Given X = {1,2,3,4} Y = { a, b , c, d, e} and

f = { (1,a) ,(2,c), (3,e) , (4,b) } Is this a function ? and what type of function is this?

This is a function . This is one to one function but not on to

---

Given X = {1,2,3,4} Y= { a, b} and f = { (1,a) , (2,a) ,(3,a) ,4,a) }

Is this a function? Mention what type of function is this ?

This is a function It is not one -to -one It is also not on to

---

X = {1,2,3,4} Y = {a, b, c, d, e} and f= {(1,a) (2,c) (3,e)} Is this a function ? Why ?

This not at all a function But only a relation.

Because 4 has no image in Y

X= { 1,2,3,4} Y = {a, b, c, d} and f ={ (1,a), (2,c), (3,b) , (4,b) Is this a function? State what type of function is this.

This is a function , This is neither one to one and (nor) on to

---

X= { 1,2,3,4,5} Y = {a} f { (1,a), (2,a),(3,a), (4,a) } Is this a function Mention what type of function is this ?

This is a function. This is not one to one function but it is on to

---

X = {1,2,3,4} Y = a ,b, c, d} and f= { (1,a) (2,c) (3,d) (4,b) } Is this a function ? State what type of function is this?

This is a function. This function is both one to one and on to

---

Given A = {1,2,3,4} and f : A $\rightarrow$ A defined by f= { (1,2),(2,4),(3,1), (4,3)} find f^{-1}

Inverse of f is = f^{-1}= { (2,1) (4,2) (1,3) ,(3,4)

Write the working rule to find the inverse of a function

1) Write y = f(x)
2) Write x in terms of y
3) Write $f^{-1}(y)$ = the expression in y
4) Replace y as x

Find the largest possible domain of f(x) = $\frac{1}{x-1}$

The largest possible domain is $\mathbb{R} - \{1\}$

EXAMPLE 1.14 (1)

Check whether the function $f : \mathbb{N} \to \mathbb{N}$ defined by f(n) = n+2 is one to ton and on to

This is one to one function because n+2 =m+2 $\Rightarrow$ n=m

But has no pre image i.e, y = n+2 $\Rightarrow$ n = y-2

$\Rightarrow$ n= 1-2 = -1 $\notin$ N hence it is not on to function

Co domain ≠ range

({1,2,3,4......) ≠ {3,4,5,.......}

EXAMPLE :14 (2)

Check whether the function $f : \mathbb{N} \cup \{-1,0\} \to \mathbb{N}$ defined by f(n) = n+2 is one to ton and on to

The function is one to one

Y = n+2 When n= -1 y= -1+2 =1 When n=0 y = 0+2 = 2

Therefore every element in the co domain has pre image

Hence this function is on to

EXAMPLE : 15 (i)

Check the function $f : \mathbb{N} \to \mathbb{N}$ defined by f(n) = n^2 one to oneness and on to ness

f(1) =1 f(2) =4 f(3=9 and so on

therefore every element in the co-domain (N) has unique image in the codomain (N)

Hence it is one to one function

But not on to because {1,4,9.....} ≠ (1,2,3,4,......} =N

EXAMPLE 15 (ii)

Check the function f : R →R defined by f(n) = n^2 one to oneness and on to ness

In the Domain if we consider two elements 2 and -2 2≠-2

But f(2) = 2^2 =4 and f(-2) = $(-2)^2$ =4

Since 2≠-2 $\Rightarrow$ f(2) = f(-2) therefore it is not one to one

It is also not on to (because some element in the codomain has no pre image.

EXAMPLE 16 (i)

Check whether f: R →R defined by f(x) = $\frac{1}{x}$ for one to oneness and on to oness

This is not a function because 0 ∈R f(x) = $\frac{1}{x}$ is not defined (at x= 0)

Example 16 (ii)

Check whether f: R –{0} → R defined by f(x) = $\frac{1}{x}$ for one to oneness and on to oness

This is a function one to one

But it is not on to 0 has no preimage

Check whether f: R –{0} → R –{0} defined by f(x) = $\frac{1}{x}$ one to one and on to

This is a function one to one and also on to

This function is a bijection function

EXAMPLE :17

If f : R - {-1,1} → R is defined by f(x) = $\frac{x}{x^2-1}$, verify whether f is one to one (or) not

f(x) = f(y)

$$\frac{x}{x^2-1} = \frac{y}{y^2-1}$$

$x(y^2-1) = y (x^2-1)$

$xy^2-x = yx^2-y$

$xy^2 – x – yx^2+y =0$

$xy^2-yx^2 +y-x =0$

xy (y-x) + 1 (y-x)=0

(y-x) (xy+1) =0

This imply that x =y or (xy=-1)

If we select two number in x and y such that xy=-1

(2,-1/2) (7,-1/7) (-2, ½) and so one we may have infinitely many possible pairs

That is f(x) = f(y) does not imply x=y

Hence it is not one to one function

EXAMPLE 18

If f: $\mathbb{R} \to \mathbb{R}$ is defined as f(x) = $2x^2$-1 Find the preimage of 17, 4 and -2

Since f(x) = $2x^2-1 = 17$

$2x^2 = 17+1 = 18$

$X^2 = 9 \Rightarrow x = \pm 3$

Hence the preimage of 17 are 3 and -3

Since f(x) = $2x^2-1 = 4$

$2x^2 = 4+1 = 5$

$X^2 = \frac{5}{2} \Rightarrow x = \pm \sqrt{\frac{5}{2}}$

Hence the pre image of 4 are $\sqrt{\frac{5}{2}}$ and $-\sqrt{\frac{5}{2}}$

Since f(x)= $2x^2-1 = -2$

$2x^2 = -2+1 = -1$

$X^2 = \pm \frac{-1}{2} \Rightarrow X = \pm \sqrt{\frac{-1}{2}}$

The values of x are imaginary values Hence -2 has no pre image under f

---

EXAMPLE 19

If f: [-2, 2} $\to$ B is given by f(x) = $2x^2$ then find B so that f is on to

Since f(x) = $2x^3$

Since The domain is $-2 \leq x \leq 2$

Cube on both sides

$(-2)^3 \leq x^3 \leq (2)^3$

$-8 \leq x \leq 8$

(Multiply with 2) $-16 \leq x \leq 16$

There fore the co domain must be B = [-16 , 16]

It is on to function because every element has pre image in the domain

EXAMPLE 20

Check whether the function f(x) = x $|x|$ defined on [-2.2] is one to one or not If it is one to one find a suitable codomain so that the function becomes bijection

When x= 0 f(o) = 0 and when y = 0 f(0) = 0

ie., x = y $\Rightarrow$ f(x) = f(y)

If x $\neq$ 0 and y$\neq$ 0

f(x) = f(y)

x $|x|$ = $y|y|$

$\frac{|x|}{|y|}=\frac{y}{x}$ Here both y is +ve and x is +ve (or)

Both y is –ve and x is –ve Since $\frac{|x|}{|y|}\succ 0$

Squaring on both sides we have

$\frac{|x|^2}{|y|^2}=\frac{y^2}{x^2}$

$\frac{x^2}{y^2}=\frac{y^2}{x^2}$ (by cross multiplication)

$X^4 = y^4 \Rightarrow x^4-y^4=0$

$(x^2+y^2)(x^2-y^2)=0$

Hence $x^2 = y^2 \Rightarrow x = y$

Yes the given function is one to one

When x = -2 f (-2)= $|-2|(-2)=-4$

When x = 2 f(2) = $|2|(2)=4$

Hence the codomain is [-4 , 4]= range (ie., on to) the function f becomes bijection

EXAMPLE : 21

Find the largest possible domain for the real valued function f defined by f(x) = $\sqrt{x^2-5x+6}$

f(x) = $\sqrt{x^2-5x+6}$

we must have x^2-5x+6 ≥ 0

(x-3) (X-2) ≥ 0

Therefore the value of x lies out side of 2 and 3

Ie., x ∈ (-∞ 2] U [3, ∞)

Therefore the possible domain is x ∈ (-∞ 2] U [3, ∞)

EXAMPLE 22

Find the domain of f(x) = $\frac{1}{1-2\cos x}$

The function is defined except 1-2cos x =0

1 = 2 cosx

½ = cos x

⇒ cos 60° = ½ = cos π/3 (π = 180 π /3 = 60°)

The general solution for cos α = cos θ ⇒ θ = 2nπ ± α , n ∈ Z

Therefore the value of x = 2nπ ± π/3

Hence the domain is R − {2nπ ± π/3 }, n ∈ Z

EXAMPLE 23

Find the range of the function f(x) = $\frac{1}{1-3\cos x}$

Cos x curve lies between - and 1 ie., -1 ≤ cos x ≤ 1

Multiply with 3 throughout -3 ≤ 3cos x ≤ 3

(multiply with - sign 3 ≥ -3 cosx ≥ -3 (inequality reversed)

Add 1 on both sides 1+3 ≥ 1-3 cos x ≥ 1-3

4 ≥ 1-3cos x ≥ -2

Taking reciprocal $\frac{1}{4} \leq 1 - 3\cos x \leq -\frac{1}{2}$ (inequality reversed)

Hence the range of f is $(-\infty, -\frac{1}{2}]$ U $[\frac{1}{4}, \infty)$

EXAMPLE 24

Find the largest possible domain for the real valued function given by f(x) = $\frac{\sqrt{9-x^2}}{\sqrt{x^2-1}}$

$9-x^2 \geq 0$ only if x lies between -3 and 3 i.e., $x \in [-3, 3]$

$X^2 - 1 > 0$ only when x must be lie out side

The largest possible domain for f is [-3,-1) U (1,3]

EXAMPLE ; 25

Let f = { (1,2) , (3,4), (2,2) } and g : { (2,1), (3,1), (4,2) } find f o g and gof

First verify whether the range of f in the domain of g

Range of f is {2,4} which is contained in the domain of g= { 2,3,4}

Therefore gof can be calculated (possible to find gof)

gof (x) = gof(1) = g(2) =1 (1,1)

go f(2) =g(2) =1 (2,1)

go f(3) = g(4) =2 (3,2)

hence gof = { (1,1) (2,1), (3,2) }

==

Similarly

Range of g is {1,2} which is contained in the domain of f = {1,2,3}

Therefore fog can be calculated (possible to find fog

fog(2) = f(2)=2 (2,2,)

fog(3) = f(1) =2 (3,2)

fog(4) = f(2) =2 (4,2)

hence fog = { (2,2) , (3,2) , (4,2) }

EXAMPLE : 26

Let f = { (1,4) , (2,5) , (3,5) and g = { (4,1), (5,2) (6,4) } find gof Can you find fog ?

gof = { (1,1) (2,2) (3,2) }

but fog cannot find (not defined)

because the range of g = { 1,2,4} is not contained in the domain of f = {1,2,3}

EXAMPLE 27

Let f and g be the two functions from $\mathbb{R}$ to $\mathbb{R}$ defined by f(x)= 3x-4 and g(x) =x^2+3

Find fog and gof

$gof(x) = g\{ f(x) \} = g(3x-4) = (3x-4)^2 +3 = 9x^2-24x+19$

$fog(x) = f\{ g(x) \} = f(x^2+3) = 3 (x^2+3) -4 = 3x^2+9-4 = 3x^2+5$

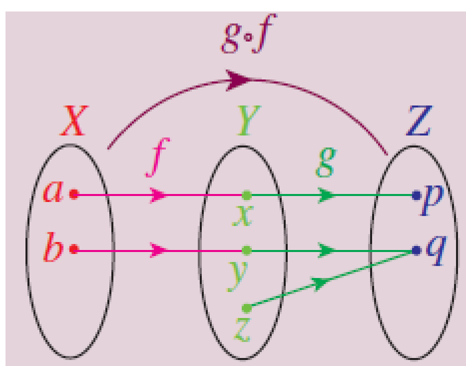
Theorem Let $f : A \to B$ and $g : B \to C$ be two functions. If f and g are one-to-one, then $g \circ f$ is one-to-one.

Proof. Let $x \neq y$ in A. Since f is one-to-one, $f(x) \neq f(y)$. Since g is one-to-one, $g(f(x)) \neq g(f(y))$. That is, $x \neq y \Rightarrow (g \circ f)(x) \neq (f \circ g)(y)$. Hence $g \circ f$ is one-to-one.

Theorem

Show that the statement

" if f and gof are one to one , then g is one to one is not true"



From the above diagram we see that f is one to one

And also gof is one to one

But g is not one to one

(because y and z has same image q) More over y and z has not two different images

EXAMPLE 29

Let $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x - |x|$ and $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = 2x + |x|$

Find fog

We know that $|x| = \begin{cases} x & if \quad x > 0 \\ -x & if \quad x \le 0 \end{cases}$

f(x) = $2x - |x|$ =2x – (-x) = 3x if x≤ 0

f(x) = $2x - |x|$ = 2x -(x) = x if x>0

g(x) = $2x + |x|$ =2x+ (-x) = x

g(x) = $2x + |x|$ = 2x + (x) = 3x

(gof) (x) = g(f(x)) = g(3x) = 3x when x ≤ 0

(gof) (x) = g(f(x)) = g(x) = 3x when x >0

EXAMPLE : 30

If $f : \mathbb{R} \to \mathbb{R}$ is defined by f(x) = 2x-3 Prove that f is a bijection function and also find it inverse.

The function <u>f and g are inverse to each other</u> we can say <u>that f and g are bijective functions</u>

Let y = f(X) = 2x-3 then x = $\frac{y+3}{2}$ = g(y)

therefore fog = $2\left(\frac{y+3}{2}\right) - 3 = y$

therefore gof = $\frac{(2x-3)+3}{2} = x$

hence fog = I_x and gof is also = I_y

f and g are inverse to each other

therefore they are bijective functions

EXERCISE PROBLEMS 1.3

1. Suppose that 120 students are studying in 4 sections of eleventh standard in a school. Let A denote the set of students and B denote the set of the sections. Define a relation from A to B as "*x related to y if the student x belongs to the section y*". Is this relation a function? What can you say about the inverse relation? Explain your answer.

The relation from A to B defined as x R y

If the student (x) belongs to the section (y)

A = school = 120 students B = number of sections = four sections

Every student in the school (A) there corresponds to unique section in (class)(B)

Yes this relation is a function from $A \rightarrow B$

==

But inverse relation is not exists

Because for every section (B) will be related to more than one student in the set (A)

Therefore inverse relation is not a function

2. Write the values of f at $-4, 1, -2, 7, 0$ if

$$f(x) = \begin{cases} -x+4 & \text{if } -\infty < x \leq -3 \\ x+4 & \text{if } -3 < x < -2 \\ x^2 - x & \text{if } -2 \leq x < 1 \\ x - x^2 & \text{if } 1 \leq x < 7 \\ 0 & \text{otherwise} \end{cases}$$

$f(-4) = -x+4 = -(-4)+4 = 8$ $f(-4) = 8$

$f(1) = x - x^2 = 1 - 1^2 = 1-1 = 0$ $f(1) = 0$

$f(-2) = x^2 - x = (-2)^2 - (-2) = 4+2 = 6$ $f(-2) = 6$

$f(7) = 0$ (since otherwise it is zero)

$f(0) = x^2 - x = 0^2 - 0 = 0$ $f(0) = 0$

3. Write the values of f at $-3, 5, 2, -1, 0$ if

$$f(x) = \begin{cases} x^2 + x - 5 & \text{if } x \in (-\infty, 0) \\ x^2 + 3x - 2 & \text{if } x \in (3, \infty) \\ x^2 & \text{if } x \in (0, 2) \\ x^2 - 3 & \text{otherwise} \end{cases}$$

f(-3) = x^2+x-5 = $(-3)^2$+(-3) -5 = 9-3-5 =1   f(-3) =1

f(5) = x^2+3x-2 = $(5)^2$+3(5)-2 = 25+15 -2 = 40-2 =38   f(5) =38

f(2) = x^2-3 (other wise) = 2^2-3 = 4-3=1   f(2) = 1

f(-1) = x^2+x-5 = $(-1)^2$ +(-1)-5= 1-1-5 = -5   f(-1) = -5

f(0) = x^2 -3 = 0-3 =-3   f(0) = -3

---

4. State whether the following relations are functions or not. If it is a function check for one-to-oneness and ontoness. If it is not a function, state why?

(i) If $A = \{a, b, c\}$ and $f = \{(a, c), (b, c), (c, b)\}$; $(f : A \to A)$.

It is a function

But It is not one to one and also not on to

(ii) If $X = \{x, y, z\}$ and $f = \{(x, y), (x, z), (z, x)\}$; $(f : X \to X)$.

It is not one to one function

In the domain y has no image in the codomain

---

5. Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}$. Give a function from $A \to B$ for each of the following:

(i) neither one-to-one nor onto. (ii) not one-to-one but onto.
(iii) one-to-one but not onto. (iv) one-to-one and onto.

(i) Let f : { (1,a) , (2,a) , (3,a), (4,a) }
Neither one to one nor on to
f is not on to
Since b , c and d does not have any pre image in A
f is not one to one
Since a has four (4) pre images in A

(ii) Not one to one but on to
It is possible to define a many one on to function from a set A to B
If n(A) >n(B)
Since n(A) = n(B) it is not possible to define function which is one to one but on to

(iii) One to one but not on to

It is possible to define a one to one in to function from a set A to B

If n(A) < n(B)

Here n(A) = n(B)

Hence it is not possible to define function which is one to one but not on to

(iv) One to one and on to

Let f = { (1.a), (2,b) ,(3,c), (4,d) }

Hence it is one to one but on to

---

6. Find the domain of $\frac{1}{1-2\sin x}$.

When the denominator is = 0

1- 2sin x =0

1 = 2 sin x

½ = sin x (since sin 30° = ½) here 30° = $\frac{\pi}{6}$

The general solution to sin α = sinθ

$$\theta = n\pi + (-1)^n \frac{\pi}{6}$$

Hence the Domain of the function is $\mathbf{R} - \{n\pi + (-1)^n \frac{\pi}{6}\}$ n ∈ $\mathbf{Z}$

---

7. Find the largest possible domain of the real valued function $f(x) = \frac{\sqrt{4-x^2}}{\sqrt{x^2-9}}$.

When x =-2 and x = 2 f(x) = 0

When x =-3 and x = 3 f(x) is not defined

Since $4-x^2$ is negative if x<-2 and x>2

Since x^2 -9 is negative if x> -3 and x<3

Hence for <u>no real values of x</u> f(x) is defined

Therefore the domain <u>R is empty set</u> (ie,, ϕ)

---

8. Find the range of the function $\frac{1}{2\cos x - 1}$.

We know that -1 ≤ cos x ≤ 1

Multiply with 2 $-2 \leq 2\cos x \leq 2$

Add (-1) on both sides $-2-1 \leq 2\cos x \leq 2-1$

$-3 \leq 2\cos x \leq 1$

Taking reciprocals on both sides

$$-\frac{1}{3} \geq \frac{1}{2\cos x - 1} \geq 1$$

Hence the range of the function is $\left(-\infty, -\frac{1}{3}\right] \cup [1, \infty)$

---

9. Show that the relation $xy = -2$ is a function for a suitable domain. Find the domain and the range of the function.

Since $xy = -2$

$Y = f(x) = -\frac{2}{x}$

The above function is defined for all values of x except at x=0

Hence the domain of the function is R - {0}

Range is also R – {0}

---

10. If $f, g : \mathbb{R} \to \mathbb{R}$ are defined by $f(x) = |x| + x$ and $g(x) = |x| - x$, find $g \circ f$ and $f \circ g$.

$f(x) = |x| + x$

$= -x + x = 0$ if $x \leq 0$

$= x + x = 2x$ if $x > 0$

$g(x) = |x| - x,$

$= -x - x = -2x$ if $x < 0$

$= x - x = 0$ if $x \geq 0$

fog(x) = f(-2x) =0

gof (x) = f(0) =0

---

11. If f, g, h are real valued functions defined on $\mathbb{R}$, then prove that $(f + g) \circ h = f \circ h + g \circ h$. What can you say about $f \circ (g + h)$? Justify your answer.

LHS: [(f+g)o h](x) = f+ g(h(x))

= f(h(x)) + g(h(x))

= foh (x) + goh(x)= foh+goh

Now fo(g+h) (x) = f((g+h) (x)) = f[g(x) + h(x)] = fog(x) + foh(x)

12. If $f:\mathbb{R}\to\mathbb{R}$ is defined by $f(x)=3x-5$, prove that f is a bijection and find its inverse.

Let y = 3x-5 $\Rightarrow$ y+5 =3x $\Rightarrow$ x = $\frac{y+5}{3}$

$$g(y)=\frac{y+5}{3}$$

fog(y) = f(g(y)) = $f\left(\frac{y+5}{3}\right)=3\left(\frac{y+5}{3}\right)-5=y$

gof(x) = g(f(x)) = g(3x-5) = $\frac{3x-5+5}{3}=\frac{3x}{3}=x$

hence f and g are inverse to each other

therefore f and g are bijective function

---

13. The weight of the muscles of a man is a function of his body weight x and can be expressed as $W(x)=0.35x$. Determine the domain of this function.

X denotes the body weight of a man

Weight of a man is not zero

For all values of x>0 the weight can be defined

Hence the Domain of the function is $(0,\infty)$

---

14. The distance of an object falling is a function of time t and can be expressed as $s(t)=-16t^2$. Graph the function and determine if it is one-to-one.

$s(t_1)=s(t_2)$

$-16\,t_1^2 = -16\,t_2^2$

$t_1^2 = t_2^2$

$t_1^2 - t_2^2 = 0$

$(t_1+t_2)(t_1-t_2) = 0$

$t_1 = -t_2 \quad t_1=t_2$

Since time cannot be negative therefore $t_1=t_2$

Hence it is one to one function

---

15. The total cost of airfare on a given route is comprised of the base cost C and the fuel surcharge S in rupee. Both C and S are functions of the mileage m; $C(m) = 0.4m + 50$ and $S(m) = 0.03m$. Determine a function for the total cost of a ticket in terms of the mileage and find the airfare for flying 1600 miles.

Cost function = C = 0.4m+50

Fuel surcharge function = S = 0.03

Total cost of the function = T = C+S

=(0.4m+50) + (0.03)

T = 0.43m + 50

Given m = 1600 miles

T = 0.43(1600) +50

= 688 +50 = Rs.738

---

16. A salesperson whose annual earnings can be represented by the function $A(x) = 30,000 + 0.04x$, where x is the rupee value of the merchandise he sells. His son is also in sales and his earnings are represented by the function $S(x) = 25,000 + 0.05x$. Find $(A + S)(x)$ and determine the total family income if they each sell Rupees 1, 50, 00, 000 worth of merchandise.

Annual earnings =A (x) = 30,000n+0.04x (x = rupee value)

Sales earnings = S(x) = 25,000 +0.05x

Total income = A + S

=30,000n+0.04x + 25,000 +0.05x

= 55,000 + 0.09x

Given x= 1,50,00,000

Total income = 55,000+0.09 (1,50,00,000)

= 55,000 + 13,50,000

= 14,05,000

---

17. The function for exchanging American dollars for Singapore Dollar on a given day is $f(x) = 1.23x$, where x represents the number of American dollars. On the same day the function for exchanging Singapore Dollar to Indian Rupee is $g(y) = 50.50y$, where y represents the number of Singapore dollars. Write a function which will give the exchange rate of American dollars in terms of Indian rupee.

Given x is number of American dollar

y is number of Singapore dollar

f(x) = 1.23x = Singapore dollar

g(x) = 50.50y = Indian rupee

Now the function which will give the exchange rate of American dollars in terms of Indian rupee

gof(x) = g(f(x))

= 50.50 (1.23x) = 62.115 x

---

18. The owner of a small restaurant can prepare a particular meal at a cost of Rupees 100. He estimates that if the menu price of the meal is x rupees, then the number of customers who will order that meal at that price in an evening is given by the function $D(x) = 200 - x$. Express his day revenue, total cost and profit on this meal as functions of x.

Number of customers = 200 –x

Cost of one meal = Rs.100

Total Cost = (Cost) x (number of customers)

= 100 (200-x)

Revenue on one meal = x

Total Revenue = (Revenue on one meal) x (number of customer)

= x (200-x)

Profit = (Revenue)- (Cost)

= x (200-x) - 100 (200-x)

Taking common term (200-x)

Profit = (200-x) (x-100)

---

19. The formula for converting from Fahrenheit to Celsius temperatures is $y = \frac{5x}{9} - \frac{160}{9}$. Find the inverse of this function and determine whether the inverse is also a function.

$$y = \frac{5x}{9} - \frac{160}{9}$$

$$y = \frac{5x - 160}{9} = f(x)$$

$9y = 5x - 160$

$5x = 9y + 160$ = g(y)

We see that fog = y= I_y

gof = x =I_x

therefore the functions are inverse to each other

hence f and g are bijective functions

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20. A simple cipher takes a number and codes it, using the function $f(x) = 3x - 4$. Find the inverse of this function, determine whether the inverse is also a function and verify the symmetrical property about the line $y = x$ (by drawing the lines).

f(x) = 3x-4

y = 3x -4 $\Rightarrow$ y+4 = 3x $\Rightarrow$ x= $\frac{y+4}{3}$

We have to draw the graph of y = 3x-4 and y = $\frac{x+4}{3}$

These two lines are symmetric with respect to the line y = x
