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11th PHYSICS -

UNIT -1

NATURE OF PHYSICAL WORLD AND MEASUREMENT

- MR. ThiviyaRaj V .,
M.,Sc.,M.Phill.,B.Ed

SCIENCE- INTRODUCTION

Root in the Latin verb Scientia.

MEANING

'to know'

IN TAMIL

'அறிவியல்'

MEANING

' Knowing the truth '

Science is the systematic organisation of knowledge gained through observation , experimentation and logical reasoning.

The knowledge of science dealing with non-living living things is **Physical science** [Physics and Chemistry]

The knowledge of science dealing with the living things is called the **biological science** [Botany and Zoology]

The word ' science ' was kind only in 19th century . Natural philosophy was the earlier name given to science.

SCIENTIFIC METHOD [3 MARK]

The scientific method is a step-by-step approach in studying natural phenomena and establishing laws which govern these phenomena.

GENERAL FEATURES

(i) Systematic observation
(ii) Controlled experimentation
(iii) Qualitative and quantitative reasoning
(iv) Mathematical modelling
(v) Prediction and verification or falsification of theories

PHYSICS- INTRODUCTION

The word ' **Physics** ' is derived from the Greek word **'Fusis'**

MEANING : NATURE

The study of nature and natural phenomena is deal within physics

APPROACHES IN STUDYING PHYSICS

1. Unification
2. Reductionism

UNIFICATION [2 MARK]

Attempting to explain diverse physical phenomena with a few concepts and laws in unification

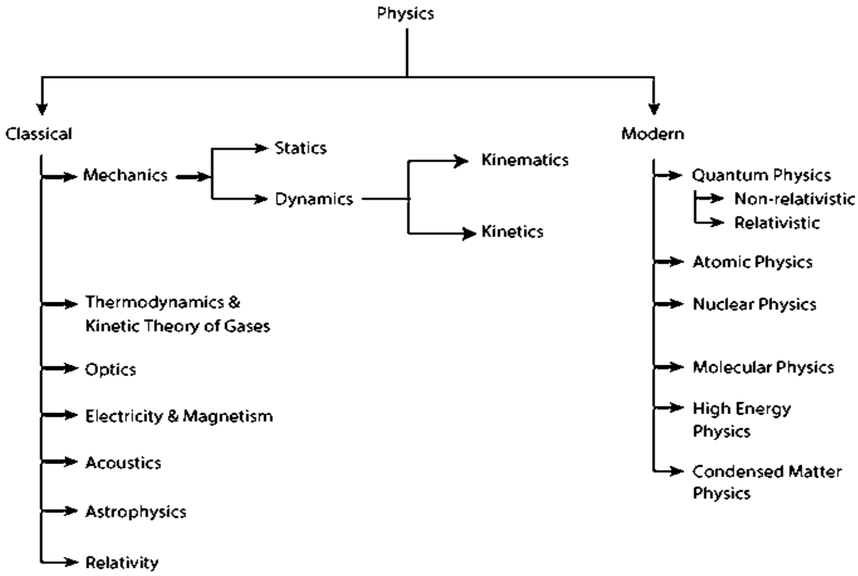
EX: Newton's universal law of gravitation explains the motion of a freely falling body towards the earth. Motion of planets around the sun , motion of moon around the earth does unifying the fundamental forces of nature .

REDUCTION [2 MARK]

An attempt to explain a macroscopic system in terms of this microscopic constituency is reductionism .

EX: thermodynamics was developed to explain macroscopic properties like temperature , entropy ,etc.. of bulk systems. The above properties have been interrupted in terms of the molecular constituents [microscopic]of the bulb system by kinetic theory statistical mechanics .

BRANCH OF PHYSICS



MEASUREMENT

The comparison of any physical quantity with its standard unit is known as measurement.

Measurement is the basis of all scientific studies and experimentation.

PHYSICAL QUANTITY

Quantities that can be measured and in terms of which ,Law of Physics are described are called **physical quantities** .

EX: Length, mass, force , energy etc.,

TYPES OF PHYSICAL QUANTITIES [3 MARK]

1. Fundamental quantities
2. Derived quantities6

Fundamental or base quantities are quantities with cannot be expressed in terms of any other physical quantities

EX : Length, mass , time , electric current , temperature, luminous intensity and amount of substance

Quantities that can be expressed in terms of fundamental quantities are called derived quantities.

EX : Area ,volume, velocity ,acceleration, force

UNIT AND ITS TYPES

UNIT OF THE QUANTITY [2 MARK]

An arbitrary choose standard of measurement of a quantity , which is accepted internationally is called unit of the quantity .

FUNDAMENTAL UNITS [2 MARKS]

The units in which the fundamental quantities are measured are called fundamental or base units

DERIVED UNITS

The units of measurement of all other physical quantities ,which can be obtained by a Suitable multiplication or division of powers of fundamental units are called a derived units.

DIFFERENT TYPES OF MEASUREMENT SYSTEMS

A complete set of units which is used to measure all kinds of fundamental and derived quantities is called a System of units.

a) f.p.s system

The British engineering system of which uses foot , pound and second as the three basic unit of measuring length , mass and time respectively.

b) c.g.s system

The Gaussian system which uses centimetre ,gram and second.

c) m.k.s system

Based on metre, kilogram and second as the three basic units for measuring length, mass and time respectively

SI UNIT SYSTEM [2 MARK]

[SYSTEMATIC INTERNATIONAL]

The System of units used by scientist and engineers around the world is commonly called the metric system but, since 1960 it has been officially as the international system or SI

ADVANTAGES [3 MARK]

1. This system makes use of only one unit for one physical quantity, which means rational System of units.
2. In this system, all the derived units can be easily obtained from basic and supplementary units which means it is a coherent System of units .
3. It is a metric system which means that multiplies and submultiplies can be expressed as power of 10

ONE METRE [2 MARK]

One metre is the length of the path travelled by light in vacuum in 1/299,792,458 of a second

ONE KILOGRAM [2 MARK]

The mass of the prototype cylinder of platinum iridium alloy, preserved at the International Bureau of Weights and Measures at Serves, near Paris, France.

ONE SECOND [2 MARK]

The duration of 9,192,631,770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of Cesium-133 atom.

ONE AMPERE [2 MARK]

The constant current, which when maintained in each of the two straight parallel conductors of infinite length and negligible cross section, held one metre apart in vacuum shall produce a force per unit length of 2×10^{-7} N/m between them.

ONE KELVIN [2 MARK]

The fraction $\frac{1}{273.16}$ of the thermodynamic temperature of the triple point of the water.

ONE MOLE [2 MARK]

The amount of substance which contains as many elementary entities as there are atoms in 0.012 kg of pure carbon-12.

ONE CANDELA [2 MARK]

The luminous intensity in a given direction, of a source that emits monochromatic radiation of frequency 5.4×10^{14} Hz and that has a radiant intensity of $\frac{1}{683}$ watt / steradian in that direction.

TRIPLE POINT OF WATER [2 MARK]

Triple point of water at the temperature at which a saturated vapour, pure and melting ice are all in equilibrium. The triple point temperature of water 273.16 K

RADIAN [rad] [2 MARK]

One Radian is the angle subtended at the centre of a circle by an are equal in length to the radius of the circle.

$$\pi \text{ radian} = 180°$$

$$1 \text{ radian } = \frac{180°}{\pi} = \frac{180° \ X \ 7}{\pi} = 57.27\ °$$

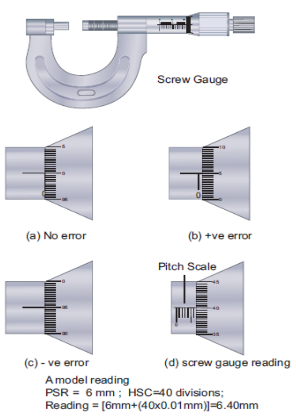
1 = 60° ; 1' = 60" [seconds of arc]

STERADIAN [sr] [2 MARK]

One Steradian is the solid angle subtended at the centre of a sphere, by the surface of the sphere, which is equal in area, to the square of radius of the sphere.

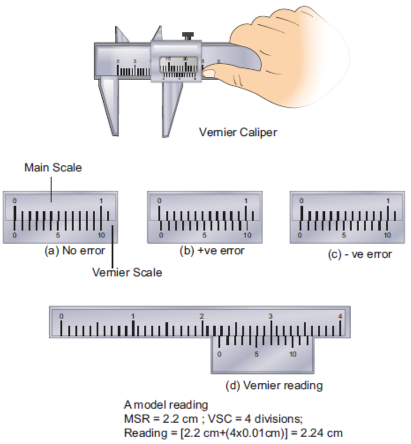
SCREW GAUGE [3 MARK]

Screw gauge is an instrument used for measuring accurately the dimensions of objects up to a maximum of about 50 mm. The principle of the instrument is the magnification of linear motion using the circular motion of a screw full stop the least count is 0.01 mm.



VERNIER CALIPER [3 MARK]

A vernier caliper is a versatile instrument for measuring the dimension of an object namely diameter of a hole or a depth of a hole . The least count is 0.1 mm.



TRIANGULATION METHOD [3 MARK]

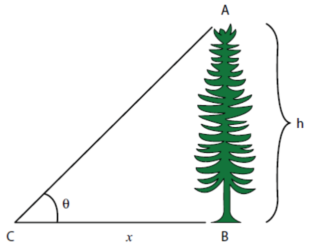
Let AB=h be the height of the tree or Tower to be measured. Let C be the point of observation at a distance x from B. Place a range finder at c and measure the angle of elevation. <ACB =θ

From right angel triangle ABC,

$$\tan\theta = \frac{AB}{BC} = \frac{h}{x}$$

height h = x $\tan\theta$

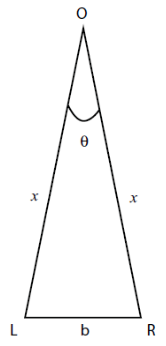
Knowing the distance 'x' , the height 'h' can be determined.



PARALLAX METHOD [3 MARK]

Very large distance, such as the distance of a planet or a star from the Earth can be measured by the Parallax method is the name given to the Apparent change in the position of an object with respect to the background when the object is seen from two different positions. The distance between the two positions is called the basis(b)

For example,



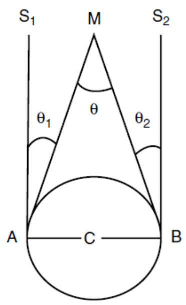
An observer is specified by the position o. The observer is holding a pen before him , against the background of a wall. When the pen is looked at first by our left eye L and then by our right eye R, the position of the pen changes with respect to the background of the wall. The shift in the position of an object when viewed with two eyes keeping one eye closed at a time is known as parallax. The distance between the left eye [L] and the right eye [R] in this case is the basis.

<LOC is called the parallax angle or parallatic angle. LR= b; LO = RO=x

We get $\theta = \frac{b}{x}$; b- basis, x- unknown distance

Knowing 'b' and measuring θ ,we can calculate 'x'

DETERMINATION OF DISTANCE OF MOON FROM EARTH [3 MARK]



C is the centre of the earth, A and B are two diametrically opposite place on the surface of the earth. From A and B, The parallexes θ_1 *and* θ_2 respectively of moon with respect to some distant star are determined with the help of an astronomical telescope. Does the total parallax of the moon subtended on earth.

$$\angle AMB = \theta_1 + \theta_2 = \theta$$

If θ is measured in radians, then

$$\theta = \frac{AB}{AM} \; ; \; AM \approx MC$$

$\theta = \frac{AB}{MC}$ or $MC = \frac{AB}{\theta}$ Knowing the values of AB and θ

We can calculate the distance MC of moon from the earth .

RADAR METHOD [3 MARK]

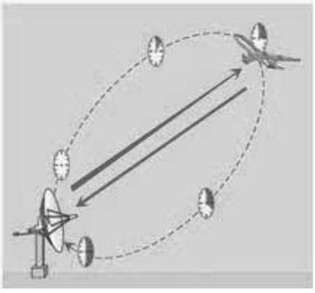
RADAR - Radio Detection and Ranging. A radar can be used to measure accurately the distance of a nearby planet such as mass.

In this method, radio waves are send from transmitters which, after reflecting from the planet ,are detected by the receiver. By measuring, the time interval (t) between the instants the radio waves are sent and received the distance of the planet can be determined as,

$$\text{speed} = \frac{distance}{time}$$

$$\text{Distance } [\ d\ ] = \frac{v \, X \, t}{2}$$

Time taken (t) is for the distance covered during the forward and backward path of the radio waves. This method can also be used to determine the height, at which an aeroplane flies from the ground.



MEASUREMENT OF MASS

Mass of a body is defined as the quantity of matter contained in a body.

SI unit ⟶ Kilogram [kg]

It does not depend on temperature , pressure and location of the body in space.

Ordinarily, the mass of an object is vitamin m kilograms using a common balance.

Large masses ⟶ Gravitational methods

Small masses ⟶ mass spectrograph

MEASUREMENT OF TIME

An atomic standard of time, is based on the periodic vibration produced in cesium atom.

A clock is used to measure the time interval. Some of the clocks developed later or electric oscillators, electronic oscillators, solar clock, Quartz crystal clock ,atomic clock, decay of Elementary particle, radioactive dating etc.,

THEORY OF ERRORS

ERROR

The result obtained for any measurement will contain some uncertainty . Such uncertainty is termed as error .

In measurements ,two different terms accuracy and precision are used.

ACCURACY [2 MARK]

Accuracy refers to how far we are from the true value.

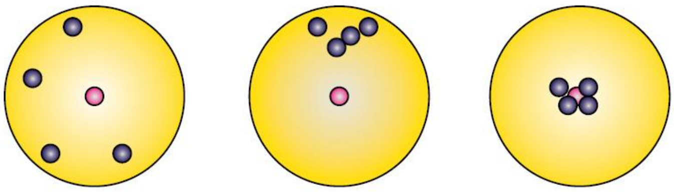
PRECISION [2 MARK]

Precision refers to how well we measure

EX: Real temperature inside the refrigerator is 9 °C

Let the temperature of a refrigerator measured by thermometer given as 10.4 °C , 10.2 °C, 10.3 °C, 10.1°C, 10.2 °C, 10.1 °C ,10.1 °C,10.1 °C. The thermometer is not accurate but since all measured values are closed to 10°C hence it is precise.

VISUAL EXAMPLE



(a) Not accurate Not precise

(b) Not accurate Precise

(c) Accurate Precise

ERRORS IN MEASUREMENT [5 MARK]

The uncertainty in measurement is called an error .

Three are three possible errors

1. Systematic errors
2. random errors
3. gross errors

i. SYSTEMATIC ERRORS

=> Reproducible inaccuracy that are consistently in the same direction.

=> These occurs after due to a problem that it persists throughout the experiment.

CLASSIFIED

1. **INSTRUMENTAL ERRORS.**

When an instrument is not calibrated properly at the time of manufacture, instrumental errors may arise.

These errors can be corrected by using the instrument carefully.

2. **IMPERFECTIONS IN EXPERIMENTAL TECHNIQUE OR PROCEDURE**

These errors arise due to the limitation in the experimental arrangement.

These errors can be corrected by necessary collection has to be applied.

3. **PERSONAL ERRORS**

These errors are due to individuals performing the experiment, may be due to incorrect initial setting up of the experiment or carelessness of the individual making the observation due to improper precautions.

4. ERRORS DUE TO EXTERNAL CAUSES

The change in the external conditions during an experiment can cause error in measurement.

Ex: change in temperature, humidity pressure.

5. **LEAST COUNT ERROR**

Least count is the smallest value that can be measured by the measuring instrument, and the error due to this measurement is least count error. The instrument's resolution hence is the cause of this error. Least count error can be reduced by using a high precision instrument for the measurement.

ii. RAMDOM ERRORS

=> Arise due to random and unpredictable variations in experimental conditions like pressure ,temperature, voltage ,supply etc..

=> Errors may also be due to personal errors by the observer who performs the experiment

=> "Chance error"

Ex:

The thickness of a wire measured using a screw gauge.

n=> number of trial readings are taken in an experiment, readings are a_1 , a_2, a_3, ... a_n

The Arithmetic mean is,

$$a_m = \frac{a_1 + a_2 + a_3 \ldots\ldots + a_n}{n}$$

$$a_m = \frac{1}{n} \sum_{i=1}^{n} a_i$$

Arithmetic mean is taken as the best possible true value of the quantity .

iii. GROSS ERROR

The error caused due to the shear careless of an observer.

Ex:

1. Reading an instrument without settings in properly
2. Taking observation in a wrong manner without to bothering about the sources of errors and precautions
3. Recording wrong observations
4. Using wrong values of observations in calculations

ERROR ANALYSIS

i. ABSOLUTE ERROR [2 MARK]

The magnitude of difference between the true value and the measured value of a quantity.

If a_1, a_2, a_3, ... a_n are the measured values of any quantity 'a' in an experiment performed 'n' times , then the Arithmetic mean of the value is called true value of the quantity

$$a_m = \frac{a_1 + a_2 + a_3 \ldots\ldots + a_n}{n} \quad [\text{ or}]$$

$$a_m = \frac{1}{n}\sum_{i=1}^{n} a_i$$

The absolute error in measured values is given by

$$|\Delta a_1| = |a_m - a_1|$$

$$|\Delta a_2| = |a_m - a_2|$$

$$\vdots$$

$$|\Delta a_n| = |a_m - a_n|$$

ii. MEAN ABSOLUTE ERROR [2 MARK]

The Arithmetic mean of absolute error in all the measurements is called the mean absolute error.

$$\Delta a_m = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| \ldots\ldots + |\Delta a_n|}{n} \quad [\text{ or}]$$

$$\Delta a_m = \frac{1}{n}\sum_{i=1}^{n} |\Delta a_n|$$

a_m => true value , Δa_m =>Mean absolute error

The magnitude of the quantity may lie between $a_m + \Delta a_m$ and $a_m - \Delta a_m$

iii. RELATIVE ERROR [2 MARK]

The ratio of the mean absolute error to the mean value is called the relative error.This is also called as fractional error.

$$\text{Relative error} = \frac{Mean \text{ absolute error}}{\text{mean value}}$$

$$= \frac{\Delta a_m}{a_m}$$

iv. PERCENTAGE ERROR [2 MARK]

The relative error expressed as a percentage is called percentage error.

$$\text{Percentage error} = \frac{\Delta a_m}{a_m} \text{ x } 100\%$$

A percentage error very close to the Zero means one is close to the targeted value, which is good and acceptable.

PROPOGATION OF ERRORS [3 MARK]

The various possibilities of the propagation of combination of errors in different mathematical operations are discussed below

(i) ERROR IN THE SUM OF TWO QUANTITIES

Let ΔA and ΔB => absolute errors in the quantities A and B respectively.

Measured value of A = A ± ΔA

Measured value of B = B ± ΔB

Sum ,Z = A + B

The error ΔZ in Z,

$$Z \pm \Delta Z = [A \pm \Delta A] + [B \pm \Delta B]$$

$$= [A + B] \pm [\Delta A \pm \Delta B]$$

$$= Z \pm [\Delta A \pm \Delta B]$$

$$\Delta Z = \Delta A + \Delta B$$

The maximum possible error in the sum of the two quantities is equal to the sum of the absolute errors in the individual quantities.

(ii) ERROR IN THE DIFFERENT OF TWO QUANTIES

Let ΔA & ΔB-> Absolute errors in the two quanties A and B

Measured value of A = $A \pm \Delta A$

Measured value of B = $B \pm \Delta B$

Difference ,Z = A - B

The error ΔZ in Z,

$$Z \pm \Delta Z = [A \pm \Delta A] - [B \pm \Delta B]$$

$$= [A - B] \pm [\Delta A \mp \Delta B]$$

$$= Z \pm [\Delta A \mp \Delta B]$$

$$\Delta Z = \Delta A + \Delta B$$

The maximum error in difference of two quantities is equal to the sum of the absolute difference in the individual quantities.

(iii) ERROR IN THE PRODUCT OF TWO QUANTIES

Let ΔA & ΔB-> Absolute errors in the two quanties A and B

The product ,Z= AB

The error ΔZ in Z,

$$Z \pm \Delta Z = [A \pm \Delta A] . [B \pm \Delta B]$$

$$= [AB] \pm [A\Delta B] \pm [B \Delta A] \pm [\Delta A . \Delta B]$$

Dividing L.H.S by Z and R.H.S by AB.

$$1 \pm \frac{\Delta Z}{z} = 1 \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \pm \frac{\Delta A}{A}.\frac{\Delta B}{B}$$

$\frac{\Delta A}{A}.\frac{\Delta B}{B}$ is neglected [$\frac{\Delta A}{A}, \frac{\Delta B}{B}$ $are\ small\ quanties$]

The maximum fractional error in Z is,

$$\frac{\Delta Z}{z} = \pm \left[\frac{\Delta A}{A} + \frac{\Delta B}{B} \right]$$

The maximum fractional error in the product of two quantities is equal to the sum of the fractional errors in the individual quantities.

(iv) ERROR IN THE DIVISION OR QUOTIENT OF TWO QUANTITIES

Let ΔA & ΔB-> Absolute errors in the two quanties A and B

The quantities , $Z = \frac{A}{B}$

The error ΔZ in Z,

$$Z \pm \Delta Z = \frac{A \pm \Delta A}{B \pm \Delta B} = \frac{A\,[\,1 \pm \frac{\Delta A}{A}]}{B\,[\,1 \pm \frac{\Delta B}{B}]}$$

$$= \frac{A}{B}\,[\,1 \pm \frac{\Delta A}{A}]\ [\,1 \pm \frac{\Delta B}{B}]^{-1}$$

$$Z \pm \Delta Z = Z\,[\,1 \pm \frac{\Delta A}{A}]\ [\,1 \mp \frac{\Delta B}{B}]$$

Dividing both sides by Z

$$1 \pm \frac{\Delta Z}{Z} = [\,1 \pm \frac{\Delta A}{A}]\ [\,1 \mp \frac{\Delta B}{B}]$$

$$= 1 \pm \frac{\Delta A}{A} \mp \frac{AB}{B} \pm \frac{\Delta A}{A}.\frac{\Delta B}{B}$$

$\frac{\Delta A}{A}$ and $\frac{\Delta B}{B}$ are small, their product term can be neglected .

$$\frac{\Delta Z}{Z} = \left[\frac{\Delta A}{A} + \frac{\Delta B}{B} \right]$$

The maximum fractional error in the quotient of two quantities is equal to the sum of their individual fractional errors .

(v) ERROR IN THE POWER OF A QUANTITY

The n^{th} power of A, Z $= A^n$

The error ΔZ in Z,

$$Z \pm \Delta Z = [A \pm \Delta A]^n$$

$$= A^n [1 \pm \frac{\Delta A}{A}]^n$$

$$= Z [1 \pm n \frac{\Delta A}{A}]$$

Dividing both sides by Z

$$1 \pm \frac{\Delta Z}{Z} = [1 \pm n \frac{\Delta A}{A}]$$

$$\frac{\Delta Z}{Z} = n \frac{\Delta A}{A}$$

The fractional error in the n^{th} power of a quantity in n times the fractional error in that quantity

SIGNIFICANT FIGURES [2 MARK]

The number of meaningful digits which contain numbers that are known reliably and first uncertain number.

RULES FOR COUNTING SIGNIFICANT FIGURES [5 MARK]

1. All non-zero digits are significant.

   EX: 1342 has four significant figures

2. All zeros between two non zero digits are significant figures.

   EX: 2008 has four significant figures.

3. All zeros to the right of a non zero digit but to the left of a decimal point are significant.

   EX: 30700 has five significant figures

4. a) For the number without a decimal point ,the terminal or trailing zero (s) or not significant.

EX: 30700 has three significant figures.

b) All zeros are significant if they come from a measurement.

EX: 30700 has five significant figures.

5. If a number is less than 1, the zero (s) on the right of the decimal point but to left of the first non zero digit are not significant.

EX: 0.00345 has a significant figures.

6. All zeros to the right of a decimal point and to the right of non zero digit are significant.

EX: 40.00 have most difficult figures.

0.030400 as five significant figures.

7. The number of significant figures does not depend on the System of units used.

EX: 1.53 cm, 0.153 m, 0.0000153 km all have three significant figures

ROUNDING OFF [2 MARK]

The result of calculation with numbers containing more than one uncertain digit should be rounded off .

RULES FOR ROUNDING OFF [3 MARK]

1. If the digit to be dropped is smaller than 5, then the preceding digit should be left unchanged.

EX: i) 7.32 is rounded off to 7.3
ii) 8.94 is rounded off to 8.9

2. If the digit to be dropped is greater than 5, thenthe preceding digit should be increased by 1

EX: i) 17.26 is rounded off to 17.3
ii) 11.89 is rounded off to 11.9

3. If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit should be raised by 1

EX: i) 7.352, on being rounded off to first decimal becomes 7.4
ii) 18.159 on being rounded off to first decimal, become 18.2

4. If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is not changed if it is even

EX: i) 3.45 is rounded off to 3.4
ii) 8.250 is rounded off to 8.2

5. If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by 1 if it is odd

EX: i) 3.35 is rounded off to 3.4
ii) 8.350 is rounded off to 8.4

ARITHEMETICAL OPERATION WITH SIGNIFICANT FIGURES [3 MARK]

(i) ADDITION AND SUBTRATION

The final result should retain as many decimal places as there are in the number with the smallest number of decimal places.

Example:

1. 3.1 + 1.780 + 2.046 = 6.926

The least number of significant digits after the decimal is one. Hence the result will be 6.9.

2. 12.637 – 2.42 = 10.217

Hence the result will be 10.22

(ii) MULTIPLICATION AND DIVISION

The result should retain as many significant figures as there are in the original number with smallest number of significant figures.

Example:

1. 1.21 × 36.72 = 44.4312 = 44.4
2. 36.72 . 1.2 = 30.6 = 31

DIMENSIONAL ANALYSIS

DIMENSION OF PHYSICAL QUANTITIES

Any physical qualities which is expressed in terms of base quantities whose exponent represents the dimension of the physical quantity.

EX: [length] $= M^0LT^0 = L$
[Area] $= M^0L^2T^0 = L^2$
[Volume] $= M^0L^3T^0 = L^3$

The base quantity L, is same but exponent or different which means dimensions are different.

For a pure number , exponent of this quantity is zero.

EX: The number 2 ,=> no dimension.

[2] $= M^0L^0T^0$ [dimensionless]

Speed ,s $= \frac{distance}{time\ taken}$ => [S] $= \frac{L}{T} = LT^{-1}$

Velocity, $\vec{v} = \frac{displacement}{time\ taken}$ => $[\vec{v}] = \frac{L}{T} = LT^{-1}$

Acceleration, $\vec{a} = \frac{velocity}{time\ taken}$ => $[\vec{a}] = \frac{LT^{-1}}{T} = LT^{-2}$

DIMENSIONAL QUANTITIES , DIMENSIONLESS QUATITIES

1. DIMENSIONAL VARIABLES [2 MARK]

Physical quantities ,which posses dimensions and have variable values.

EX: length ,velocity , acceleration etc..

2. DIMENSIONLESS VARIBLES [2 MARK]

Physical quantities which have no dimensions ,but have variable values .

EX: specific gravity, strain ,refractive index

3. DIMENSIONAL CONSTANT [2 MARK]

Physical quantities which posses dimension and have constant values .

EX: gravitational constant, Planck's Constant etc..

4. DIMENSIONLESS CONSTANT [2 MARK]

Quantities which have constant values and also have no dimensions .

EX: [Euler's numbers], numbers, etc...

PRINCIPLE OF HOMOGENEITY OF DIMENSIONS [2 MARK]

The dimensions of all the terms in a physical expression should be same.

EX: $V^2 = U^2 + 2as$.

The dimensions of V^2 ,U^2 and 2as are the same and equal to [$L^2 T^{-2}$]

APPLICATION OF THE METHOD OF DIMENSION ANALYSIS [2 MARK]

This method is used to

(i) Convert a physical quantity from one system of units to another.

(ii) Check the dimensional correctness of a given physical equation.

(iii) Establish relations among various physical quantities.

(i) To convert a physical quantity from one system of units to another

This is based on the fact that the product of the numerical values (n) and its corresponding unit (u) is a constant.

i.e, n [u] = constant (or)

$n_1[u_1] = n_2[u_2]$.

Consider a physical quantity which has dimension '*a*' in mass, '*b*' in length and '*c*' in time. If the fundamental units in one system are M_1, L_1 and T_1 and the other system are M_2, L_2 and T_2 respectively

$$n_1 [M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$$

EXAMPLE 1.12 [2 MARK]

Convert 76 cm of mercury pressure into Nm^{-2} using the method of dimensions.

SOLUTION

In cgs system 76 cm of mercury pressure = 76X13.6X 980 dyne cm^{-2}

The dimensional formula of pressure P is $[ML^{-1}T^{-2}]$

$$P_1[M_1^a L_1^b T_1^c] = P_2[M_2^a L_2^b T_2^c]$$

We have ,

$$P_2 = P_1 = \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

M_1 =1g , M_2 = 1kg

L_1 =1Cm , L_2 = 1m

T_1 =1s , T_2 = 1s

So a =1, b =-1 and c = -2

Then

$$P_2 = 76 \text{ x } 13.6 \text{ x } 980 \left[\frac{1g}{1kg}\right]^1 \left[\frac{1\,cm}{1\,m}\right]^{-1} \left[\frac{1\,s}{1\,s}\right]^{-2}$$

$$= 76 \text{ x } 13.6 \text{ x } 980 \left[\frac{10^{-3}g}{1kg}\right]^1 \left[\frac{10^{-2}m}{1\,m}\right]^{-1} \left[\frac{1\,s}{1\,s}\right]^{-2}$$

$$= 76 \text{ x } 13.6 \text{ x } 980 \text{ x } 10^{-3} \text{ x } 10^2$$

$P_2 = 1.01 \text{ x } 10^5\ Nm^{-2}$

EXAMPLE 1.13 [2 MARK]

If the value of universal gravitational constant in SI is $6.6 \text{x} 10^{-11}\ Nm^2\ kg^{-2}$, then find its value in CGS System?

SOLUTION

Let G_{SI} be the gravitational constant in the SI system and G_{cgs} in the *cgs* system. Then

G_{SI} = 6.6 x 10^{-11}Nm^2 kg^{-2}
G_{cgs} =?

$$n_2 = n_1 = \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$G_{cgs} = G_{SI} = \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

M_1 =1kg , M_2 = 1g

L_1 =1m , L_2 = 1cm

T_1 =1s , T_2 = 1s

The dimensional formula of pressure G is [$M^{-1}L^3T^{-2}$]

a =-1, b = 3 and c = -2

$$G_{cgs} = 6.6 \times 10^{-11} \left[\frac{1kg}{1g}\right]^{-1} \left[\frac{1\,m}{1\,cm}\right]^3 \left[\frac{1\,s}{1\,s}\right]^{-2}$$

$$= 6.6 \times 10^{-11} \left[\frac{1kg}{10^{-3}g}\right]^{-1} \left[\frac{1\,m}{10^{-2}m}\right]^3 \left[\frac{1\,s}{1\,s}\right]^{-2}$$

$$= 6.6 \times 10^{-11} \times 10^{-3} \times 10^6 \times 1$$

G_{cgs} = 6.6 x 10^{-8} dyne cm^2 g^{-2}

(ii) To check the dimensional correctness of a given physical equation

Let us take the equation of motion

v = u + at

Apply dimensional formula on both sides

[LT^{-1}] = [LT^{-1}] + [LT^{-1}]

The dimensions of both sides are same. Hence the equation is dimensionally correct.

(iii) To establish the relation among various physical quantities

If the physical quantity Q depends upon the quantities Q_1, Q_2 and Q_3.

$$(i.e)\ Q \ \alpha \ Q_1^a Q_2^b \ Q_3^c$$

$$Q = k \ Q_1^a Q_2^b \ Q_3^c$$

Where k is a dimensionless constant.

EXAMPLE 1.15 [2 MARK]

Obtain an expression for the time period T of a simple pendulum. The time period T depends on (i) mass '*m*' of the bob (ii) length '*l*' of the pendulum and (iii) acceleration due to gravity *g* at the place where the pendulum is suspended. (Constant k = 2π)

SOLUTION

$$T \propto m^a l^b g^c$$

$$T = k\, m^a l^b g^c$$

Here k is the dimensionless constant. Rewriting the above equation with dimensions

$$[T^1] = [M^a]\,[L^b]\,[LT^{-2}]^c$$
$$[M^0L^0T^1] = [M^a\, L^{b+c}\, T^{-2c}]$$

Comparing the powers of M, L and T on both sides, a=0, b+c=0, -2c=1

Solving for a,b and c a = 0, b = 1/2, and c = −1/2

From the above equation, $T = k\, m^0\, l^{\frac{1}{2}}\, g^{\frac{-1}{2}}$

$$\mathbf{T = k\left[\frac{l}{g}\right]^{\frac{1}{2}} = k\sqrt{\frac{l}{g}}}$$

Experimentally k = 2π, hence $T = 2\pi\sqrt{\frac{l}{g}}$

Limitations of Dimensional analysis [2 MARK]

1. This method gives no information about the dimensionless constants in the formula like 1, 2, ……..π,e, etc.
2. This method cannot decide whether the given quantity is a vector or a scalar.
3. This method is not suitable to derive relations involving trigonometric, exponential and logarithmic functions.
4. It cannot be applied to an equation involving more than three physical quantities.
5. It can only check on whether a physical relation is dimensionally correct but not the correctness of the relation. For example using dimensional analysis, s = ut +$\frac{1}{3}at^2$ is dimensionally correct whereas the correct relation is s = ut +$\frac{1}{3}at^2$

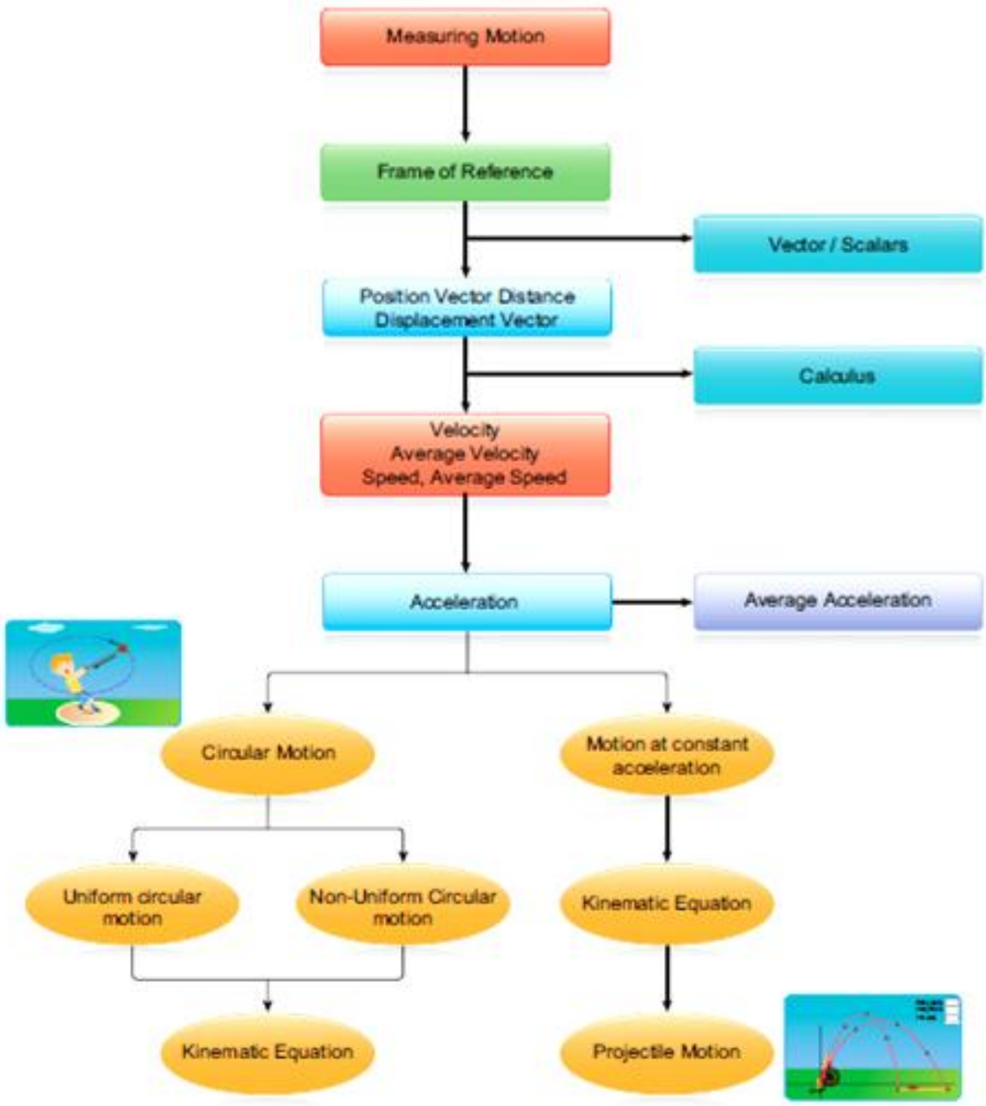
★★★

11TH PHYSICS

UNIT 2 - KINEMATICS

-MR. THIVIYARAJ V.,M.SC.,M.PHILL.,B.ED.,

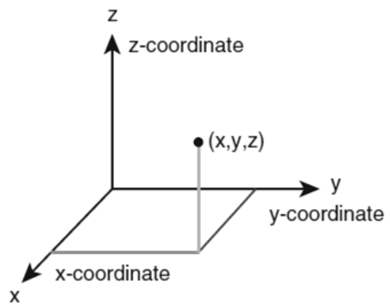
LECTURE VIDEOS



1. FRAME OF REFRENCE **[2 MARK]**

If we imagine a coordinate system and the position of an object is described relative to it, then such a coordinate system is called frame of reference.

2. CARTESIAN COORDINATE SYSTEM **[2 MARK]**



3. POINT MASS **[2 MARK]**

Let the mass of any object be assumed to be concentrated at a point. Then this idealized mass is called "point mass".

4. TYPES OF MOTION **[3 MARK]**

a) LINEAR MOTION

An object is said to be in linear motion if it moves in a straight line.

Examples

1. An athlete running on a straight track
2. A particle falling vertically downwards to the Earth.

b) CIRCULAR MOTION

Circular motion is defined as a motion described by an object traversing a circular path.

Examples

1. The whirling motion of a stone attached to a string
2. The motion of a satellite around the Earth

c) ROTIONAL MOTION

If any object moves in a rotational motion about an axis, the motion is called 'rotation'. During rotation every point in the object transverses a circular path about an axis.

Examples

1. Rotation of a disc about an axis through its center
2. Spinning of the Earth about its own axis.

d) VIBRATORY MOTION [OSCILLATORY MOTION]

If an object or particle executes a *to–and– fro* motion about a fixed point, it is said to be in vibratory motion. This is sometimes also called oscillatory motion.

Examples

1. Vibration of a string on a guitar
2. Movement of a swing

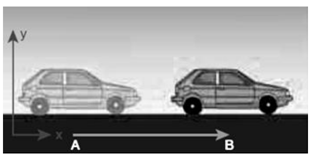
5. MOTION IN ONE ,TWO AND THREE DIMENSIONS [3 MARKS]

Let the position of a particle in space be expressed in terms of rectangular coordinates x, y and z. When these coordinates change with time, then the particle is said to be in motion. However, it is not necessary that all the three coordinates should together change with time. Even if one or two coordinates changes with time, the particle is said to be in motion.

i. MOTION IN ONE DIMENSION

The motion of a particle moving along a straight line.

In this motion, only one coordinate specifying the position of the object changes with time



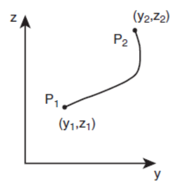
Examples

i. Motion of a train along a straight railway track.
ii. An object falling freely under gravity close to earth

ii. MOTION IN TWO DIMENSION

If a particle is moving along a curved path in a plane, then it is said to be in two dimension motion

In this motion, two of the three rectangular coordinates specifying the position of object change



Examples

i. Motion of a coin on a board.
ii. An insect crawling over the floor on the room

iii. MOTION IN THREE DIMENSION

A particle moving in usual three dimensional space has three dimensional motion.

In this motion, all the three coordinates specifying the position of an object change with respect to time

Examples

i. A bird flying in the sky.
ii. Random motion of a gas molecule.
iii. Flying of a kite on a windy day

6. ELEMENTARY CONCEPTS OF VECTOR ALGEBRA [2 MARK]

SCALAR

It is a property which can be described only by magnitude. In physics a number of quantities can be described by scalars.

EXAMPLE: Distance , mass, temperature , speed and energy

VECTOR

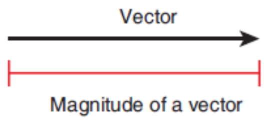
It is a quantity which is described by both magnitude and direction. Geometrically a vector is a directed line segment.

EXAMPLE:

Force, velocity, displacement, position vector, acceleration ,linear momentum and angular momentum

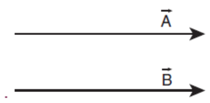
7. MAGNITUDE OF A VECTOR [NORM OF A VECTOR] [2 MARK]

The length of a vector is called magnitude of the vector. It is always a positive quantity. Sometimes the magnitude of a vector is also called 'norm' of the vector. For a vector $\vec{A}$ the magnitude or norm is denoted by $|\vec{A}|$ or A



8. DIFFERENT TYPES OF VECTORS [3 MARK]

1. Equal vectors: Two vectors $\vec{A}$ and $\vec{B}$ said to be equal when they have equal magnitude and same direction and represent the same physical quantity

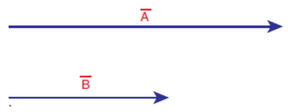


(a) Collinear vectors:

Collinear vectors are those which act along the same line. The angle between them can be 0° or 180°.

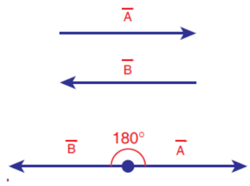
(i) Parallel Vectors:

If two vectors $\vec{A}$ *and* $\vec{B}$ act in the same direction along the same line or on parallel lines, then the angle between them is 0° .



(ii) Anti–parallel vectors:

Two Vectors $\vec{A}$ and $\vec{B}$ are said to be anti–parallel when they are in opposite directions along the same line or on parallel lines. Then the angle between them is 180°



2. UNIT VECTOR

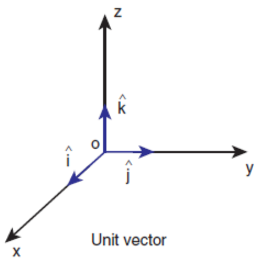
A vector divided by its magnitude is a unit vector. The unit vector for $\vec{A}$is denoted by $\hat{A}$. It has a magnitude equal to unity or one.

$$\hat{A} = \frac{\vec{A}}{A} \quad ; \qquad \vec{A} = A\hat{A}$$

Unit vector specifies only the direction of the vector quantity.

3. ORTHOGONAL UNIT VECTORS

Let $\hat{\imath}, \hat{\jmath}$ *and* $\hat{k}$ be three unit vectors which specify the directions along positive *x*–axis, positive *y*–axis and positive *z*–axis respectively. These three unit vectors are directed perpendicular to each other, the angle between any two of them is 90°.$\hat{\imath}, \hat{\jmath}$ *and* $\hat{k}$ are examples of orthogonal vectors. Two vectors which are perpendicular to each other are called orthogonal vectors



ADDITION OF VECTORS

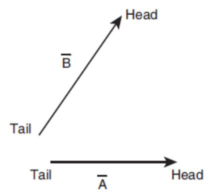
Vectors can be added geometrically or analytically using certain rules called 'vector algebra'.The sum of two vectors, which are inclined to each other, we use

(i) Triangular law of addition method[or]

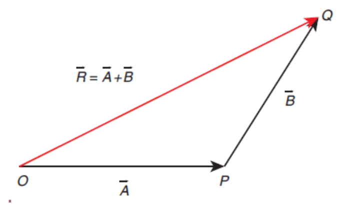
(ii) Parallelogram law of vectors.

9. TRIANGULAR LAW OF ADDITION METHOD [5 MARK]

Let us consider two vectors $\vec{A}$ and $\vec{B}$



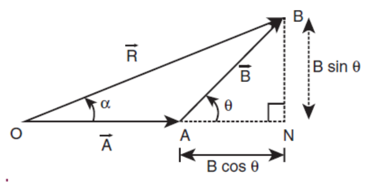
The vectors $\vec{A}$ and $\vec{B}$ by the two adjacent sides of a triangle taken in the same order. Then the resultant is given by the third side of the triangle taken in the reverse order .



The head of the first vector $\vec{A}$is connected to the tail of the second vector $\vec{B}$. Let θ be the angle Between $\vec{A}$ and $\vec{B}$. Then $\vec{R}$is the resultant vector connecting the tail of the first vector $\vec{A}$to the head of the second vector $\vec{B}$. The magnitude of $\vec{R}$ is given geometrically by the length of $\vec{R}$ (OQ) and the direction of the resultant vector is the angle between $\vec{R}$ and $\vec{A}$

$$\vec{R} = \vec{A} + \vec{B}.$$
$$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}.$$

MAGNITUDE OF RESULTANT VECTOR



Consider the triangle ABN, which is obtained by extending the side OA to ON. ABN is a right angled triangle.

$$\text{Cos }\theta = \frac{AN}{B}\ ;\ AN = B\text{ Cos }\theta$$

$$\sin\theta = \frac{BN}{B}\ ;\ AN = B\text{ Sin }\theta$$

For ΔOBN ,

$$OB^2 = ON^2 + BN^2$$

$R^2 = [A + B\cos\theta]^2 + [B\sin\theta]^2$

$R^2 = A^2 + B^2 + 2AB\cos\theta + B^2\sin^2\theta$

$R^2 = A^2 + B^2[\cos^2\theta + \sin^2\theta] + 2AB\cos\theta$

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

DIRECTION OF RESULTANT VECTORS

The angle between $\vec{A}$ and $\vec{B}$, then

$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$\vec{R}$ makes an angle α with $\vec{A}$, then in ΔOBN,

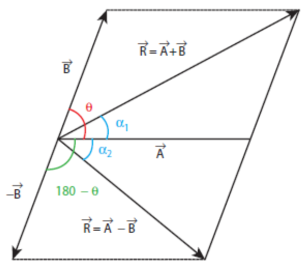
$\tan\alpha = \frac{BN}{ON} = \frac{BN}{OA+AB}$

$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$

$\alpha = \tan^{-1}\left[\frac{B\sin\theta}{A + B\cos\theta}\right]$

10. SUBTRACTION OF VECTORS **[3 MARKS]**

Two non-zero vectors $\vec{A}$ and $\vec{B}$ Which are inclined to each other at an angle θ, the difference $\vec{A}$ -$\vec{B}$ is obtained as follows. First obtained -$\vec{B}$. The anglebetween $\vec{A}$ and -$\vec{B}$is 180–θ.



The difference $\vec{A}$ -$\vec{B}$ is the same as the resultant of $\vec{A}$ and -$\vec{B}$.

$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

MAGNITUDE

$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos[180° - \theta]}$

DIRECTION

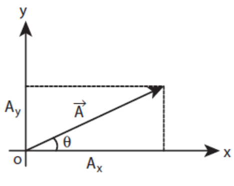
$\tan\alpha_2 = \frac{B\sin[180° - \theta]}{A + B\cos[180° - \theta]}$

$\sin[180° - \theta] = \sin\theta$

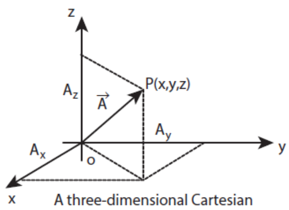
$\tan\alpha_2 = \frac{B\sin\theta}{A - B\cos}$

11. COMPONENTS OF A VECTOR [2 MARK]

In the Cartesian coordinate system any vector $\vec{A}$ can be resolved into three components along *x*, *y* and *z* directions.



A two-dimensional Cartesian coordinate system



A three-dimensional Cartesian coordinate system

Consider a 3–dimensional coordinate system with respect to this a vector can be written in component form as

$$\vec{A} = A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}$$

Here

A_x => x – component of $\vec{A}$

A_y => y – component of $\vec{A}$

A_z = > z – component of $\vec{A}$

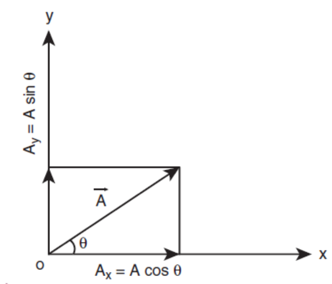
In a 2–dimensional Cartesian coordinate system the vector $\vec{A}$ is given by

$$\vec{A} = A_x\,\hat{i} + A_y\,\hat{j}$$

If $\vec{A}$ makes an angle θ with x axis, and A_x and A_y are the components of $\vec{A}$ along *x*–axis and *y*–axis respectively,

$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$



A =>Magnitude of the vector $\vec{A}$

$$A = \sqrt{A_x^2 + A_y^2}$$

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12. VECTOR ADDITION USING COMPONENTS [2 MARK]

The two vectors $\vec{A}$ and $\vec{B}$ in a Cartesian coordinate system

$$\vec{A} = A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}$$

$$\vec{B} = B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}$$

Then the addition of two vectors is equivalent to adding their corresponding x, y and z components.

$$\vec{A} + \vec{B} = [A_x + B_x]\,\hat{i} + [A_y + B_y]\,\hat{j} + [A_z + B_z]\,\hat{k}$$

Similarly the subtraction of two vectors

$$\vec{A} - \vec{B} = [A_x - B_x]\,\hat{i} + [A_y - B_y]\,\hat{j} + [A_z - B_z]\,\hat{k}$$

The above rules form an analytical way of adding and subtracting two vectors.

13. MULTIPLICATION OF VECTOR BY A SCALAR [2 MARK]

A vector $\vec{A}$ multiplied by a scalar λ results in another vector, $\lambda\,\vec{A}$

If λ is a positive number then $\lambda\,\vec{A}$ is also in the direction of $\vec{A}$

If λ is a negative number then $\lambda\,\vec{A}$ is the opposite direction to the vector $\vec{A}$

SCALAR PRODUCT OF TWO VECTORS

14. DOT PRODUCT [2 MARK]

The scalar product of two vectors is defined as the product of the magnitudes of both the vectors and the cosine of the angle between them.

$$\vec{A}.\vec{B} = AB\cos\theta$$

15. PROPERTIES [3 MARK]

(i) The product quantity $\vec{A}.\vec{B}$ is always a scalar. It is positive if the angle between the vectors is acute ($\theta < 90°$) and negative if the angle between them is obtuse (i.e. $90° < \theta < 180°$).

(ii) The scalar product is commutative,

$$\vec{A}.\vec{B} = \vec{B}.\vec{A}$$

(iii) The vectors obey distributive law

$$\vec{A}.[\vec{B} + \vec{C}] = \vec{A}.\vec{B} + \vec{A}.\vec{C}$$

(iv) The angle between the vectors

$$\theta = \cos^{-1}\frac{[\vec{A}.\vec{B}]}{AB}$$

(v) The scalar product of two vectors will be maximum when cos θ = 1, i.e. θ = 0°, i.e., when the vectors are parallel;

$$(\vec{A}.\vec{B})_{MAX} = AB$$

(vi) The scalar product of two vectors will be minimum, when cos θ = –1, i.e., θ = 180°

$$(\vec{A}.\vec{B})_{MIN} = -AB$$

when the vectors are anti-parallel.

(vii) If two vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other then their scalar product $\vec{A}.\vec{B}= 0$, because cos 90°=0. Then the vectors $\vec{A}$ and $\vec{B}$ are said to be mutually orthogonal.

(viii) The scalar product of a vector with itself is termed as self–dot product and is given by

$$[\vec{A}.]^2 = \vec{A}.\vec{A} = AA\cos\theta = A^2.$$

Here angle θ = 0°

The magnitude or norm of the vector

$\vec{A}$ is $|\vec{A}| = A = \sqrt{\vec{A}\vec{A}}$

(ix) A unit vector $\hat{n}$

$$\hat{n}.\hat{n} = |x| \text{ x } \cos 0 = 1 \text{ . For eg : } \hat{\imath}.\hat{\imath}=\hat{\jmath}.\hat{\jmath}=\hat{k}.\hat{k} =1$$

(x) Orthogonal unit vectors $\hat{\imath},\hat{\jmath}$ and $\hat{k}$

$$\hat{\imath}.\hat{\jmath}=\hat{\jmath}.\hat{k}=\hat{k}.\hat{\imath} =1.1= \cos 90° = 0$$

(xi) Components the scalar product

$$\vec{A}.\vec{B}=[\ A_x\hat{\imath}\ + A_y\hat{\jmath}\ + A_z\hat{k}\ ].[\ B_x\hat{\imath}\ + B_y\hat{\jmath}\ + B_z\hat{k}\ ]$$

$$=A_xB_x + A_yB_y + A_zB_z$$

MAGNITUDE OF VECTOR $|\vec{A}|$:-

$$|\vec{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

VECTOR PRODUCT OF TWO VECTORS

16. CROSS PRODUCT **[2 MARK]**

The vector product of two vectors is defined as another vector having a magnitude equal to the product of the magnitudes of two vectors and the sine of the angle between them. The direction of the product vector is perpendicular to the plane containing the two vectors, in accordance with the right hand screw rule or right hand thumb rule

$$\vec{C} = \vec{A} \times \vec{B} = [AB\sin\theta]\, \hat{n}$$

17. PROPERTIES **[3 MARK]**

1. The vector product of any two vectors is always another vector whose direction is perpendicular to the plane containing these two vectors, i.e., orthogonal to both the vectors $\vec{A}$ and $\vec{B}$, even though the vectors $\vec{A}$ and $\vec{B}$ may or may not be mutually orthogonal.
2. The vector product of two vectors is not commutative,

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

But , $\vec{A} \times \vec{B} = -[\vec{B} \times \vec{A}]$

$$|\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}| = AB\sin\theta$$

The magnitudes are equal but directions are opposite to each other.

3. The vector product of two vectors will have maximum magnitude

$\sin\theta = 1$; $\theta = 90°$

$\vec{A}$ and $\vec{B}$ are orthogonal to each other.

$$(\vec{A} \times \vec{B})_{Max} = AB\, \hat{n}$$

4. The vector product of two non–zero vectors will be minimum

$|\sin\theta| = 0$; $\theta = 0°$ or $180°$

$$(\vec{A} \times \vec{B})_{Min} = 0$$

The vector product of two non–zero vectors vanishes, if the vectors are either parallel or antiparallel.

5. The self–cross product, i.e., product of a vector with itself is the null vector

$$\vec{A} \times \vec{A} = AA\sin°\, \hat{n} = \vec{0}$$

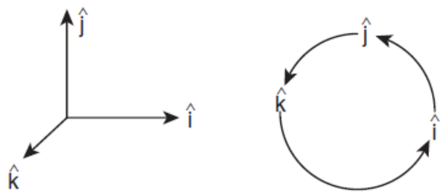
In physics the null vector $\vec{0}$ is simply denoted as zero.

6. The self–vector products of unit vectors are thus zero.

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = \vec{0}$$

7. The orthogonal unit vectors $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ in accordance with the right hand screw rule

$$\hat{\imath} \times \hat{\jmath} = \hat{k};\ \hat{\jmath} \times \hat{k} = \hat{\imath};\ \hat{k} \times \hat{\imath} = \hat{\jmath}$$



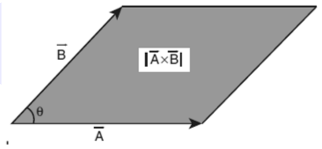
$$\hat{\jmath} \times \hat{\imath} = -\hat{k};\ \hat{k} \times \hat{\jmath} = -\hat{\imath};\ \hat{\imath} \times \hat{k} = -\hat{\jmath}$$

8. In terms of components, the vector product of two vectors $\vec{A}$ X $\vec{B}$ is,

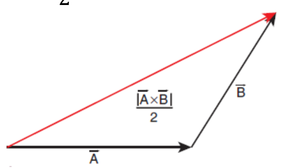
$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{\imath}\,[A_y B_z - A_z B_y] + \hat{\jmath}\,[A_z B_x - A_x B_z] + \hat{k}\,[A_x B_y - A_y B_x]$$

9. If two vectors form $\vec{A}$ and $\vec{B}$ adjacent sides in a parallelogram, then the magnitude of $\vec{A}$ X $\vec{B}$ will give the area of the parallelogram



10. Since we can divide a parallelogram into two equal triangles, the area of a triangle with $\vec{A}$ and $\vec{B}$ as sides is $\frac{1}{2}|\vec{A} \times \vec{B}|$



A number of quantities used in Physics are defined through vector products. Particularly physical quantities representing rotational effects like torque, angular momentum, are defined through vector products.

EXAMPLES

1. Torque $\vec{\tau} = \vec{r} \times \vec{F}$
2. Angular momentum $\vec{L} = \vec{r} \times \vec{P}$
3. Linear velocity $\vec{V} = \vec{w} \times \vec{r}$

Here ,

$\vec{r}$ = positional vector of a particle

$\vec{F}$ = Force

$\vec{P}$ = Linear momentum

$\vec{w}$ = angular velocity

18. PROPERTIES OF THE COMPONENTS OF VECTORS [2 MARK]

If two vectors $\vec{A}$ and $\vec{B}$ are equal, then their individual components are also equal.

Let $\vec{A} = \vec{B}$

Then ,

$$A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

$$A_x = B_x \; ; A_y = B_y \; ; A_z = B_z$$

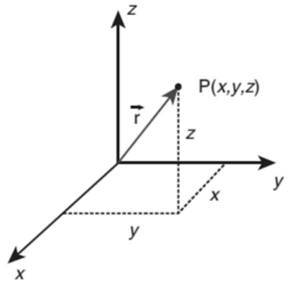
19. POSITION VECTOR [2 MARK]

It is a vector which denotes the position of a particle at any instant of time, with respect to some reference frame or coordinate system.

The position vector $\hat{r}$ of the particle at a point P is given by

$$\hat{r} = x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}$$

Where x ,y and z are components of $\hat{r}$



20. DISTANCE AND DISPLACEMENT [2 MARK]

Distance is the actual path length travelled by an object in the given interval of time during the motion. It is a positive scalar quantity.

Displacement is the difference between the final and initial positions of the object in a given interval of time.

The shortest distance between these two positions of the object and its direction is from the initial to final position of the object, during the given interval of time. It is a vector quantity.

21. DISPLACEMENT VECTOR IN CARTESIAN [2 MARK]

Let us consider a particle moving from a point P_1 having position vector

$\vec{r_1} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$ to point P_2

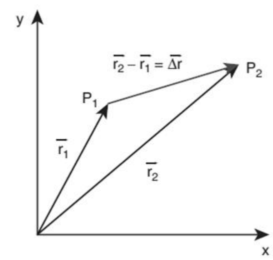
where its position vector is ,

$$\vec{r_2} = x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}$$

The displacement vector is given by,

$$\Delta\vec{r} = \vec{r_2} - \vec{r_1}$$

$$= [x_2 - x_1]\,\hat{\imath} + [y_2 - y_1]\,\hat{\jmath} + [z_2 - z_1]\,\hat{k}$$



DIFFERENTIAL CALCULAS FORMULAE

1. $\frac{d}{dx}[c] = 0$ [c ia a constant]
2. Y =cu [c ia a constant ,u is a function of x]

$$\frac{dy}{dx} = \frac{d}{dx}[cu] = c\frac{du}{dx}$$

3. Y = u $\pm v \pm w$ [u,v and w => function of x]

$$\frac{dy}{dx} = \frac{d}{dx}[u \pm v \pm w] = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx}$$

4. Y=x^n [n=> real numbers]

$$\frac{dy}{dx} = \frac{d}{dx}[x^n] = nx^{n-1}$$

5. Y=uv [u and v => function of x]

$$\frac{dy}{dx} = \frac{d}{dx}[uv] = u\frac{dv}{dx} + v\frac{du}{dx}$$

6. $\frac{d}{dx}[e^x] = e^x$
7. $\frac{d}{dx}[\ln x] = \frac{1}{x}$
8. $\frac{d}{dx}[\sin\theta] = \cos\theta$
9. $\frac{d}{dx}[\cos\theta] = -\sin\theta$

10. [Y => trigonometric function of θ and

θ => function of t]

$$\frac{d}{dx}[\sin\theta] = \cos\theta \, . \frac{d\theta}{dt}$$

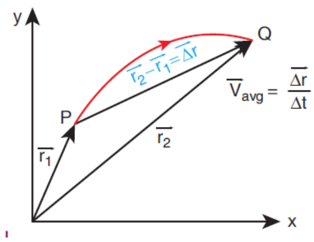
$$\frac{d}{dx}[\cos\theta] = -\sin\theta \, . \frac{d\theta}{dt}$$

INTEGRAL FORMULAE

1. $\int dx = x \, ; \frac{d}{dx}[x] = 1$
2. $\int x^n dx = \frac{x^{n+1}}{n+1} \, ; \quad \frac{d}{dx}\left[\frac{x^{n+1}}{n+1}\right] = x^n$
3. $\int cudx = c\int udx$ [c ia a constant]
4. $\int[u \pm v \pm w]dx = \int udx \pm \int vdx \pm \int wdx$
5. $\int \frac{1}{x}.dx = \ln x + c$
6. $\int e^x . dx = e^x + c$
7. $\int \cos\theta . d\theta = sin\theta + c$
8. $\int \sin\theta . d\theta = -cos\theta + c$

22. AVERAGE VELOCITY **[3 MARK]**

Consider a particle located initially at point P having position vector $\vec{r}_1$. In a time interval Δt the particle is moved to the point Q having position vector $\vec{r}_2$. The displacement vector is $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$



Ratio of the displacement vector to the corresponding time interval is known as the average velocity

$$\vec{V}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$$

The direction of average velocity is in the direction of the displacement vector[$\Delta\vec{r}$]

23. AVERAGE SPEED **[2 MARK]**

The ratio of total path length travelled by the particle in a time interval.

$$\text{Average speed} = \frac{\text{total path length}}{total\ time}$$

24. INSTANTEANOUS VELOCITY OR VELOCITY **[3 MARK]**

As limiting value of the average velocity as $\Delta t \to 0$, evaluated at time t.

In other words, velocity is equal to rate of change of position vector with respect to time. It is a vector quantity.

$$\vec{V} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

In component form,

$$\vec{V} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left[x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right]$$

$$= \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k}$$

Here , $\frac{dx}{dt} = V_x$ = x – component of velocity

$\frac{dy}{dt} = V_y$ = y – component of velocity

$\frac{dz}{dt} = V_z$ = z – component of velocity

The magnitude of velocity 'V' is called speed is ,

$$V = \sqrt{V_x^2 + V_y^2 + V_z^2}$$

Speed is always a positive scalar. The unit of speed is also meter per second.

25. MOMENTUM **[2 MARK]**

The product of mass with velocity.

$$\vec{P} = m\vec{V} \text{ [or] } p = mv$$

Direction is the direction of velocity ,

In component ,

$$P_x\hat{\imath} + P_y\hat{\jmath} + P_z\hat{k} = mV_x\hat{\imath} + mV_y\hat{\jmath} + mV_y\hat{k}$$

P_x = x component of momentum = mV_x

P_y = y component of momentum = mV_y

P_z = z component of momentum = mV_y

UNIT => kg ms^{-1}

EXAMPLE

Consider a butterfly and a stone, both moving towards you with the same velocity 5 ms^{-1}. If both hit your body, the effects will not be the same. The effects not only depend upon the velocity, but also on the mass. The stone has greater mass compared to the butterfly. The momentum of the stone is thus greater than the momentum of the butterfly.

MOTION ALONG ONE DIMENSION

26. AVERAGE VELOCITY [3 MARK]

If a particle moves in one dimension, say for example along the x direction, then

$$\text{The average velocity} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

It is a vector quantity. But in one dimension we have only two directions (positive and negative x direction), hence we use positive and negative signs to denote the direction.

The instantaneous velocity or velocity is,

$$V = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Graphically the slope of the position-time graph will give the velocity of the particle. At the same time, if velocity time graph is given, the distance and displacement are determined by calculating the area under the curve.

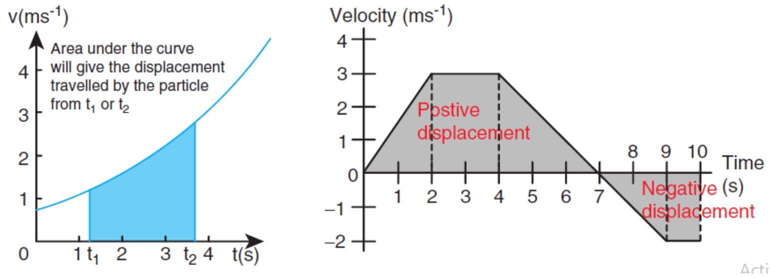
$$V = \frac{dx}{dt}$$

$$dx = V.dt$$

$$\int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} V.dt$$

$\int_{t_1}^{t_2} V.dt$ represents the area under the curve v as a function of time.

$\int_{x_1}^{x_2} dx$ => represents the displacement travelled by the particle from time $t1$ to $t2$, the area under the velocity time graph will give the displacement of the particle. If the area is negative, it means that displacement is negative, so the particle has travelled in the negative direction.



27. RELATIVE VELOCITY IN ONE AND TWO DIMENSIONAL MOTION

[5 MARK]

Two objects A and B are moving with different velocities, then the velocity of one object A with respect to another object B is called relative velocity of object A with respect to B.

CASE 1

Consider two objects A and B moving with uniform velocities V_A and V_B, , along straight tracks in the same direction $\vec{V}_A, \vec{V}_B$
with respect to ground.

The relative velocity of object A with respect to object B is,

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

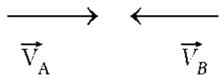
The relative velocity of object B with respect to A is,

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$

The magnitude of relative velocity of one object with respect to another is equal to the difference in magnitude of two velocities.

CASE 2

Consider two objects A and B moving with uniform velocities V_A and V_B along the same straight tracks but opposite in direction



The relative velocity of object A *with respect to* object B is

$$\vec{V}_{AB} = \vec{V}_A - [-\vec{V}_B] = \vec{V}_A + \vec{V}_B$$

The relative velocity of object B *with respect to* object A is

$$\vec{V}_{BA} = -\vec{V}_B - \vec{V}_A = -[\vec{V}_A + \vec{V}_B]$$

The magnitude of relative velocity of one object with respect to other is equal to the sum of magnitude of their velocities.

CASE 3

Consider the velocities $\vec{V}_A$ *and* $\vec{V}_B$ at an angle θ between their directions.

The relative velocity of A *with respect to* B is

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

Then , the magnitude and direction of $\vec{V}_{AB}$ is ,

$$V_{AB} = \sqrt{V_A^2 + V_B^2 + 2V_A V_B cos\theta}$$

$\tan \beta = \frac{V_B sin\theta}{V_B - V_B cos\theta}$ (Here β is angle between $\vec{V}_{AB}$ and $\vec{V}_B$)

i. The bodies move along parallel straight lines in the same direction,

$\theta = 0°$,

$V_{AB} = (V_A - V_B)$ in the direction of $\vec{V}_A$.

Obviously ,

$V_{BA} = (V_B - V_A)$ in the direction of $\vec{V}_B$

ii. $\theta = 180°$,In the opposite directions

$V_{AB} = (V_A + V_B)$ in the direction of $\vec{V}_A$.

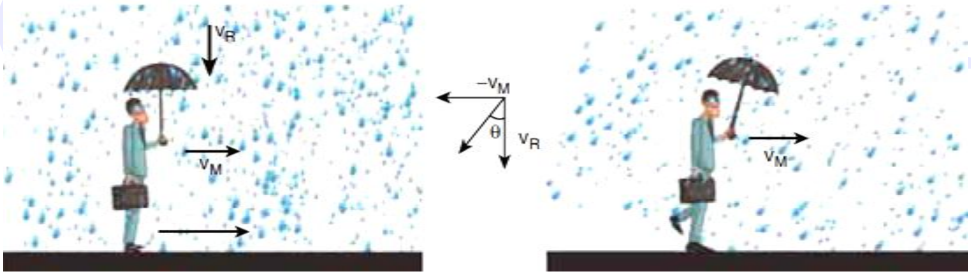
Similarly ,

$V_{BA} = (V_B + V_A)$ in the direction of $\vec{V}_B$

iii. If two bodies are moving at right angles to each other, then $\theta = 90°$.

$V_{AB} = \sqrt{V_A^2 + V_B^2}$

iv. Consider a person moving horizontally with velocity $\vec{V}_M$. Let rain fall vertically with velocity $\vec{V}_R$. An umbrella is held to avoid the rain. Then the relative velocity of the rain with respect to the person



$$\vec{V}_{RM} = \vec{V}_R - \vec{V}_M$$

Magnitude ,

$$V_{RM} = \sqrt{V_R^2 + V_M^2}$$

Direction ,

$$\theta = \tan^{-1}\left[\frac{V_M}{V_R}\right]$$

In order to save himself from the rain, he should hold an umbrella at an angle θ with the vertical.

28. ACCELERATED MOTION **[2 MARK]**

During non-uniform motion of an object, the velocity of the object changes from instant to instant i.e., the velocity of the object is no more constant but changes with time. Such a motion is said to be an accelerated motion.

UNIFORM ACCELERATED MOTION

In accelerated motion, if the change in velocity of an object per unit time is same then the object is said to be
moving with uniformly accelerated motion.

NON-UNIFORM ACCELERATED MOTION

If the change in velocity per unit time is different at different times, then the object is said to be moving with non-uniform accelerated motion.

29. AVERAGE ACCELERATION **[2 MARK]**

The ratio of change in velocity over the time interval $\Delta t = t_2 - t_1$

$$\vec{a}_{AVG} = \frac{\vec{V}_2 - \vec{V}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}$$

Average acceleration is a vector quantity in the same direction as the vector $\Delta\vec{v}$

30. INSTANTANEOUS ACCELERATION **[3 MARK]**

The ratio of change in velocity over Δt, as Δt approaches zero.

$$\text{Acceleration } \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

the acceleration of the particle at an instant t is equal to rate of change of velocity.

It is a vector quantity .

UNIT=> ms^{-2}

DIMENSIONAL FORMULA => $M^0L^1T^{-2}$

Acceleration is positive if its velocity is increasing, and is negative if the velocity is decreasing. The negative acceleration is called Retardation or deceleration.

In terms of components,

$$\vec{a} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k} = \frac{d\vec{v}}{dt}$$

$a_x = \frac{dv_x}{dt}, a_y = \frac{dv_y}{dt}, a_z = \frac{dv_z}{dt}\hat{k}$ are the components of instantaneous acceleration. Since each component of velocity is the derivative of the corresponding coordinate, the components are a_x, a_y and a_z .

$$a_x = \frac{d^2x}{dt^2} ; a_y = \frac{d^2y}{dt^2} ; a_z = \frac{d^2z}{dt^2}$$

The acceleration ,

$$\vec{a} = \frac{d^2x}{dt^2}\hat{\imath} + \frac{d^2y}{dt^2}\hat{\jmath} + \frac{d^2z}{dt^2}\hat{k} = \frac{d^2\vec{r}}{dt^2}$$

Acceleration is the second derivative of position vector with respect to time.

Graphically the acceleration is the slope in the velocity-time graph.

If the acceleration-time graph is given, then the velocity can be found from the area under the acceleration-time graph.

From $\frac{dv}{dt}$ = a ,

dv = a.dt

hence , V = $\int_{t_1}^{t_2} a.dt$

t_2 => Initial time

t_1 => Final time

31. EQUATIONS OF UNIFORMLY ACCELERATED MOTION BY CALCULAS METHOD [5 MARK]

Consider an object moving in a straight line with uniform or constant acceleration 'a'.

Let u be the velocity of the object at time t = 0, and v be velocity of the body at a later time t.

VELOCITY- TIME RELATION

(i) The acceleration of the body at any instant is given

a = $\frac{dv}{dt}$ [or] dv =a.dt

Integrating both sides ,

time changes from 0 to t,

velocity changes from u to v.

For the constant acceleration,

$$\int_u^v dv = \int_0^t a.dt = a\int_0^t dt$$

$$[\text{ V }]_u^v = a\,[\text{ t }]_o^t$$

V – u =at [or] V = u + at ⟶ (1)

DISPLACEMENT - TIME RELATION

(ii) The velocity of the body is

V = $\frac{ds}{dt}$ [or] ds =V.dt

Since v = u + at,

ds = [u +at] dt

Time t = 0, the particle started from the origin. At a later time t, the particle displacement is s.

$$\int_0^s ds = \int_0^t u.dt = \int_0^t at.dt$$

$$s = ut + ½ at^2 \longrightarrow (2)$$

VELOCITY- DISPLACEMENT RELATION

(iii) The acceleration is

$$a = \frac{dv}{dt}$$

$$= \frac{dv}{dt} \cdot \frac{ds}{ds} = \frac{dv}{ds} \cdot \frac{ds}{dt} = \frac{dv}{ds}.V \qquad [\ v = \frac{ds}{dt}]$$

a.ds = V.dv

Integrating the above equation,

velocity changes from u to v,

displacement changes from 0 to s,

$$a\int_0^s ds = \int_u^v v.dv$$

$$a\ [s - 0] = \left[\frac{v^2}{2}\right]_u^v$$

$$as = \frac{v^2}{2} - \frac{u^2}{2}$$

$$2as = v^2 - u^2$$

$$v^2 = u^2 + 2as$$

From equ (1)

at = V- u

$$s = ut + ½ at^2$$

s = ut + ½ [V- u]t

$$s = \frac{[u+v]t}{2}$$

NOT FOR EXAMINATION

s = ut + ½ [V- u]t

s = ut + ½ vt - ½ ut

s = ut - ½ ut + ½ vt

s = ½ ut + ½ vt

s = ½ t [u + v]

$$s = \frac{[u+v]t}{2}$$

KINEMATICS EQUATION

V = u +at

$$s = ut + ½ at^2$$

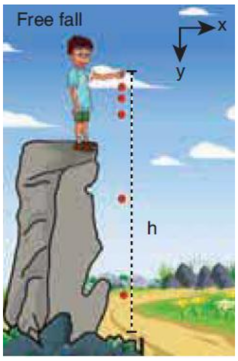
$$v^2 = u^2 + 2as$$

$$s = \frac{[u+v]t}{2}$$

These above equations are valid only if the motion is in a straight line with constant acceleration. Not valid in circular motion and oscillatory motion

32. EQUATION OF MOTION UNDER GRAVITY [5 MARK]

CASE (i) A BODY FALLING FROM A HEIGHT 'H'



Consider an object of mass m falling from a height h. Assume there is no air resistance. For convenience, let us choose the downward direction as positive y-axis. The object experiences acceleration 'g' due to gravity which is constant near the surface of the Earth.

The acceleration $\vec{a} = g\vec{j}$

By comparing the components, we get

$a_x = 0$; $a_z = 0$; $a_y = g$

$a_y = a = g$

If the particle is thrown with initial velocity 'u' downward which is in negative y axis, then velocity and position at of the particle any time t is

$V = u + gt$

$Y = ut + \frac{1}{2} gt^2$

$v^2 = u^2 + 2gy$

Suppose the particle starts from rest.

$u=0$

$V = gt$

$Y = \frac{1}{2} gt^2$

$v^2 = 2gy$

The time (t = T) taken by the particle to reach the ground (y = h)

$h = \frac{1}{2} gt^2$

$T = \sqrt{2h/g}$

Greater the height(h), particle takes more time(T) to reach the ground. For lesser height, it takes lesser time to reach the ground.

The speed of the particle ,(y = h)

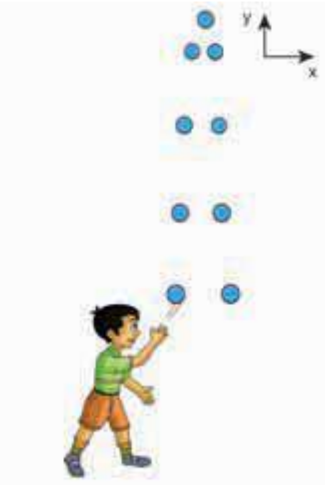
$$V_{ground} = \sqrt{2gh}$$

The body falling from greater height(h) will have higher velocity when it reaches the ground.

FREE FALL [2 MARK]

The motion of a body falling towards the Earth from a small altitude (h << R), purely under the force of gravity

CASE (ii) A BODY THROWN VERTICALLY UPWARDS



Consider an object of mass m thrown vertically upwards with an initial velocity u. Let us neglect the air friction. In this casewe choose the vertical direction as positive y axis, then the acceleration a = -g and g points towards the negative y axis.

The velocity and position of the object at any time t are,

$$V = u - gt$$

$$S = ut - ½ gt^2$$

$$v^2 = u^2 - 2gy$$

33. PROJECTILE MOTION [2 MARK]

When an object is thrown in the air with some initial velocity, and then allowed to move under the action of gravity alone, the object is known as a projectile. The path followed by the particle is called its **trajectory.**

Examples of projectile are

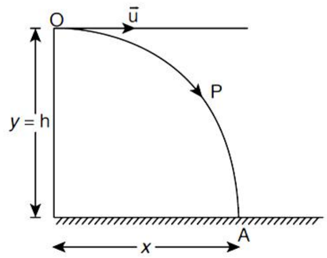
1. An object dropped from window of a moving train.
2. A bullet fired from a rifle.
3. A ball thrown in any direction.
4. A javelin or shot put thrown by an athlete.
5. A jet of water issuing from a hole near the bottom of a water tank.

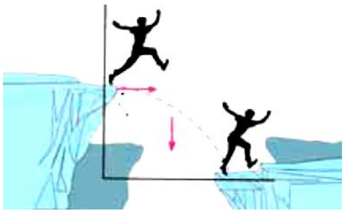
34. TYPES OF PROJECTILE MOTION [2 MARK]

i. Projectile given an initial velocity in the horizontal direction (horizontal projection)
ii. Projectile given an initial velocity at an angle to the horizontal (angular projection)

35. PROJECTILE IN HORIZONTAL PROJECTION [5 MARK]

Consider a projectile, say a ball, thrown horizontally with an initial velocity $\vec{u}$ from the top of a tower of height h





As the ball moves, it covers a horizontal distance due to its uniform horizontal velocity u, and a vertical downward distance because of constant acceleration due to gravity g. Thus, under the combined effect the ball moves along the path OPA.

The motion is in a 2-dimensional plane. Let the ball take time t to reach the ground at point A, Then the horizontal distance travelled by the ball is x (t) = x, and the vertical distance travelled is y (t) = y

We can apply the kinematic equations along the x direction and y direction separately. Since this is two-dimensional motion, the velocity will have both horizontal component u_x and vertical component u_y.

MOTION ALONG HORIZONTAL DIRECTION

The particle has zero acceleration along x direction. So, the initial velocity u_x remains constant throughout the motion.

The distance traveled by the projectile at a time t is given by the equation

$$x = u_x t + \frac{1}{2} at^2$$

Since a = 0 along x direction,

$$x = u_x t$$

MOTION ALONG DOWNWARD DIRECTION

Here $u_y = 0$; a= g and distance Y at time t.

$$Y = u_y t + \frac{1}{2} gt^2$$

$$Y = \frac{1}{2} gt^2$$

$$Y = \frac{1}{2} gt^2 = \frac{1}{2} g\frac{x^2}{u_x^2} = \left[\frac{g}{2u_x^2}\right] x^2 \qquad \left\{ \begin{matrix} x = u_x t \\ t = x/u_x \end{matrix} \right\}$$

$$Y = Kx^2 \ ; \quad K = \frac{g}{2u_x^2} \text{ is constant}$$

Thus, the path followed by the projectile is a parabola (curve OPA)

- **TIME OF FLIGHT**

The time taken for the projectile to complete its trajectory or time taken by the projectile to hit the ground is called time of flight.

Consider the example of a tower and projectile. Let h be the height of a tower. Let T be the time taken by the projectile to hit the ground, after being thrown horizontally from the tower.

$S_y = u_y t + \frac{1}{2} at^2$ for vertical motion.

Here , $S_y = h$, $t=T$, $u_y = 0$

$$h = \frac{1}{2} gt^2$$

$$T = \sqrt{\frac{2h}{g}}$$

➢ HORIZANTAL RANGE

The horizontal distance covered by the projectile from the foot of the tower to the point where the projectile hits the ground is called **horizontal range.**

$$S_x = u_x t + \frac{1}{2} a t^2$$

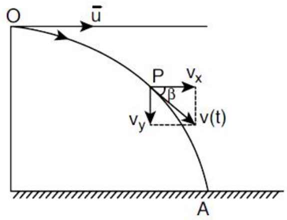
Here , S_x =R , u_x = u ,a=0 ;t =T

$$R = uT \qquad \left[T = \sqrt{\frac{2h}{g}}\right]$$

$$R = u\sqrt{\frac{2h}{g}} \qquad R \propto u \ ; R \propto \frac{1}{g}$$

➢ RESULTANT VELOCITY

At any instant t, the projectile has velocity components along both x-axis and y-axis. The resultant of these two components gives the velocity of the projectile at that instant t,



The velocity component at any t along horizontal (x-axis) is $V_x = u_x + a_x t$

$$u_x = 0 \ ; a_x = 0,$$

$$V_x = u$$

The component of velocity along vertical direction (y-axis) is $V_y = u_y + a_y t$

$$\text{Since, } u_y = u, \ a_y = g$$

$$V_y = gt$$

The velocity of the particle at any instant is

$$\vec{V} = u\hat{i} + g + \hat{j}$$

The speed of the particle at any instant t is given by

$$V = \sqrt{u_x^2 + v_y^2}$$

$$V = \sqrt{u^2 + g^2 t^2}$$

➢ SPEED OF PROJECTILE WHEN IT HITS THE GROUND

The time of flight is

$$T = \sqrt{\frac{2h}{g}}$$

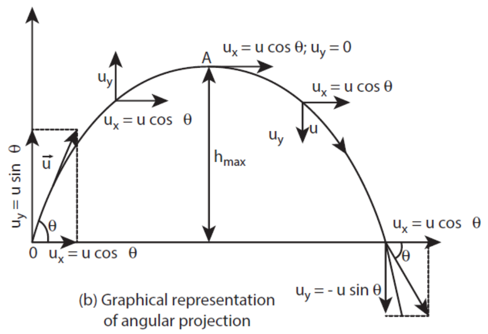
$V_x = u$, The vertical component velocity of the projectile at time T is

$$V_y = gT = g\sqrt{\frac{2h}{g}} = \sqrt{2gh}$$

$$V = \sqrt{u^2 + 2gh}$$

36. PROJECTILE UNDER AN ANGULAR PROJECTILE [5 MARK]

This projectile motion takes place when the initial velocity is not horizontal, but at some angle with the vertical.

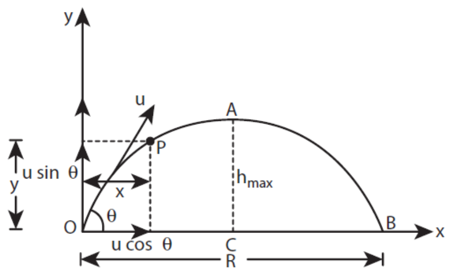


(b) Graphical representation of angular projection

Examples:

1. Water ejected out of a hose pipe held obliquely.
2. Cannon fired in a battle ground.

Consider an object thrown with initial velocity $\vec{u}$ at an angle θ with the horizontal.



$$\vec{u} = u_x\hat{i} + u_y\hat{j}$$

The horizontal component of velocity. $u_x = u\cos\theta$

The vertical component of velocity. $u_y = u\sin\theta$

Since the acceleration due to gravity is in the direction opposite to the direction of vertical component u_y, this component will gradually reduce to zero at the maximum height of the projectile. At this maximum height, the same gravitational force will push the projectile to move downward and fall to the ground. There is no acceleration along the x direction throughout the motion. So, the horizontal component of the velocity ($u_x = u\cos\theta$) remains the same till the object reaches the ground.

The horizontal motion,

$$V_x = u_x + a_x t = u_x = u\cos\theta$$

The horizontal distance travelled by projectile in time t is , $S_x = u_x t + ½ a_x t^2$

$$x = u\cos\theta \quad [\text{ or}] \quad t = \frac{x}{u\cos\theta}$$

Next, the vertical motion $v_y = u_y + a_y t$

$$u_y = u\sin\theta,\ a_y = -g$$

$$v_y = u\sin\theta - gt$$

The vertical distance travelled by the projectile in the same time *t* is

$$S_Y = u_Y t + ½ a_Y t^2$$

$$S_Y = y,\ u_y = u\sin\theta,\ a_y = -g$$

$$Y = u\sin\theta\, t - ½ gt^2 \qquad \left\{t = \frac{x}{u\cos\theta}\right\}$$

$$Y = u\sin\theta \frac{x}{u\cos\theta} - ½ gt^2 \frac{x^2}{u^2\cos^2\theta}$$

$$Y = x\tan\theta - ½ gt^2 \frac{x^2}{u^2\cos^2\theta}$$

Thus the path followed by the projectile is an inverted parabola.

MAXIMUM HEIGHT [h_{MAX}]

The maximum vertical distance travelled by the projectile during its journey

The vertical part of the motion,

$$V_y^2 = u_y^2 + 2 a_Y s$$

$$u_y = u\sin\theta t,\ a_y = -g,\ s = h_{MAX};\ V_y = 0$$

$$0 = u^2\sin^2\theta - 2g\, h_{MAX}$$

$$h_{MAX} = \frac{u^2\sin^2\theta}{2g}$$

TIME OF FLIGHT [T_f]

The total time taken by the projectile from the point of projection till it hits the horizontal plane.

This time taken by the projectile to go from point O to B via point A

$$S_Y = u_Y t + ½ a_Y t^2$$

$$S_Y = y = 0,\ u_y = u\sin\theta,\ a_y = -g,\ t = T_f$$

$$0 = u\sin\theta\, T_f - ½ g T_f^{\ 2}$$

$$T_f = 2u\frac{\sin\theta}{g}$$

HORIZONTAL RANGE [R]

Th e maximum horizontal distance between the point of projection and the point on the horizontal plane where the projectile hits the ground

R = Horizontal component of velocity x time of flight

= ucosθ X T_f

$$R = u\cos\theta \times 2u\frac{\sin\theta}{g} = \frac{2u^2\sin\theta\cos\theta}{g}$$

$$R = \frac{u^2\sin 2\theta}{g}$$

R directly depends on the initial speed (u) and the sine of angle of projection (θ). It inversely depends onacceleration due to gravity ‘g’

For a given initial speed u, the maximum possible range is reached when sin2θ is maximum, sin2θ = 1. This implies $2\theta = \frac{\pi}{2}$

$$\theta = \frac{\pi}{4}$$

The particle is projected at 45 degrees with respect to horizontal, it attains maximum range,

$$R_{max} = \frac{u^2}{g}$$

37. DEGREES AND RADIANS [2 MARKS]

In measuring angles, there are several possible units used, but the most common units are degrees and radians.

Radians are used in measuring area, volume, and circumference of circles and surface area of spheres.

RADIAN

The angle subtended at the center of a circle by an arc that is equal in length to the radius of the circle

DEGREE

The unit of measurement which is used to determine the size of an angle.When an angle goes all the way around in a circle

A circle has 360°, the full circle has 2π radian.

$360° = 2\pi$ radian.

π => irrational number
3.14 (or) $\frac{22}{7}$. This approximation to π and not equal to π

$$1 \text{ radian} = \frac{180}{\pi} \text{ degree}$$

1 rad =57.27°

38. ANGULAR DISPLACEMENT **[2 MARK]**

The angle described by the particle about the axis of rotation a given time

$$\theta = \frac{S}{r} \quad [\text{or}] \quad s = \theta r$$

39. ANGULAR VELOCITY [$\vec{\omega}$] **[2 MARK]**

The rate of change of angular displacement is called angular velocity.

$$\omega = \frac{d\theta}{dt} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

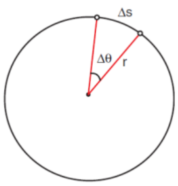
40. ANGULAR ACCELERATION[∝] **[2 MARK]**

The rate of change of angular velocity is called angular acceleration.

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

41. TANGENTIAL ACCELATION **[2 MARK]**

Consider an object moving along a circle of radius r. In a time Δt, the object travels an arc distance Δs. The corresponding angle subtended is $\Delta\theta$



$$\Delta s = r\,\Delta\theta$$

In a time Δt,

$$\frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$$

In the limit $\Delta t \to 0$,

$$\frac{ds}{dt} = r\,\omega$$

$\frac{ds}{dt} \to$ linear speed which is tangential to the circle

$V = r\,\omega$ $\omega \to$ Angular speed

This is true only for circular motion.

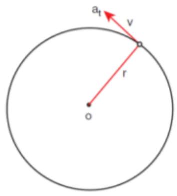
$$\vec{V} = \vec{\omega} \times \vec{r}$$

Differentiating the equation $V = \omega r$

$$\frac{dV}{dt} = r\,\frac{d\omega}{dt}$$

$$a_t = r\,\alpha$$

$a_t \to$ tangential acceleration



The tangential acceleration is in the direction of linear velocity.

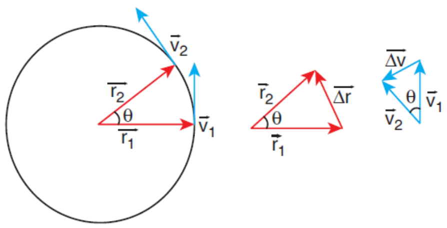
42. CIRCULAR MOTION [2 MARK]

When a point object is moving on a circular path with a constant speed, it covers equal distances on the circumference of the circle in equal intervals of time. Then the object is said to be in uniform circular motion.

43. CENTRIPETAL ACCELERATION [3 MARK]

The velocity is tangential at every point in the circle, the acceleration is acting towards the center of the circle. This is called centripetal acceleration. It always points towards the center of the circle.

The centripetal acceleration is derived from a simple geometrical relationship between position and velocity vectors.



For uniform circular motion,

$$r = |\vec{r}_1| = |\vec{r}_2|$$

$$v = |\vec{v}_1| = |\vec{v}_2|$$

If the particle moves from position vector $\vec{r}_1$ and $\vec{r}_2$. The displacement is,

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

The change in velocity from $\vec{v}_1$ to $\vec{v}_2$

$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$$

$$\frac{\Delta r}{r} = -\frac{\Delta v}{v} = 0$$

The negative sign implies that Δv points radially inward, towards the center of the circle.

$$\Delta v = -v\left[\frac{\Delta r}{r}\right]$$

$$a = \frac{\Delta v}{\Delta t} = -\frac{v}{r}\left[\frac{\Delta r}{\Delta t}\right] = -\frac{v^2}{r}$$

V = ωr ω → Angular velocity

The centripetal acceleration ,

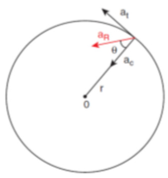
$a = -\omega^2 r$

44. NON- UNIFORM CIRCULAR MOTION

If the speed of the object in circular motion is not constant, then we have non-uniform circular motion.

EX;

The bob attached to a string moves in vertical circle



The resultant acceleration is obtained by vector sum of centripetal and tangential acceleration.

$$a_R = \sqrt{a_t^2 + \left[\frac{v^2}{r}\right]^2}$$

This resultant acceleration makes an angle θ with the radius vector,

This angle is, $\tan\theta = \frac{a_t}{\left[\frac{v^2}{r}\right]}$

45. KINEMATICS EQUATONS OF LINEAR MOTION AND ANGULAR MOTION [2 MARK]

Kinematic equations for linear motion	Kinematic equations for angular motion
$v = u + at$	$\omega = \omega_0 + \alpha t$
$s = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = u^2 + 2as$	$\omega^2 = \omega_0^2 + 2\alpha\theta$
$s = \frac{(v+u)t}{2}$	$\theta = \frac{(\omega_0 + \omega)t}{2}$

ω_0 => Initial angular velocity

ω => Final angular velocity

$\propto$ => Angular acceleration