moving point P is

(i) $(9\cos\alpha, 9\sin\alpha)$

(ii) $(9\cos\alpha, b\sin\alpha)$

Soln

(i) Let $p(h,k)$ be a point on the locus

from the given data

$h = 9\cos\alpha$ | $k = 9\sin\alpha$

$\frac{h}{9} = \cos\alpha \rightarrow (1)$ | $\frac{k}{9} = \sin\alpha$

$(1)^2 + (2) \Rightarrow$

$$\frac{h^2}{9^2} + \frac{k^2}{9^2} = \cos^9 + \sin^2\alpha$$

$$\frac{h^2}{9^2} + \frac{k^2}{9^2} = 1$$

$$h^2 + k^2 = 9^2$$

put $h = x$, $k = y$

$$x^2 + y^2 = 9^2$$

it is the required locus of P it is a circle centred at 'o'

(ii) Let $p(h,k)$ be the point on the locus.
from the given data

$$h = 9\cos\alpha \qquad k = 6\sin\alpha$$

$$\frac{h}{9} = \cos\alpha \rightarrow (1) \qquad \frac{k}{6} = \sin\alpha \rightarrow (2)$$

$(1)^2 + (2)^2 \Rightarrow$

$$\frac{h^2}{81} + \frac{k^2}{36} = \cos^2\alpha + \sin^2\alpha$$

$$\frac{h^2}{81} + \frac{k^2}{36} = 1$$

put $h = x$, $k = y$

$$\frac{x^2}{81} + \frac{y^2}{36} = 1$$ it is the required locus of 'p'.

5. If θ is a parameter, find the equ. of the locus of a moving point, whose coordinates are $(a\sec\theta, b\tan\theta)$

Soln
Let $P(h,k)$ be a point of the locus

$$\therefore h = a\sec\theta \qquad k = b\tan\theta$$

$$\frac{h}{a} = \sec\theta \rightarrow (1) \qquad \frac{k}{b} = \tan\theta \rightarrow (2)$$

$(1)^2 - (2)^2 \Rightarrow$

$$\frac{h^2}{a^2} - \frac{k^2}{b^2} = \sec^2\theta - \tan^2\theta$$

$$\frac{h^2}{a^2} - \frac{k^2}{b^2} = 1$$

put $h = x$, $k = y$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

it is the requ. locus of 'p'

Ex-3

6. If θ is a parameter, find the locus of a moving point, whose coordinates are $x = a\cos^3\theta$, $y = a\sin^3\theta$

Soln Let $p(h,k)$ be a point on the locus.

from the given data

$$h = a\cos^3\theta \qquad k = a\sin^3\theta$$

$$\frac{h}{a} = \cos^3\theta \qquad \frac{k}{a} = \sin^3\theta$$

$$\left(\frac{h}{a}\right)^{1/3} = \cos\theta \rightarrow ① \qquad \left(\frac{k}{a}\right)^{1/3} = \sin\theta \rightarrow ②$$

$①^2 + ②^2 \Rightarrow$

$$\left(\frac{h}{a}\right)^{2/3} + \left(\frac{k}{a}\right)^{2/3} = \cos^2\theta + \sin^2\theta$$

$$\left(\frac{h}{a}\right)^{2/3} + \left(\frac{k}{a}\right)^{2/3} = 1$$

$$h^{2/3} + k^{2/3} = a^{2/3}$$

put $h = x$, $k = y$

$$x^{2/3} + y^{2/3} = a^{2/3}$$

it is required locus of the point 'p'

Ex: 4

7. Find the value of k and b if the points $P(-3,1)$ and $Q(2,b)$ lie on the locus of $x^2 - 5x + ky = 0$.

Soln The point $P(-3,1)$ lies on the locus of $x^2 - 5x + ky = 0$

$$\therefore \quad 9 + 15 + k = 0$$

$$24 + k = 0$$

$$\boxed{k = -24}$$

$\therefore$ locus is $x^2 - 5x - 24y = 0$

$Q(2,b)$ is on $x^2 - 5x - 24y = 0$

$$\therefore \quad 4 - 10 - 24b = 0$$

$$-6 - 24b = 0$$

$$-24b = 6$$

$$b = \frac{-6}{24} = -\frac{1}{4}$$

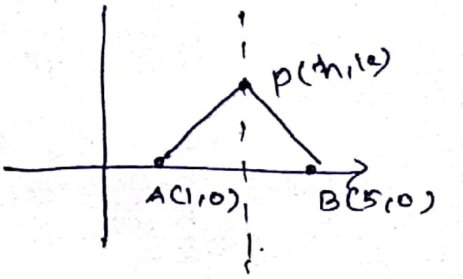
$\therefore$ $k = -24$

$b = -\frac{1}{4}$

8. Find the locus of a point P moves such that its distances from two fixed points A (1,0) & B (5,0), are always equal.

Soln

Let P(h,k) be the point on the locus.



From the given data

$$PA = PB$$

$$(PA)^2 = (PB)^2$$

$$(h-1)^2 + (k-0)^2 = (h-5)^2 + (k-0)^2$$

$$(h-1)^2 + k^2 = (h-5)^2 + k^2$$

$$(h-1)^2 = (h-5)^2$$

$$h^2 + 1 - 2h = h^2 + 25 - 10h$$

$$-24 = -8h$$

$$8h = 24$$

$$\boxed{h = 3}$$

PA = PB (given)

$PA = \sqrt{(h-1)^2 + (k-0)^2}$

$PB = \sqrt{(h-5)^2 + (k-0)^2}$

put $h = 3$

$\therefore \quad x = 3$

It is the required locus of P.

Ex-6

9 Find the equ. of the locus of the point P such that ~~the line segment AB, joining the points~~ the sum of the squares of the distance from the points A (3, 5) & B(~~4, 2~~ (1, −1)) is equal to 20.

Soln Let P(h,k) be any pt on the locus

From the given data

$$(PA)^2 + (PB)^2 = 20.$$

$$\{(h-3)^2 + (k-5)^2\} + \{(h-1)^2 + (k+1)^2\} = 20$$

$$h^2 + 9 - 6h + k^2 + 25 - 10k + h^2 + 1 - 2h + k^2 + 1 + 2k = 20$$

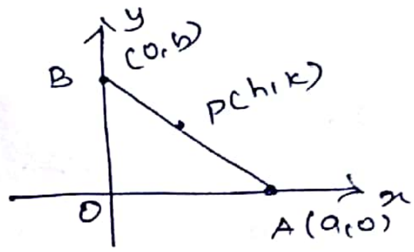
$$2h^2 + 2k^2 - 8h - 8k + 16 = 0$$

$$h^2 + k^2 - 4h - 4k + 8 = 0$$

$$x^2 + y^2 - 4x - 4y + 8 = 0$$ this circle is equ. locus.

10. A straight rod of length 8 units slides with its ends A & B always on the axis x & y axis res. Find the locus of the ~~mid~~ mid point of the line segment AB.

Soln



Let P(h,k) be a pt on the locus

Let the co-ordinates of A & B be A(a,0) & B(0,b)

From the data

P(h,k) is mid point of A & B.

$OA = a$
$OB = b$
$AB = 8$

$\therefore \quad h = \frac{a+0}{2}$ | $k = \frac{0+b}{2}$

$2h = a$ | $2k = b$

$\triangle$ OAB

$$OA^2 + OB^2 = AB^2$$
$$a^2 + b^2 = 8^2$$
$$4h^2 + 4k^2 = 8^2 = 64$$
$$h^2 + k^2 = 16$$

put $h = x$, $k = y$

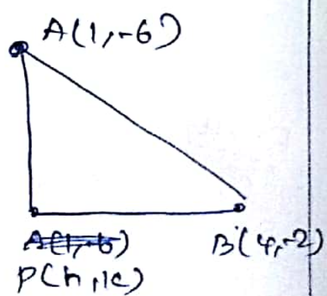
$$x^2 + y^2 = 16$$

it is the req. locus of the point P.

Ex-7

11. Find the equation of the locus of the point P such that the line segment AB, joining the points A(1,-6) & B(4,-2) subtends a right angle at P.

Soln



Let P(h,k) be a pt on the locus

from the given data & the ~~figur~~ picture

$$AB^2 = AP^2 + BP^2$$

$$(1-4)^2 + (-6+2)^2 = \left((h-1)^2 + (k+6)^2\right) + (h-4)^2 + (k+2)^2$$

$$9+16 = h^2 + 1 - 2h + k^2 + 36 + 12k + h^2 + 16 - 8h + k^2 + 4 + 4k$$

$$2h^2+2k^2-10h+16k+32=0$$

$\div 2$ $\quad h^2+k^2-5h+8k+16=0$

put $h=x$, $k=y$

$$x^2+y^2-5x+8y+16=0.$$

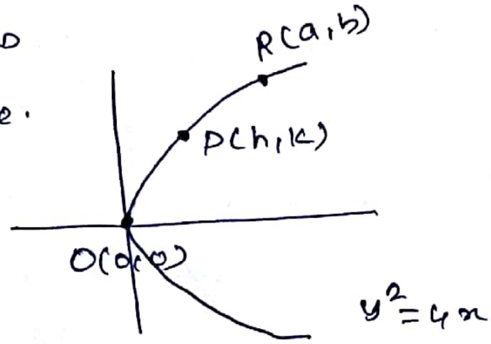
Ex 8

12. If O is origin and R is a variable pt on $y^2=4x$ then find the equ. of the locus of the mid-point of the line segment OR.

soln

Let p be a point on the locus

from the given data & picture.



P(h,k) is mid pt of R & O

$\therefore h=\dfrac{a+0}{2}$ | $k=\dfrac{0+b}{2}$

$2h=a$ | $2k=b$

R(a,b) is the pt on $y^2=4x$

$$b^2=4a$$

$\therefore$

$$4k^2=8h$$

$$k^2=2h$$

put $h=x$, $k=y$

$$y^2=2x$$

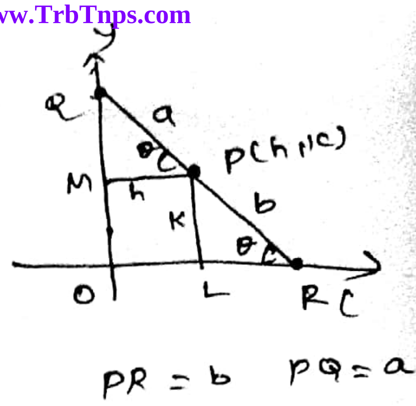
it is the required locus of the point P.

Ex 4

13 If R is any point on the x axis and Q is any point on the y axis and P is a variable point on RQ with RP = b, PQ = a then find the equ. of locus of P.

Soln

Let $P(h,k)$ be the point on RQ such that $\angle ORQ = \theta$



ΔPLR $\quad \sin\theta = \frac{k}{b}$

ΔPMR $\quad \cos\theta = \frac{h}{a}$

$\cos^2\theta + \sin^2\theta = 1$

$\frac{h^2}{a^2} + \frac{k^2}{b^2} = 1$

$\frac{h^2}{a^2} + \frac{k^2}{b^2} = 1$

put $h = x$, $k = y$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

It is the equ. locus of P.

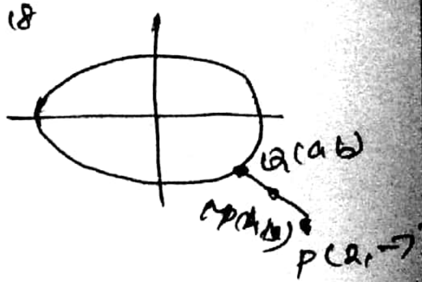
(ii)

Ex-10

14. If $P(2,-7)$ is a given point and Q is the point on $2x^2+9y^2=18$ then find the equ. of locus of the mid-point of PQ.

Soln

Let $Q(a,b)$ a point on the $2x^2+9y^2=18$



$M(h,k)$ be the mid point of PQ.

$\therefore h = \frac{a+2}{2}$ $\quad\Big|\quad$ $k = \frac{b-7}{2}$

$\boxed{2h+2 = a}$ $\quad\Big|\quad$ $\boxed{2k+7 = b}$

$Q(a,b)$ lies on

$2x^2 + 9y^2 = 18$

$2(2h-2)^2 + 9(2k+7)^2 = 18$

$2(4h^2+4-8h) + 9(4k^2+49+28k) = 18$

$$8h^2+36k^2-16h+252k+431=0.$$

put $h=x$, $k=y$

$$8x^2+36y^2-16x+252y+431=0$$

it is the requ. locus of the point M.

EX-15

15. The sum of the distance of moving point from the points (4,0) and (-4,0) is always 10 units. find the equ. of to the locus of the moving point.

Soln

Let $P(h,k)$ be a point on the locus.

$$\therefore \quad PA+PB = 10.$$

$$\sqrt{(h-4)^2+(k-0)^2}+\sqrt{(h+4)^2+(k-0)^2}=10$$

$$\sqrt{(h-4)^2+k^2} = 10-\sqrt{(h+4)^2+k^2}$$

squ.

$$(h-4)^2+k^2 = 100+(h+4)^2+k^2-20\sqrt{(h+4)^2+k^2}$$

$$\cancel{h^2}+\cancel{16}-8h+\cancel{k^2} = 100+\cancel{h^2}+\cancel{16}+8h+\cancel{k^2}-20\sqrt{h^2+16+8h+k^2}$$

$$-16h-100 = -20\sqrt{h^2+k^2+8h+16}$$

$\div 4$

$$4h+25 = 5\sqrt{h^2+k^2+8h+16}$$

squ.

$$16h^2+625+200h = 25(h^2+k^2+8h+16)$$

$$9h^2+25k^2=225$$

$\div 225$

$$\frac{h^2}{25}+\frac{k^2}{9}=1$$

put $h=x$, $k=y$

$$\frac{x^2}{25}+\frac{y^2}{9}=1$$

it is the locus of req. point.

Ex-13

16 If Q is the point on the locus of $x^2+y^2+4x-3y+7=0$ then find the equ. of locus of P which divides segment OQ externally in the ratio 3:4 where O is origin.

Soln

Let Q(a,b) be a point on $x^2+y^2+4x-3y+7=0$.

$$(x,y) = \left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$$

Let P(h,k) be a point on locus

Given P divides OQ externally in the ratio. 3:4

$$\therefore (h,k) = \left(\frac{3a-4(0)}{-1}, \frac{3b-4(0)}{-1}\right)$$

$h=-3a$ | $k=-3b$

$a=-\frac{h}{3}$ | $b=-\frac{k}{3}$

Q(a,b) lies on $x^2+y^2+4x-3y+7=0$

$$\therefore \left(\frac{h}{3}\right)^2+\left(\frac{k}{3}\right)^2-\frac{4h}{3}+\frac{3k}{3}+7=0$$

$$\frac{h^2}{9}+\frac{k^2}{9}-\frac{4}{3}h+k+7=0$$

$$\times 9 \Rightarrow \quad h^2+k^2-12h+9k+63=0$$

put $h=x$, $k=y$

$$x^2+y^2-12x+9y+63=0$$

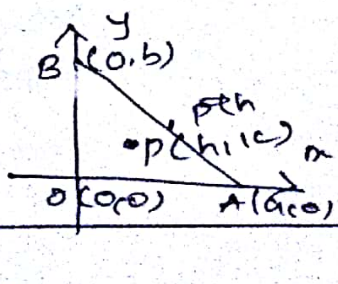
it is the required locus of given pt 'P'.

Ex-12

17 A straight rod of length 6 units, slides with its ends A and B always on the x & y axis respect. If O is the origin, find the locus of the centroid of $\triangle$ OAB

Soln

Let the coordinates of O, A & B are (0,0), (a,0), (0,b)



Let $p(h,k)$ is the centroid of Δ OAB

$\therefore \quad h = \frac{0+a+0}{3} \qquad k = \frac{0+0+b}{3}$

$\boxed{3h = a} \qquad \boxed{3k = b}$

Δ OAB

$OA^2 + OB^2 = AB^2$

$a^2 + b^2 = 36 \qquad AB = 6$

$9h^2 + 9k^2 = 36$

$h^2 + k^2 = 4$

put $h = x$, $k = y$

$x^2 + y^2 = 4$

it is the requ. locus of given pt $P(h,k)$.

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EX -12

18 If the point $P(6,2)$ & $Q(-2,1)$ and R the vertices of a Δ PQR & R is the point on the locus $y = x^2 - 3x + 4$, then find the equation of the locus of the centroid of Δ PQR.

Soln

let $p(h,k)$ be the centroid of Δ PQR

let $P(6,2)$, $Q(-2,1)$, $R(a,b)$ are vertices of Δ PQR.

$\therefore \quad h = \frac{6-2+a}{3} \quad \Big| \quad k = \frac{2+1+b}{3}$

$\boxed{3h-4 = a} \qquad \boxed{3k-3 = b}$

$R(a,b)$ is the pt on the locus $y = x^2 - 3x + 4$

$3k-3 = (3h-4)^2 - 3(3h-4) + 4$

$3k-3 = 9h^2 - 24h + 16 - 9h + 12 + 4$

$9h^2 - 33h - 3k + 35 = 0$

$9x^2 - 33x - 3y + 35 = 0$ it is the required locus.

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Ex 9 19. The coordinates of a point moving P are $\left(\frac{a}{2}(\operatorname{cosec}\theta+\sin\theta), \frac{b}{2}(\operatorname{cosec}\theta-\sin\theta)\right)$ where θ is the parameter. Show that the equ. of locus of P is $b^2x^2-a^2y^2=a^2b^2$

Soln: Let $P(h,k)$ be the point on the locus

$\therefore h=\frac{a}{2}(\operatorname{cosec}\theta+\sin\theta)$ $k=\frac{b}{2}(\operatorname{cosec}\theta-\sin\theta)$

$\frac{2h}{a}=\operatorname{cosec}\theta+\sin\theta \rightarrow (1)$ $\frac{2k}{b}=\operatorname{cosec}\theta-\sin\theta \rightarrow (2)$

$(1)^2-(2)^2 \Rightarrow$

$\frac{4h^2}{a^2}-\frac{4k^2}{b^2}=\operatorname{cosec}^2\theta+\sin^2\theta+2\operatorname{cosec}\theta\sin\theta - \operatorname{cosec}^2\theta-\sin^2\theta+2\operatorname{cosec}\theta\sin\theta$

$=4\operatorname{cosec}\theta\sin\theta$

$\frac{4h^2}{a^2}-\frac{4k^2}{b^2}=4$

$\frac{h^2}{a^2}-\frac{k^2}{b^2}=1$

$b^2h^2-a^2k^2=a^2b^2$

put $h=x, k=y$

$b^2x^2-a^2y^2=a^2b^2$

it is the required locus of the pt P.

20. If θ is the parameter, find the equ. of the locus of moving point, whose coordinates are $a(\theta-\sin\theta), a(1-\cos\theta)$

Soln: Let $P(h,k)$ be the point on the locus

$\therefore h=a(\theta-\sin\theta) \rightarrow (1)$ $k=a(1-\cos\theta)$

$\frac{h}{a}=\theta-\sin\theta$ $\frac{k}{a}=1-\cos\theta$

$\sin\theta=\theta-\frac{h}{a}$ $\cos\theta=1-\frac{k}{a}$

$\cos\theta=\frac{a-k}{a}$

 $\cos\theta = \dfrac{a-k}{a}$ $\Rightarrow$ $\theta = \cos^{-1}\left(\dfrac{a-k}{a}\right)$

$$\sin\theta = \sqrt{1-\cos^2\theta}$$

$$= \sqrt{1-\frac{(a-k)^2}{a^2}}$$

$$= \frac{\sqrt{a^2-(a^2+k^2-2ak)}}{a}$$

$$= \frac{\sqrt{2ak-k^2}}{a}$$

From (1)

$$h = a\cos^{-1}\left(\frac{a-k}{a}\right) - \sqrt{2ak-k^2}$$

put $h = x$, $k = y$

$$x = a\cos^{-1}\left(\frac{a-y}{a}\right) - \sqrt{2ay-y^2}$$

it is the required locus of the point

All the Best

it is useful for students.