Unit - 5.

Binomial Theorem, Sequence and Series.

$$nC_r = \frac{n(n-1)(n-2)\cdots(n-(r-1))}{r(r-1)(r-2)\cdots !}$$

$$= \frac{n!}{(n-r)!\,r!}$$

$$nC_r = \frac{n!}{(n-r)!\,r!}$$

Binomial Theorem:

$$(a+b)^n = nC_0 a^n b^0 + nC_1 a^{n-1} b^1 + nC_2 a^{n-2} b^2 + nC_3 a^{n-3} b^3 + \cdots + nC_r a^{n-r} b^r + \cdots + nC_n a^0 b^n.$$

Particular cases of Binomial Theorem:

$$(a-b)^n = nC_0 a^n b^0 - nC_1 a^{n-1} b^1 + nC_2 a^{n-2} b^2 - nC_3 a^{n-3} b^3 + nC_4 a^{n-4} b^4 + \cdots$$

For example $(100-2)^4$

$a = 100$ $b = 2$ $n = 4$ (or)

$$(a+b)^n = nC_0 a^n b^0 + nC_1 a^{n-1} b^1 + nC_2 a^{n-2} b^2 + nC_3 a^{n-3} b^3 + nC_4 a^{n-4} b^4 + \cdots$$

For example $(100-2)^4$

$a = 100$ $b = -2$ $n = 4$

Example 5.1

Find the expansion of $(2x+3)^5$.

Sol:

Given that

$= (2x+3)^5$

$a = 2x \quad b = 3 \quad n = 5$

$(a+b)^n = nC_0\, a^n b^0 + nC_1\, a^{n-1} b^1 + nC_2\, a^{n-2} b^2 +$
$nC_3\, a^{n-3} b^3 + nC_4\, a^{n-4} b^4 + nC_5\, a^{n-5} b^5.$

$(2x+3)^5 = 5C_0 (2x)^5 3^0 + 5C_1 (2x)^{5-1} 3 +$
$5C_2 (2x)^{5-2} (3)^2 + 5C_3 (2x)^{5-3} (3)^3 + 5C_4 (2x)^{5-4} (3)$
$+ 5C_5 (2x)^{5-5} (3)^5$

$= (1)\, 32x^5 (1) + 5(16x^4)\, 3 + 10(8x^3)\, 9$
$+ 10(4x^2)\, 27 + 5(2x)\, 81 + 243$

$= 32x^5 + 240x^4 + 720x^3 + 108x^2 + 810x +$
$243.$

$nC_r = \frac{n!}{(n-r)!\,r!}$ $\qquad nC_r = \frac{n!}{(n-r)!\,r!}$

$nC_0 = \frac{5}{(5-0)!\,0!}$
$= \frac{\cancel{5}}{\cancel{5}}$
$= 1$

$= \frac{\cancel{1\times2\times3\times4\times5}}{\cancel{4}!\;1!}$
$= \frac{5}{1}$
$= 5$

$n = 10$

$nC_3 = \frac{5}{2!\,3!}$
$= \frac{\cancel{3}\times4\times5}{2} = \frac{20}{2}$
$= 10$

$nC_1 = \frac{5!}{(5-1)!\,1!}$
$= \frac{5!}{4!\,1!}$

$nC_2 = \frac{5}{3!\,2!}$
$= \frac{\cancel{1\times2\times3}\times4\times5}{}$

Example: 5.2.

Evaluate 98^4.

$nCr = \frac{n!}{(n-r)!r!}$

$4C_1 = \frac{4}{3!1!} = 4$

Sol:

Given that

$98^4 = (100-2)^4$

$a = 100 \quad b = 2 \quad n = 4$

$(a-b)^n = nC_0 a^n b^0 - nC_1 a^{n-1} b^1 + nC_2 a^{n-2} b^2 - nC_3 a^{n-3} b^3 + nC_4 a^{n-4} b^4$

$(100-2)^4 = 4C_0 (100)^4 (2)^0 - 4C_1 (100)^{4-1} (2)^1 + 4C_2 (100)^{4-2} (2)^2 + 4C_3 (100)^{4-3} (2)^3 + 4C_4 (100)^{4-4} (2)^4$

$= 100000000 - 4(1000000)2 + 6(10000)4 - 4(100)8 + 1(16)$

$= 100000000 - 8000000 + 240000 - 3200 + 16$

$= 92236816.$

Exercise: 5.1

2. compute

i) 102^4.

Sol:

Given that

$102^4 = (100+2)^4$

$a = 100 \quad b = 2 \quad n = 4$

Lesson - 5.

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5. Binomial Theorem

Exercise: 5.2.

1. Write the n^{th} term of the following sequences.

i) 2, 2, 4, 4, 6, 6.

Sol:

Given that

odd terms are

even terms are 2, 4, 6

$$\therefore a_n = \begin{cases} n+1 & \text{if } n \text{ is odd} \\ n & \text{if } n \text{ is even.} \end{cases}$$

ii) $1/2, 2/3, 3/4, 4/5, 5/6 \ldots\ldots$

Sol:

Given that

Nr:

1, 2, 3, 4, 5 ... is an A.P

$$c_n = a + (n-1)d$$
$$= 1 + (n-1)1$$
$$= 1 + n - 1$$
$$= n$$

Dr:

2, 3, 4, 5 ....

$t_n = a + (n-1)d$

$= 2 + (n-1) 1$

$= 2 + n - 1$

$t_n = n + 1$

$$a_n = \frac{n}{n+1}, \forall n \in N$$

iii) $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}$ ....

Sol:

Nr:

1, 3, 5, 7, 9 ....

$a = 1$ $d = t_2 - t_1$

$d = 3 - 1$

$d = 2$

$t_n = a + (n-1)d$

$= 1 + (n-1) 2$

$= 1 + 2n - 2$

$t_n = 2n - 1$

Dr:

2, 4, 6, 8, 10 .....

$a = 2$ $d = 2$

$t_n = a + (n-1)d$

$= 2 + (n-1)2$

$= 2 + 2n - 2$

$t_n = 2n$

$$a_n = \frac{2n-1}{2n} \quad \forall n \in N.$$

iv) 6, 10, 4, 12, 2, 14, 0, 16, -2 ......

Sol:

odd terms 6, 4, 2, 0, -2

even terms 10, 12, 14, 16 ....

$t_n = 6 + (n-1)(-2)$

$= 6 - 2n + 2$

$= 8 - 2n$

$t_n = 10 + (n-1)2$

$= 10 + 2n - 2$

$= 8 + 2n$

$$\therefore a_n = \begin{cases} 8-2n & \text{if } n \text{ is odd} \\ 8+2n & \text{if } n \text{ is even.} \end{cases}$$

4. The product of three increasing number in G.P is 5832. If we add 6 to the second number and 9 to the third number then resulting numbers form an A.P. Find the number in G.P.

Sol:

Let $\frac{a}{r}, a, ar$

Product $= 5832$

$\Rightarrow \left(\frac{a}{r}\right)(a)(ar) = 5832$

$\Rightarrow \frac{a}{\cancel{r}} \times a \times a\cancel{r} = 5832$

$a^3 = 5832$

$a = \sqrt[3]{5832}$

$a = \sqrt{2\times2\times2\times9\times9\times9}$

$a = 2\times9$

$\boxed{a = 18}$

$$\begin{array}{r|l} 2 & 5832 \\ 2 & 2916 \\ 2 & 1458 \\ 9 & 729 \\ 9 & 81 \\ 9 & 9 \\ & 1 \end{array}$$

$\frac{a}{r}, a+6, ar+9$ is an A.P.

$t_2 - t_1 = t_3 - t_2$

$(a+6) - \frac{a}{r} = (ar+9) - (a+6)$

$(a+6) + (a+6) = a/r + ar + 9$

$2(a+6) = \frac{a}{r} + ar + 9$

$2(18+6) = \frac{18}{r} + 18 + 9$

$2\times24 = \frac{18}{r} + 18 + 9$

$$48 = \frac{18 + 18r^2 + 9r}{r}$$

$$48r = 18 + 18r^2 + 9r$$

$$\Rightarrow 18r^2 + 9r - 48r + 18 = 0$$

$$18r^2 - 39r + 18 = 0$$

324

-27 -12

-39

$$18r^2 - 27r - 12r + 18 = 0$$

$$9r(2r-3) - 6(2r-3) = 0$$

$$(2r-3)(9r-6) = 0$$

$$2r - 3 = 0 \qquad 9r - 6 = 0$$

$$2r = 3 \qquad 9r = 6$$

$$r = 3/2 \qquad r = 6/9 = 2/3$$

$\Rightarrow a = 18$

$r = 3/2$

$\frac{a}{r}, a, ar$

$\frac{18}{3/2}, 18, \cancel{18}^{9} \times 3/\cancel{2}_1$

$\cancel{18}^{6} \times 2/\cancel{3}_1, 18, 27$

$12, 18, 27$

$\Rightarrow a = 18$

$r = 2/3$

$\frac{a}{r}, a, ar$

$\frac{18}{2/3}, 18, \cancel{18}^{6} \times \frac{2}{\cancel{3}_1}$

$\cancel{18}^{9} \times \frac{3}{\cancel{2}_1}, 18, 12$

$27, 18, 12.$

5. Write the n^{th} term of the sequences $\frac{3}{1^2 2^2}, \frac{5}{2^2 3^2}, \frac{7}{3^2 4^2}$ ...... as differences of two terms.

Given that

$$\frac{3}{1^2 2^2}, \frac{5}{2^2 3^2}, \frac{7}{3^2 4^2} \ldots$$

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Nr :

3, 5, 7 in an A.P.

$a = 3$

$d = 2$

$t_n = a + (n-1)d$

$= 3 + (n-1)2$

$= 3 + 2n - 2$

$t_n = 1 + 2n$

Dr :

$1^2 2^2, \ 2^2 3^2, \ 3^2 4^2 \ldots\ldots$

$t_n = [n(n+1)]^2$

$$a_n = \frac{2n+1}{[n(n+1)]^2} = \frac{2n+1}{n^2(n+1)^2}$$

$$= \frac{n^2 + 2n + 1 - n^2}{n^2(n+1)^2}$$

$$= \frac{(n+1)^2 - n^2}{n^2(n+1)^2}$$

$$= \frac{\cancel{(n+1)^2}}{n^2 \cancel{(n+1)^2}} - \frac{\cancel{n^2}}{\cancel{n^2}(n+1)^2}$$

$a_n = \frac{1}{n^2} - \frac{1}{(n+1)^2}$

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6. If t_k is the k^{th} term of a G.P then show that t_{n-k}, t_n, t_{n+k} also form a G.P for any positive integer k.

Sol:

Given that

t_k is the k^{th} term of a G.P

$t_2 = t_n = a \times r^{n-1}$

$t_1 = t_{n-k} = a \times r^{n-k-1}$

$t_3 = t_{n+k} = a \times r^{n+k-1}$

common ratio $R = \frac{t_2}{t_1}$

$= \frac{ar^{n-1}}{ar^{n-k-1}}$

$= r^{\cancel{n}\cancel{-1}\cancel{-n}+k\cancel{+1}}$

$R. = r^k$

common ratio $R = \frac{t_3}{t_2}$

$= \frac{ar^{n+k-1}}{ar^{n-1}}$

$= r^{\cancel{n}+k\cancel{-1}\cancel{-n}\cancel{+1}}$

$= r^k$

$R = r^k$

Since the common ratio is same

$\therefore t_{n-k}, t_n, t_{n+k}$ from a G.P.

4. If a, b, c are in geometric progression and if $a^{1/x} = b^{1/y} = c^{1/z}$ then prove that x, y, z are in arithmetic progression.

Sol:

Given that

If a, b, c are in A.P

and if $a^{1/x} = b^{1/y} = c^{1/z}$

x, y, z are in A.P

$$a^{1/x} = b^{1/y} = c^{1/z} = k$$

Also,

$a^{1/x} = k \quad b^{1/y} = k \quad c^{1/z} = k$

$a = k^x \quad b = k^y \quad c = k^z$

Given a, b, c are in G.P

$$\frac{t_2}{t_1} = \frac{t_3}{t_2}$$

$$\frac{b}{a} = \frac{c}{b}$$

$$b^2 = ac$$

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$$b^2 = ac$$

$$(k^y)^2 = k^x k^z$$

$$k^{2y} = k^{x+z}$$

$$2y = x + z$$

$$y + y = x + z$$

$$y - x = z - y$$

$$t_2 - t_1 = t_3 - t_2$$

$\therefore$ x, y, z are in A.P.

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8. The AM of two numbers exceeds their GM by 10 and HM by 16. Find the number.

Sol:

Given that

Let the numbers a and b

$$A.M = \frac{a+b}{2} \qquad G.M = \sqrt{ab}$$

$$H.M. H = \frac{2ab}{a+b}$$

$$A - G = 10 \Rightarrow G = A - 10$$

$$A - H = 16 \Rightarrow H = A - 16$$

Formula:

$$G^2 = AH$$

$$(A-10)^2 = A(A-16)$$

$$A^2 - 100 - 20A = A^2 - 16A$$

$\Rightarrow A^2 + 100 - 2A - A^2 - 16A$

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$100 = 20A - 16A$

$100 = 4A$

$A = \frac{100}{4}$

$A = 25$ (or) $A \cdot M = 25$

$\Rightarrow A \cdot M = \frac{a+b}{2}$

$25 = \frac{a+b}{2}$

$25(2) = a+b$

$a+b = 50 \longrightarrow (1)$

$\Rightarrow G = A - 10$

$G = 25 - 10$

$G = 15$ (or) $G \cdot M = 15$

$\Rightarrow G \cdot M = \sqrt{ab}$

$15 = \sqrt{ab}$

$\sqrt{ab} = 15$

$ab = 15^2$

$ab = 225 \longrightarrow (2)$

$b = \frac{225}{a}$

$a + b = 50$

$$a + \frac{225}{a} = 50$$

$$\frac{a^2 + 225}{a} = 50$$

$$a^2 + 225 = 50a$$

225

-45 -5

-50

$$\Rightarrow a^2 - 50a + 225 = 0$$

$$(a-45)(a-5) = 0$$

$$a = 45 . a = 5$$

$$\Rightarrow b = \frac{225}{a} = \frac{225}{45} = 5$$

$$\Rightarrow b = \frac{225}{5} = 45.$$

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9. If the root of the equation $(q-r)x^2 + (r-p)x + p-q = 0$ are equal then show that p, q and r are in A.P.

Sol:

Given that

$$(q-r)x^2 + (r-p)x + p-q = 0$$

$\therefore$ Roots are equal

$$b^2 - 4ac = 0$$

$$A = q-r \quad B = r-p \quad C = p-q$$

$$\Rightarrow (r-p)^2 - 4(q-r)(p-q) = 0$$

$$\Rightarrow (r^2 + p^2 - 2rp) - 4[pq - q^2 - rp + rq] = 0$$

$$\Rightarrow (r^2 + p^2 - 2rp - 4pq + 4q^2 + 4rp - rp = 0$$

$\Rightarrow r^2+p^2+4q^2+2rp-4pq-4qr=0.$

$\Rightarrow p^2+(-2q)^2+r^2+2(p)(r)+2(-2q)p+2(-2q)r=0$

$\Rightarrow [\because (a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ac]$

$(p-2q+r)^2=0$

$p-2q+r=0$

$p+r=2q$

$p+r=q+q$

$p-q=q-r$

$\therefore p, q, r$ are in A.P.

10. If a, b, c are respectively the p^{th}, q^{th} and r^{th} terms of a G.P show that $(q-r)\log a+(r-p)\log b+(p-q)\log c=0$.

Sol:

$m\log a=\log a^m$

$\log A+\log B=\log AB$

Given that.

$t_p=a \quad t_q=b \quad t_r=c$

$t_p=a$

$a=t_p \quad b=t_q \quad c=t_r$

$g_n=ar^{n-1}$

$a=xy^{p-1} \quad b=xy^{q-1} \quad c=xy^{R-1}$

L.H.S

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$\Rightarrow (q-r)\log a + (r-p)\log b + (p-q)\log c$

$\Rightarrow \log a^{q-r} + \log b^{r-p} + \log c^{p-q}$

$\Rightarrow \log\left[a^{q-r} \times b^{r-p} \times c^{p-q}\right] \rightarrow (1)$

$= a^{q-r} \times b^{r-p} \times c^{p-q}$

$= (xy^{p-1})^{q-r} \times (xy^{q-1})^{r-p} \times (xy^{r-1})^{p-q}$

$= x^{q-r+r-p+p-q} \quad y^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)}$

$= x^0 \quad y^0$

$= 1$

$\therefore \log\left[a^{q-r} \times b^{r-p} \times c^{p-q}\right]$

$= \log(1) \qquad (\because \log 1 = 0)$

$= 0$

$= R.H.S$

Finite series.

Exercise 5.3.

1. Find the sum of the first 20-terms of the arithmetic progression having the sum of first 10 terms as 52 and the sum of the first 15 terms as 77.

Sol:

Given that

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$n=10$

$$S_{10} \Rightarrow \frac{10}{2}[2a+(10-1)d] = 52$$

$$\Rightarrow 5(2a+9d) = 52$$

$$2a+9d = \frac{52}{5} \rightarrow (1)$$

$n=15$

$$S_{15} \Rightarrow \frac{15}{2}[2a+(15-1)d] = 77$$

$$2a+14d = \frac{77\times 2}{15}$$

$$2a+14d = \frac{154}{15} \rightarrow (2)$$

(2) − (1)

$$\cancel{2a}+14d = \frac{154}{15}$$

$$\cancel{2a}+9d = \frac{52}{5}$$

$$(-) \quad (-) \quad (-)$$

$$5d = \frac{154}{15} - \frac{52}{5}$$

$$5d = \frac{154-52\times 3}{15}$$

$$5d = \frac{154-156}{15}$$

$$5d = \frac{-2}{15}$$

$$d = \frac{-2}{15+5}$$

$$d = \frac{-2}{75}$$

① $\Rightarrow$ $$2a + 9d = \frac{52}{5}$$

$$2a + 9\left(-2/75\right) = \frac{52}{5}$$

$$2a - \frac{18}{75} = \frac{52}{5}$$

$$2a = \frac{52}{5} + \frac{18}{75}$$

$$2a = \frac{52 \times 15 + 18}{75}$$

$$= \frac{780 + 18}{75}$$

$$= \frac{\cancel{798}^{266}}{\cancel{75}_{25}}$$

$$2a = \frac{266}{25}$$

$$a = \frac{\cancel{266}^{133}}{25 \times \cancel{2}_{1}}$$

$$a = \frac{133}{25}$$

$$S_{20} = \frac{n}{2}\left[2a + (n-1)d\right]$$

Sol:

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$$S_n = 8[1+11+111+\dots+n\ term]$$

$$= \frac{8}{9}[9+99+999+\dots+n\ term]$$

$$= \frac{8}{9}[(10-1)+(100-1)+(1000-1)+\dots+n]$$

$$= \frac{8}{9}[(10+10^2+10^3+\dots+n\ term)-n]$$

$$\Rightarrow \frac{8}{9}\left[\frac{10(10^n-1)}{9}-n\right] = \frac{8}{81}[10(10^n-1)$$

ii) $6+66+666+\dots\dots n$ Terms.

Sol:

$$S_n = 6[1+11+111+\dots+n\ terms]$$

$$= \frac{6}{9}[9+99+999+\dots+n\ terms]$$

$$= \frac{6}{9}[(10-1)+(100-1)+(1000-1)+\dots$$

$$= \frac{6}{9}[(10+10^2+10^3+\dots+n\ terms)-n]$$

$$= \frac{6}{9}\left[\frac{10(10^n-1)}{9}-n\right] = \frac{6}{81}[10(10^n$$

4. compute the sum of first n terms of $1+(1+4)+(1+4+4^2)+(1+4+4^2+4^3)+\dots\dots$

Sol: Given that

Let T_n be the n^{th} term of

The given series.

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Then,

$$1+4+4^2+4^3+\cdots\cdots$$

$$T_n = 1\left[\frac{4^k-1}{4-1}\right]$$

$$T_n = \frac{4^k-1}{3}$$

$$S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n} \frac{4^k-1}{3}$$

$$S_n = \frac{1}{3}\left[\sum_{k=1}^{n} 4^k - \sum_{k=1}^{n} 1\right]$$

$$S_n = \frac{1}{3}\left[(4^1+4^2+4^3+\cdots+4^n] - n\right]$$

$$= \frac{1}{3}\left[4(\frac{4^n-1)}{3} - n\right]$$

$$= \frac{4}{9}\left[(4^n-1)\right] - (n/3]$$

$$S_n = \frac{4}{9}(4^n-1) - n/3$$

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$t_n = a + (n-1)d$

$t_n = 1 + (n-1)3$

$= 1 + 3n - 3$

$t_n = 3n - 2$

Dr:

$\Rightarrow \frac{1}{1}, \frac{1}{3}, \frac{1}{3^2}, \frac{1}{3^3} \dots\dots$

$a = 1 \quad r = 1/3$

The given steps are:

$a, (a+d)r, (a+2d)r^2, \dots\dots$

$\therefore$ This is an A.G.P

$T_n = [a + (n-1)d]\, r^{n-1}$

$= [1 + (n-1)3]\,(1/3)^{n-1}$

$= [1 + 3n - 3]\,(1/3)^{n-1}$

$= 3n - 2 \left(\frac{1}{3^{n-1}}\right)$

$T_n = \frac{3n-2}{\cancel{3n+2}}\left(\frac{1}{3^{n-1}}\right)$

$T_n = \frac{3n-2}{3^{n-1}}$

Sn be the sum of the n term of given sequences

$$S_n = \sum_{k=1}^{n} \frac{3k-2}{3^{k-1}}$$

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$$S_n = \sum_{k=1}^{n} (3k-2) \sum \frac{1}{3^{k-1}}$$

$$= \left[3(1+2+3+\cdots n) - 2n\right]\left(\frac{1}{3^0+3^1+3^2+\cdots 3^{n-1}}\right)$$

$$= \left[\frac{3n(n+1)}{2} - 2n\right]\left[\frac{1}{1\left(\frac{3^n-1}{3-1}\right)}\right]$$

$$= \left[\frac{3n^2+3n}{2} - 2n\right]\left[\frac{2}{3^n-1}\right]$$

$$= \left[\frac{3n^2+3n-4n}{\cancel{2}}\right]\left[\frac{\cancel{2}}{3^n-1}\right]$$

$$S_n = \frac{3n^2-n}{3^n-1} = \frac{n(3n-1)}{3^n-1}$$

6. Find the value of n if the sum to n terms of the series $\sqrt{3} + \sqrt{75} + \sqrt{243} + \cdots 435\sqrt{3}$

Sol:

Given that

$$= \sqrt{3} + \sqrt{75} + \sqrt{243} + \cdots \text{ is an A.P}$$

$$= \sqrt{3} + \sqrt{25\times 3} + \sqrt{81\times 3} + \cdots$$

$$= \sqrt{3} + 5\sqrt{3} + 9\sqrt{3} + \cdots$$

$a = \sqrt{3}$ $\quad d = t_2 - t_1$

$= 5\sqrt{3} - \sqrt{3}$ $\quad d = 4\sqrt{3}$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

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$$= \frac{n}{2}[2(\sqrt{3})+(n-1)4\sqrt{3})] = 435\sqrt{3}$$

$$= n[\sqrt{3}+(n-1)2\sqrt{3}] = 435\sqrt{3}$$

$$= \sqrt{3}n + 2\sqrt{3}(n-1)n = 435\sqrt{3}$$

$\div \sqrt{3}$

$$\Rightarrow n + 2n(n-1) = 435$$

$$\Rightarrow n + 2n^2 - 2n = 435$$

$$\Rightarrow 2n^2 - n - 435 = 0$$

-870

$-30 \quad 29$

-1

$$\Rightarrow 2n^2 - 30n + 29n - 435 = 0$$

$$\Rightarrow 2n(n-15)\ 29(n-15) = 0$$

$$(2n+29)(n-15) = 0$$

$$2n+29 = 0 \qquad n-15 = 0$$

$$2n = -29 \qquad n = 15.$$

$$n = \frac{-29}{2}$$

7. Show that the sum of $(m+n)^{th}$ and $(m-n)^{th}$ term of an A.P is equal to twice the m^{th} term.

Sol:- Given.

$$t_{m+n} = t_{m-n} = 2t_m$$

$t_m = a+(m-1)d \rightarrow (1)$

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$t_{m+n} = a+(m+n-1)d \rightarrow (2)$

$t_{m-n} = a+(m-n-1)d \rightarrow (3)$

$(2)+(3) \Rightarrow (t_{m+n}) + (t_{m-n})$

$= a+(m+n-1)d + a+(m-n-1)d$

$= a+(md+nd-d)+a+(md-nd-d)$

$= a+md+nd-d+a+md-nd-d$

$= 2a+2md-2d$

$= 2a+2(m-1)d$

$= 2(a+2(m-1)d)$

$= 2(a+(m-1)d)$

$= 2t_m$

$t_{m+n} + t_{m-n} = 2t_m$

$\therefore$ Hence proved.

8. A man repays an amount of Rs: 3250 by paying Rs. 20 in the first month and then increases the payment by Rs. 15 per month. How long will it take him to clear the amount?

Sol:

Given that,

$= 20+35+50+\dots\dots+3250$

This is an A.P.

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$a = 20 \quad d = t_2 - t_1$

$= 35 - 20$

$d = 15$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \frac{n}{2}[2(20) + (n-1)15] = 3250$

$\Rightarrow \frac{n}{2}[40 + (n-1)15] = 3250$

$\frac{n}{2}[40 + 15n - 15] = 3250$

$\frac{n}{2}[15n + 25] = 3250$

$n[15n + 25] = 3250 \times 2$

$\Rightarrow 15n^2 + 25n = 6500$

$\div 5$

$3n^2 + 5n = 1300$

$3n^2 + 5n - 1300 = 0$

$3n^2 - 60n + 65n - 1300 = 0$

$3n(n - 20) + 65(n-20) = 0$

$(3n + 65)(n - 20) = 0$

$3n + 65 = 0 \quad n - 20 = 0$

$3n = -65 \quad \boxed{n = 20}$

$n = \frac{-65}{3}$

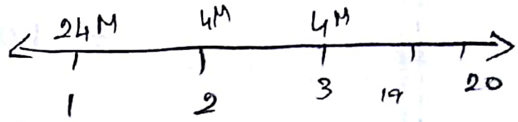
-3900

$65 \quad -60$

5

9. In a race 20 balls are placed in a line at intervals of 4 meters with the first ball 24 meters away from the starting point. A contestant is required to bring the balls back to the starting place one at a time. How far would the contestant run to bring back all balls?

Sol:



Given that

$\Rightarrow$ Distance travelled to bring 1^{st} ball

$= 2(24)$

$= 48M$

$\Rightarrow$ Distance travelled to bring 2^{nd} ball

$= 2(24+4)$

$= 2(28)$

$= 56M$

$\Rightarrow$ Distance travelled to bring 3^{rd} ball

$= 2(24+4+4)$

$= 2(32)$

$= 64M.$

The series $48, 56, 64, \ldots\ldots$ in an A.P

$a = 48$ $\quad n = 20$

$d = t_2 - t_1$

$= 56 - 48$

$= 8$

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$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$S_{20} = \frac{20}{2}[2(48)+(20-1)8]$$

$$= 10[96+19\times 8]$$

$$= 10[96+152]$$

$$= 10[248]$$

$$S_{20} = 2480\,M.$$

10 The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally how many bacteria will be present at the end of 2^{nd} hour, 4^{th} hour and n^{th} hour?

Sol:

Given that

The number of bacteria at the end of different hours from an G.P.

$a = 30$

The number of 1^{st} hours.

$\Rightarrow t_2 = 2(30)$

$t_2 = 60$

The number of 2^{nd} hour

$t_3 = ar^2$

$= 30(2)^2$

$= 30 \times 4$

$t_3 = 120$

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At the end of 4^{th} hour

$t_5 = ar^4$

$= 30(2^4)$

$= 30(16)$

$t_5 = 480$

At the end of n^{th} hour

$\Rightarrow t_{n+1} = ar^n$

$= 30(2)^n$

$t_{n+1} = 30(2)^n$.

11\. What will Rs. 500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually?

Sol:

Given that.

Amount $\Rightarrow A = P(1+i)^n$ $\quad i = 10\%$

$n = 10 \quad P = 500 \quad i = \frac{10}{100}$

$A = 500\left[1 + \frac{10}{100}\right]^n$

$= 500\left[\frac{100+10}{100}\right]^n$

$= 500\left[\frac{110}{100}\right]^n$

$A = 500\left[\frac{11}{10}\right]^n$

$A = 500\left(\frac{11}{10}\right)^n$ (or) $A = 500(1.1)^n$.

12. In a certain town a viral disease caused severe health hazards upon its people disturbing their normal life. It was found that on each day the virus which caused the disease spread in G.P. The amount of infectious virus particle gets doubled each day, being 5 particles on the first day. Find the day when the infectious virus particles just grow over 1,50,000 times?

Sol:

Given that,

$$a = 5 \quad r = 2$$

$$5, 10, 20, 40 \ldots\ldots 1,50,000$$

$$t_n = ar^{n-1}.$$

$$t_n = 5(2)^{n-1} > 1,50,000$$

$$2^{n-1} > \frac{1,50,000}{5} = 30000$$

$$2^{n-1} > 2^4 \times 1875$$

$$2^{n-1-4} > 1875$$

$$2^{n-5} > 1875$$

$$2^{n-5} > 2^{10} + \text{decimal}$$

2 | 30,000
2 | 15,000
2 | 7500
2 | 3750
2 | 1875
2 | 937.5
2 | 468.75
2 | 234.375
2 | 117
2 | 58
2 | 29
2 | 14

2 | 4
2 | 3
2 | 1

$2^n > 2^{15}$ + decimal

$n = 15$

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Exercise : 5.4.

1. Expand the following in ascending powers of x and find the condition on x for which the binomial expansion is valid.

i) $\frac{1}{5+x}$.

Sol:

$$\Rightarrow \frac{1}{5+x} = \frac{1}{5\left(1+\frac{x}{5}\right)}$$

$$= \frac{1}{5}\left(1+\frac{x}{5}\right)^{-1}$$

Formula: $(1+x)^{-1} = 1-x+x^2-x^3+\cdots$

$$= \frac{1}{5}\left[1-\left(\frac{x}{5}\right)+\left(\frac{x}{5}\right)^2-\left(\frac{x}{5}\right)^3+\cdots\right]$$

$$= \frac{1}{5}\left[1-\frac{x}{5}+\frac{x^2}{25}-\frac{x^3}{125}+\cdots\right]$$

$\therefore$ This expansion valid only

$$\left|\frac{x}{5}\right| < 1$$

$$|x| < 5.$$

ii) $\frac{2}{(3+4x)^2}$.

Sol:

Given that $= \frac{2}{3^2\left(1+\frac{4x}{3}\right)^2}$

$$= \frac{2}{9} \frac{1}{\left(1+\frac{4x}{3}\right)^2}$$

$$= \frac{2}{9}\left[1+\frac{4x}{3}\right]^{-2}$$

$$= \frac{2}{9}\left[1+y\right]^{-2} \quad \because y = \frac{4x}{3}$$

$$= \frac{2}{9}\left[1-2y+3y^2-4y^3+\cdots\cdots\right]$$

$$= \frac{2}{9}\left[1-2\left(\frac{4x}{3}\right)+3\left(\frac{4x}{3}\right)^2-4\left(\frac{4x}{3}\right)^3+\cdots\right]$$

$\therefore$ The expansion is valid only if $|y|<1$

But $y = \frac{4x}{3}$

$$\therefore \left|\frac{4x}{3}\right| < 1 = |x| < +\frac{3}{4}.$$

iii) $(5+x^2)^{2/3}$

Sol:

Given that

$$= 5^{2/3}\left(1+\frac{x^2}{5}\right)^{2/3}$$

$$(1+x)^{p/q} = 1+\frac{p}{q}x + \frac{p(p-q)}{q^2\,2!}x^2+\cdots\cdots$$

$$5^{2/3}\left(1+\frac{x^2}{5}\right)^{2/3} = 5^{2/3}\left[1+\frac{2}{3}\left(\frac{x^2}{5}\right)+\frac{2(2-\cdots}{9\cdot 2}\left(x^2/5\right)^2+\cdots\right.$$

$$= 5^{2/3}\left[1+\frac{2x^2}{15}-\frac{1}{9}\left(\frac{x^4}{25}\right)+\cdots\right]$$

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∴ The expansion is valid only if $\left|\frac{x^2}{5}\right| < 1$

$$= |x^2| < 5.$$

iv) $(x+2)^{-2/3}$

Sol:

Given that

$$= (2+x)^{-2/3}$$

$$= 2^{-2/3}\left[1+\frac{x}{2}\right]^{-2/3}$$

$$(1+x)^{-p/q} = \left[1-\left(\frac{p}{q}\right)x+\frac{p(p-q)}{q^2-2!}\left(\frac{x}{2}\right)^2+\cdots\right]$$

$$= 2^{-2/3}\left[1-\frac{2}{3}\left(\frac{x}{2}\right)+\frac{2(2-3)}{9-2!}\left(\frac{x}{2}\right)^2+\cdots\right]$$

$$= 2^{-2/3}\left[1-\frac{x}{3}-\frac{1}{9}\left(\frac{x^2}{4}\right)+\cdots\right]$$

---

d. Find $\sqrt[3]{1001}$ approximately (two decimal places).

Sol:

Given that

$$= \sqrt[3]{1001}$$

$$(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\cdots\cdots$$

$$\sqrt[3]{1001} = (1000+1)^{1/3}$$

$$= (1000)^{1/3}\left(1+\frac{1}{1000}\right)^{1/3}$$

$$= (10^3)^{1/3}\left(1+\frac{1}{1000}\right)^{1/3}$$

$$= 10\left[1+\frac{1}{3}\left(\frac{1}{1000}+\cdots\right)\right]$$

$$= 10\left[1+\frac{1}{3000}+\cdots\right]$$

$$= 10[1+0.0033]$$

$$= 10+0.00333$$

$$= 10.00333$$

3. Prove that $\sqrt[3]{x^3+6} - \sqrt[3]{x^3+3}$ is approximately equal to $\frac{1}{x^2}$ when x is sufficiently large.

Sol:

$$\sqrt[3]{x^3+6} - \sqrt[3]{x^3+3} = (x^3+6)^{1/3} - (x^3+3)^{1/3}$$

$$= x\left(1+6/x^3\right)^{1/3} - x\left(1+3/x^3\right)^{1/3}$$

$$(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\cdots$$

$$= x\left(1+\frac{1/3}{2}\left(6/x^3\right)+\cdots\right) - x\left[1+1/3\left(3/x^3\right)\right]$$

$$= \left(x+\frac{2x}{x^3_2}+\cdots\right) - \left(x+\frac{x}{x^3_2}+\cdots\right)$$

$$= \left(x + \frac{2}{x^2} + \cdots\right) - \left(x + \frac{1}{x^2} + \cdots\right)$$

$$= x + \frac{2}{x^2} + \cdots x - \frac{1}{x^2}$$

$$= \frac{2}{x^2} - \frac{1}{x^2}$$

$$= \frac{2-1}{x^2}$$

$$= \frac{1}{x^2} \text{ (approximately)}.$$

Formula:

$\Rightarrow (1+x)^n = 1 + nx + \frac{n(n-1)}{\lfloor 2}x^2 + \frac{n(n-1)(n-2)}{\lfloor 3}x^3 + \cdots$

$\Rightarrow (1-x)^n = 1 - nx + \frac{n(n-1)}{\lfloor 2}x^2 - \frac{n(n-1)(n-2)}{\lfloor 3}x^3 + \cdots$

$\Rightarrow (1+x)^{-n} = 1 - nx + \frac{n(n+1)}{\lfloor 2}x^2 - \frac{n(n+1)(n+2)}{\lfloor 3}x^3 + \cdots$

$\Rightarrow (1-x)^{n} = 1 + nx + \frac{n(n+1)}{\lfloor 2}x^2 + \frac{n(n+1)(n+2)}{\lfloor 3}x^3 + \cdots$

$\Rightarrow (1+x)^{+1} = 1 - x + x^2 - x^3 + \cdots$

$\Rightarrow (1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots$

$\Rightarrow (1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots$

$\Rightarrow (1-x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \cdots$

4. Prove that $\sqrt{\frac{1-x}{1+x}}$ is approximately equal to $1 - x + \frac{x^2}{2}$ when x is very small.

Sol:

If x is so small

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Prove:

$$\sqrt{\frac{1-x}{1+x}} = \frac{1-x+x^2}{x}$$

L.H.S:

$$= \sqrt{\frac{1-x}{1+x}}$$

$$= (1-x)^{1/2}(1+x)^{-1/2}$$

$$= \left(1 - \frac{\frac{1}{2}x}{1!} + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)x^2}{2!} \cdots\cdots\right)$$

$$\left(1 - \frac{1}{2}x + \frac{1}{2}\frac{\left(\frac{1}{2}+1\right)x^2}{2!}\right)$$

$$= \left(1 - \frac{x}{2} + \frac{1}{2}\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)x^2\right)$$

$$= \left[1 - \frac{x}{2} - \frac{1}{8}x^2 + \cdots\right]\left[1 - \frac{x}{2} + \frac{3}{8}x^2 + \cdots\right]$$

$$= 1 - \frac{x}{2} - \frac{x}{2} + \frac{x^2}{4} - \frac{1}{8}x^2 + \frac{3}{8}x^2$$

$$= \left(1 - \frac{x}{2} + \left(\frac{1}{2}\right)\left(\frac{3}{8}\right)\left(\frac{1}{2}\right)x^2\right)$$

$$= 1 - \frac{x}{2} + \frac{3}{4}x^2 - \frac{x}{2} + \frac{x^2}{4} - \frac{x^2}{8}$$

$$= 1 - x + x^2\left(\frac{3}{8} - \frac{1}{8} + \frac{1}{4}\right)$$

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$$= 1 - \frac{2x}{2} + \frac{1}{2}x^2$$

$$\sqrt{\frac{1-x}{1+x}} = 1 - x + \frac{x^2}{2}$$

Exponential Series:

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$$

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!}$$

Logarithmic Series.

$$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} \ldots$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$$

$$\log\left(\frac{1+x}{1-x}\right) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\right)$$

5. Write the first 6 terms of the exponential series.

i) e^{5x},

Sol:

Given that $= e^{5x}$

$$= 1 + \frac{5x}{1!} + \frac{(5x)^2}{2!} + \frac{(5x)^3}{3!} + \frac{(5x)^4}{4!} + \ldots$$

$= 1 + \frac{5x}{1!} + \frac{25x^2}{2!} + \frac{125x^3}{3!} + \frac{625x^4}{4!} +$

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$\frac{3125}{5!} x^5.$

b) e^{-2x}.

Sol:

$\frac{1-(2x)}{1!} + \frac{(2x)^2}{2!} - \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} + \frac{(2x)^5}{5!}$

$e^{-2x} = 1 - 2x + \frac{4x^2}{2} - \frac{8x^3}{6} + \frac{16x^4}{24} - \frac{32x^5}{120} + \dots$

c) $e^{-\frac{1}{2}x} - e^{x/2}$.

$= 1 + \frac{x/2}{1!} + \frac{(x/2)^2}{2!} + \frac{(x/2)^3}{3!} + \frac{(x/2)^4}{4!}$

$\frac{(x/2)^5}{5!}$

$e^{x/2} = 1 + \frac{x}{2} + \frac{x^2}{4\times 2} + \frac{x^3}{8\times 6} + \frac{x^4}{16\times 24} +$

$\frac{x^5}{32\times 120}$

$= 1 + \frac{x}{2} + \frac{x^2}{8} + \frac{x^3}{48} + \frac{x^4}{384} + \frac{x^5}{3840}.$

6. Write the first 4 terms of the logarithmetic series

i) $\log(1+4x)$,

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Sol:

using

$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4}$

Given that

$\log(1+4x)$

put $4x = y$

$\log(1+y) = y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \cdots$

$= 4x - \frac{(4x)^2}{2} + \frac{(4x)^3}{3} - \frac{(4x)^4}{4} + \cdots$

$\therefore$ The series is valid only when

$|4x| < 1$ $\quad |x| < \frac{1}{4}$

$\Rightarrow -\frac{1}{4} < x < \frac{1}{4}$

ii) $\log(1-2x)$

Sol:

Given that

$\log(1-2x)$

put $2x = y$

$\log(1-y) = -y - \frac{y^2}{2} - \frac{y^3}{3} - \frac{y^4}{4}$

$\log(1-2x) = -2x - \frac{(2x)^2}{2} - \frac{(2x)^3}{3} - \frac{(2x)^4}{4}$

$$= -2x - \frac{4x^2}{2} - \frac{8x^3}{3} - \frac{16x^4}{4}$$

This series is valid only when

$|2x| < 1$

$= |x| < 1/2$

$= -\frac{1}{2} < x < 1/2.$

c) $\log \frac{(1+3x)}{(1-3x)}$

Sol:

Put $3x = y$

$$\log\left(\frac{1+y}{1-y}\right) = 2\left[y + \frac{y^3}{y} + \frac{y^5}{y} + \frac{y^7}{y} + \cdots\right.$$

$$\log\left(\frac{1+3x}{1-3x}\right) = 2\left[3x + \frac{(3x)^3}{3} + \frac{(3x)^5}{3} + \frac{(3x)}{3}\right.$$

This series is valid only when

$= |3x| < 1$

$= |x| < 1/3$

d) $\log\left(\frac{1-2x}{1+2x}\right)$

Sol:

put $2x = y$

$$\log\left(\frac{1-y}{1+y}\right) = -2\left[y + \frac{y^3}{3} + \frac{y^5}{5} + \frac{y^7}{7} \cdots\right]$$

$$= -\left(2x + \frac{(2x)^3}{3} + \frac{(2x)^5}{5} + \frac{(2x)^7}{7} + \cdots\right)$$

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This expansion of valid only when

$$|2x| < 1$$

$$= |x| < 1/2$$

$$= -\frac{1}{2} < x < \frac{1}{2}$$

7. If $y = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots$ then show that $x = y - \frac{y^2}{2!} + \frac{y^3}{3!} - \frac{y^4}{4!} + \cdots$

Sol:

Given

$$y = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots$$

$$y = -\log(1-x)$$

multiply by (-1) both sides

$$-y = \log(1-x)$$

Taking exponential both sides

$$e^{-y} = e^{\log(1-x)}$$

$$e^{-y} = 1-x$$

$$x = 1-e^{-y}$$

$$x = 1-\left[1-\frac{y}{1!} + \frac{y_2}{2!} - \frac{y^3}{3!} + \cdots\right]$$

$$= 1-1+\frac{y}{1!} - \frac{y^2}{2!} + \frac{y^3}{3!} - \cdots$$

$$x = y - \frac{y^2}{2!} + \frac{y^3}{3!} - \frac{y^4}{4!}$$

8. If $p-q$ is small compared to either to p or q then show that $\sqrt[n]{\frac{p}{q}} \simeq \frac{(n+1)p+(n-1)q}{(n-1)p+(n+1)q}$

Sol:

Given.

$p-q$ is small compared to either p or q.

$$\frac{p-q}{p+q} = x \text{ (say)}$$

$$p-q = x(p+q)$$

$$p-q = xp+qx$$

$$p-px = qx+q$$

$$p(1-x) = q(x+1)$$

$$\frac{p}{q} = \frac{1+x}{1-x}$$

$$\left(\frac{p}{q}\right)^{1/n} = \left(\frac{1+x}{1-x}\right)^{1/n}$$

$$= \frac{(1+x)^{1/n}}{(1+x)^{1/n}}$$

$$= \frac{1+\frac{1}{n}x+\cdots\cdots}{1-\frac{1}{n}x+\cdots\cdots}$$

$$= \frac{1+\frac{1}{n}x}{1-\frac{1}{n}x}$$

$$= \frac{1 + \frac{x}{n}}{1 - \frac{x}{n}}$$

$$= \frac{n + \frac{x}{\cancel{n}}}{n - \frac{x}{\cancel{n}}}$$

$$= \frac{n+x}{n-x}$$

$$= \frac{n + \frac{p-q}{p+q}}{n - \frac{p-q}{p+q}}$$

$$= \frac{\frac{n(p+q) + (p-q)}{\cancel{p+q}}}{\frac{n(p+q) - (p-q)}{(\cancel{p+q})}}$$

$$= \frac{n(p+q) + (p-q)}{n(p-q) - (p-q)}$$

$$= \frac{(n+1)p + (n-1)q}{(n-1)p + (n+1)q}$$

$\therefore$ Hence Proved.

$n = 8 \qquad p = 5 \qquad q = 16$

$$\sqrt[8]{\frac{15}{16}} = \frac{(n+1)p + (n-1)q}{(n-1)p + (n+1)q}$$

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$$= \frac{(8+1)^{15} + (8-1)^{16}}{(8-1)^{15} + (8+1)^{16}}$$

$$= \frac{9\times15 + 7\times16}{7\times15 + 9\times16}$$

$$\sqrt[8]{\frac{15}{16}} = \frac{247}{249}$$

9. Find the coefficient of x^4 in the expansion of $\frac{3-4x+x^2}{e^{2x}}$.

Sol:

Given that,

$$\frac{3-4x+x^2}{e^{2x}} = (3-4x+x^2)e^{-2x}$$

$\Rightarrow (3-4x+x^2) \cdot e^{-2x}$

$\Rightarrow (3-4x+x^2)\left[1 - \frac{(2x)}{1!} + \frac{(2x)^2}{2!} - \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} \ldots\right.$

$\Rightarrow (3-4x+x^2)\left[1 - \frac{2x}{1} + \frac{4x^2}{2} - \frac{8x^3}{6} + \frac{16x^4}{24}\ldots\right.$

coefficient of x^4 is

$$= \cancel{3}\left(\frac{\overset{2}{\cancel{16}}}{\underset{8}{\cancel{24}}}\right) - \cancel{4}_{2}\left(-\frac{\cancel{8}}{\underset{3}{\cancel{6}}}\right) + 1\left(\frac{\overset{2}{\cancel{4}}}{\cancel{2}}\right)$$

$= 2 + \frac{16}{3} + 2$

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$= \frac{6+16+6}{3}$

$= \frac{28}{3}$

10. Find the value of $\sum_{n=1}^{\infty} \frac{1}{2^{n-1}} \left(\frac{1}{9^{n-1}} + \frac{1}{9^{2n-1}} \right)$.

sol:

Given that,

$$\sum_{n=1}^{\infty} \frac{1}{2^{n-1}} \left(\frac{1}{9^{n-1}} + \frac{1}{9^{2n-1}} \right)$$

$$= \frac{1}{2^0} \left(\frac{1}{9^0} + \frac{1}{9^{2(1)-1}} \right)$$

$$S_\infty = 1\left(1+\frac{1}{9}\right) + \frac{1}{2}\left(\frac{1}{9}+\frac{1}{9^3}\right) + \frac{1}{2^2}\left(\frac{1}{9^2}+\frac{1}{9^5} + \cdots\right)$$

$$= \left(1+\frac{1}{9}\right) + \frac{1}{2}\left(\frac{1}{9}+\frac{1}{9^3}\right) + \frac{1}{2^2}\left(\frac{1}{9^2}+\frac{1}{9^5}\right) \cdots$$

Separating the 1^{st} and 2^{nd} term in each bracket.

$$= \left[1+\frac{1}{2}\left(\frac{1}{9}\right)+\frac{1}{2^2}\left(\frac{1}{9^2}\right)+\cdots\right] + \left[\frac{1}{9}+\frac{1}{2}\left(\frac{1}{9^3}\right)+\frac{1}{2^2}\left(\frac{1}{9^5}\right)+\cdots\right]$$

$$= \left[1+\frac{1}{2}\left(\frac{1}{9}\right)+\frac{1}{2^2}\left(\frac{1}{9^2}\right)+\cdots\right] + \frac{1}{9}\left[1+\frac{1}{2}\left(\frac{1}{9^2}\right)+\frac{1}{2^2}\left(\frac{1}{9^4}\right)+\cdots\right]$$

(1)

consider

i) $1 + \frac{1}{2}\left(\frac{1}{9}\right) + \frac{1}{2^2}\left(\frac{1}{9^2}\right) + \cdots$

$a = 1 \quad r = \frac{1}{18}$

$S_\infty = \frac{a}{1-r} = \frac{1}{1-1/18}$

$= \frac{1}{\frac{18-1}{18}} = \frac{1}{\frac{17}{18}}$

$S_\infty = \frac{18}{17}$

ii) consider.

$1 + \frac{1}{2}\left(\frac{1}{9^2}\right) + \frac{1}{2^2}\left(\frac{1}{9^3}\right) + \cdots$

$a = 1 \quad r = \frac{1}{162}$

$S_\infty \Rightarrow \frac{a}{1-r} = \frac{1}{1-\frac{1}{162}}$

$\Rightarrow \frac{1}{\frac{162-1}{162}} = \frac{1}{\frac{161}{162}}$

$S_\infty \Rightarrow \frac{162}{161}$

(1) $\Rightarrow S_\infty = \frac{18}{17} + \frac{1}{9}\left(\frac{162}{161}\right)$

$= \frac{18}{17} + \frac{162}{1449}$

$$= \frac{18 \times 1449 + 162 \times 17}{17 \times 1444}$$

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$$= \frac{28836}{24632}$$

$$S_\infty = 1.170.$$

Example: 5.19.

Find the sum: $1 + \frac{4}{5} + \frac{7}{25} + \frac{10}{125} + \ldots\ldots$

Sol:

Given that

$$1 + \frac{4}{5} + \frac{7}{25} + \frac{10}{125} + \ldots\ldots$$

Nr:

1, 4, 7, 10 .... is an A.P

$a = 1$ $\quad d = t_2 - t_1$

$= 4 - 1$

$\therefore d = 3$

Dr:

$1, \frac{1}{5}, \frac{1}{25}, \frac{1}{125} \ldots\ldots$

$a = 1$ $\quad r_1 = \frac{t_2}{t_1} = \frac{1/5}{1} = \frac{1}{5}$

Given series is A.G.P

$$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$$

$$= \frac{1}{1-\frac{1}{5}} + \frac{3 \times 1/5}{(1 - 1/5)^2}$$

$$= \frac{1}{\frac{5-1}{5}} + \frac{3/5}{\left(1-\frac{1}{5}\right)^2}$$

$$= \frac{1}{4/5} + \frac{3/5}{\left(\frac{5-1}{5}\right)^2}$$

$$= \frac{1}{4/5} + \frac{3/5}{\left(4/5\right)^2}$$

$$= \frac{5}{4} + \frac{3}{\cancel{5}} \times \frac{5^{\cancel{2}}}{4^2}$$

$$= \frac{5}{4} + \frac{15}{16}$$

$$= \frac{20+15}{16}$$

$$S_\infty = \frac{35}{16}$$

Example : 5.20.

Find $\sum_{n=1}^{\infty} \frac{1}{n^2+5n+6}$

Sol :

Given that

$$\frac{1}{n^2+5n+6}$$

$$\frac{1}{n^2+5n+6} = \frac{1}{(n+2)(n+3)}$$

$$\frac{1}{(n+2)(n+3)} = \frac{A}{(n+2)} + \frac{B}{(n+3)}$$

$$\frac{1}{(n+2)(n+3)} = \frac{A(n+3)+B(n+2)}{(n+2)(n+3)}$$

$$1 = A(n+3)+B(n+2)$$

Put $n=-3$

$$1 = A(-3+3)+B(-3+2)$$

$$1 = A(0)+B(-1)$$

$$B=-1$$

Put $n=-2$

$$1 = A(-2+3)+B(-2+2)$$

$$1 = A(1)+B(0)$$

$$A=1$$

$$\frac{1}{(n+2)(n+3)} = \frac{1}{n+2} - \frac{1}{n+3}$$

$$a_n = \frac{1}{n+2} - \frac{1}{n+3}$$

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n$$

$$S_n = \left(\frac{1}{3}-\frac{1}{4}\right) + \left(\frac{1}{4}-\frac{1}{5}\right) + \left(\frac{1}{5}-\frac{1}{6}\right) + \cdots + \left(\frac{1}{n+2}-\frac{1}{n+3}\right)$$

$$= \frac{1}{3} - \frac{1}{n+3}$$

$$\infty = \frac{\text{anything}}{0} = \frac{1}{0}$$

$$\frac{1}{\infty} = \frac{1}{1/0} = \frac{1\times 0}{1} = \frac{0}{1} = 0$$

n tends to ∞

$$= \frac{1}{3} - \frac{1}{\infty}$$

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$$= \frac{1}{3} - 0$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2+5n+6} = \frac{1}{3}$$

Example: 5.21.

Expand $(1+x)^{2/3}$ up to four terms for $|x| < 1$.

Sol:

Given that

$$= (1+x)^{2/3}$$

Formula:

$$(1+x)^n = 1 + \frac{nx}{\lfloor 1} + \frac{n(n-1)}{\lfloor 2} x^2 + \frac{n(n-1)(n-2)}{\lfloor 3} x^3 + \cdots$$

$$(1+x)^{2/3} = 1 + \frac{2}{3}x + \frac{2/3(2/3-1)}{1\times2}x^2 + \frac{2/3(2/3-1)(2/3-2)}{1\times2\times3}x^3$$

$$= 1 + \frac{2}{3}x + \frac{\frac{2}{3}\left(\frac{2-3}{3}\right)}{2}x^2 + \frac{2/3\left(\frac{2-3}{3}\right)\left(\frac{2-6}{3}\right)}{6}$$

$$= 1 + \frac{2}{3}x + \frac{\frac{2}{3}(-1/3)}{2}x^2 + \frac{2/3(-1/3)(-4/3)}{6}x^3$$

$$= 1 + \frac{2}{3}x + \frac{\cancel{2}}{3}\left(-\frac{1}{3}\right) \times \frac{1}{\cancel{2}} x^2 + \frac{2}{3}\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right) \times \frac{1}{6} x^3 + \ldots$$

$$(1+x)^{2/3} = 1 + \frac{2}{3}x - \frac{1}{9}x^2 + \frac{4}{81}x^3 + \ldots\ldots$$

Example : 5.22.

Expand $\frac{1}{(1+3x)^2}$ in powers of x. Find a condition on x for which the expansion is valid.

Sol:

Given that

$$\frac{1}{(1+3x)^2} = (1+3x)^{-2}$$

valid

$|3x| < 1$

$|x| < 1/3$

Expansion valid only $|x| < \frac{1}{3}$

$$(1+x)^{-n} = 1 - \frac{nx}{\lfloor 1} + \frac{n(n+1)}{\lfloor 2}x^2 - \frac{n(n+1)(n+2)}{\lfloor 3}x^3 + \ldots$$

$$(1+3x)^{-2} = 1 - \frac{2}{\lfloor 1}(3x) + \frac{2(2+1)}{\lfloor 2}(3x)^2 - \frac{2(2+1)(2+2)}{\lfloor 3}(3x)^3 + \ldots$$

$$\frac{1}{(1+3x)^2} = 1 - 6x + \frac{\cancel{2}(3)}{\cancel{2}}9x^2 - \frac{\cancel{2}(3)(4)}{\cancel{6}}27x^3 + \ldots$$

$$\frac{1}{(1+3x^2)} = 1 - 6x + 27x^2 - 108x^3 \ldots \ldots |x| < 1/3$$

(81)

Put $y = 3x$

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$$(1+3x)^{-2} = (1+y)^{-2}$$

Formula:

$$(1+y)^{-2} = 1 - 2y + 3y^2 - 4y^3 + \cdots$$

$$(1+3x)^{-2} = 1 - 2(3x) + 3(3x)^2 - 4(3x)^3 + \cdots$$

$$= 1 - 6x + 3(9x^2) - 4(27x^3) + \cdots$$

$$= 1 - 6x + 27x^2 - 108x^3 + \cdots,$$

$|3x| < 1$

$|x| < 1/3$

$\therefore 1 - 6x + 27x^2 - 108x^3 + \cdots, |x| < \frac{1}{3}$ //.

Example: 5.23

Expand $\frac{1}{(3+2x)^2}$ in power of x.

Find a condition on x for which the expansion is valid.

Sol:

$$\frac{1}{(3+2x)^2} = \frac{1}{3^2\left(1+\frac{2x}{3}\right)^2}$$

$$= \frac{1}{9}\left(1+\frac{2x}{3}\right)^{-2}$$

$$= \frac{1}{9}(1+y)^{-2}$$

$$= \frac{1}{9}\left[1 - 2y + 3y^2 - 4y^3 + \cdots\right]$$

$$= \frac{1}{9}\left[1 - 2\left(\frac{2x}{3}\right) + 3\left(\frac{2x}{3}\right)^2 - 4\left(\frac{2x}{3}\right)^3 + \cdots\right]$$

$$= \frac{1}{9}\left[1 - \frac{4x}{3} + 3\left(\frac{4x^2}{9}\right) - 4\left(\frac{8x^3}{27}\right) + \cdots\right]$$

$$= \frac{1}{9}\left[1 - \frac{4x}{3} + \frac{4x^2}{3} - \frac{32x^3}{27} + \cdots\right]$$

$$= \frac{1}{9} - \frac{4x}{9\times 3} + \frac{4x^2}{9\times 3} - \frac{32x^3}{27\times 9} + \cdots$$

$$= \frac{1}{9} - \frac{4x}{27} + \frac{4}{27}x^2 - \frac{32x^3}{243} + \cdots$$

valid $\left|\frac{2x}{3}\right| < 1$

$$= \frac{1}{9} - \frac{4x}{27} + \frac{4}{27}x^2 - \frac{32}{243}x^3 + \cdots \text{ valid } |x| < \frac{3}{2}$$

Example: 5.24.

Find $\sqrt[3]{65}$.

Sol:

Given that

$$\sqrt[3]{65} = 65^{1/3}$$
$$= (1+64)^{1/3}$$
$$= (64+1)^{1/3}$$
$$= (64)^{1/3}\left(1 + \frac{1}{64}\right)^{1/3}$$
$$= (4^3)^{1/3}\left(1 + \frac{1}{64}\right)^{1/3}$$
$$\sqrt[3]{65} = 4\left(1 + 1/64\right)^{1/3}$$

Formula

$$(1+x)^n = 1 + nx + \frac{n(n+1)}{\lfloor 2}x^2 + \frac{n(n-1)(n-2)}{}$$

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$$= 4\left[1 + \left(\frac{1}{3}\right)\left(\frac{1}{64}\right) + \frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)}{\lfloor 2}\left(\frac{1}{64}\right)^2 + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{\lfloor 3}\left(\frac{1}{64}\right)^3 \cdots\right]$$

$$= 4\left[1 + \frac{1}{3\times 64} + \left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)\left(\frac{1}{2}\right)\left(\frac{1}{64}\right)^2 + \cdots\right]$$

$$= 4 + \frac{4}{3\times 64} - 4\times\frac{1}{3}\times\frac{2}{3}\times\frac{1}{2}\times\frac{1}{64}\times\frac{1}{64}\cdots$$

$$= 4 + \frac{1}{48} + \cdots$$

$$= 4 + 0.02$$

$$= 4.02 \text{ (approximately).}$$

Example: 5.25

Prove that $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

Sol:

Given that

$$= \sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$$

$$\sqrt[3]{x^3+7} = (x^3+7)^{1/3}$$

$$= \left[(x^3)^{1/3}\left(1+\frac{7}{x^3}\right)^{1/3}\right], \quad \left|\frac{7}{x^3}\right| < 1 \text{ as } x \text{ is large}$$

$$= x\left(1+\frac{7}{x^3}\right)^{1/3}$$

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Formula

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{\lfloor 2}x^2 + \frac{n(n-1)(n-2)}{\lfloor 3}x^3 + \cdots$$

$$= x\left[1 + \left(\tfrac{1}{3}\right)\left(\tfrac{7}{x^3}\right) + \cdots\right]$$

$$= x\left[1 + \frac{7}{3x^3} + \cdots\right]$$

$$= x\left[1 + \frac{7}{3x^3} + \cdots\right]$$

$$= x + \frac{7\cancel{x}^1}{3x^{\cancel{3}2}} + \cdots$$

$$= x + \frac{7}{3x^2} \longrightarrow (1)$$

$$\sqrt[3]{x^3+4} = (x^3+4)^{1/3}$$

$$= x\left(1 + \tfrac{4}{x^3}\right)^{1/3}$$

$$= x\left[1 + \left(\tfrac{1}{3}\right)\left(\tfrac{4}{x^3}\right) + \cdots\right]$$

$$= x + \frac{4\cancel{x}^1}{3x^{\cancel{3}2}} + \cdots$$

$$= x + \frac{4}{3x^2} \longrightarrow (2)$$

(1) - (2)

$$\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4} = \left(x + \frac{7}{3x^2}\right) - \left(x + \frac{4}{3x^2}\right)$$

$$= \cancel{x} + \frac{7}{3x^2} - \cancel{x} - \frac{4}{3x^2}$$

$$= \frac{7-4}{3x^2}$$

$$= \frac{3}{3x^2}$$

$$\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4} = \frac{1}{x^2}$$

Example : 5.16.

Find the sum up to n terms of the series : $1 + \frac{6}{7} + \frac{11}{49} + \frac{16}{343} + \ldots\ldots$

Sol:

Given that.

$$1 + \frac{6}{7} + \frac{11}{49} + \frac{16}{343} + \ldots$$

Nr :

$1, 6, 11, 16 \ldots$

$a = 1 \quad d = 6-1 = 5$

Dr :

$1, \frac{1}{7}, \frac{1}{49}, \frac{1}{343}$

$a = 1 \quad r = 1/7$

$$S_n = \frac{a - (a+(n-1)d)r^n}{1-r} + dr\left(\frac{1-r^{n-1}}{(1-r)^2}\right)$$

$$= \frac{1 - (1+(n-1)5)(1/7)^n}{1-1/7} + 5 \times 1/7 \left[\frac{1-(1/7)^{n-1}}{(1-1/7)^2}\right]$$

$$= \frac{1 - \left[\frac{1+5n-5}{7^n}\right]}{6/7} + 5/7\left[\frac{1-1/7^{n-1}}{(6/7)^2}\right]$$

$$= \frac{\dfrac{7^n - 5n - 4}{7^n}}{6/7} + \frac{5}{7}\left[\frac{7^{n-1}-1}{7^{n-1}\left(6/7\right)^2}\right]$$

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$$= \frac{7^n - 5n + 4}{7^{n-1} \times 6/\cancel{7}} + \frac{5}{7}\,\frac{(7^{n-1}-1)}{7^{n-2} \times \frac{36}{\cancel{7^2}}}$$

$$= \frac{7^n - 5n + 4}{7^{n-1} \times 6} + \frac{5(7^{n-1}-1)}{7^{n-2}(36)}$$

Example: 5.18.

Find $\sum_{k=1}^{n} \frac{1}{k(k+1)}$.

Sol: Given that

$$\sum_{k=1}^{n} \frac{1}{k(k+1)}$$

$$\frac{1}{k(k+1)} = \frac{A}{k} + \frac{B}{k+1}$$

$$\frac{1}{\cancel{k(k+1)}} = \frac{A(k+1) + B(k)}{\cancel{k(k+1)}}$$

$$1 = A(k+1) + B(k)$$

Put $k = -1$

$$1 = A(-1+1) + B(-1)$$

$$1 = A(0) + B(-1)$$

$$B = -1$$

Put $k = 0$

$$1 = A(0+1) + B(0)$$

$$1 = A(1)$$

$$A = 1$$

$$\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$$

$$\sum \frac{1}{k(k+1)} = \sum \left(\frac{1}{k} - \frac{1}{k+1} \right)$$

$$\Rightarrow t_1 + t_2 + t_3 + \dots + t_n$$

$$= \left(1 - \cancel{\tfrac{1}{2}}\right) + \left(\cancel{\tfrac{1}{2}} - \cancel{\tfrac{1}{3}}\right)\left(\cancel{\tfrac{1}{3}} - \cancel{\tfrac{1}{4}}\right) + \dots + \left(\cancel{\tfrac{1}{n}} - \frac{1}{n+1}\right)$$

$$= 1 - \frac{1}{n+1} \,//.$$

Example: 5.17.

Find the sum of the first n terms of the series $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}}, \dots$

Sol:

Given that.

$$t_k = \frac{1}{\sqrt{k} + \sqrt{k+1}}$$

$$= \frac{1}{\sqrt{k} + \sqrt{k+1}} \cdot \frac{\sqrt{k} - \sqrt{k+1}}{\sqrt{k} - \sqrt{k+1}}$$

$$= \frac{\sqrt{k} - \sqrt{k+1}}{(\sqrt{k})^2 - (\sqrt{k+1})^2}$$

$$= \frac{\sqrt{k} - \sqrt{k+1}}{k - (k+1)}$$

$$= \frac{\sqrt{k} - \sqrt{k+1}}{\cancel{k} - \cancel{k} + 1}$$

$= -(\sqrt{k} - (\sqrt{k+1})$

$= \sqrt{k+1} - \sqrt{k}$

Thus,

$= t_1 + t_2 + t_3 + \dots t_n$

$= (\cancel{\sqrt{2}} - \sqrt{1}) + (\sqrt{3} - \cancel{\sqrt{2}}) + \dots + (\sqrt{n+1} - \sqrt{n})$

$= \sqrt{n+1} - 1.$

Example : 5.14.
Find seven numbers $A_1, A_2, \dots A_7$ so that the sequence $4, A_1, A_2, \dots A_7, 7$ is in A.P and also 4 number G_1, G_2, G_3, G_4 so that the sequences $12, G_1, G_2, G_3, G_4, 3/8$ in G.P.

Sol::

Given that $4, A_1, A_2, A_3, A_4, A_5, A_6, A_7, 7$ in A.P

$a = 4 \qquad t_9 = 7$

$a + (9-1)d = 7$

$4 + 8d = 7$

$8d = 7 - 4$

$8d = 3$

$d = 3/8$

$A_1 = 4 + \frac{3}{8} = \frac{32+3}{8} = \frac{35}{8} = 4\frac{3}{8}$

$A_2 = \frac{35}{8} + \frac{3}{8} = \frac{38}{8} = 4\frac{6}{8}$

$A_3 = \frac{38}{8} + \frac{3}{8} = \frac{41}{8} = 5\frac{1}{8}$

$A_4 = \frac{41}{8} + \frac{3}{8} = \frac{44}{8} = 5\frac{4}{8}$

$A_5 = \frac{44}{8} + \frac{3}{8} = \frac{47}{8} = 5\frac{7}{8}$

$A_6 = \frac{47}{8} + \frac{3}{8} = \frac{50}{8} = 6\frac{2}{8}$

$A_7 = \frac{50}{8} + \frac{3}{8} = \frac{53}{8} = 6\frac{5}{8}$

$\therefore$ 7 numbers are $4\frac{3}{8}, 4\frac{6}{8}, 5\frac{1}{8}, 5\frac{4}{8}$,

$5\frac{7}{8}, 6\frac{2}{8}, 6\frac{5}{8}, ..$

ii) $12, G_1, G_2, G_3, G_4, 3/8$

$a = 12 \quad t_6 = 3/8$

$ar^{n-1} = 3/8$

$12\, r^{n-1} = 3/8$

$12\, r^{6-1} = 3/8$

$12 r^5 = 3/8$

$r^5 = \frac{3}{8 \times 12}$

$r^5 = \frac{1}{32}$

$r = \left(\frac{1}{35}\right)^{1/5}$

$r = \left(\frac{1}{2^5}\right)^{1/5}$

$r = 1/2$

 $a = 12$, $r = 1/2$

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$G_1 = (\cancel{12})^{6} \frac{1}{\cancel{2}} = 6$

$G_2 = \cancel{6}^{3} \times \frac{1}{\cancel{2}} = 3$

$G_3 = 3 \times \frac{1}{2} = \frac{3}{2}$

$G_4 = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$

$\therefore$ 4 numbers are $6, 3, 3/2, 3/4$

Example: 5.15

If the product of the 4th, 5th, 6th terms of a G.P is 4096 and if the product of the 5th, 6th and 7th terms of it is 32768, find the sum of first 8 terms of the Geometric progression?

Sol:

Given that

$t_4 = ar^3$

$t_5 = ar^4$

$t_6 = ar^5$

Product of $t_4\, t_5\, t_6 = 4096$

$(ar^3)(ar^4)(ar^5) = 4096$

$a r^3 r^{12} = 4096 \rightarrow (1)$

$t_5 = ar^4$, $t_6 = ar^5$, $t_7 = ar^6$

Product of $t_5\, t_6\, t_7 = 32768$

$(ar^4)(ar^5)(ar^6) = 32768$

$a^3 r^{15} = 32768 \rightarrow (2)$

$$\frac{(2)}{(1)} \Rightarrow \frac{a^3 r^{15}}{a^3 r^{12}} = \frac{32768}{4096}$$

$$r^3 = 8$$

$$r = \sqrt[3]{8}$$

$$r = 2.$$

$$(1) \Rightarrow a^3 r^{12} = 4096$$

$$a^3 (2)^{12} = 4096$$

$$a^3 = \frac{4096}{(2)^{12}}$$

$$a^3 = \frac{4096}{4096}$$

$$a^3 = 1$$

$$a = 1$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_8 = \frac{1(1-2^8)}{1-2}$$

$$= \frac{1-2^8}{1-2}$$

$$= \frac{1-256}{-1}$$

$$= \frac{-255}{-1}$$

$$S_8 = 255$$

Example: 5.12

Prove that if a, b, c are in H.P if and only if $\frac{a}{c} = \frac{a-b}{b-c}$.

Sol: Given that

a, b, c are in H.P

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are A.P

$$t_2 - t_1 = t_3 - t_2$$

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$$\frac{1}{b} + \frac{1}{b} = \frac{1}{a} + \frac{1}{c}$$

$$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

$$\frac{2}{b} = \frac{c+a}{ac}$$

$$2ac = (c+a)b$$

$$2ac = cb + ab$$

$$ac + ac = cb + ab$$

$$ac - cb = ab - ac$$

$$c(a-b) = a(b-c)$$

$$\frac{a-b}{b-c} = \frac{a}{c}$$

In other hand,

$$\frac{a}{c} = \frac{a-b}{b-c}$$

$$a(b-c) = c(a-b)$$

$$ab - ac = ca - cb$$

$\div abc$ all terms.

$$\frac{ab}{abc} - \frac{ac}{abc} = \frac{ac}{abc} - \frac{bc}{abc}$$

$$\frac{1}{c} - \frac{1}{b} = \frac{1}{b} - \frac{1}{a}$$

$\therefore$ Thus $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

Example: 5.13.

If the 5^{th} and 9^{th} terms of a harmonic progression are $\frac{1}{19}$ and $\frac{1}{35}$. find the 12^{th} term of the sequences.

Sol:

Given that.

Let h_n be a H.P

Let a_n be a A.P

$$a_n = \frac{1}{h_n}$$

$a_5 = 19 \qquad a_9 = 35$

$a+(n-1)d = 19 \qquad a+(n-1)d = 35$

$a+(5-1)d = 19 \qquad a+(9-1)d = 35$

$a+4d = 19 \rightarrow (1) \qquad a+8d = 35 \rightarrow (2)$

(1) − (2)

$$\begin{array}{r} a + 4d = 19 \\ a + 8d = 35 \\ (-) \quad (-) \quad\quad (-) \\ \hline -4d = -16 \end{array}$$

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$$d = \frac{16}{4}$$

$$d = 4.$$

sub $d = 4$ in eqn (1)

$$a + 4d = 19$$

$$a + 4(4) = 19$$

$$a + 16 = 19$$

$$a = 19 - 16$$

$$a = 3$$

$\therefore a = 3 \quad d = 4$

$$t_{12} = a + (n-1)d$$

$$t_{12} = 3 + (12-1)4$$

$$= 3 + (11)4$$

$$= 3 + 44$$

$$t_{12} = 47$$

$\therefore$ The 12^{th} term of H.P is $t_{12} = \frac{1}{47}$".

Example: 5.7.

The 2^{nd}, 3^{rd} and 4^{th} terms in the binomial expansion of $(x+a)^n$ are 240, 720 and 1080 for a suitable value of x. Find x, a & n.

Sol:

Given that

Formula:

$$(x+a)^n = \overset{T_1}{nc_0 x^n a^0} + \overset{T_2}{nc_1 x^{n-1} a} + \overset{T_3}{nc_2 x^{n-2} a^2} + \overset{T_4}{nc_3 x^{n-3} a^3} + \ldots\ldots$$

$$T_2 = nc_1 x^{n-1} a = 240 \longrightarrow (1)$$

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$T_4 = nC_3\ x^{n-3} a^3 = 1080 \rightarrow (3)$

Dividing $\frac{(2)}{(1)}$ we get,

$$\Rightarrow \frac{nC_2\ \cancel{x^{n-2}}\ a^{\cancel{2}}}{nC_1\ \cancel{x^{n-1}}_{x}\ \cancel{a}} = \frac{720}{240}$$

$$\frac{a}{x} = \frac{72\cancel{0}}{24\cancel{0}} \times \frac{nC_1}{nC_2}$$

$$\frac{a}{x} = \frac{\cancel{72}^{6}}{\cancel{24}_{\cancel{12}}} \times \frac{\cancel{n}}{\frac{\cancel{n}(n-1)}{\cancel{2}}}$$

$$\frac{a}{x} = \frac{6}{n-1} \rightarrow (4).$$

Dividing $\frac{(3)}{(2)}$ we get.

$$\Rightarrow \frac{nC_3 .\ \cancel{x^{n-3}}\ a^3}{nC_2\ \cancel{x^{n-2}}_{x}\ \cancel{a^2}} = \frac{1080}{72\cancel{0}}$$

$$\frac{a}{x} = \frac{108}{72} \times \frac{nC_2}{nC_3}$$

$$\frac{a}{x} = \frac{108}{72} \times \frac{\frac{n(n-1)}{\lfloor 2}}{\frac{n(n-1)(n-2)}{\lfloor 3}}$$

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$$\frac{a}{x} = \frac{\cancel{108}^{\cancel{54}\,9}}{\cancel{72}_{\cancel{24}\,\cancel{12}\,2}} \times \frac{\cancel{n}(n-1)}{\cancel{2}_1} \times \frac{\cancel{6}^{3}}{\cancel{n}\cancel{(n-1)}(n-2)}$$

$$\frac{a}{x} = \frac{9}{2(n-2)} \rightarrow (5)$$

(4) φ (5)

$$\frac{6}{(n-1)} = \frac{9}{2(n-2)}$$

$$\cancel{12}^{4}(n-2) = \cancel{9}^{3}(n-1)$$

$$4n-8 = 3n-3$$

$$4n-3n = -3+8$$

$$n = 5$$

$n=5$ in (1) $\Rightarrow$ $nC_1\, x^{n-1} a = 240$

$$5C_1\, x^{5-1} a = 240$$

$$5x^4 a = 240 \rightarrow (6)$$

$n=5$ in (4) $\Rightarrow$ $\frac{a}{x} = \frac{6}{n-1}$

$$\frac{a}{x} = \frac{6}{5-1}$$

$$\frac{a}{x} = \frac{6}{4} \rightarrow (7)$$

$$\frac{(6)}{(7)} = \frac{5x^4 \cancel{a}}{\cancel{a}/x} = \frac{240}{6/4}$$

$$5x^5 = \frac{\cancel{240}^{40}}{\cancel{6}_1} \times 4$$

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$$x^5 = \frac{40 \times 4}{5}$$

$$x^5 = 32$$

$$x^5 = 32$$

$$x = (32)^{1/5}$$

$$x = (2^5)^{1/5}$$

$$x = 2$$

(4) $\Rightarrow \frac{a}{x} = \frac{6}{n-1}$

$$\frac{a}{2} = \frac{6}{5-1}$$

$$\frac{a}{2} = \frac{6}{4}$$

$$a = \frac{\cancel{6}^{3} \times \cancel{2}}{\cancel{4}\cancel{2}}$$

$$a = 3$$

By

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