XII-02

Unit Test

Complex Numbers

Mathematics

Time: 1.15   Marks: 50

Part - A

Answer any five questions   5 x 2 = 10

1. **Simplify:** $\displaystyle\sum_{n=1}^{102} i^n$

2. **If** $\dfrac{z+3}{z-5i} = \dfrac{1+4i}{2}$, **find the complex number z in the rectangular form.**

3. **Find the value of** $\displaystyle\sum_{k=1}^{8}\left(\cos\dfrac{2k\pi}{9} + i\sin\dfrac{2k\pi}{9}\right)$

4. **Find the square roots of** $-6+8i$

5. **Show that** $\left|2z+2-4i\right| = 2$ **represents a circle and find its centre and radius.**

6. **Simplify** $\left(\sin\dfrac{\pi}{6} + i\cos\dfrac{\pi}{6}\right)^{18}$

Part - B

Answer any five questions   5 x 3 = 15

7. **Show that** $\left(2+i\sqrt{3}\right)^{10} - \left(2-i\sqrt{3}\right)^{10}$ **is purely imaginary**

8. **If** z_1, z_2 **and** z_3 **are complex numbers such that** $\left|z_1\right| = \left|z_2\right| = \left|z_3\right| = \left|z_1+z_2+z_3\right| = 1$ **find the**

   value of $\left|\dfrac{1}{z_1} + \dfrac{1}{z_2} + \dfrac{1}{z_3}\right|$

9. **Show that the points** $1,\ \dfrac{-1}{2}+i\dfrac{\sqrt{3}}{2}$ **and** $\dfrac{-1}{2}-i\dfrac{\sqrt{3}}{2}$ **are the vertices of an equilateral triangle**

10. If $z = x + iy$ is a complex number such that $\left|\dfrac{z-4i}{z+4i}\right| = 1$, show that the locus of z is

   real axis

11. Find the fourth roots of unity

12. If $\omega \neq 1$ is a cube root of unity, show that i) $\left(1-\omega+\omega^2\right)^6 + \left(1+\omega-\omega^2\right)^6 = 128$

   ii) $\left(1+\omega\right)\left(1+\omega^2\right)\left(1+\omega^4\right)\left(1+\omega^8\right)..\left(1+\omega^{2^{11}}\right) = 1$

Part - C

Answer any five questions   **5 x 5 = 25**

13. Show that the equation $z^3 + 2\bar{z} = 0$ has five solutions

14. If $z = x + iy$ and $\arg\left(\dfrac{z-i}{z+2}\right) = \dfrac{\pi}{4}$ then show that $x^2 + y^2 + 3x - 3y + 2 = 0$

15. Simplify: $\left(-\sqrt{3} + 3i\right)^{31}$

16. Suppose z_1, z_2 and z_3 are the vertices of an equilateral triangle inscribed in the circle

   $|z| = 2$. If $z_1 = 1 + i\sqrt{3}$ then find z_2 and z_3

17. If $\dfrac{1+z}{1-z} = \cos 2\theta + i\sin 2\theta$, show that $z = i\tan\theta$

18. If $2\cos\alpha = x + \dfrac{1}{x}$ and $2\cos\beta = y + \dfrac{1}{y}$ show that

   i) $\dfrac{x^m}{y^n} - \dfrac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$ ii) $x^m y^n + \dfrac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$

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XII-03

Unit Test

Theory of Equations

Mathematics

Time: 1.15   Marks: 50

Part - A

Answer any five questions   **5 x 2 = 10**

1. If α and β are the roots of the quadratic equation $2x^2 - 7x + 13 = 0$, construct a quadratic

   equation whose roots are α^2 and β^2

2. A 12 metre tall tree was broken into two parts. It was found that the height of the part which

   was left standing was the cube root of the length of the part that was cut away. Formulate this

   into a mathematical problem to find the height of the part which was cut away.

3. If $x^2 + 2(k+2)x + 9k = 0$ has equal roots, find k

4. Discuss the nature of the roots of the polynomial $x^5 - 19x^4 + 2x^3 + 5x^2 + 11$

5. Find the sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$

6. Solve: $x^4 - 14x^2 + 45 = 0$

Part - B

Answer any five questions   **5 x 3 = 15**

7. Form a polynomial equation with integer coefficients with $\sqrt{\dfrac{\sqrt{2}}{\sqrt{3}}}$ as a root

8. If k is real, discuss the nature of the roots of the polynomial equation $2x^2 + kx + k = 0$ in

   terms of k

9. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5} - \sqrt{3}$

   as a root.

10. It is known that the roots of the equation $x^3 - 6x^2 - 4x + 24 = 0$ are in aritmetic progression. Find its roots.

11. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions.

12. Find all real numbers satisfying $4^x - 3(2^{x+2}) + 2^5 = 0$

Part - C

Answer any five questions   **5 x 5 = 25**

13. If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, show that it must

be equal to $\dfrac{pq' - p'q}{q - q'}$ or $\dfrac{q - q'}{p' - p}$

14. Prove that a straight line and parabola cannot intersect at more than two points.

15. Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that

$1 + 2i$ and $\sqrt{3}$ are two of its zeros

16. Solve: $(x - 4)(x - 7)(x - 2)(x + 1) = 16$

17. Solve the equation $7x^3 - 43x^2 = 43x - 7$

18. Solve: $8x^{\frac{3}{2n}} - 8x^{\frac{-3}{2n}} = 63$

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