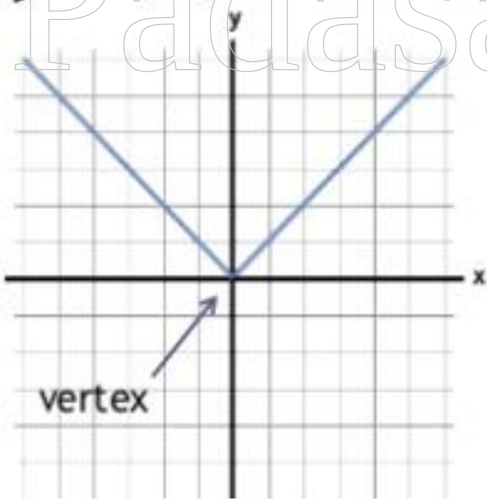
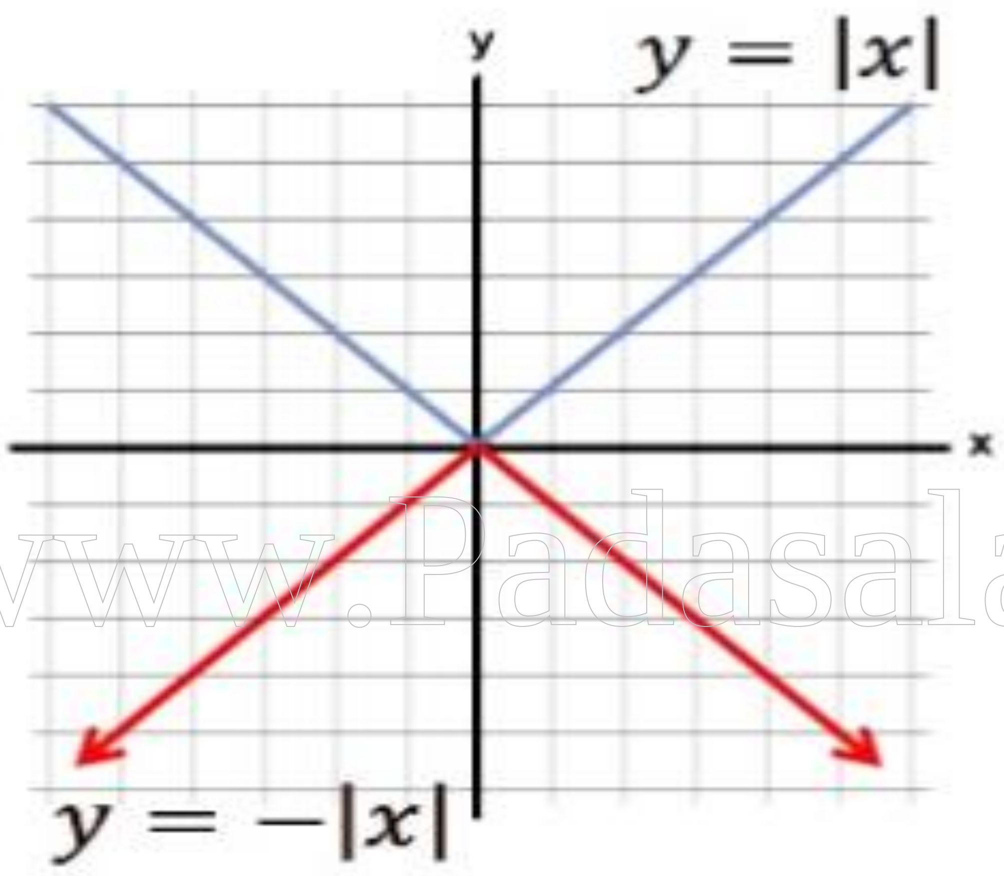
Absolute Value Functions

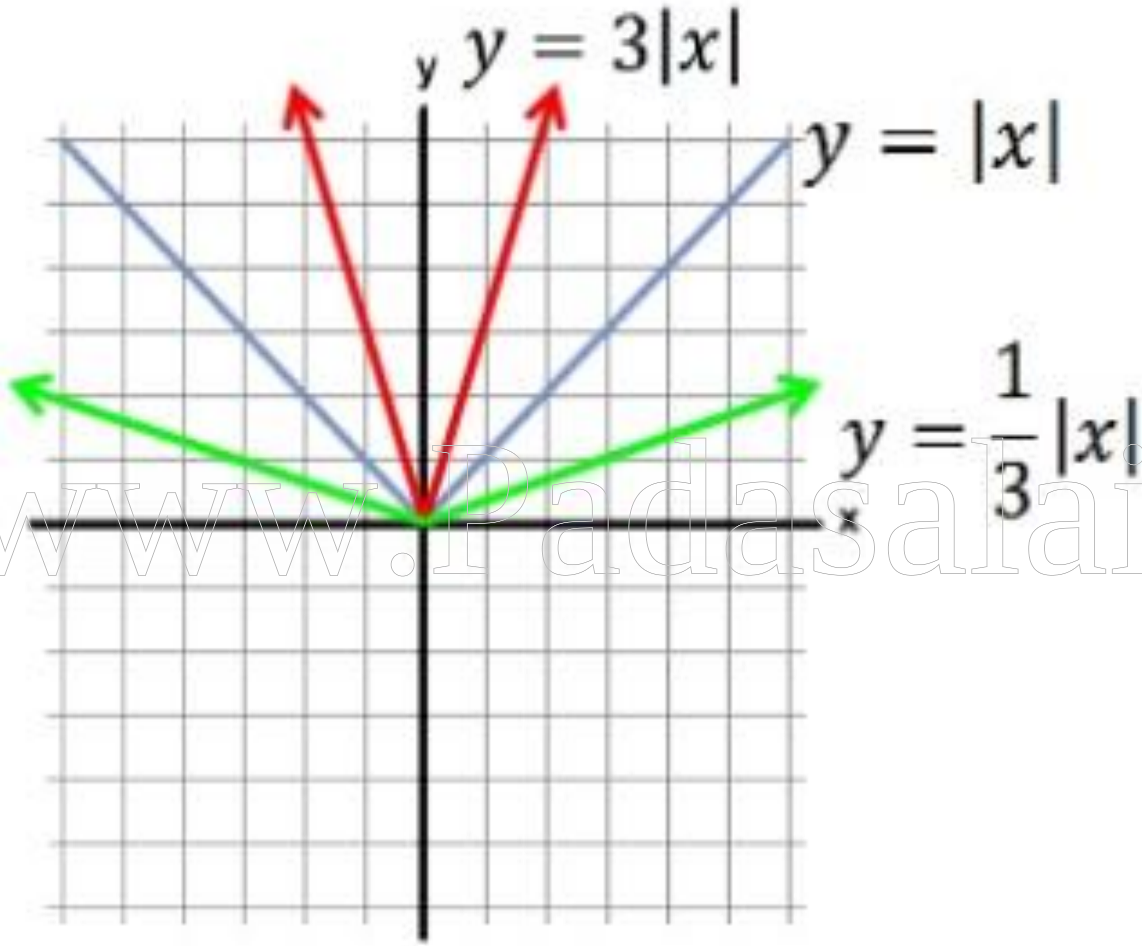
- The absolute value of x is defined by:

$$|x| = \begin{cases} x, & if\ x > 0 \\ 0, & if\ x = 0 \\ -x, & if\ x < 0 \end{cases}$$

- The graph of y = |x| looks like a v-shape.

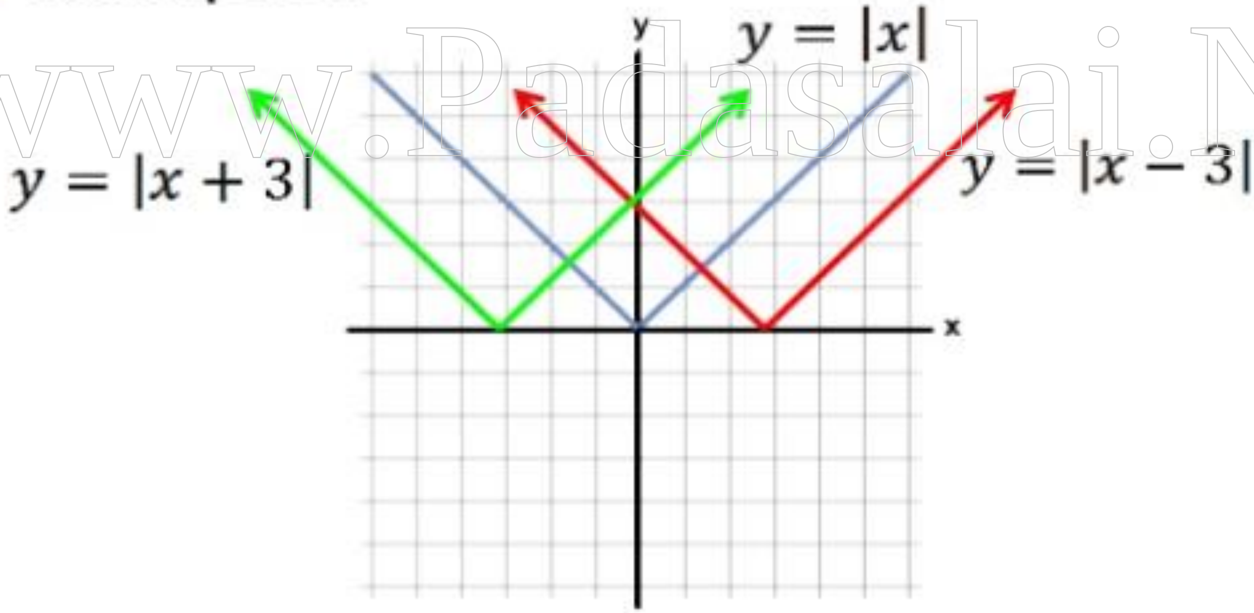



y
$y = |x|$
x
$y = -|x|$


y
$y = 3|x|$
$y = |x|$
$y = \frac{1}{3}|x|$
x

$$y = a\,|x - h| + k$$

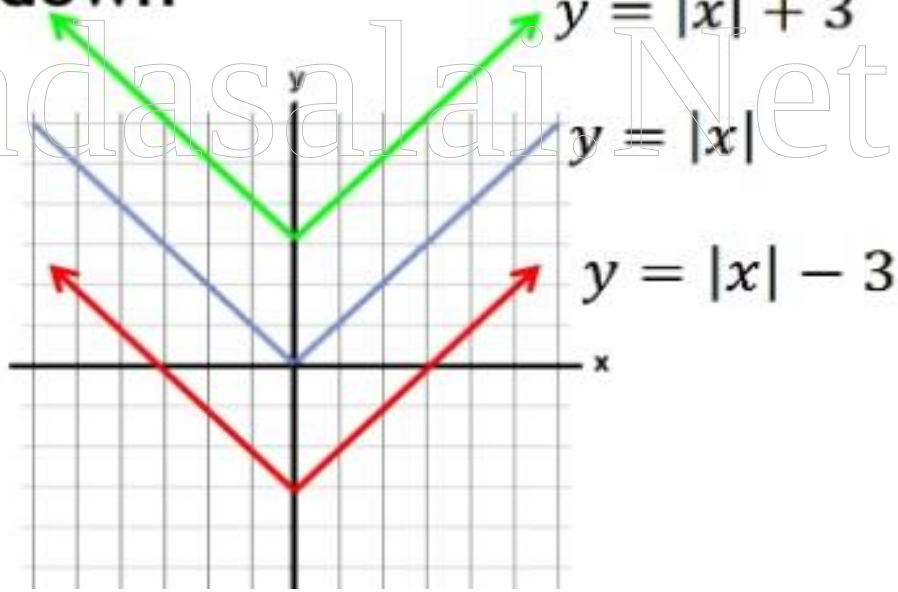
- **h** shifts the vertex left or right
- The direction is **opposite** the sign of **h**
- Examples:



Effects of **k**

$$y = a\,|x - h| + k$$

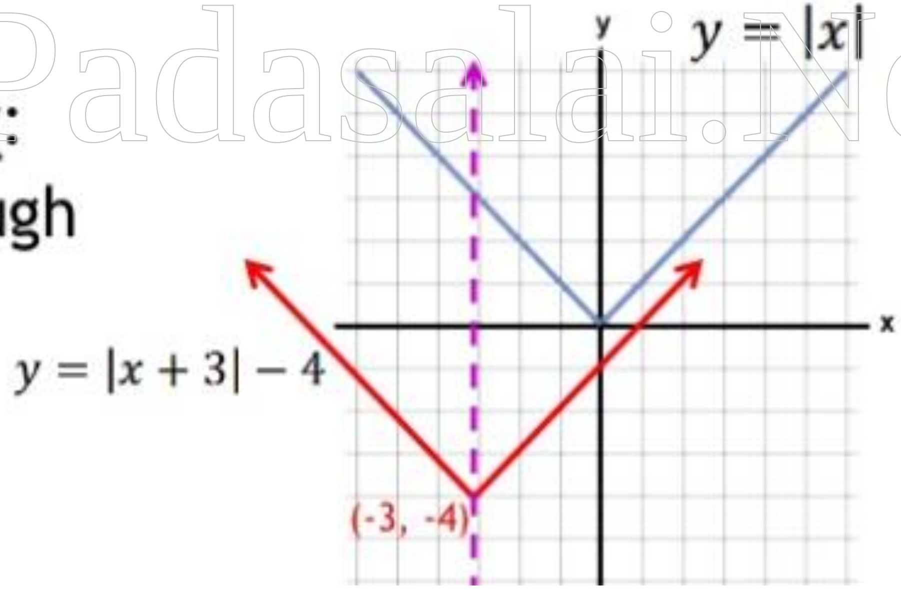
- **k** shifts the vertex up or down
- **Positive** k shifts **up**
- **Negative** k shifts **down**
- Examples:



The Vertex

- The vertex will be at (h, k).
- Example: $y = |x + 3| - 4$
- Vertex: (-3, -4)

- **Axis of symmetry:** Vertical line through the vertex

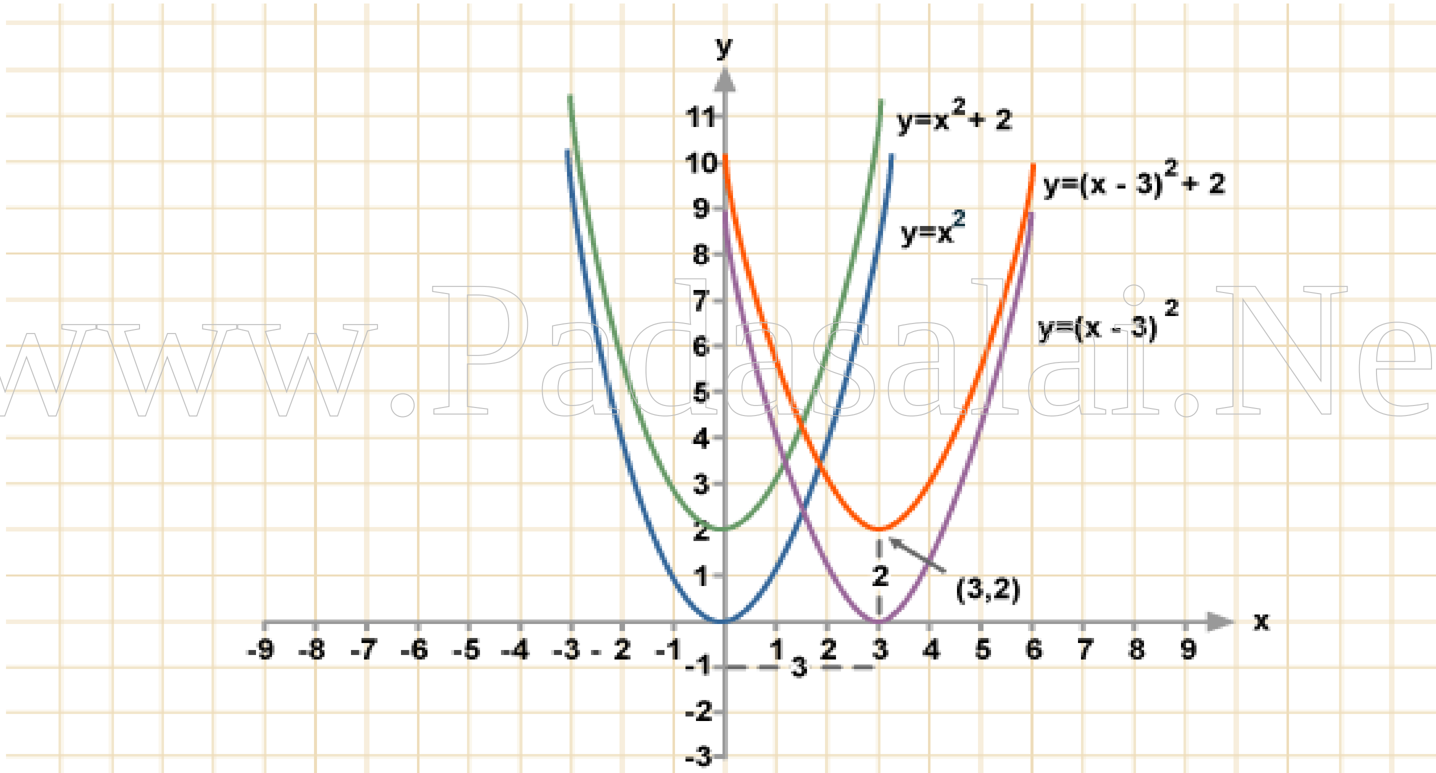


Below are the graphs of:

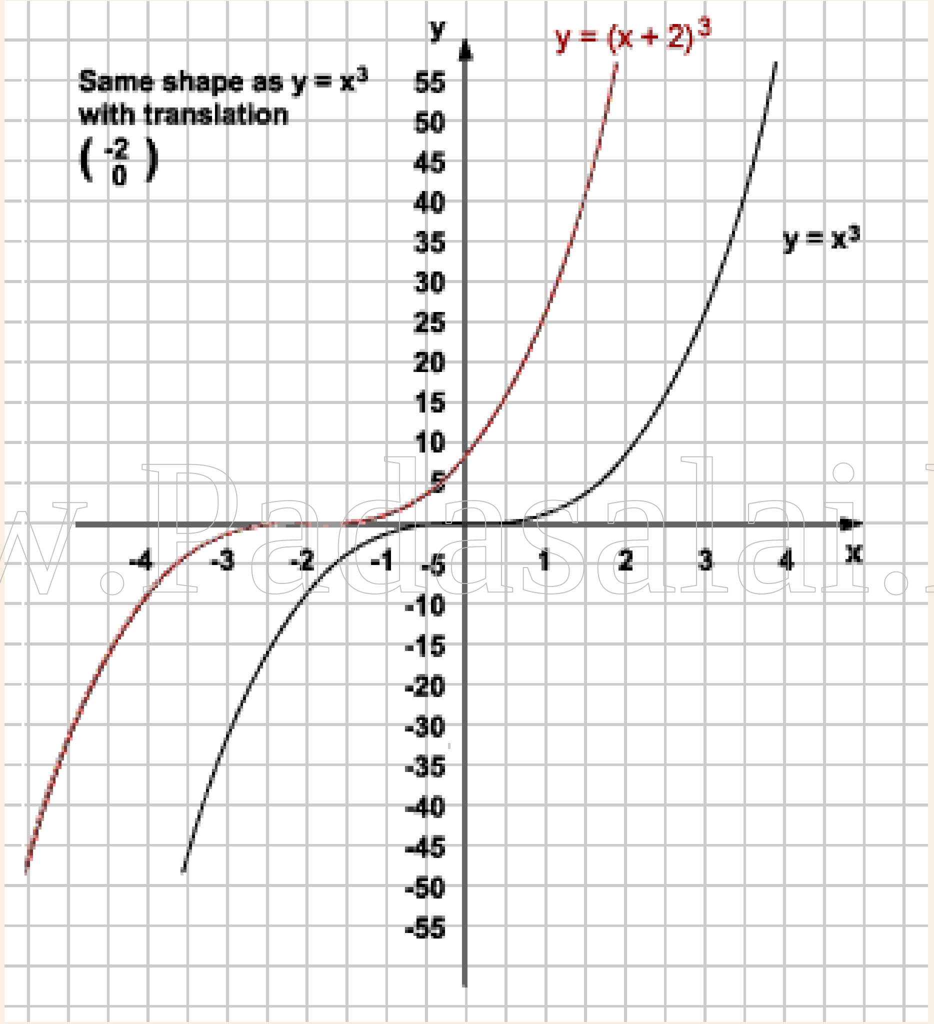
$y = x^2 + 2$

$y = (x - 3)^2$

$y = (x - 3)^2 + 2$



Same shape as $y = x^3$ with translation

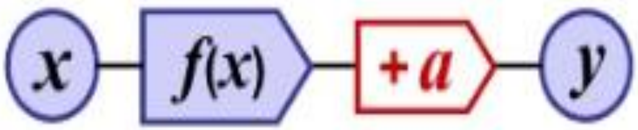


Introduction to Transformations

A transformation is an algebraic change to a function which affects the shape or position of the graph.

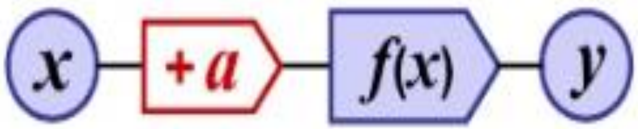
What is the difference between $y = f(x) + a$ and $y = f(x + a)$?

$y = f(x) + a$



Making a change after the function will affect only the y-coordinate.

$y = f(x + a)$



Making a change before the function will affect only the x-coordinate.

Translation

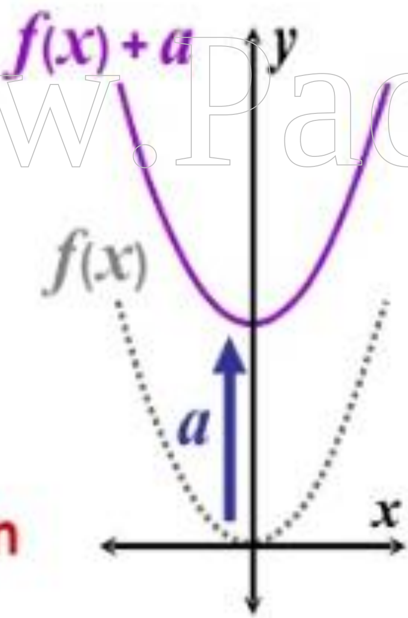
Any change to the position of a graph is called a translation.

$y = f(x) + a$

Slides the graph of $f(x)$ vertically.

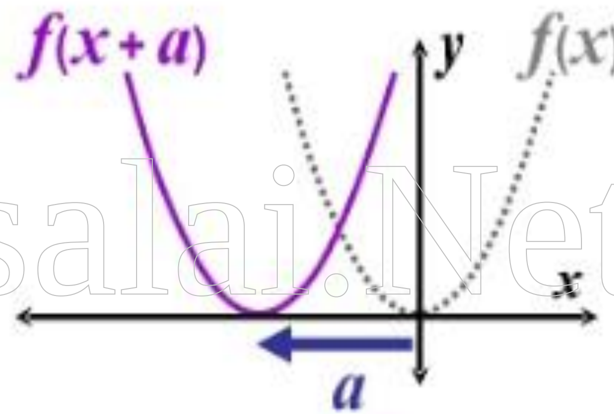
$+a$ up

$-a$ down



$y = f(x + a)$

Slides the graph of $f(x)$ horizontally.



CAREFUL!

$+a$ left

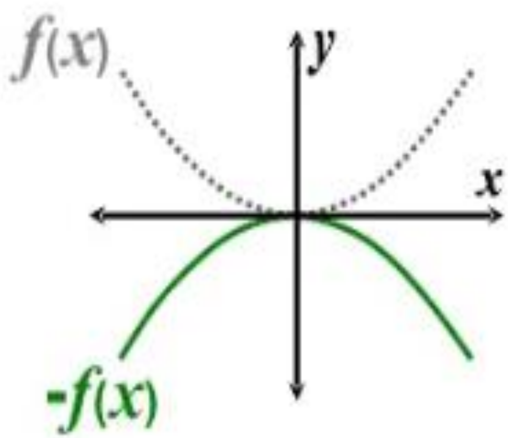
$-a$ right

Reflection

If the positive and negative coordinates of a graph are inverted, the graph will be reflected across either the x or y-axis.

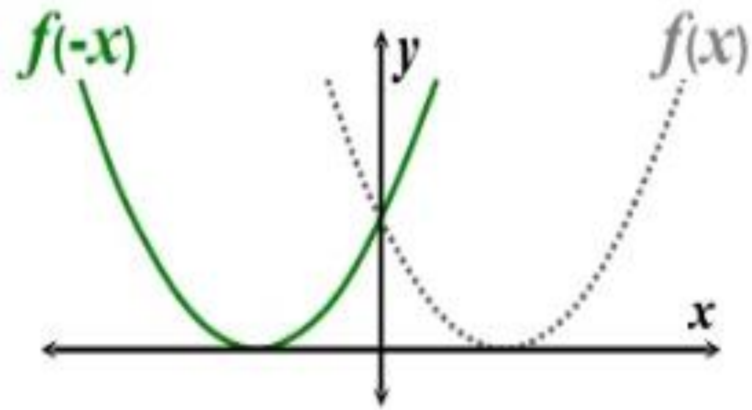
$y = -f(x)$

Reflects the graph of $f(x)$ vertically across the x-axis.



$y = f(-x)$

Reflects the graph of $f(x)$ horizontally across the y-axis.



Distortion

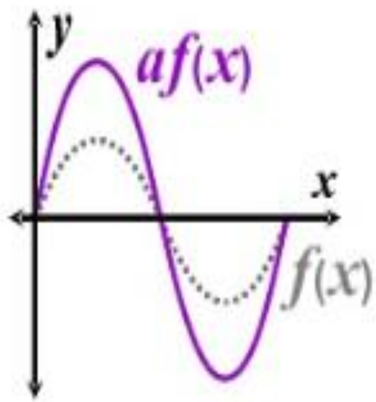
If every coordinate is multiplied by the same value, the overall shape of the graph will be distorted.

$y = af(x)$

Changes the vertical size of the graph of $f(x)$.

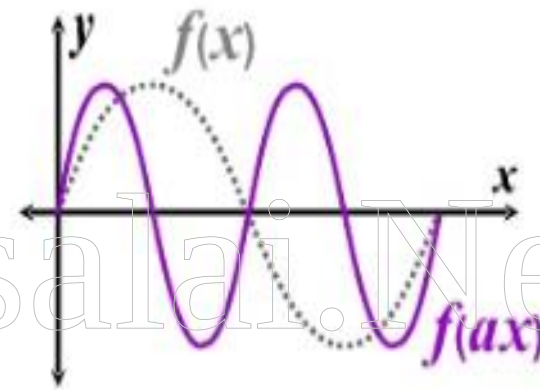
$a > 1$ 'stretch'

$a < 1$ 'compress'



$y = f(ax)$

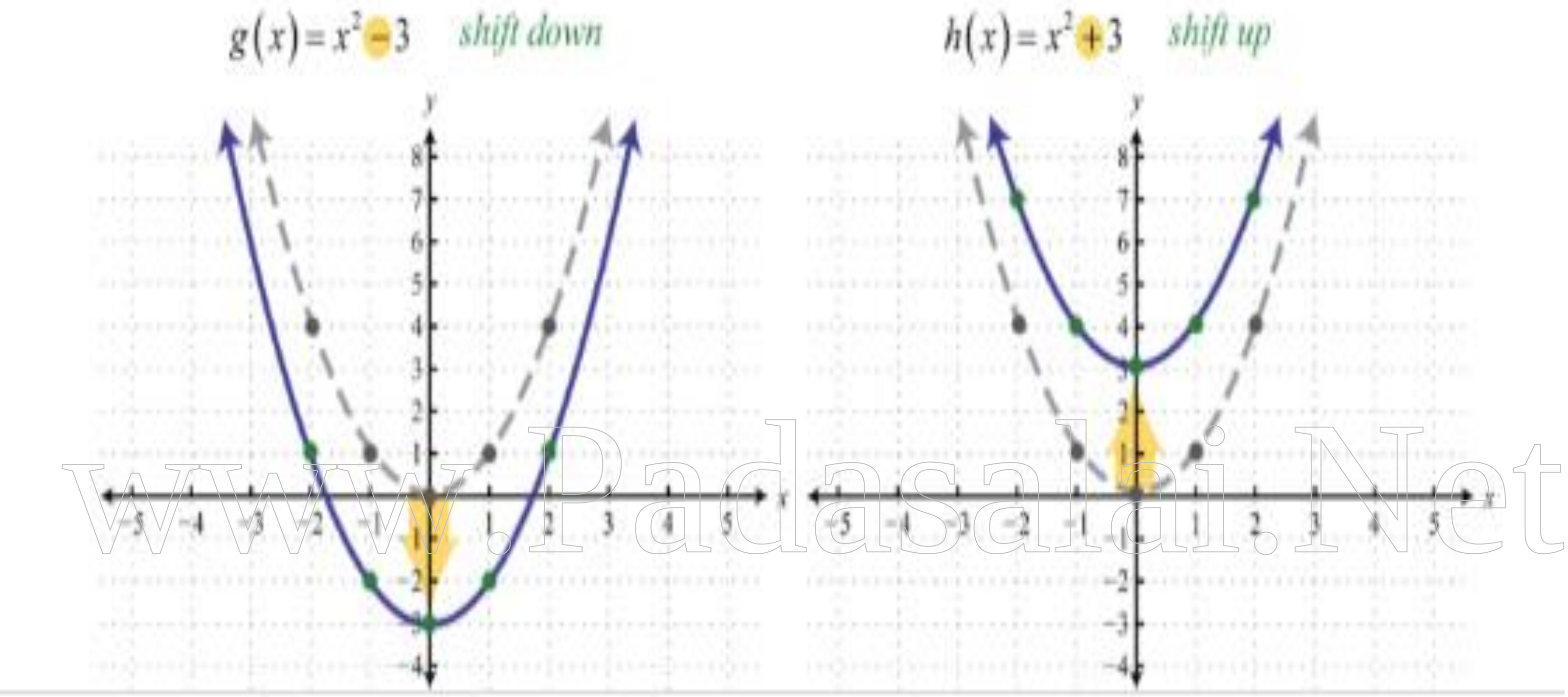
Changes the horizontal size of the graph of $f(x)$.



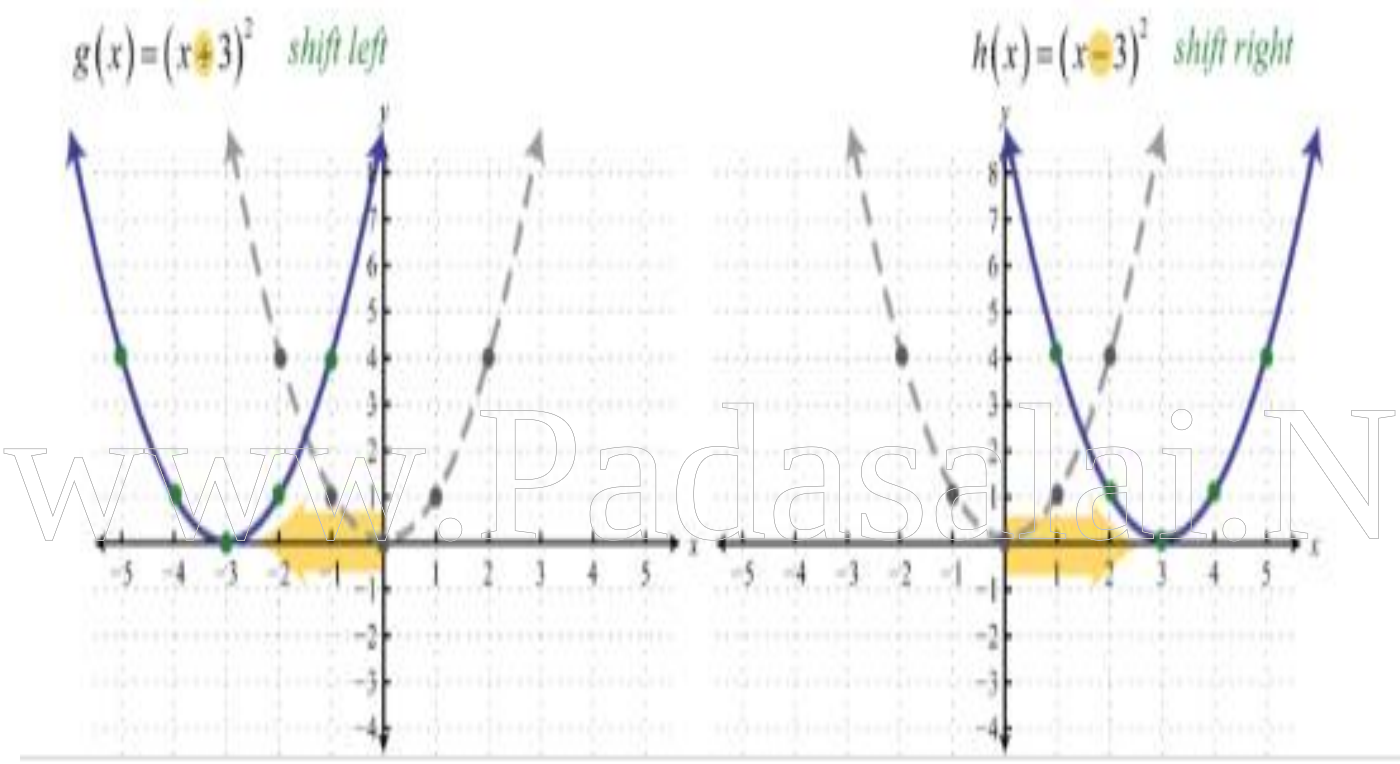
CAREFUL!

$a < 1$ 'stretch'

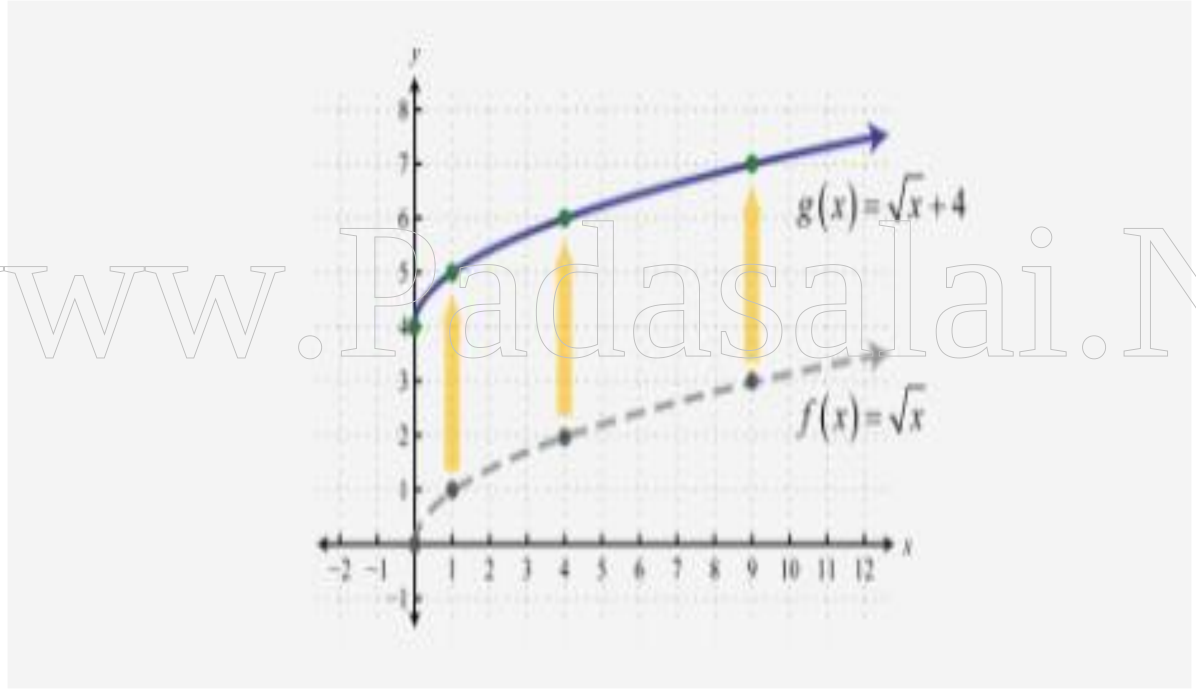
$a > 1$ 'compress'



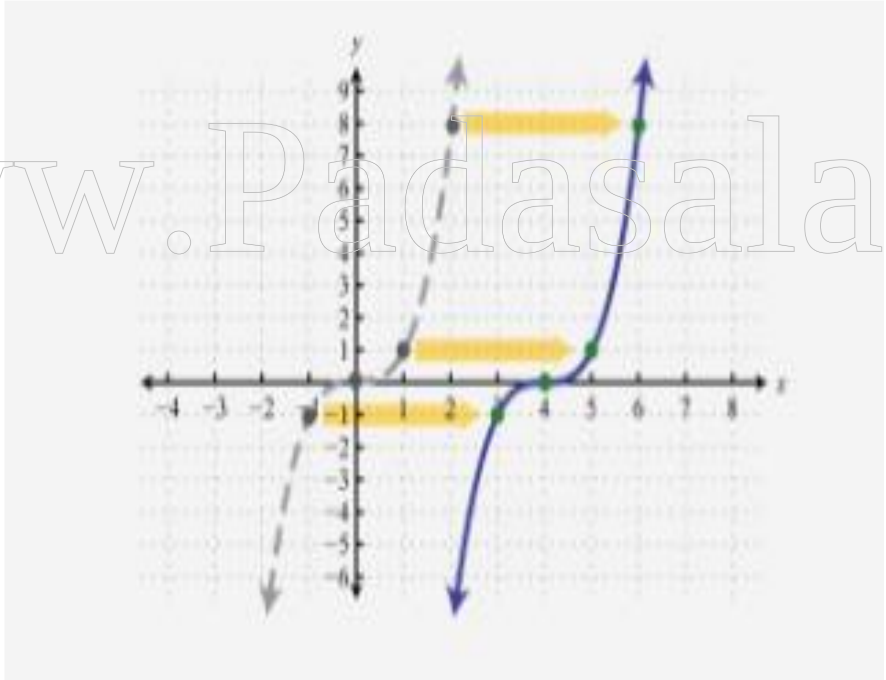
$g(x) = x^2 - 3$
shift down
$h(x) = x^2 + 3$
shift up


$g(x) = (x+3)^2$
shift left
$h(x) = (x-3)^2$
shift right

Sketch the graph of $g(x) = \sqrt{x} + 4$.

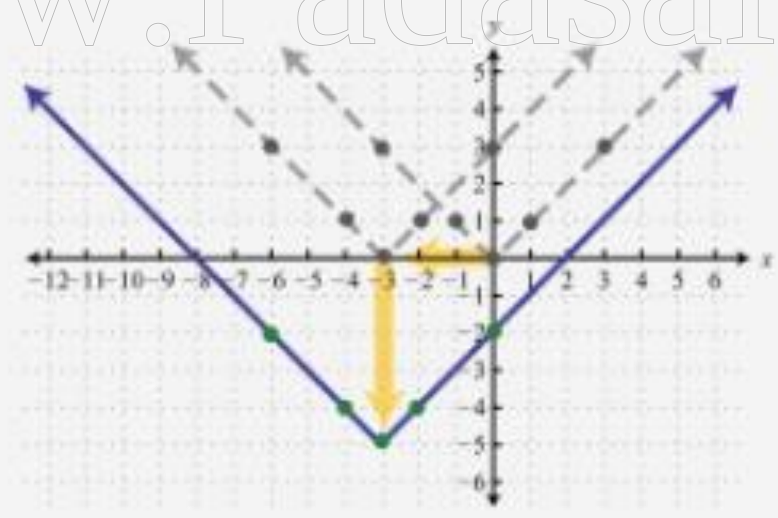


Sketch the graph of $g(x) = (x-4)^3$.

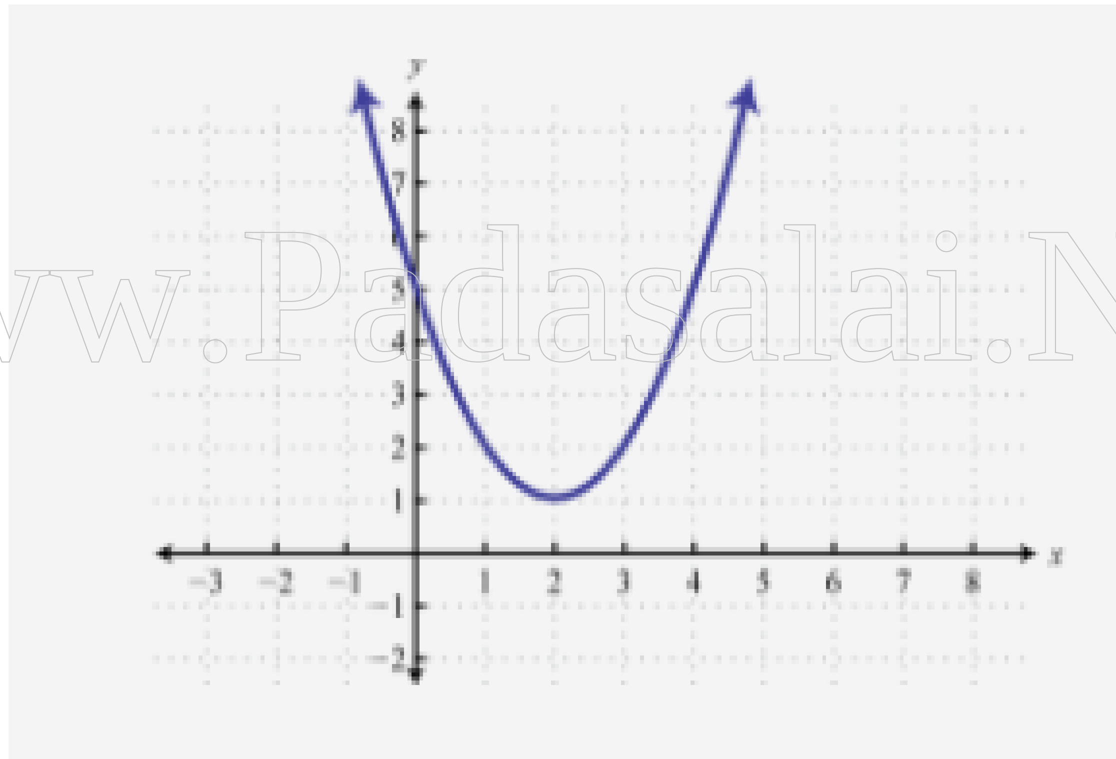


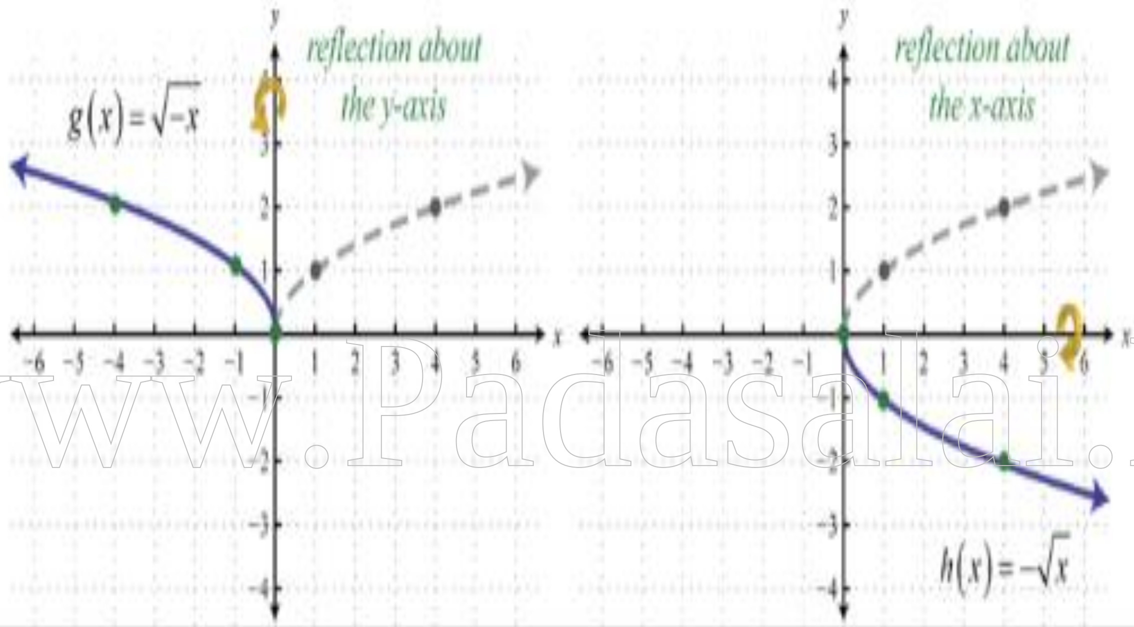
Sketch the graph of $g(x) = |x+3| - 5$.

$y = \|x\|$	*Basic function*
$y = \|x+3\|$	*Horizontal shift left 3 units*
$y = \|x+3\| - 5$	*Vertical shift down 5 units*



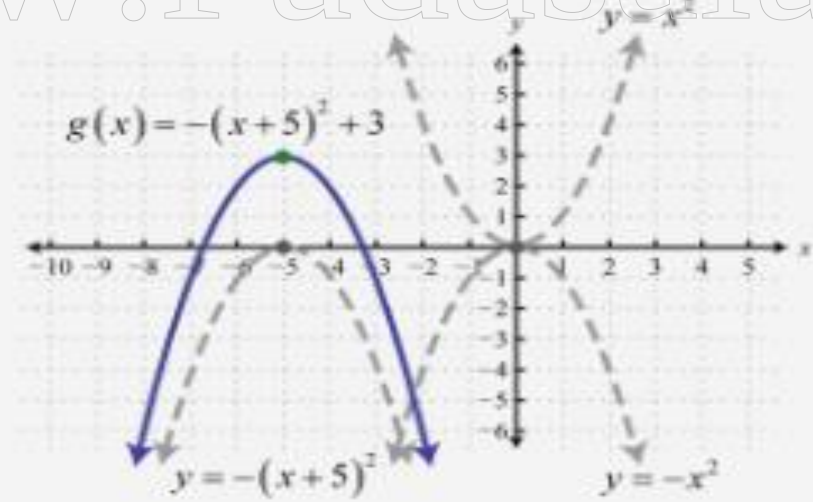
Sketch the graph of $g(x) = (x-2)^2 + 1$.

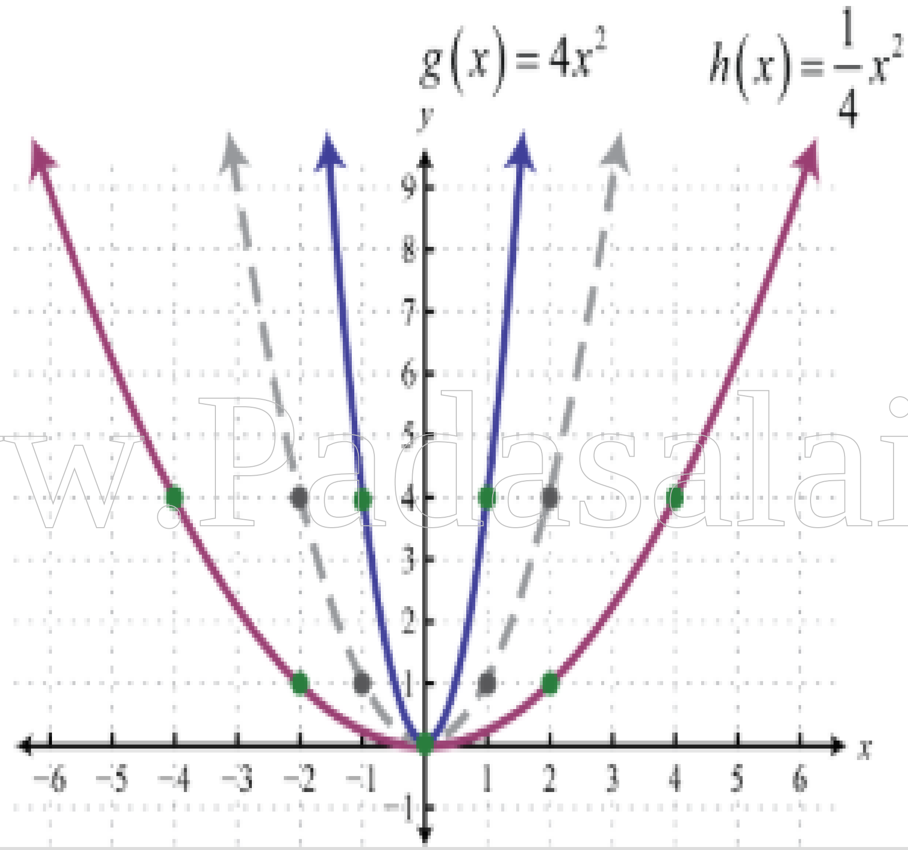



reflection about
the y-axis
$g(x)=\sqrt{-x}$
reflection about
the x-axis
$h(x)=-\sqrt{x}$

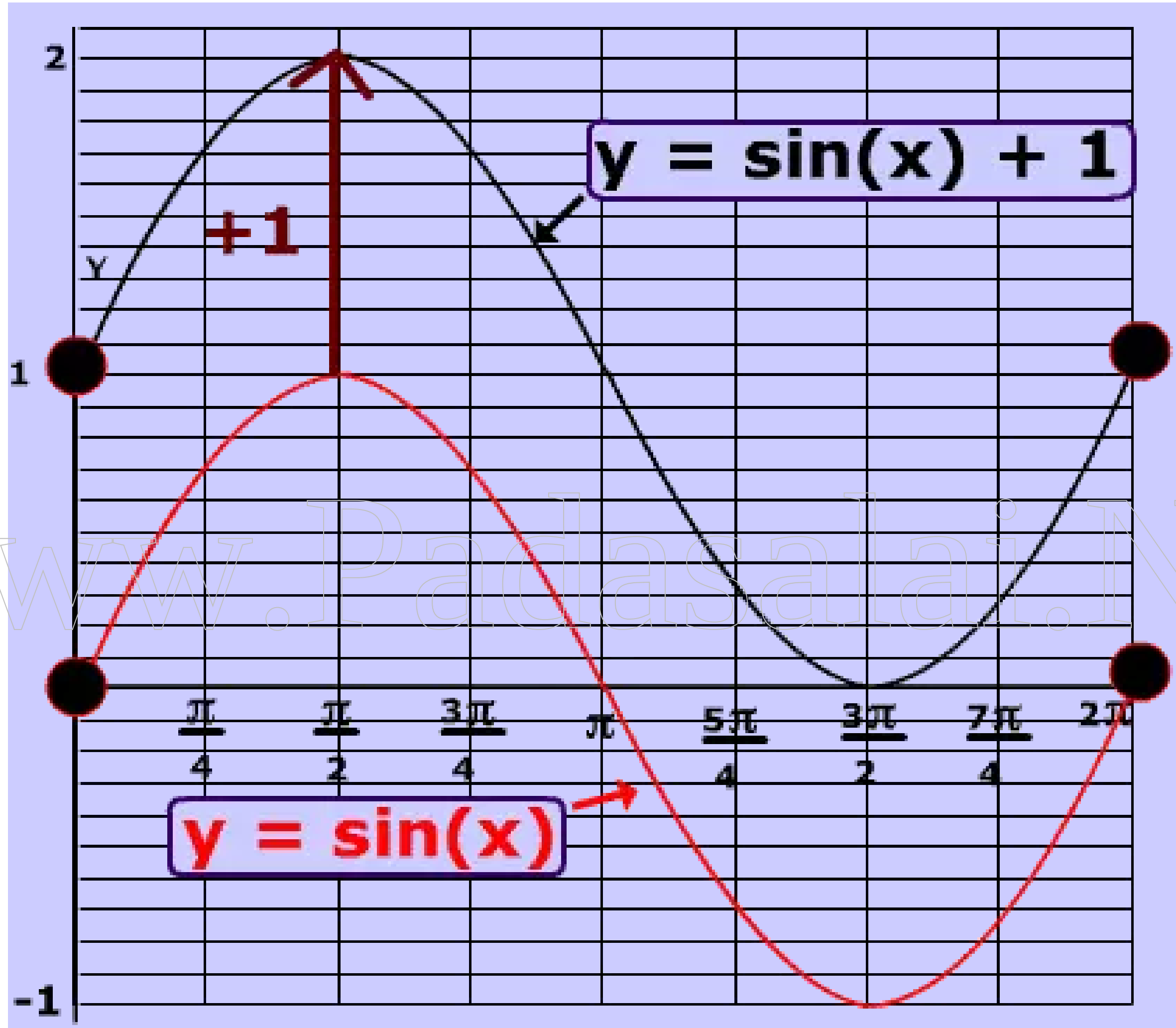
Sketch the graph of $g(x) = -(x+5)^2 + 3$.

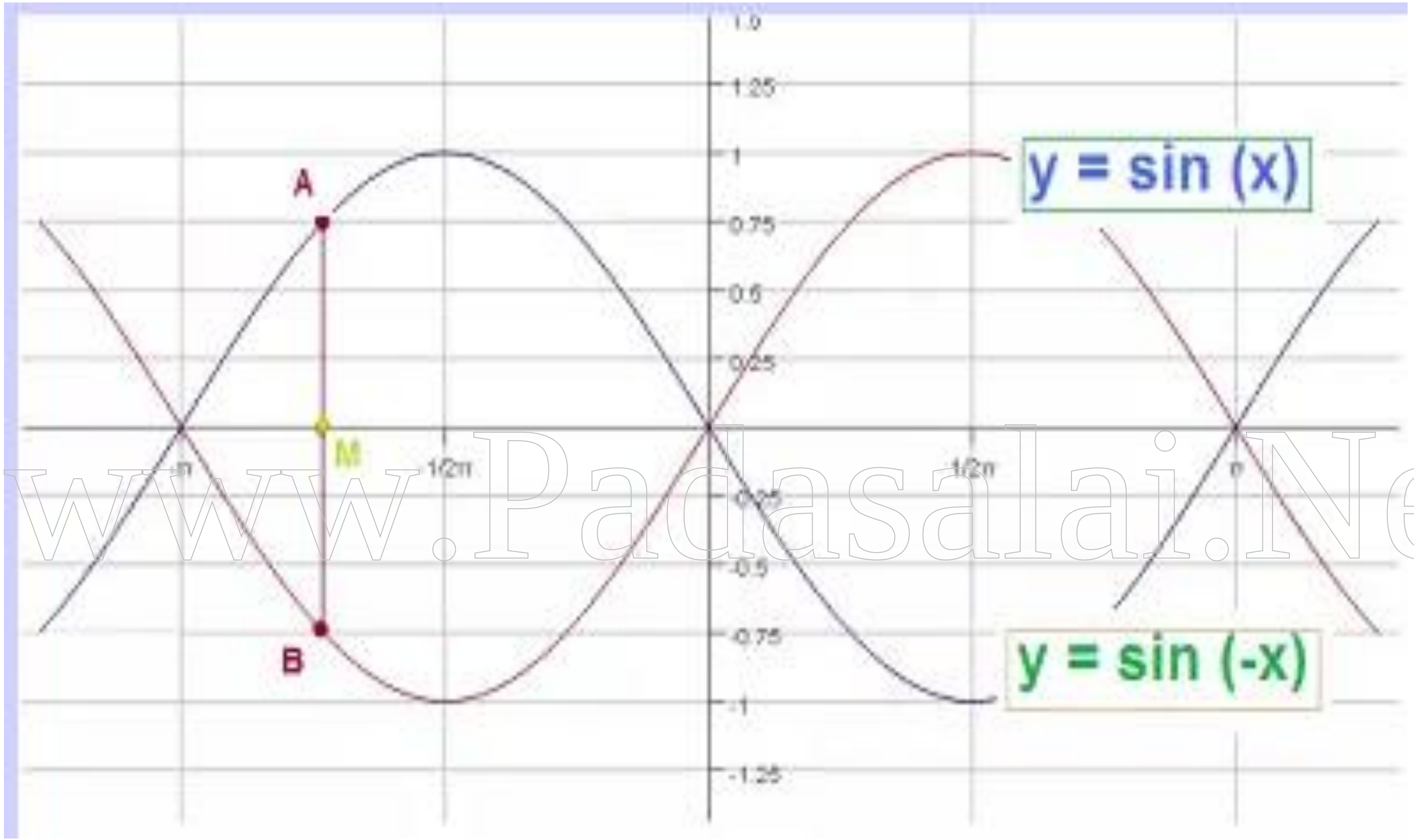
$y = x^2$	*Basic function.*
$y = -x^2$	*Reflection about the x-axis.*
$y = -(x+5)^2$	*Horizontal shift left 5 units.*
$y = -(x+5)^2 + 3$	*Vertical shift up 3 units.*





$g(x) = 4x^2$
$h(x) = \frac{1}{4}x^2$
y
x
9
8
7
6
5
4
3
2
1
−1
−6
−5
−4
−3
−2
−1
1
2
3
4
5
6

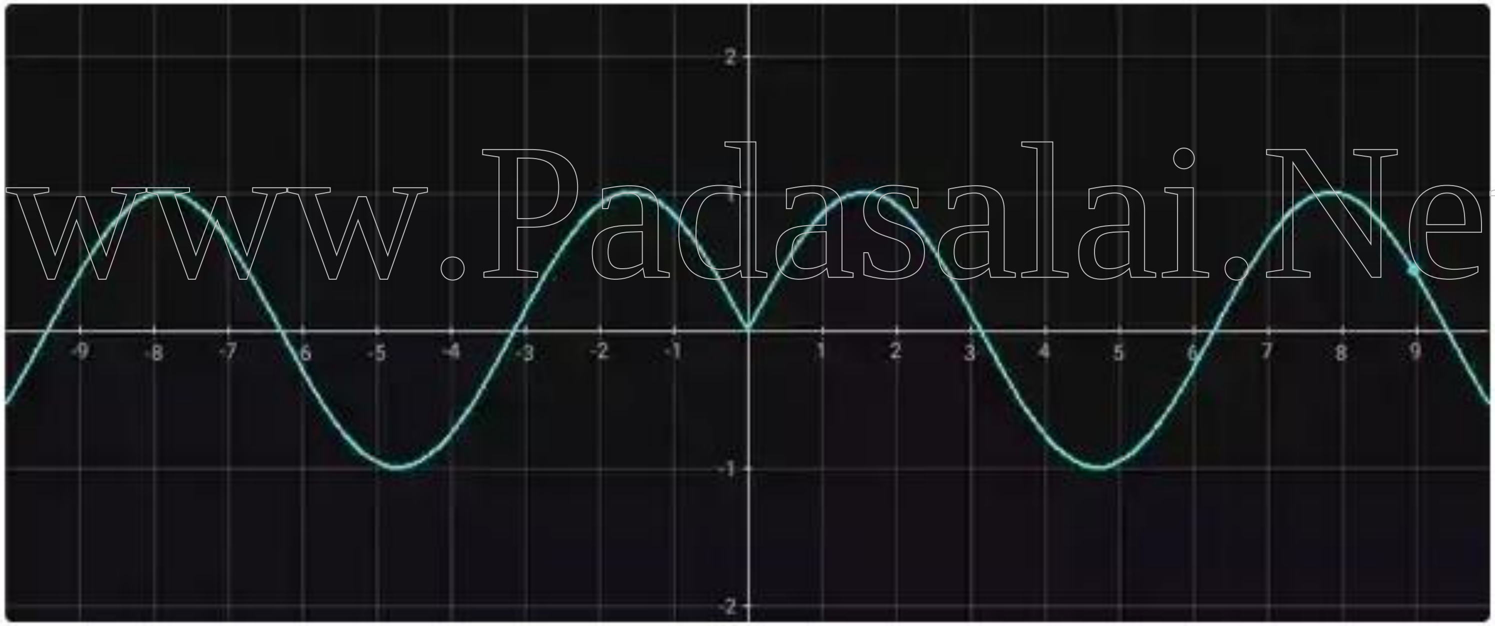

2
1
-1
Y
+1
y = sin(x) + 1
y = sin(x)
π/4
π/2
3π/4
π
5π/4
3π/2
7π/4
2π


A
B
M
y = sin (x)
y = sin (-x)
1.5
1.25
1
0.75
0.5
0.25
-0.25
-0.5
-0.75
-1
-1.25
1/2π
1/2π
π

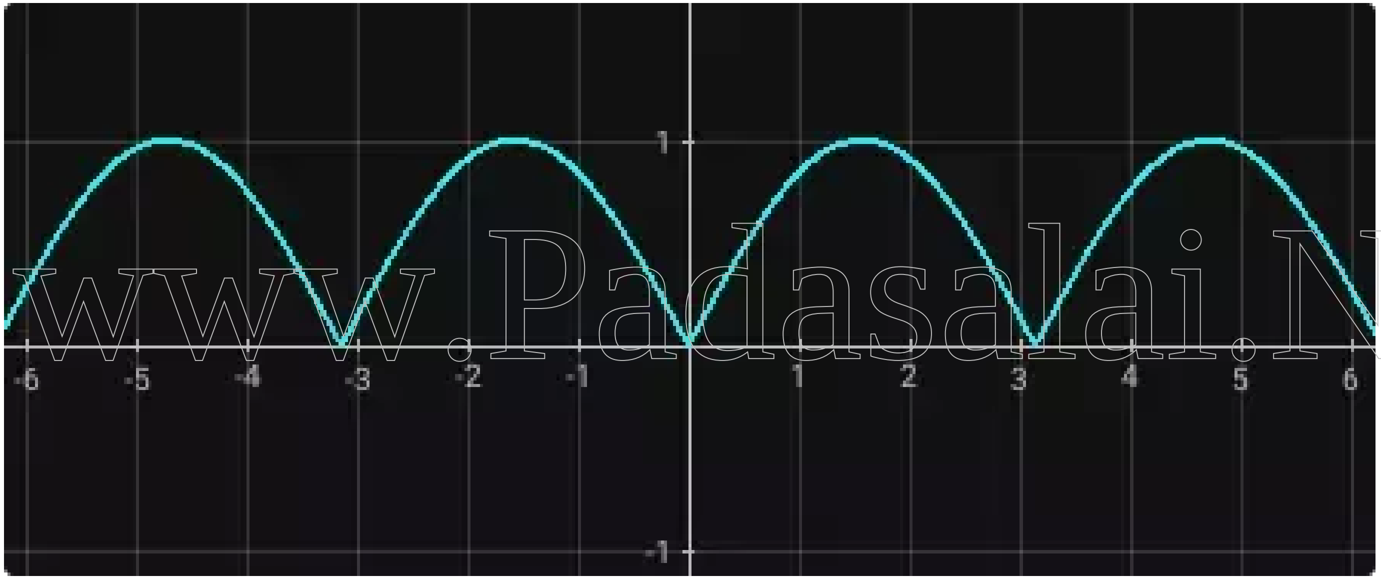
$\sin|x| = -\sin x \; ; \; x < 0$

$= \sin x \; ; \; x > 0$ &

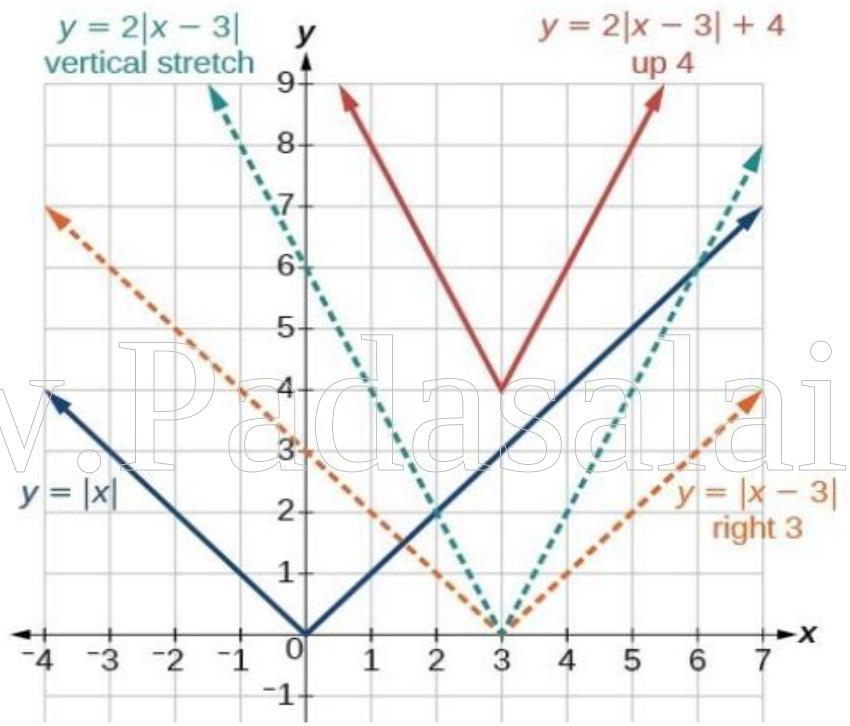
so graph of $\sin|x|$ will look like,



So graph of |sinx| will look like



the graph of $y = 2\,|x - 3| + 4$. The graph of $y = |x|$ has been shifted right 3 units, vertically stretched by a factor of 2, and shifted up 4 units. This means that the corner point is located at $(3, 4)$ for this transformed function.



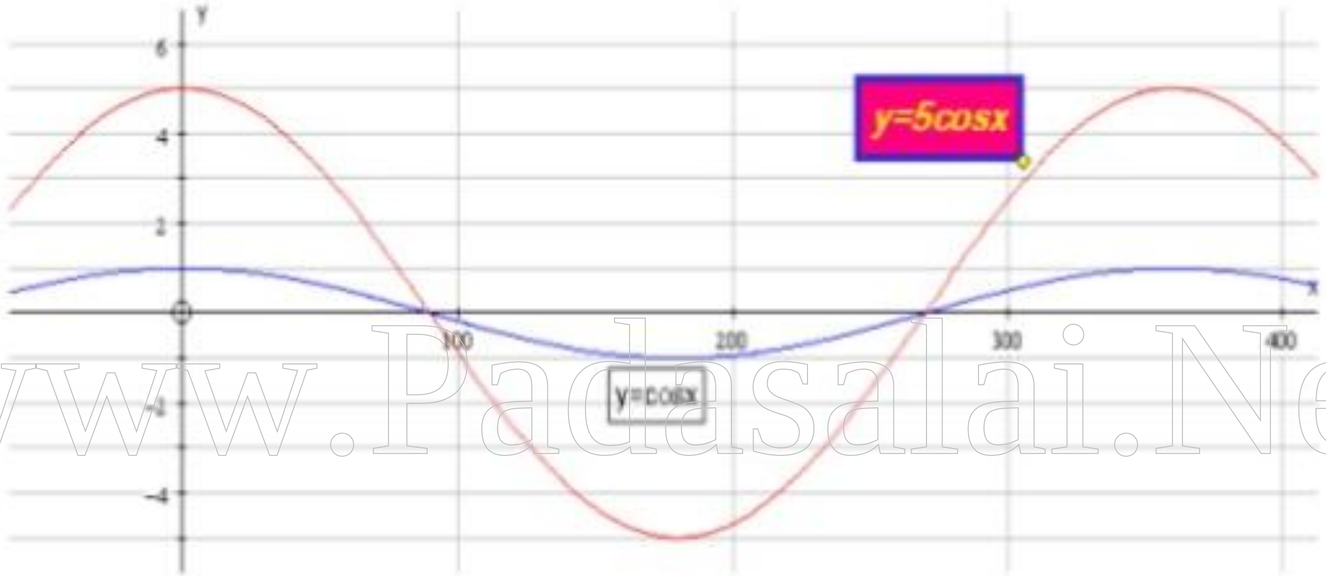

y
x
2
1
-1
-2
-90
90
180
270
360
y=sinx
y=sin2x

You should know how the following graphs differ from the basic trigonometric graphs:

Y= 2 SIN X

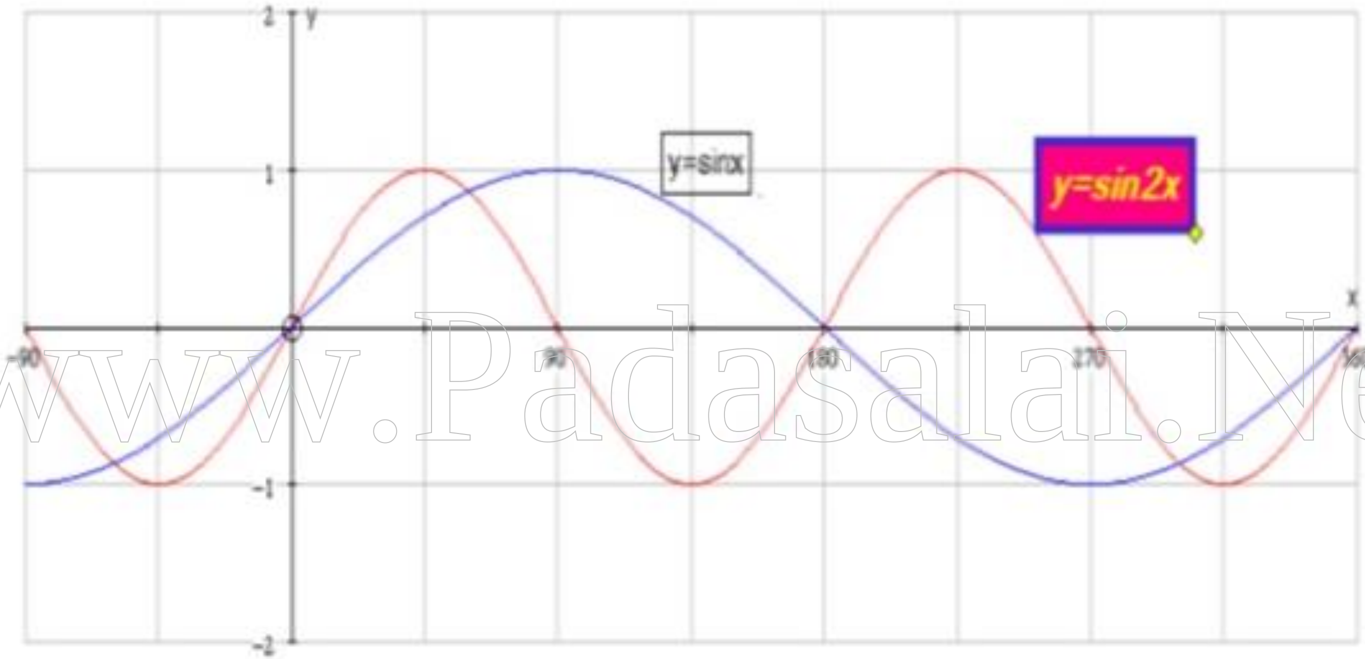
The 2 in front of the sin x changes the ***"amplitude"*** of the graph.

Y = 5 COS X



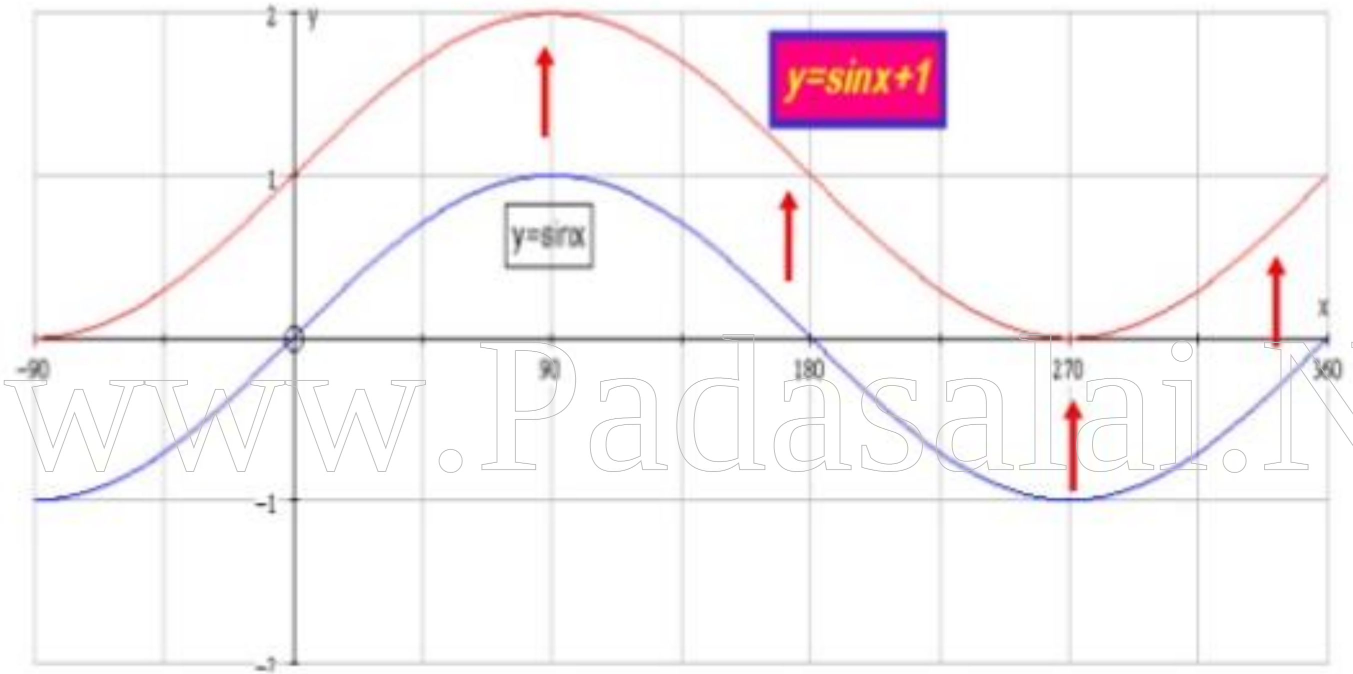
As expected the amplitude of the graph is now 5. Hence the graph has a maximum value of 5 and a minimum value of –5.

Y = SIN 2X



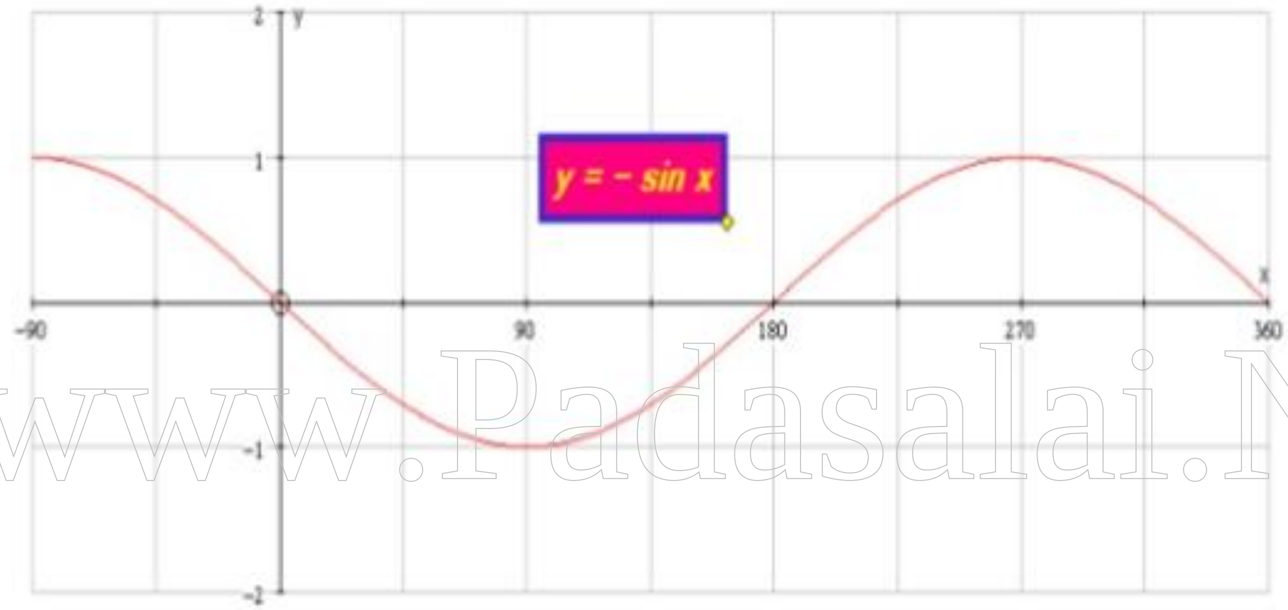
By introducing the 2 in front of the X , the "***period***" of the graph now becomes $360^{\circ} \div 2 = 180^{\circ}$.

Y= SIN X + 1



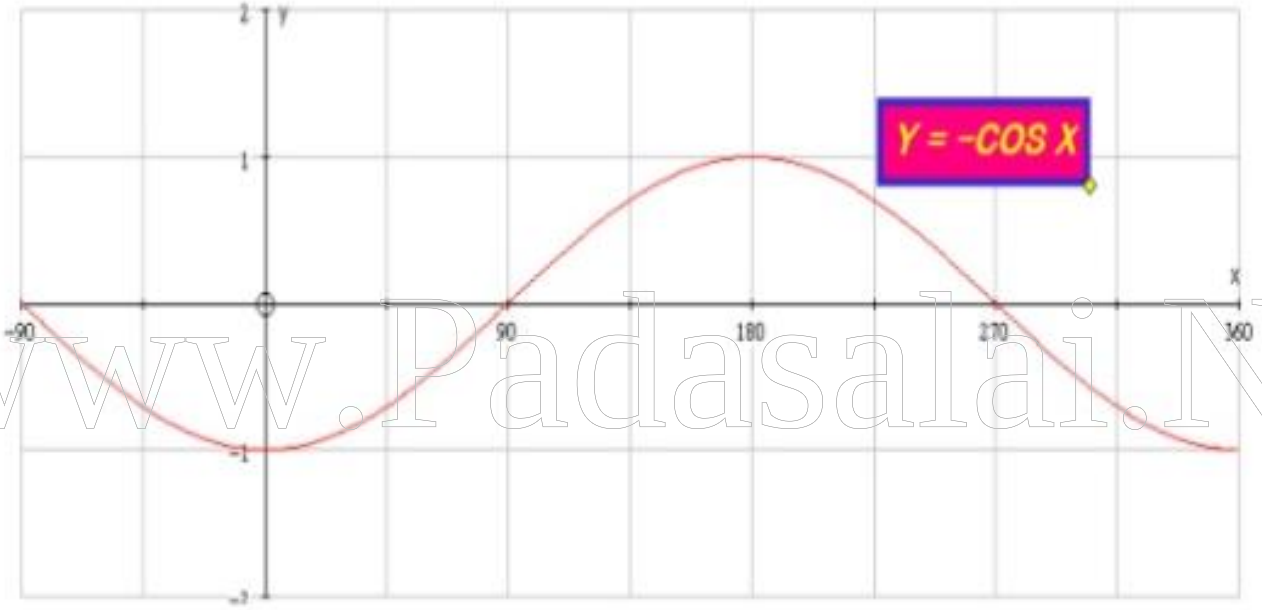
The plus 1 has the effect of "***translating***" the graph one square parallel to the y axis.

Y= – SIN X



The minus sign in front of the function "***reflects***" the whole graph in the X axis.

Y = – COS X



As expected the cosine graph is reflected in the X axis.

Example 1.

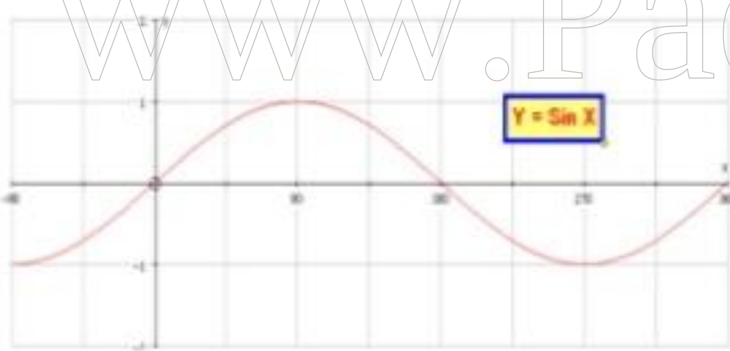
Draw the graph of :

$$y = 4\sin 2x + 3$$

Solution.

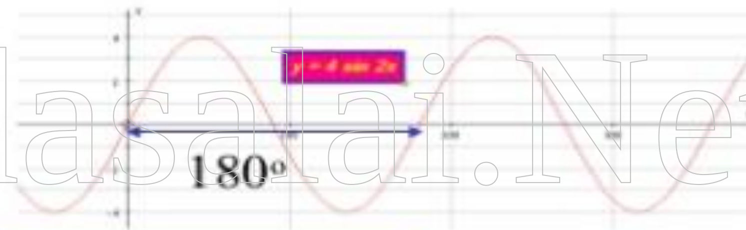
Draw the graph of :

$y = \sin x$



$y = 4\sin x$

$y = 4\sin 2x$



$y = 4\sin 2x + 3$

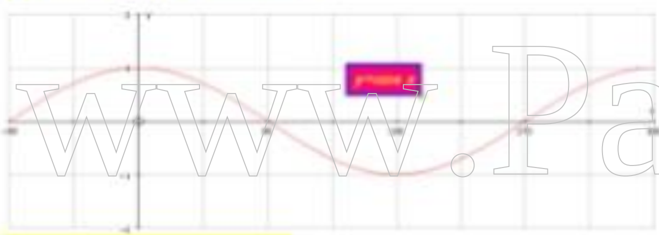
Example 2

Draw the graph $y = 2 - 6\cos 5x$

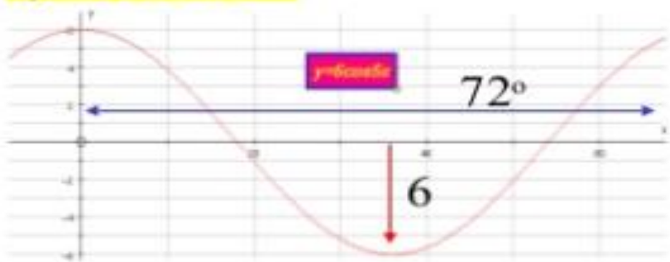
Solution.

Draw the graph of :

$y = \cos x$

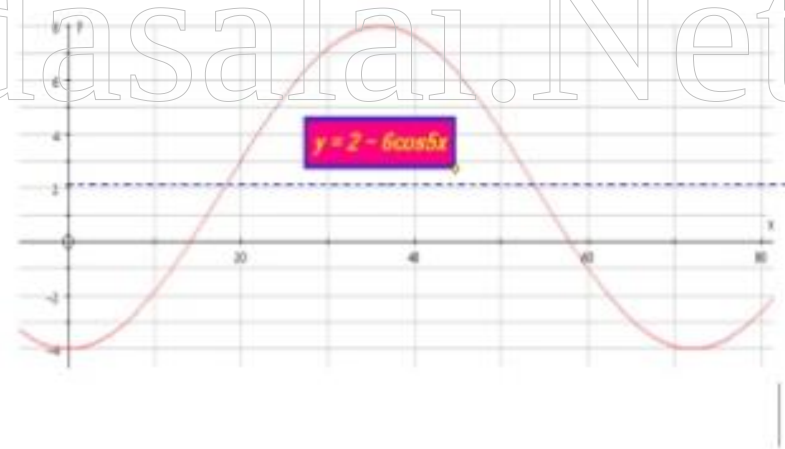


$y = 6\cos 5x$



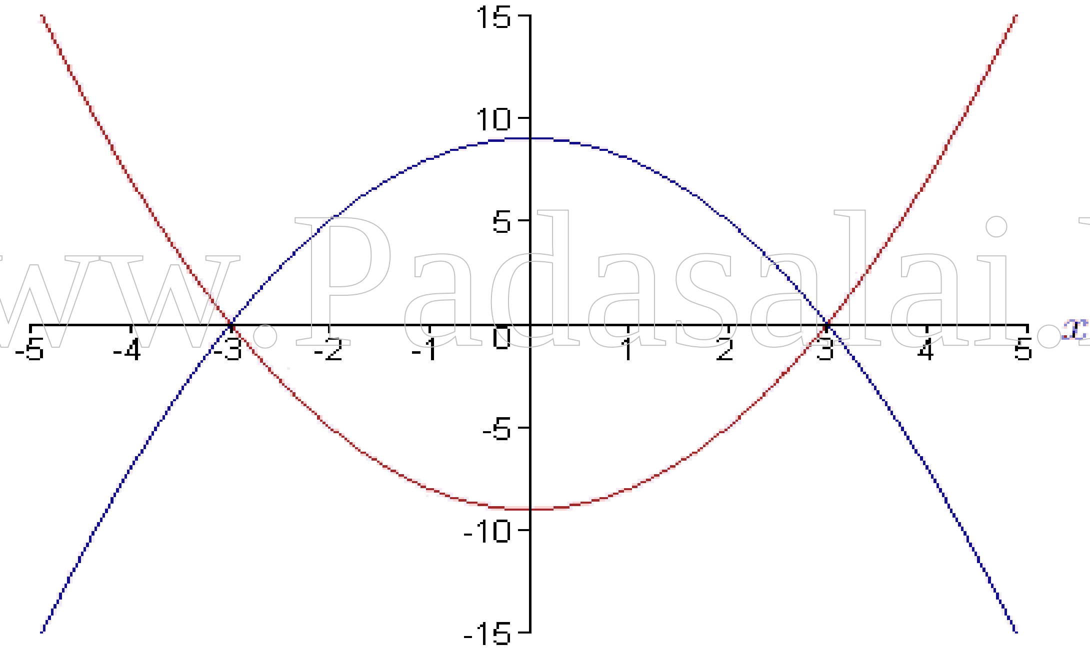
$y = -6\cos 5x$

$y = 2 - 6\cos 5x$



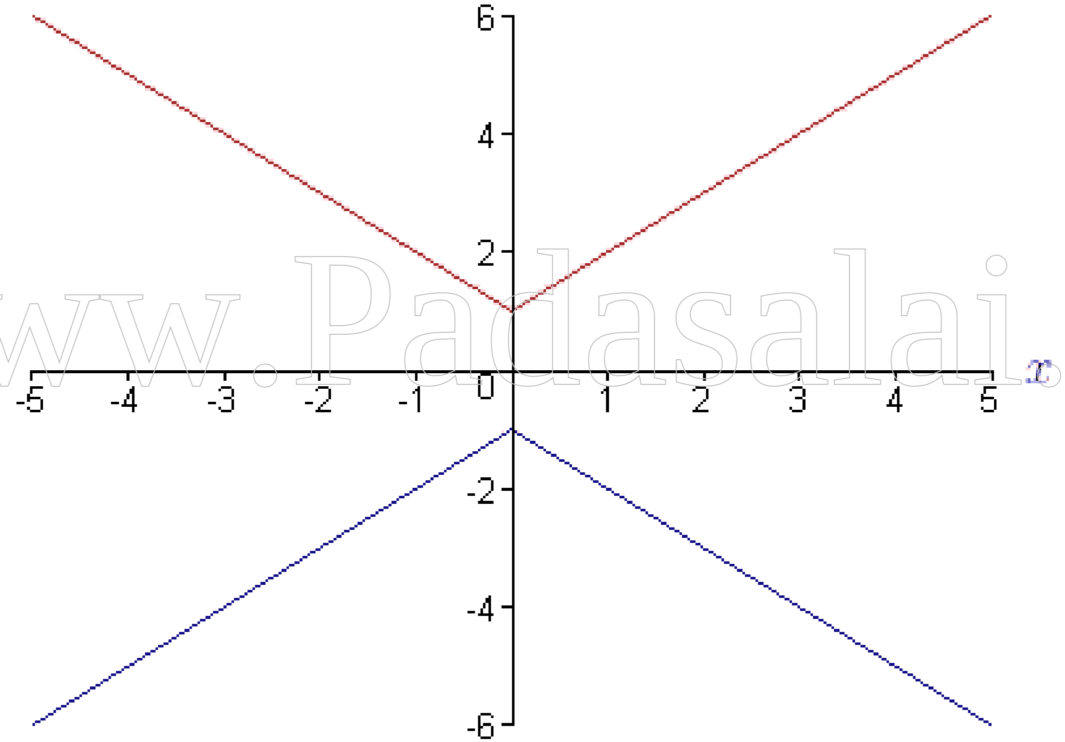
$$f(x) = x^2 - 9$$

$$y_1(x) = -x^2 + 9$$



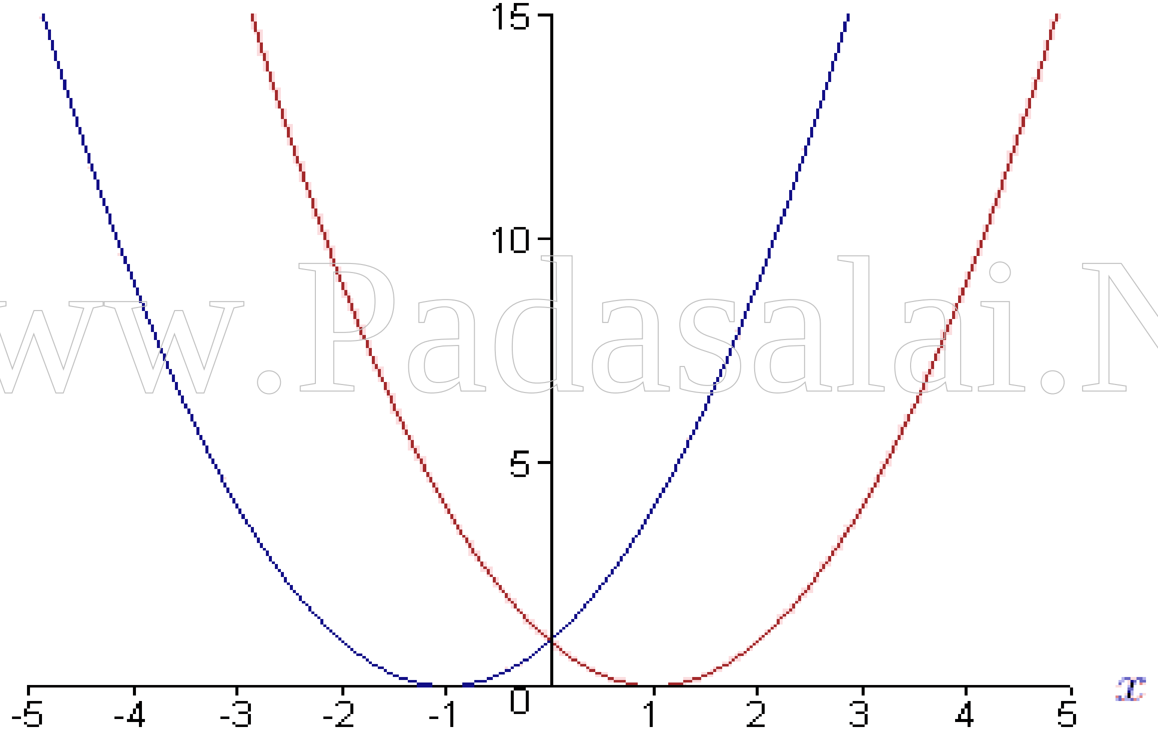
$$g(x) = |x| + 1$$

$$y_2(x) = -|x| - 1$$

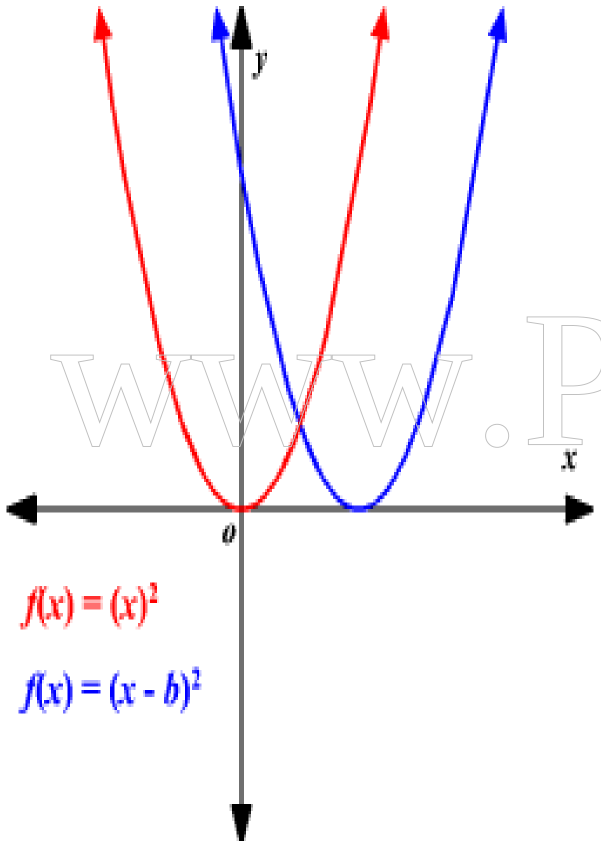


$$f(x) = (x-1)^2$$

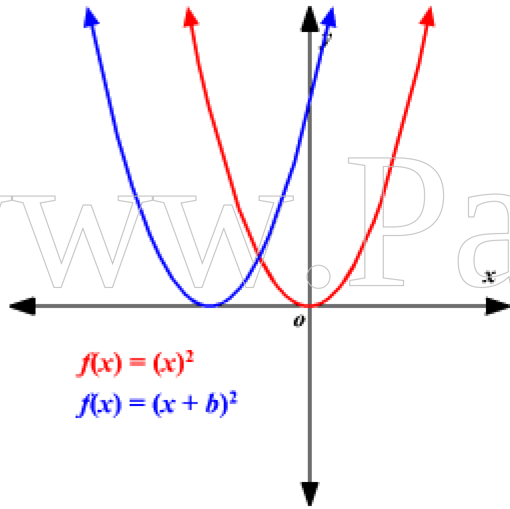
$$y_1(x) = (-x-1)^2$$



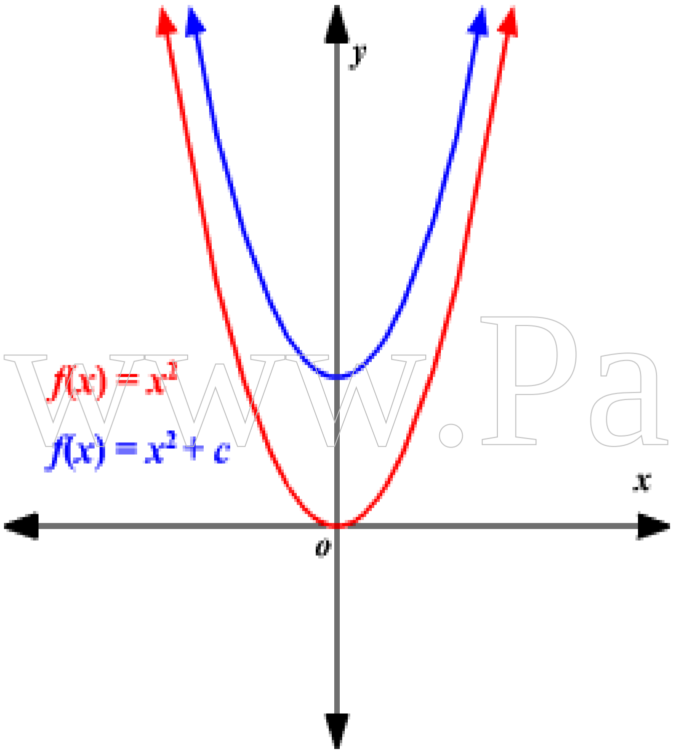
- Replacing $f(x)$ with $f(x-b)$ results in the graph being shifted b units to the right.



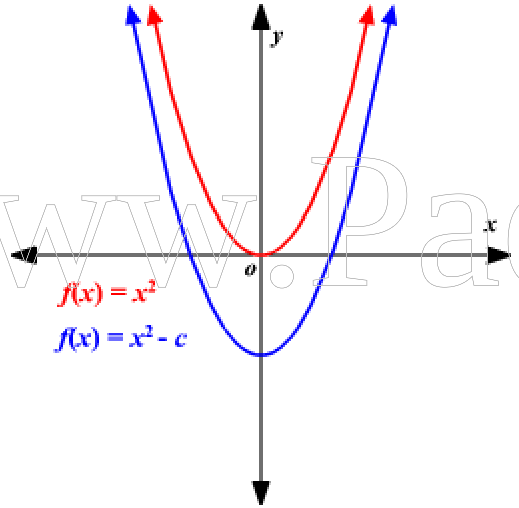
- Replacing $f(x)$ with $f(x+b)$ results in the graph being shifted b units to the left.



- Replacing $f(x)$ with $f(x)+c$ results in the graph being shifted c units up.

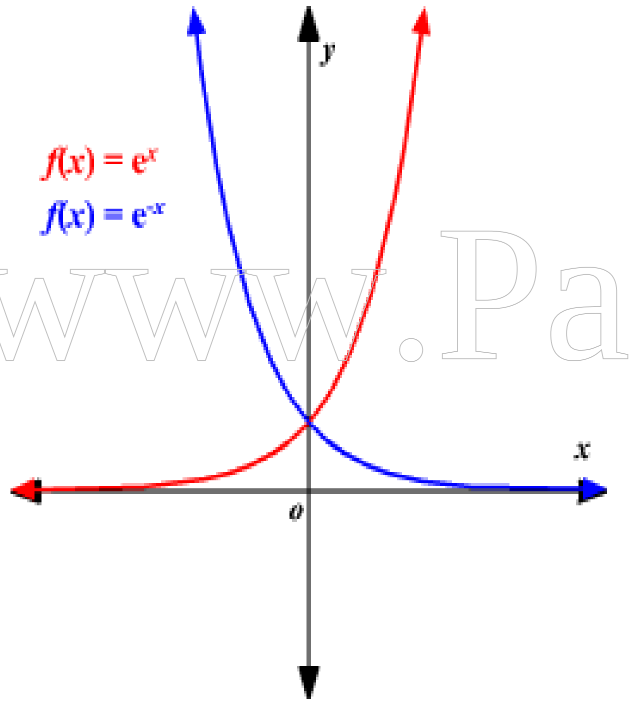


- Replacing $f(x)$ with $f(x) - c$ results in the graph being shifted c units down.

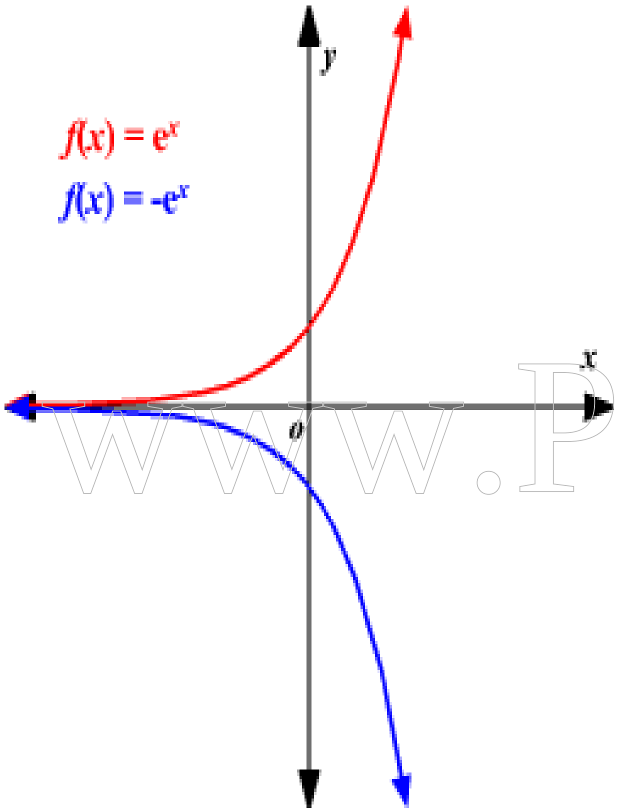


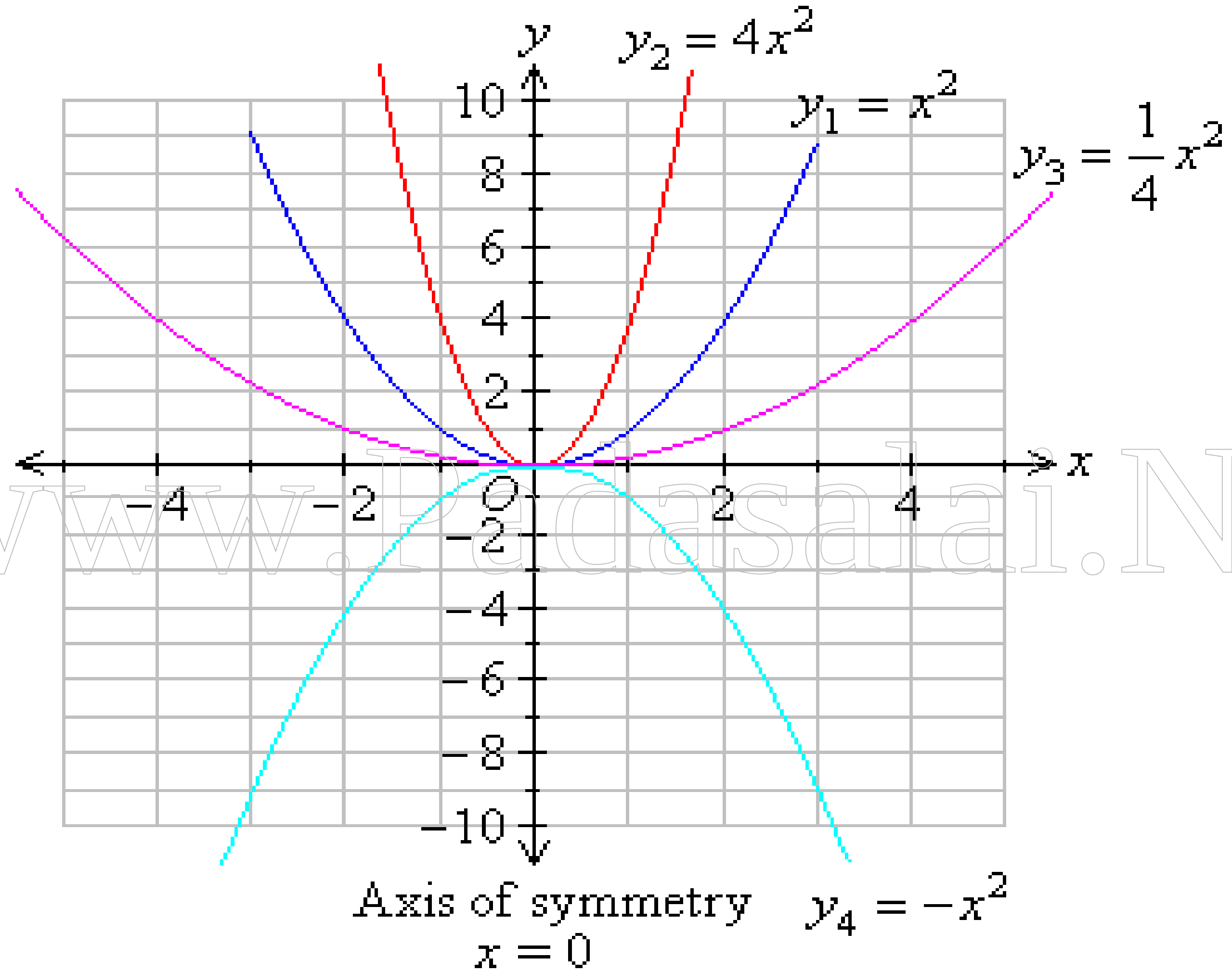
Reflection

- Replacing x with $-x$ results in the graph being reflected across the y -axis.



- Replacing $f(x)$ with $-f(x)$ results in the graph being reflected across the x -axis.



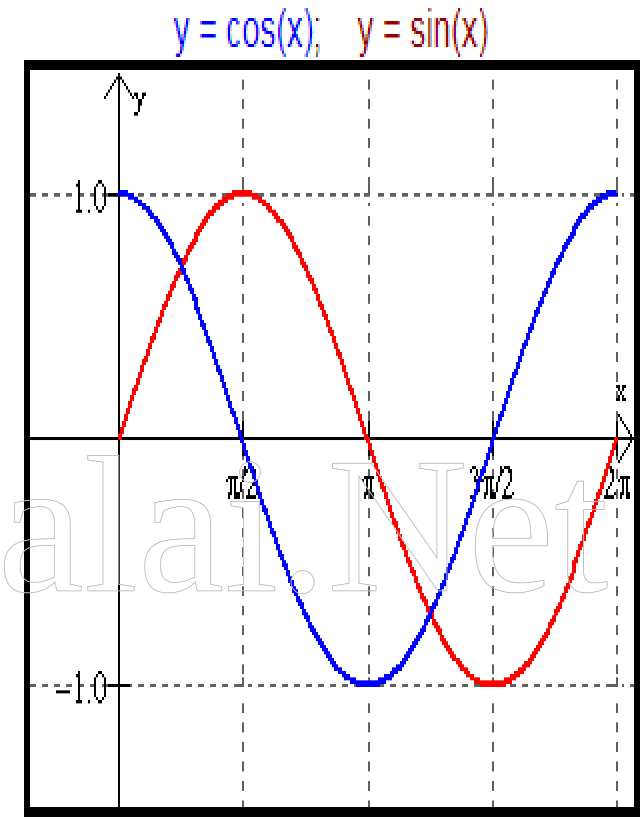


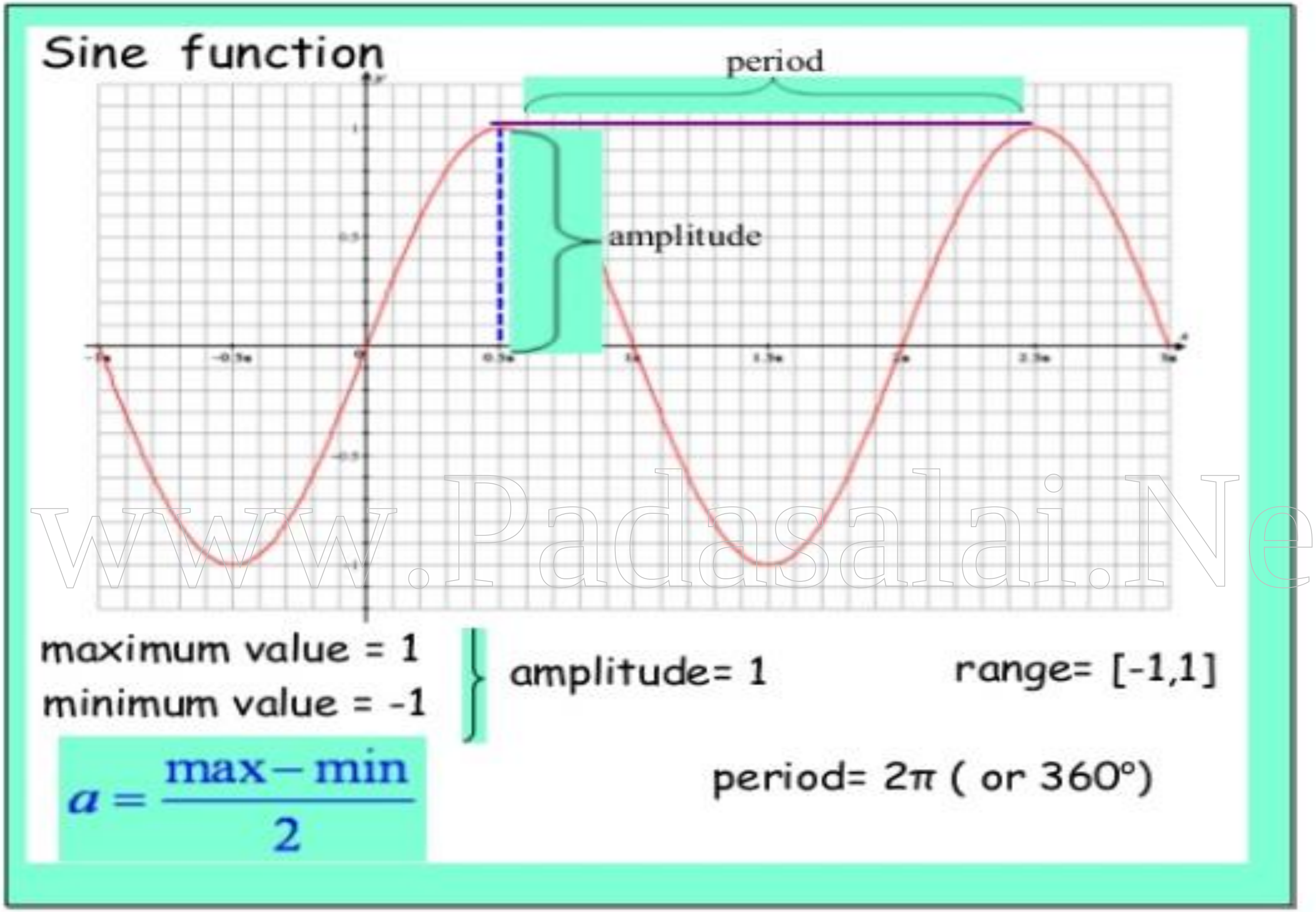
y
$y_2 = 4x^2$
$y_1 = x^2$
$y_3 = \frac{1}{4}x^2$
x
10
8
6
4
2
−2
−4
−6
−8
−10
−4
−2
0
2
4
Axis of symmetry
$x = 0$
$y_4 = -x^2$

The graphs of sine and cosine are the same when sine is shifted left by 90º or $\frac{\pi}{2}$ radians. Such a shifting is referred to as a **horizontal shift.**

$$\sin\left(x+\frac{\pi}{2}\right)=\cos(x)$$ shift sine to the left to create cosine

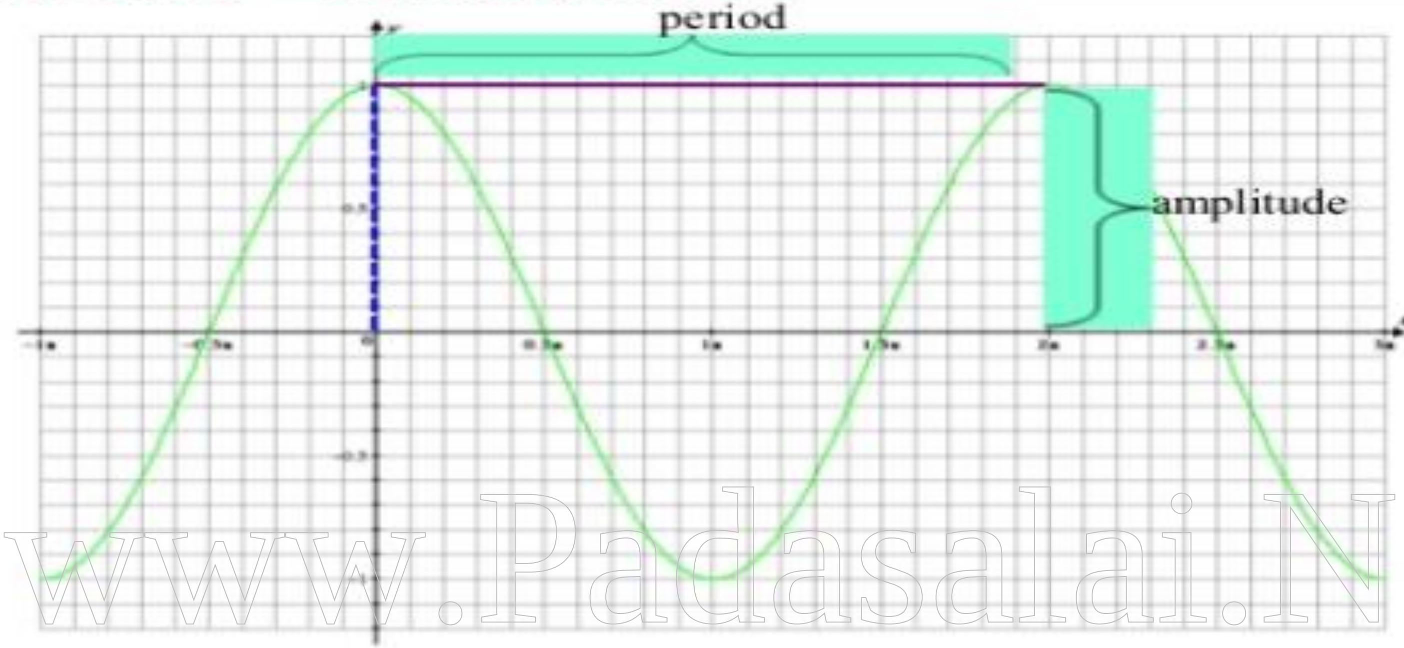
$$\cos\left(x-\frac{\pi}{2}\right)=\sin(x)$$ shift cosine to the right to create sine





Sine function
period
amplitude
maximum value = 1
minimum value = -1
amplitude= 1
range= [-1,1]
$a = \frac{max - min}{2}$
period= 2π (or 360°)

Cosine function

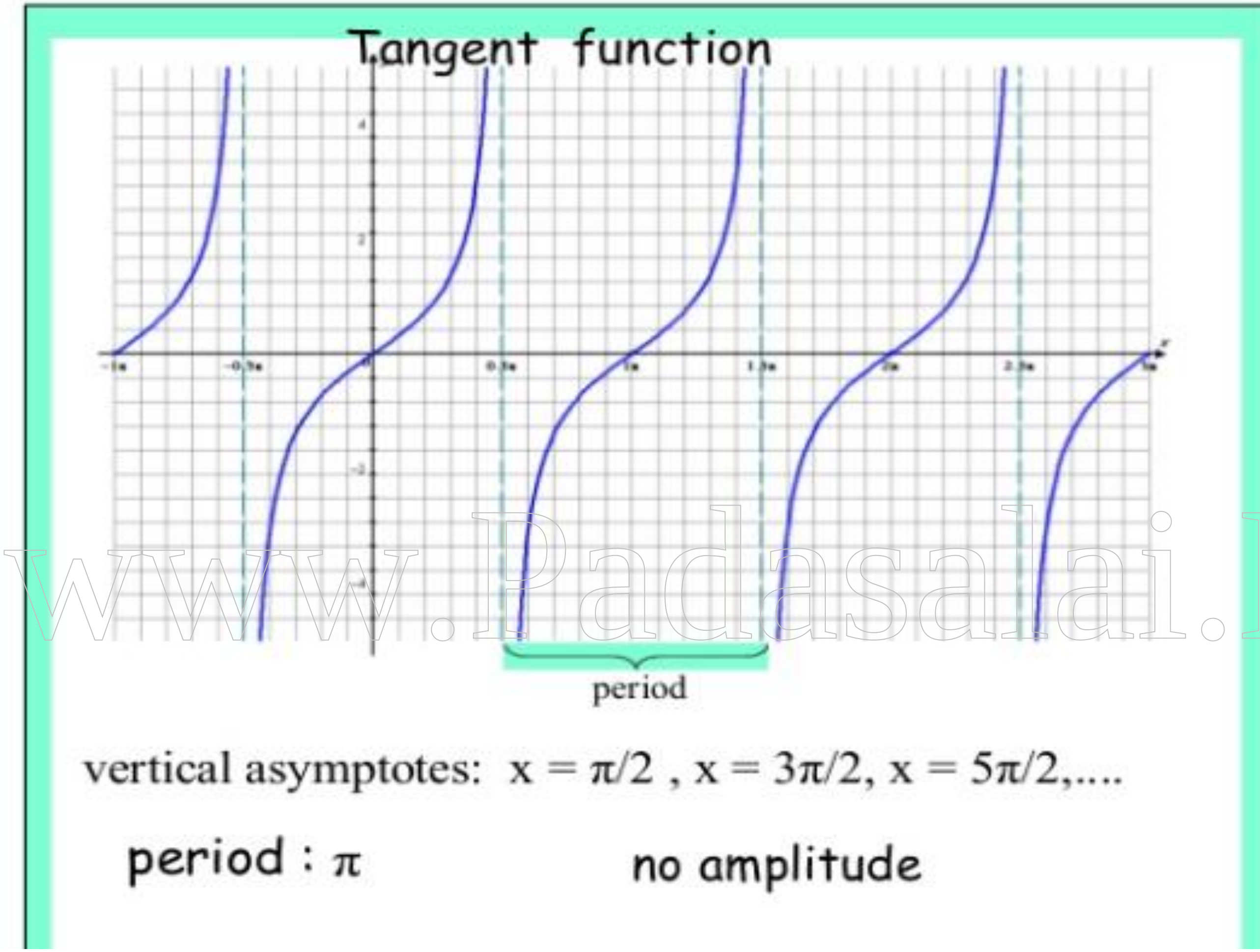


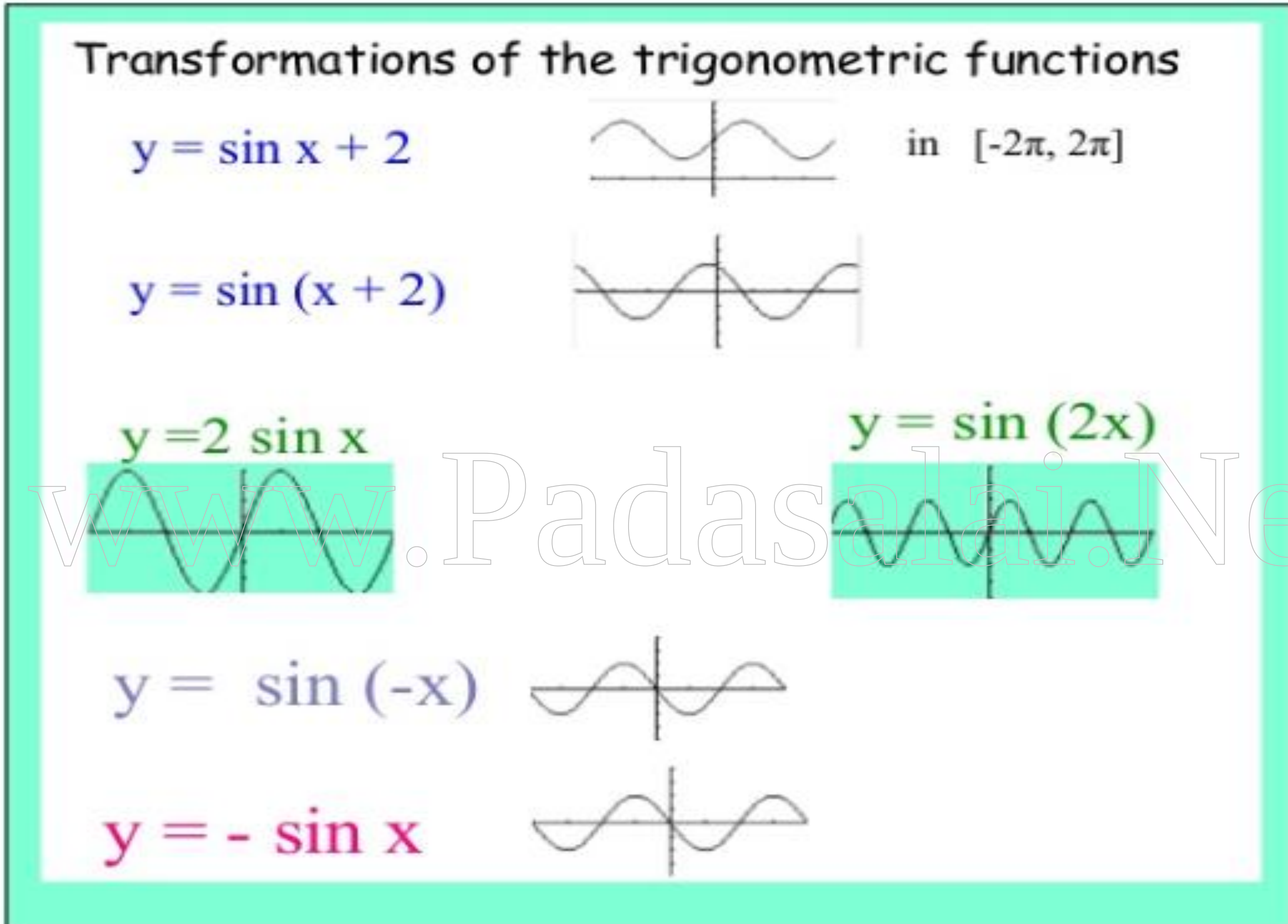
maximum value = 1
minimum value = -1

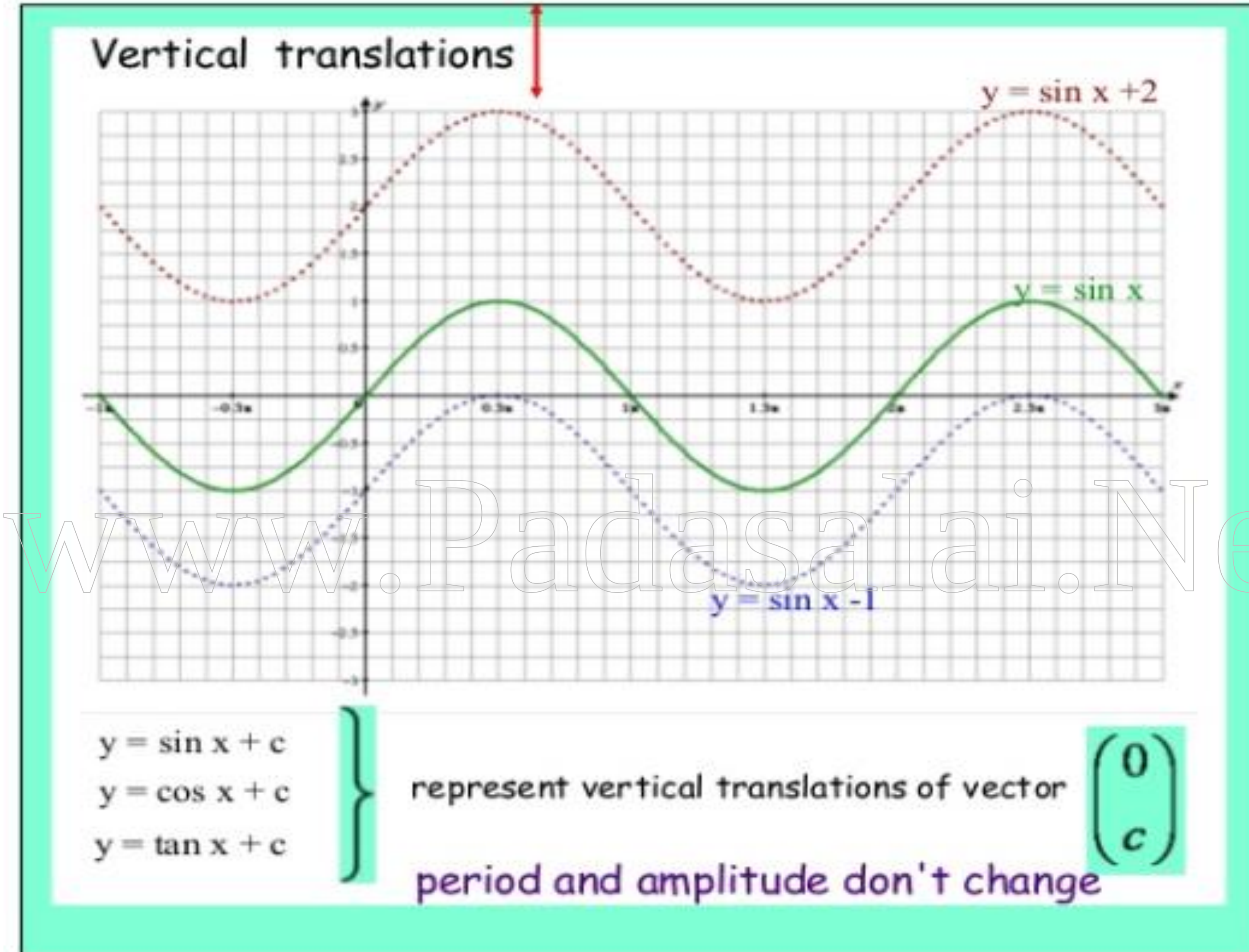
amplitude= 1

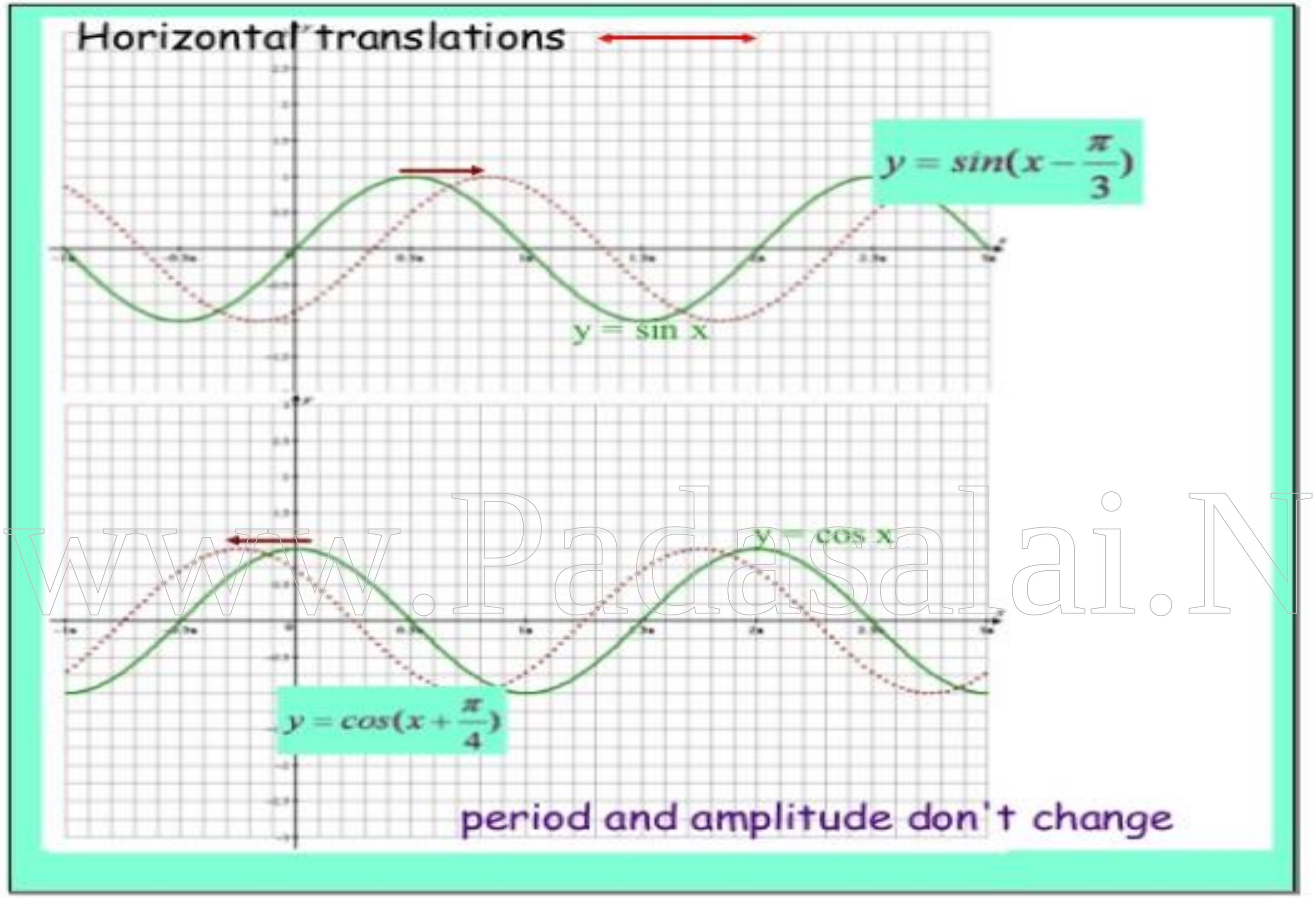
range= [-1,1]

period= 2π (or $360°$)


Tangent function
period
vertical asymptotes: x = π/2 , x = 3π/2, x = 5π/2,....
period : π
no amplitude


Transformations of the trigonometric functions
y = sin x + 2
in [-2π, 2π]
y = sin (x + 2)
y =2 sin x
y = sin (2x)
y = sin (-x)
y = - sin x


Vertical translations
y = sin x +2
y = sin x
y = sin x -1
y = sin x + c
y = cos x + c
y = tan x + c
represent vertical translations of vector
(0 c)
period and amplitude don't change


Horizontal translations
$y = sin(x - \frac{\pi}{3})$
y = sin x
y = cos x
$y = cos(x + \frac{\pi}{4})$
period and amplitude don't change

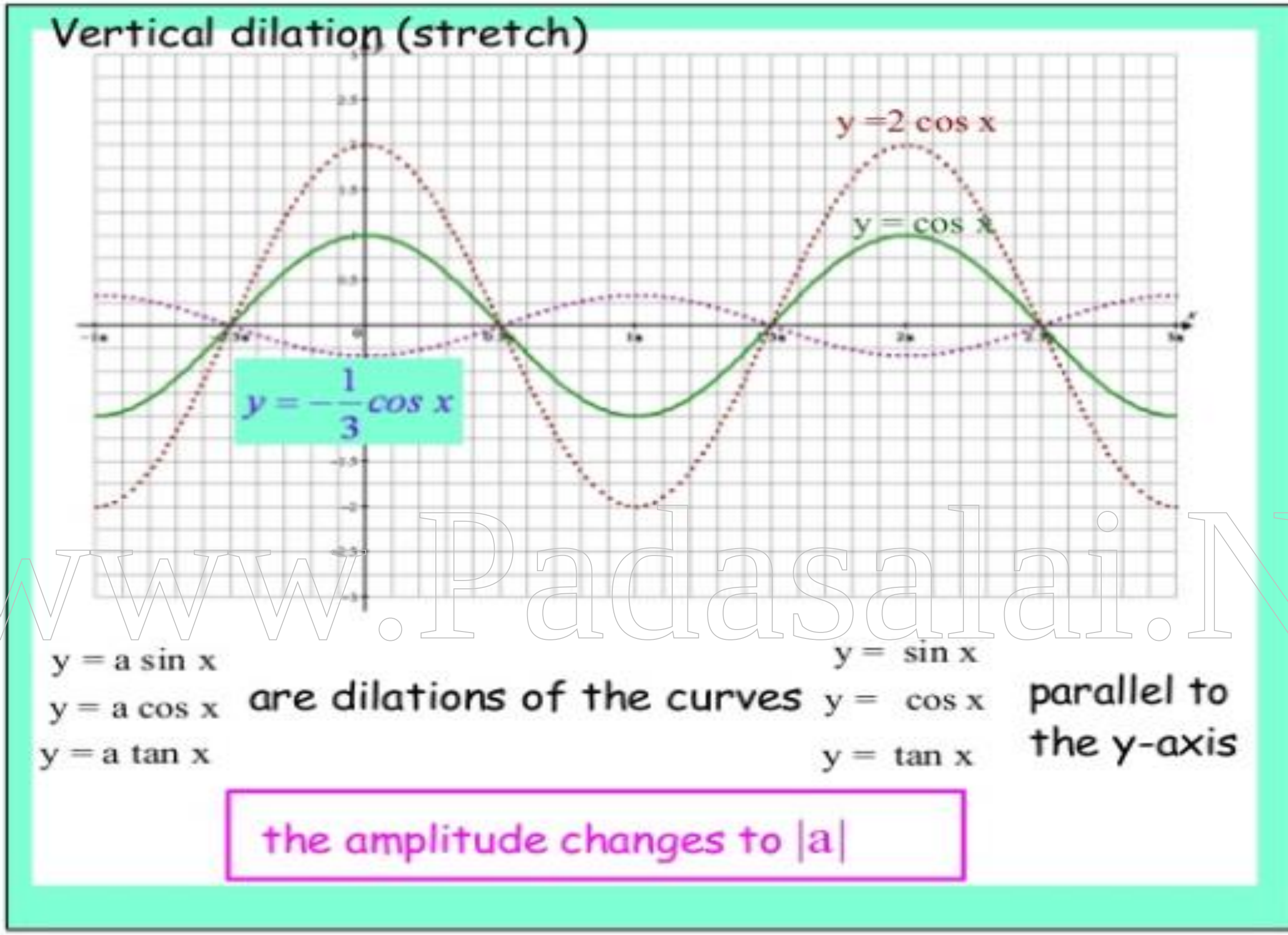
Horizontal translations

$y = \sin(x \pm a)$
$y = \cos(x \pm a)$
$y = \tan(x \pm a)$

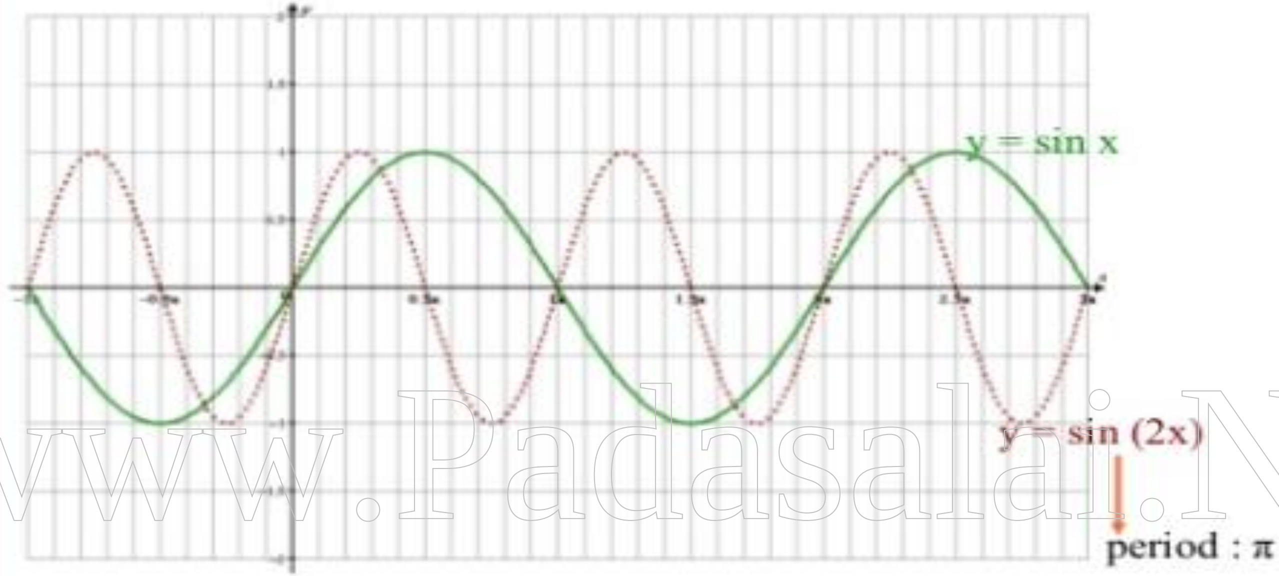
represent horizontal translations of the curves $y = \sin x$, $y = \cos x$ and $y = \tan x$

$y = \sin(x - a) \longrightarrow$ to the right

$y = \sin(x + a) \longleftarrow$ to the left

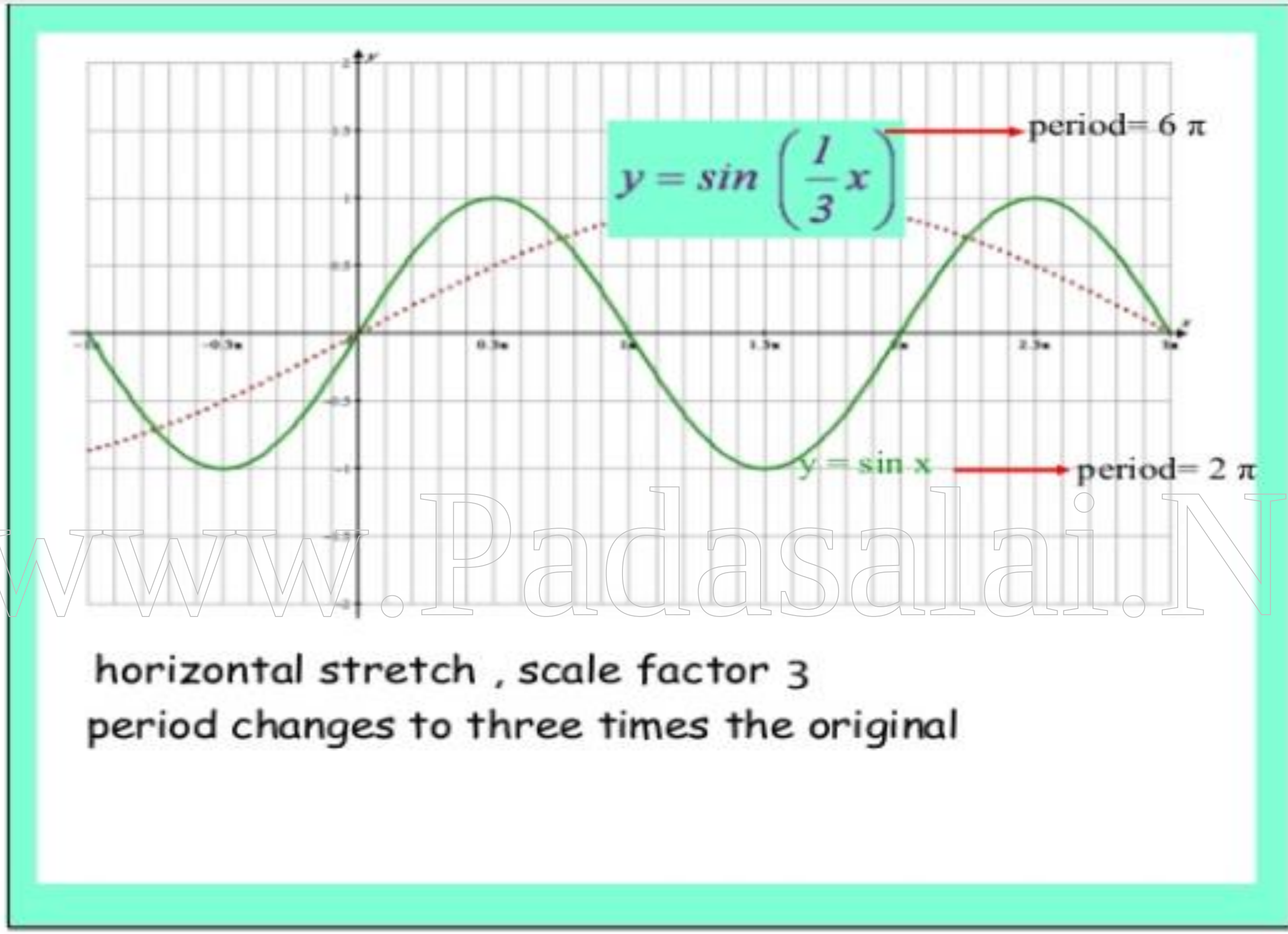

Vertical dilation (stretch)
y =2 cos x
y = cos x
y = -1/3 cos x
y = a sin x
y = a cos x
y = a tan x
are dilations of the curves
y = sin x
y = cos x
y = tan x
parallel to the y-axis
the amplitude changes to |a|

Horizontal dilation (stretch)



horizontal stretch , scale factor ½

period changes to half the original

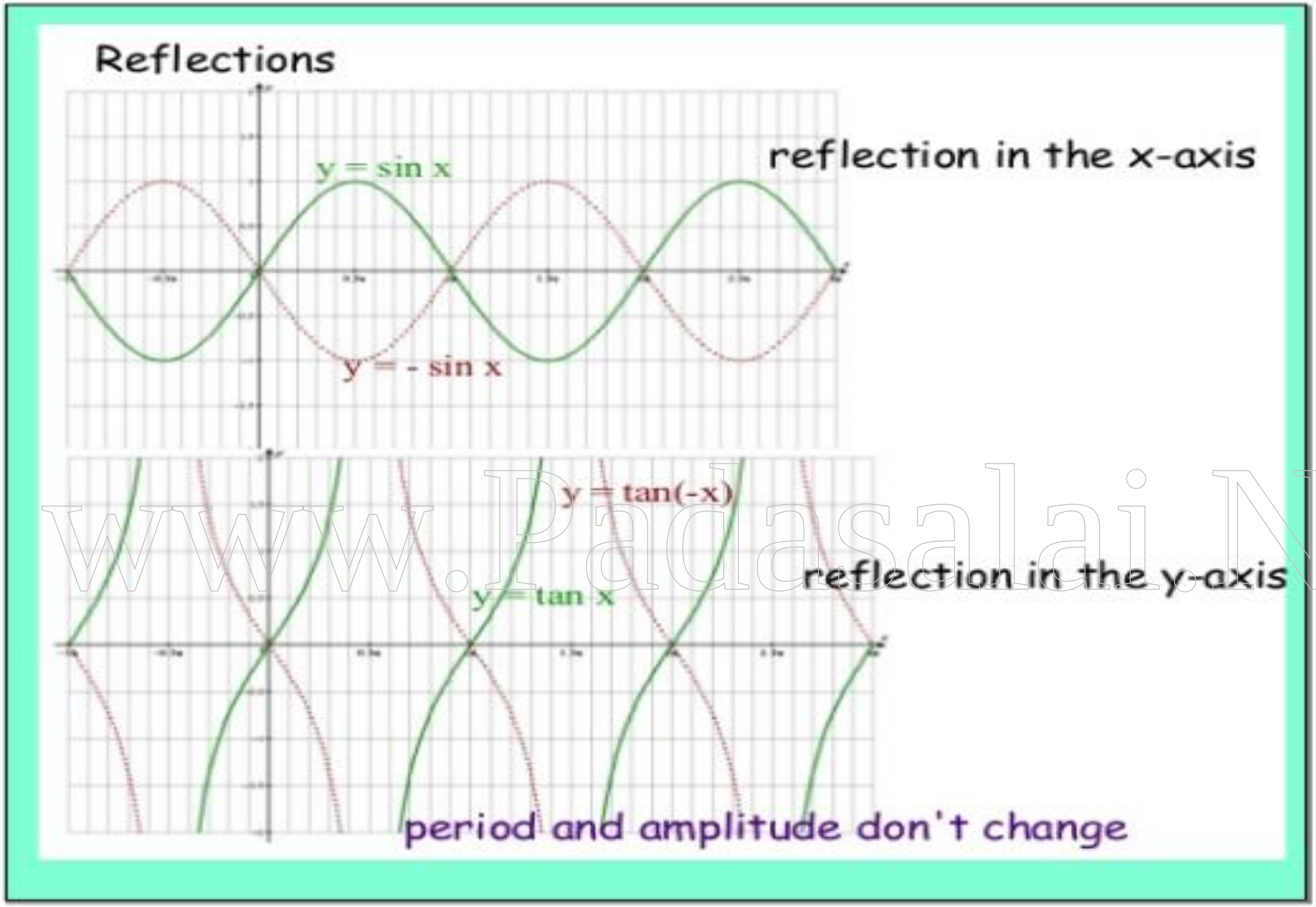

$y = sin\left(\frac{1}{3}x\right)$
period= 6 π
y = sin x
period= 2 π
horizontal stretch , scale factor 3
period changes to three times the original

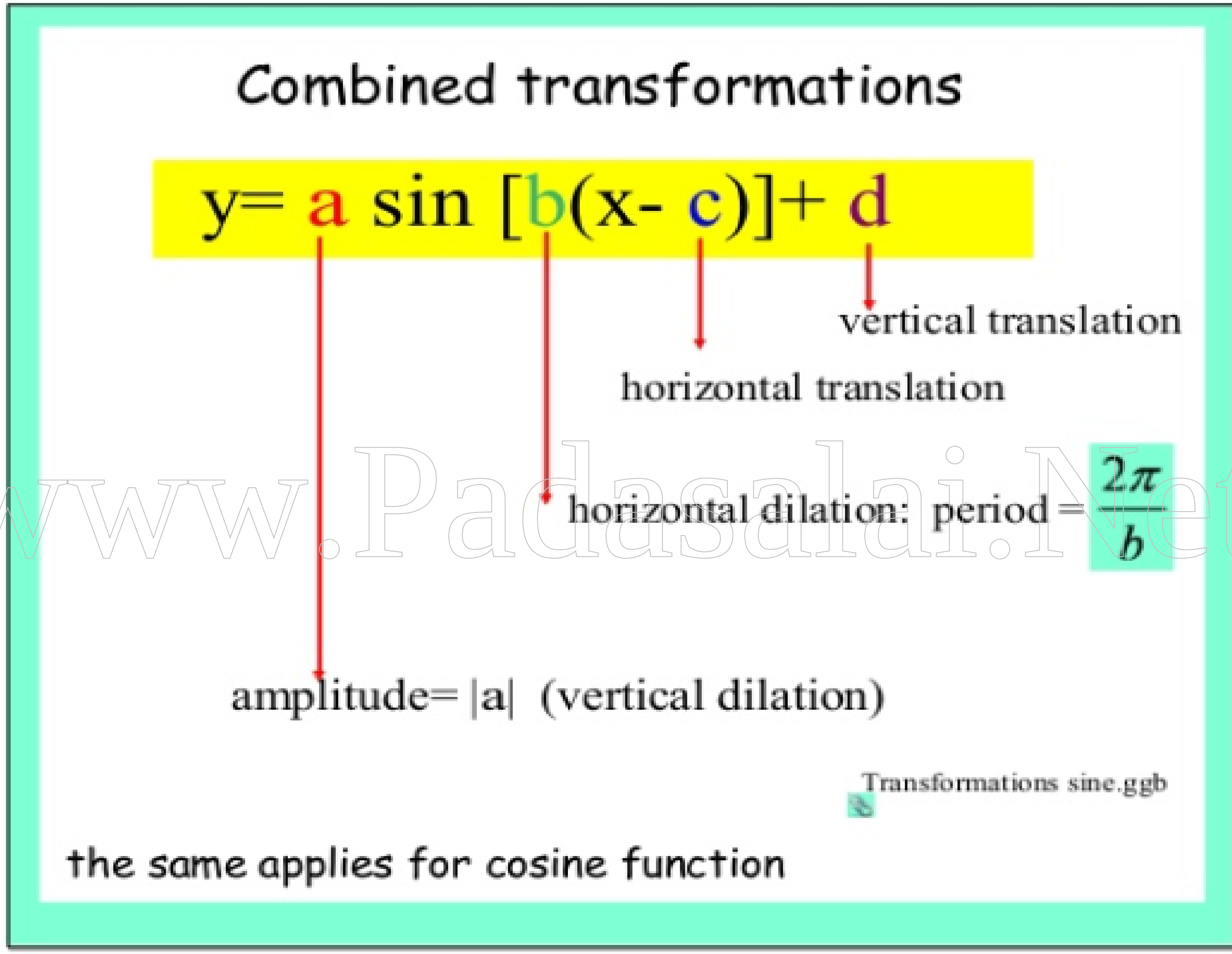
$y = \sin(bx)$, $y = \cos(bx)$ are dilations of the curves $y = \sin x$, $y = \cos x$ parallel to the x-axis

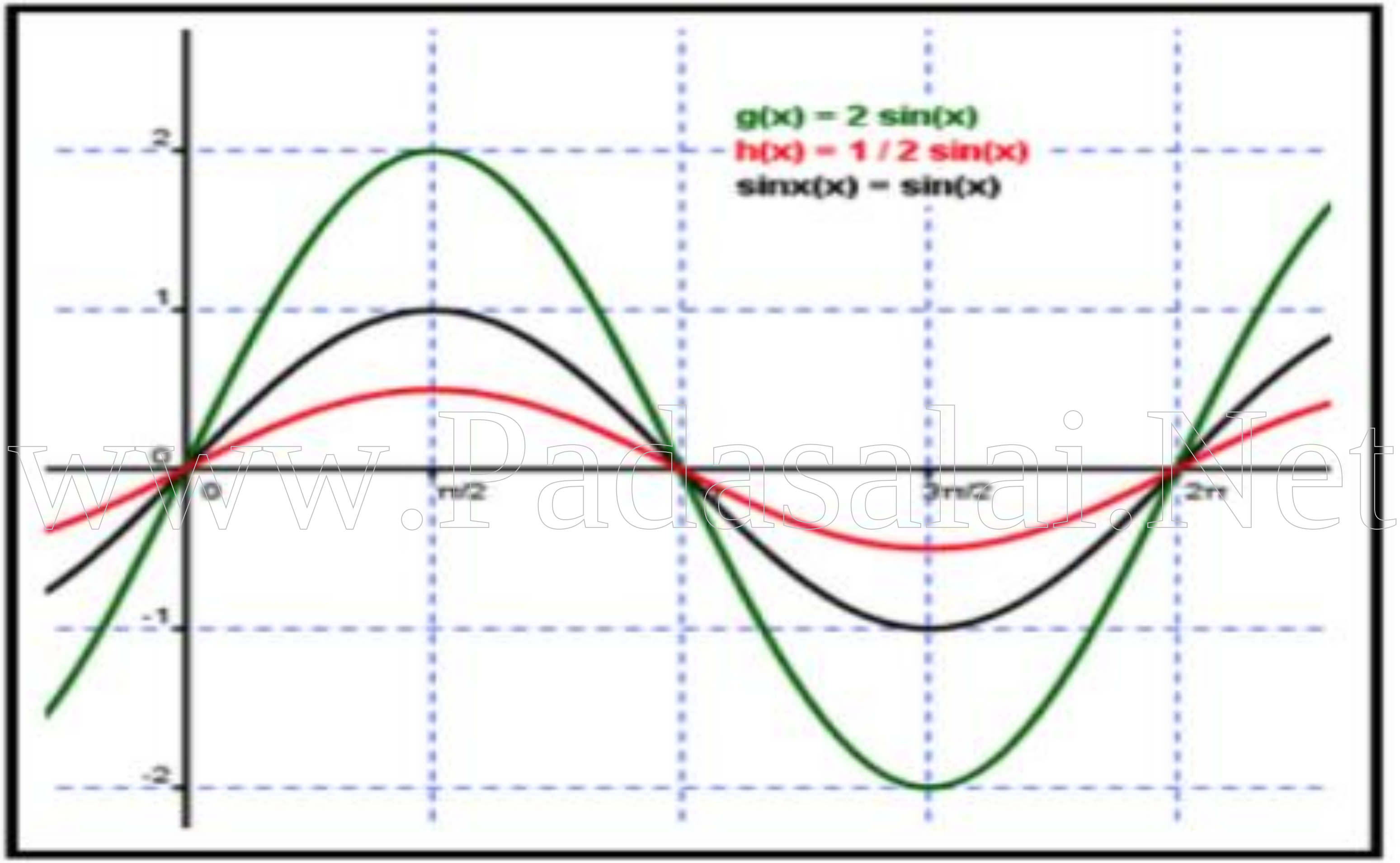
the period changes to $\frac{2\pi}{b}$

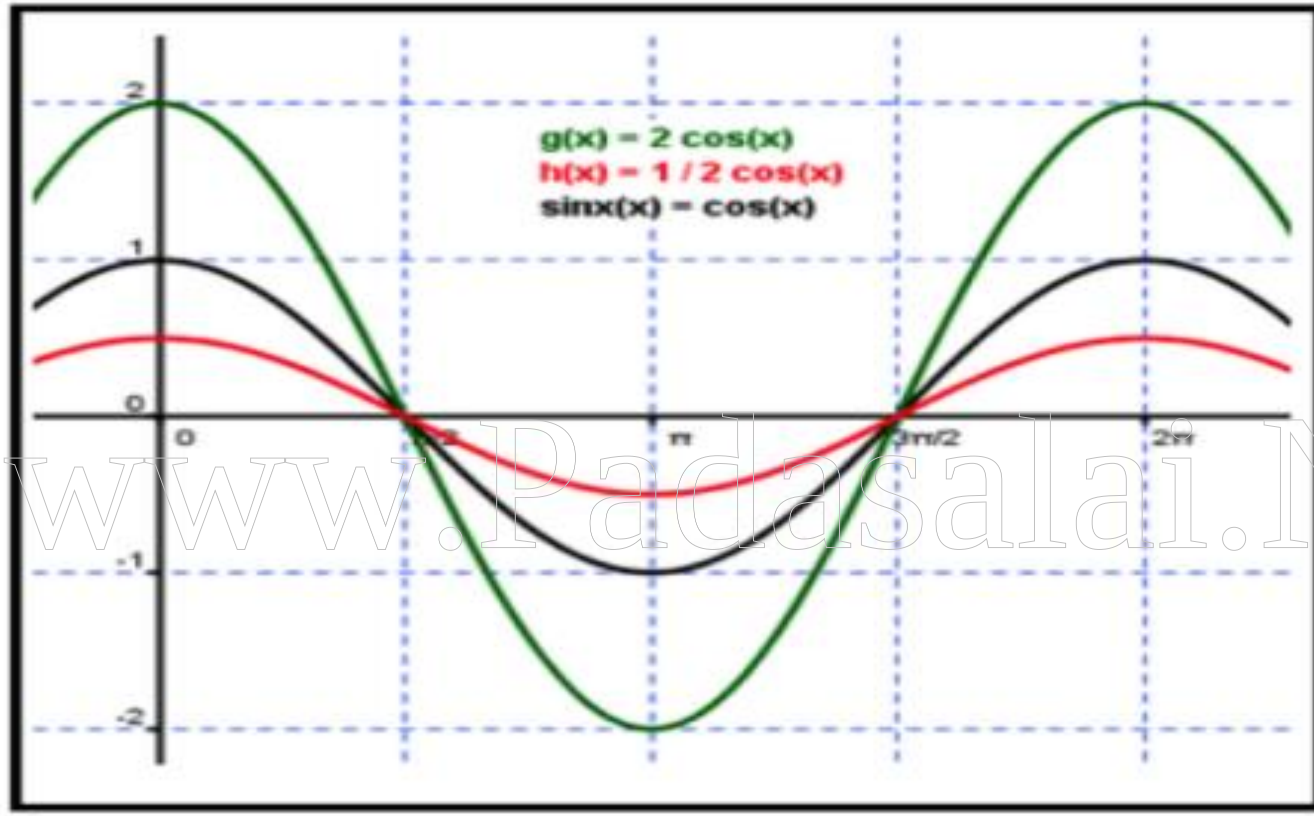
$y = \tan(bx)$ is a dilation of the curve $y = \tan x$ parallel to the x-axis

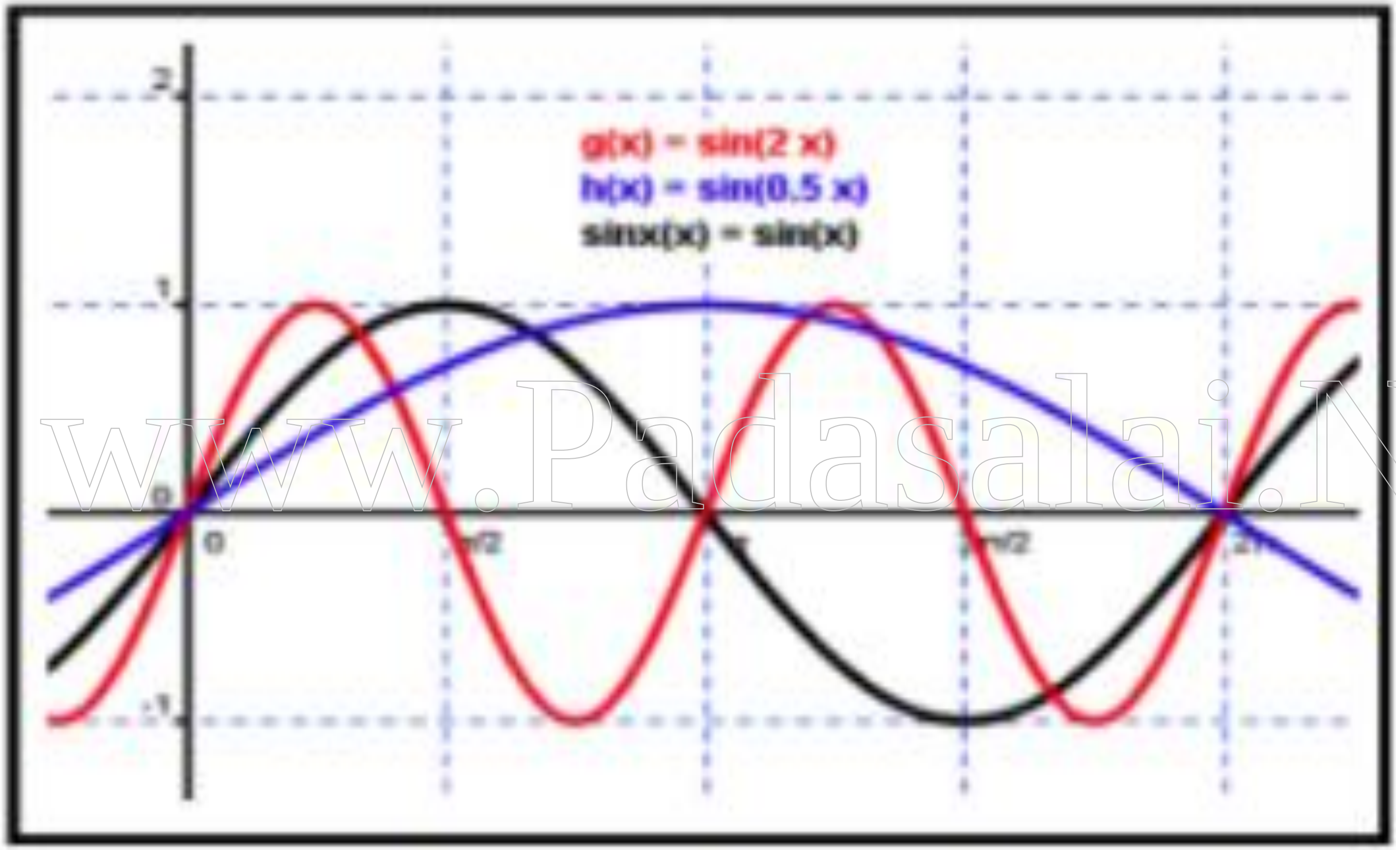
the period changes to $\frac{\pi}{b}$


Reflections
y = sin x
reflection in the x-axis
y = - sin x
y = tan(-x)
reflection in the y-axis
y = tan x
period and amplitude don't change


Combined transformations
y= a sin [b(x- c)]+ d
vertical translation
horizontal translation
horizontal dilation: period = $\frac{2\pi}{b}$
amplitude= |a| (vertical dilation)
Transformations sine.ggb
the same applies for cosine function


g(x) = 2 sin(x)
h(x) = 1 / 2 sin(x)
sinx(x) = sin(x)
2
1
0
-1
-2
0
π/2
3π/2
2π


g(x) = 2 cos(x)
h(x) = 1 / 2 cos(x)
sinx(x) = cos(x)
2
1
0
-1
-2
0
π
3π/2
2π


g(x) = sin(2 x)
h(x) = sin(0.5 x)
sinx(x) = sin(x)
2
1
0
-1
0
π/2
π
3π/2
2π