

Trigonometry XI Std 2018-19. (New)

Definitions, Results, Tips, formulae, Conditions, values

Definitions: Degree: An angle is called a right angle when its terminal side and initial sides are $\perp$ to each other. If a right angle is divided into 90 equal parts then each part is called a degree. One degree is divided into 60 equal parts and each part is called minute. One minute is further divided into 60 equal parts and each part is called 1 second.
(or) In degrees, one complete rotation is split into 360 equal parts and each part is called 1 degree.

Note: 1) Two angles that have the exact same measure are called Congruent angles.

2) Two angles that have their measures adding to 90° are called Complementary angle

3. Two angles that have their measures adding to 180° are called Supplementary angle.

4) Two angles between 0 and 360° are Congruent if their sum equal to 360° .

Coterminal angles: If the difference of two angles is $k(360)$ then they are Coterminal angle.

Radian Measure: The ^{radian} angle measure of the angle is the ratio of the arc length it subtends to the radius of the circle in which it is the central angle. $\theta = \frac{\text{arc length}}{\text{radius}} = \frac{s}{r}$ (or) $s = r\theta$.

Relationship between degree and radian.

$$\pi^\circ = 180^\circ \Rightarrow 1^\circ = \frac{\pi}{180}$$

$$1^\circ = \frac{180}{\pi}$$

Note: The ratio of the circumference of any circle to its diameter is always constant. This constant is denoted by irrational number π

$$\text{Area of the sector} = \left(\frac{\pi r^2}{360} \right) \theta \text{ in degree}$$

$$= \left(\frac{\pi r^2}{2\pi} \right) \theta = \frac{r^2 \theta}{2} \text{ in radian.}$$

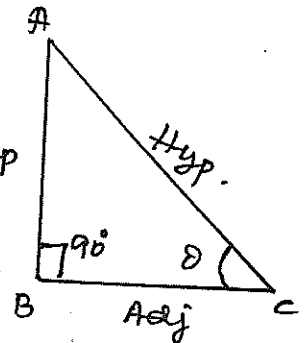
$$1^c \approx 50^{\circ} 17' 15'' \quad (\odot)$$

Radian	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	π	$3\pi/2$	2π
Degree	30	45	60	180	270	360

Basic Trigonometric ratios using right angle

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}, \quad \cos \theta = \frac{\text{adj}}{\text{hyp}}, \quad \tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \text{opp}$$



$$\csc \theta = \frac{1}{\sin \theta}, \quad \csc \theta \sin \theta = 1, \quad \frac{1}{\csc \theta} = \sin \theta$$

$$\sec \theta = \frac{1}{\cos \theta}, \quad \sec \theta \cos \theta = 1, \quad \cos \theta \sec \theta = 1$$

$$\cot \theta = \frac{1}{\tan \theta}, \quad \cot \theta \tan \theta = 1, \quad \tan \theta \cot \theta = 1$$

θ	0	30	45	60	90	180	270	360
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	$-\infty$	0

$$\sin^2 \theta + \cos^2 \theta = 1, \quad \sin^2 \theta = 1 - \cos^2 \theta, \quad \cos^2 \theta = 1 - \sin^2 \theta$$

$$\sin \theta = \sqrt{1 - \cos^2 \theta}, \quad \cos \theta = \sqrt{1 - \sin^2 \theta}$$

$$1 + \tan^2 \theta = \sec^2 \theta, \quad \sec^2 \theta - \tan^2 \theta = 1, \quad 1 - \sec^2 \theta = -\tan^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta, \quad \csc^2 \theta - \cot^2 \theta = 1, \quad 1 - \csc^2 \theta = -\cot^2 \theta$$

Note: The Trigonometric identity is an equation that is true for all values of its domain values.

$$-1 \leq \cos \theta \leq 1 \quad \text{and} \quad -1 \leq \sin \theta \leq 1 \quad \text{Always.}$$

II quadrant Silver $90 + \theta$ $180 - \theta$		I quadrant. $90 - \theta$ All $360 + \theta$	
Tea $108 + \theta$ $270 - \theta$ III quadrant		IV quadrant. cups	
$90 \pm \theta$ $270 \pm \theta$ } Corratio $\sin \leftrightarrow \cos$ $\sec \leftrightarrow \csc$ $\tan \leftrightarrow \cot$		$180 \pm \theta$ $360 \pm \theta$ } Same ratio $\sin \theta \rightarrow \sin \theta$ $\cos \theta \rightarrow \cos \theta$	

	I	II	III	IV	
	$90 - \theta$	$90 + \theta$ $180 - \theta$	$180 + \theta$ $270 - \theta$	$270 + \theta$ $360 - \theta$	
$\sin \theta$	$\cos \theta$	$\sin \theta$	$-\sin \theta$	$-\cos \theta$	
$\cos \theta$	$\sin \theta$	$-\sin \theta$	$-\cos \theta$	$\sin \theta$	
$\tan \theta$	$\cot \theta$	$-\cot \theta$	$\tan \theta$	$-\tan \theta$	
$\csc \theta$	$\sec \theta$	$\sec \theta$	$-\csc \theta$	$-\sec \theta$	
$\sec \theta$	$\csc \theta$	$-\csc \theta$	$-\sec \theta$	$\csc \theta$	
$\cot \theta$	$\tan \theta$	$-\tan \theta$	$\cot \theta$	$-\cot \theta$	

Periodic function: A function $y = f(x)$ is said to be periodic if there exists a real number ($T > 0$) s.t. $f(x+T) = f(x) \forall x$.

$$\sin(2\pi + \theta) = \sin \theta \quad \csc(2\pi + \theta) = \csc \theta$$

$$\cos(2\pi + \theta) = \cos \theta \quad \sec(2\pi + \theta) = \sec \theta$$

$$\text{But } \tan(\pi + \theta) = \tan \theta \quad \cot(\pi + \theta) = \cot \theta$$

∴ The period of $\sin \theta$, $\cos \theta$, $\csc \theta$, $\sec \theta$ are 2π whereas the period of $\tan \theta$ and $\cot \theta = \pi$.

$$1) \sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \begin{cases} \cos^2 A - \sin^2 A \\ 1 - 2 \sin^2 A \\ 2 \cos^2 A - 1 \end{cases}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$\cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$\sin A = 2 \sin \frac{A}{2} \cos \frac{A}{2}$$

$$\cos A = \cos^2 \frac{A}{2} - \sin^2 \frac{A}{2}$$

$$= 1 - 2 \sin^2 \frac{A}{2}$$

$$2 \cos^2 \frac{A}{2} - 1$$

$$\tan A = \frac{2 \tan \frac{A}{2}}{1 - \tan^2 \frac{A}{2}}$$

Trigonometric functions of Sum and difference of Two angles.

$$1) \sin(A+B) = \sin A \cos B + \cos A \sin B.$$

$$2) \sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$3) \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$4) \cos(A-B) = \cos A \cos B + \sin A \sin B.$$

$$5) \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$6) \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}.$$

$$7) \cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$$

$$8) \cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}.$$

$$9) \tan(45+A) = \frac{1 + \tan A}{1 - \tan A}$$

$$10) \tan(45-A) = \frac{1 - \tan A}{1 + \tan A}.$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\cos^2 A/2 = \frac{1 + \cos A}{2}$$

$$\sin^2 A/2 = \frac{1 - \cos A}{2}$$

$$\sin 3A = 3 \sin A - 4 \sin^3 A$$

$$\cos 3A = 4 \cos^3 A - 3 \cos A$$

$$\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}.$$

$$\text{Results! } 1) \sin(A+B) \cdot \sin(A-B) = \sin^2 A - \sin^2 B \quad \left\{ \begin{array}{l} \text{or} \cos^2 B - \cos^2 A \end{array} \right\}$$

$$2) \cos(A+B) \cos(A-B) = \cos^2 A - \sin^2 B \quad \left\{ \begin{array}{l} \text{or} \cos^2 B - \sin^2 A \end{array} \right\}.$$

Product to Sum and Sum to product.

$$1) \sin(A+B) + \sin(A-B) = 2 \sin A \cos B.$$

$$2) \sin(A+B) - \sin(A-B) = 2 \cos A \sin B.$$

$$3) \cos(A+B) + \cos(A-B) = 2 \cos A \cos B$$

$$4) \cos(A+B) - \cos(A-B) = -2 \sin A \sin B$$

$$\text{or } \cos(A-B) - \cos(A+B) = 2 \sin A \sin B.$$

$$1) \sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$2) \sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$3) \cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$4) \cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2} \quad \text{or } \cos D - \cos C = 2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$$

Results: 1) $\sin(60-A) \sin A \sin(60+A) = \frac{1}{4} \sin 3A$

2) $\cos(60-A) \cos A \cos(60+A) = \frac{1}{4} \cos 3A$

3) $\tan(60-A) \tan A \tan(60+A) = \tan 3A$.

Result: $\cos A \cos 2A \cos^2 2A \cos^3 2A \dots \cos^{2^n-1} 2A = \frac{\sin 2^n A}{2^n \sin A}$.

Special values.

$\sin 18^\circ = \cos 72^\circ = \frac{\sqrt{5}-1}{4}$	$\sin 75^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$
$\sin 54^\circ = \cos 36^\circ = \frac{\sqrt{5}+1}{4}$	$\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$
$\cos 18^\circ = \sin 72^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}$	$\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$
$\cos 54^\circ = \sin 36^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4}$	$\cos 75^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$

Working Rule For Solving the problems which are Conditional Identities involving sines and cosines of Multiples (or) Sub multiple of the angles.

- 1) Take any two of the given terms and express this as a product of by using the formula occurring $\sin C$ and $\sin D$.
- 2) In the product so obtained express the sum of the two angles in terms of the third angle by using the given condition.
- 3) Expand the third term by using one of the formula $\sin 2\theta$, or $\cos 2\theta$ whichever is applicable.
- 4) Take out the common factor and in the other factor express the $\tan$ ratios of single angle into that of the sum of angles by using the given condition.
- 5) Use one of C, D formula to change the sum of $\tan$ ratios into a Product.

Working Rule For Solving the problems which are conditional identities which involving squares of sines and cosines of Multiple (or) Sub Multiples of the angles.

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}, \quad \cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

1. Take any two of the given terms and express in the form either $\sin^2 \theta - \sin^2 \phi$ (or) $\cos^2 \theta - \cos^2 \phi$

For this purpose The formula $\sin^2 \alpha + \cos^2 \alpha = 1$ may be used.

2) Use the formula $\sin^2 \theta - \sin^2 \phi = \sin(\theta + \phi) \sin(\theta - \phi)$ or $\cos^2 \theta - \cos^2 \phi = \cos(\theta + \phi) \cos(\theta - \phi)$

3) Take out the common factor and in the other factor, express the t ratio of single angle into that the sum of angles by using the given Condition.

4) Use one of the C/D formula to change the sum of t-ratios into a product.

Working Rule to solve the problem with identities involving tangents and Cotangents of Multiples or Sub multiples of the angle.

1. Express the given condition so that LHS is the sum of two multiples (or sub multiples) of the angles occurring with identity
2. Take tangents (or cotangents) on both sides.
3. Use addition formula and cross multiply
4. Arrange the terms as given in the identity.

General Solutions of Trigonometric Equations.

$$1) \sin \theta = 0 \Rightarrow \theta = n\pi \quad n \in \mathbb{Z}$$

$$2) \cos \theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2} \quad n \in \mathbb{Z}$$

$$3) \tan \theta = 0 \Rightarrow \theta = n\pi \quad n \in \mathbb{Z}.$$

If α is some constant angle, then prove that

$$1) \sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha, \quad n \in \mathbb{Z}.$$

$$2) \cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha \quad n \in \mathbb{Z}$$

$$3) \tan \theta = \tan \alpha \Rightarrow \theta = n\pi + \alpha \quad n \in \mathbb{Z}.$$

Working rule for solving $a \cos \theta + b \sin \theta = 0$

1. Check that $|c| \leq \sqrt{a^2 + b^2}$. If $|c| > \sqrt{a^2 + b^2}$ then the equation will not have any solution
2. Put $a = r \cos \alpha$, $b = r \sin \alpha$, $r > 0$ and get $r(\cos(\theta - \alpha)) = c$
(ie) $\cos(\theta - \alpha) = \frac{c}{r}$ and a value of α is found by using the equations $\cos \alpha = \frac{a}{r}$, $\sin \alpha = \frac{b}{r}$ alternatively put $a = r \sin \alpha$, $b = r \cos \alpha$ and get $r \sin(\theta + \alpha) = c$ (ie) $\sin(\theta + \alpha) = \frac{c}{r}$

Here $r = \sqrt{a^2 + b^2}$ and a value of α is found by using the equations $\sin \alpha = \frac{a}{r}$ and $\cos \alpha = \frac{b}{r}$

3) Write the general solution of the equation $\cos(\theta - \alpha) = \frac{c}{r} \sin(\theta + \alpha) = \frac{c}{r}$ and shift the angle α on the right hand side.

Sine Formula: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$, $R = \frac{abc}{4\Delta}$

Cosine Formula: $a^2 + b^2 - c^2 = 2ab \cos C$
 $b^2 + c^2 - a^2 = 2bc \cos A$
 $c^2 + a^2 - b^2 = 2ca \cos B$

(or) $a^2 = b^2 + c^2 - 2bc \cos A$
 $b^2 = c^2 + a^2 - 2ca \cos B$
 $c^2 = a^2 + b^2 - 2ab \cos C$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
 $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$
 $\cos B = \frac{c^2 + a^2 - b^2}{2ca}$

Projection formula: $a = b \cos C + c \cos B$
 $b = c \cos A + a \cos C$
 $c = a \cos B + b \cos A$

Napier's Formula: $\tan \frac{B-C}{2} = \frac{b-c}{b+c} \cot \frac{A}{2}$
 $\tan \frac{C-A}{2} = \frac{c-a}{c+a} \cot \frac{B}{2}$
 $\tan \frac{A-B}{2} = \frac{a-b}{a+b} \cot \frac{C}{2}$

Half angle Formula

$\sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$, $\cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$, $\tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$
 $\sin \frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$, $\cos \frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$, $\tan \frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$
 $\sin \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$, $\cos \frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$, $\tan \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

$\sin A = \frac{2}{bc} \sqrt{s(s-a)(s-b)(s-c)}$, $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ Heron's Formula

Area of the $\Delta = \Delta = \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B$

(or) $\Delta = a^2 \frac{\sin B \sin C}{\sin A}$, $a \cos A + b \cos B + c \cos C = 2a \sin B \sin C$

For a fixed perimeter $2s$ the area of the Δ is maximum when $a = b = c$

and the area of the $\Delta = \frac{s^2}{3\sqrt{3}}$ sq. units, $\frac{2\Delta}{ab} = \sin C$, $\frac{2\Delta}{bc} = \sin A$, $\frac{2\Delta}{ca} = \sin B$

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2/3/5 Marks Problems: (New)

1) If $\tan^2 x = 1 - a^2$ P.T $\sec x + \tan^3 x \operatorname{cosec} x = (2 - a^2)^{3/2}$. And also find the values of a for which the above result holds good.

$$\begin{aligned}
 \text{Sol: } \sec x + \tan^3 x \operatorname{cosec} x &= \sec x \left(1 + \tan^3 x \cdot \frac{\operatorname{cosec} x}{\sec x} \right) \\
 &= \sqrt{1 + \tan^2 x} \left(1 + \tan^3 x \cdot \frac{\cos x}{\sin x} \right) \\
 &= \sqrt{1 + \tan^2 x} (1 + \tan^3 x \cot x) \\
 &= \sqrt{1 + \tan^2 x} (1 + \tan^2 x) \\
 &= (1 + \tan^2 x)^{3/2} \\
 &= (1 + 1 - a^2)^{3/2} = (2 - a^2)^{3/2}
 \end{aligned}$$

always
 $\tan^2 x \geq 0 \Rightarrow 1 - a^2 \geq 0 \Rightarrow a^2 - 1 \leq 0 \Rightarrow -1 \leq a \leq 1$

$$\begin{aligned}
 \therefore \sec x + \tan^3 x \operatorname{cosec} x &= (2 - a^2)^{3/2} \\
 \Rightarrow 2 - a^2 &\geq 0 \Rightarrow a^2 - 2 < 0 \Rightarrow -\sqrt{2} \leq a \leq \sqrt{2}
 \end{aligned}$$

2) $\frac{\cos^4 \alpha}{\cos^2 \beta} + \frac{\sin^4 \alpha}{\sin^2 \beta} = 1$ P.T 1) $\sin^4 \alpha + \sin^4 \beta = 2 \sin^2 \alpha \sin^2 \beta$
 2) $\frac{\cos^4 \beta}{\cos^2 \alpha} + \frac{\sin^4 \beta}{\sin^2 \alpha} = 1$

Sol: Given $\frac{\cos^4 \alpha}{\cos^2 \beta} + \frac{\sin^4 \alpha}{\sin^2 \beta} = 1$

$$\cos^4 \alpha \sin^2 \beta + \sin^4 \alpha \cos^2 \beta = \cos^2 \beta \sin^2 \beta$$

$$(1 - \sin^2 \alpha)^2 \sin^2 \beta + \sin^4 \alpha (1 - \sin^2 \beta) = (1 - \sin^2 \beta) \sin^2 \beta \quad (\text{change all in terms of } \sin)$$

$$(\sin^4 \alpha + 1 - 2 \sin^2 \alpha) \sin^2 \beta + \sin^4 \alpha (1 - \sin^2 \beta) = (1 - \sin^2 \beta) \sin^2 \beta$$

$$\cancel{\sin^4 \alpha \sin^2 \beta} + \sin^2 \beta - 2 \sin^2 \alpha \sin^2 \beta + \sin^4 \alpha - \cancel{\sin^4 \alpha \sin^2 \beta} = \cancel{\sin^2 \beta} - \sin^4 \beta$$

$$\sin^4 \alpha + \sin^4 \beta - 2 \sin^2 \alpha \sin^2 \beta = 0$$

$$(\sin^2 \alpha - \sin^2 \beta)^2 = 0 \Rightarrow \sin^2 \alpha = \sin^2 \beta$$

$$\begin{aligned}
 1. \sin^4 \alpha + \sin^4 \beta &= (\sin^2 \alpha - \sin^2 \beta)^2 + 2 \sin^2 \alpha \sin^2 \beta \\
 &= 2 \sin^2 \alpha \sin^2 \beta
 \end{aligned}$$

$$2) \frac{\cos^4 \beta}{\cos^2 \alpha} + \frac{\sin^4 \beta}{\sin^2 \alpha} = \frac{\cos^2 \alpha \cdot \cos^2 \alpha}{\cos^2 \alpha} + \frac{\sin^2 \beta \cdot \sin^2 \alpha}{\sin^2 \beta} = 1 \quad \because \sin^2 \alpha = \sin^2 \beta$$

Procedure: Just add by algebraic method and change all the terms in terms of $\sin$. Simplify - we get $\sin^2 \alpha = \sin^2 \beta$. By using this result

3) If $a = \frac{2 \sin x}{1 + \cos x + \sin x}$, then p.t. $\frac{1 - \cos x + \sin x}{1 + \sin x} = a$.

Sol: $a = \frac{2 \sin x}{1 + \cos x + \sin x} = \frac{2 \sin x}{1 + \cos x + \sin x} \times \frac{(1 - \cos x + \sin x)}{(1 - \cos x + \sin x)}$

Procedure:

Multiplying both Nr and Dr by $1 - \cos x + \sin x$ and simplify.

$$= \frac{2 \sin x (1 - \cos x + \sin x)}{[(1 + \sin x) + \cos x][(1 + \sin x) - \cos x]}$$

$$= \frac{2 \sin x (1 - \cos x + \sin x)}{(1 + \sin x)^2 - \cos^2 x}$$

$$= \frac{2 \sin x (1 - \cos x + \sin x)}{1 + 2 \sin x + \sin^2 x - (1 - \sin^2 x)}$$

$$= \frac{2 \sin x (1 - \cos x + \sin x)}{2 \sin x + 2 \sin^2 x}$$

$$a = \frac{2 \sin x (1 - \cos x + \sin x)}{2 \sin x (1 + \sin x)}$$

$$a = \frac{1 - \cos x + \sin x}{1 + \sin x}$$

4) If $\sec x - \sin x = a^3$, $\sec x - \cos x = b^3$ p.t. $a^2 b^2 (a^2 + b^2) = 1$

Given $\sec x - \sin x = a^3$

$$\frac{1}{\sin x} - \sin x = a^3$$

$$\frac{1 - \sin^2 x}{\sin x} = a^3$$

$$\frac{\cos^2 x}{\sin x} = a^3 \quad \text{--- (1)}$$

Given $\sec x - \cos x = b^3$

$$\frac{1}{\cos x} - \cos x = b^3$$

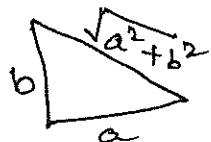
$$\frac{1 - \cos^2 x}{\cos x} = b^3$$

$$\frac{\sin^2 x}{\cos x} = b^3 \quad \text{--- (2)}$$

$$\therefore \frac{b^3}{a^3} = \frac{\sin^2 x}{\cos x} \times \frac{\sin x}{\cos^2 x} = \tan^3 x$$

$$\Rightarrow \tan x = \frac{b}{a} \quad \therefore \sin x = \frac{b}{\sqrt{a^2 + b^2}}$$

$$\cos x = \frac{a}{\sqrt{a^2 + b^2}}$$



Sub. in (1) $\frac{a^2}{(a^2 + b^2)} \times \frac{\sqrt{a^2 + b^2}}{b} = a^3 \Rightarrow a = \frac{1}{\sqrt{a^2 + b^2} \cdot b} \Rightarrow ab = \frac{1}{\sqrt{a^2 + b^2}}$
 $a^2 b^2 (a^2 + b^2) = 1$

5) * If $\cot x (1 + \sin x) = 4m$, $\cot x (1 - \sin x) = 4n$ P.T $(m^2 - n^2)^2 = mn$.

TBP

Given $\cot x (1 + \sin x) = 4m$

$$\frac{\cos x}{\sin x} (1 + \sin x) = 4m$$

$$\cot x + \cos x = 4m \quad \text{--- ①}$$

$$\cot x - \cos x = 4n \quad \text{--- ②}$$

$$\text{①}^2 - \text{②}^2$$

$$(\cot x + \cos x)^2 - (\cot x - \cos x)^2 = 16m^2 - 16n^2$$

$$4 \cot x \cos x = 16(m^2 - n^2)$$

$$\frac{\cos^2 x}{\sin x} = 4(m^2 - n^2) \quad \text{--- ③}$$

$$\text{①} \times \text{②} (\cot x + \cos x)(\cot x - \cos x) = 16mn$$

$$\cot^2 x - \cos^2 x = 16mn$$

$$\frac{\cos^2 x}{\sin^2 x} - \cos^2 x = 16mn$$

$$\frac{\cos^2 x - \sin^2 x \cos^2 x}{\sin^2 x} = 16mn$$

$$\frac{\cos^2 x (1 - \sin^2 x)}{\sin^2 x} = 16mn$$

$$\Rightarrow \frac{\cos^2 x \cdot \cos^2 x}{\sin^2 x} = 16mn \Rightarrow \frac{\cos^2 x}{\sin x} = 4\sqrt{mn} \quad \text{--- ④}$$

From ③ and ④ $4(m^2 - n^2) = 4\sqrt{mn}$
 $(m^2 - n^2)^2 = mn$.

6 Eliminate θ from the equation $a \sec \theta - c \tan \theta = b$
 $b \sec \theta + d \tan \theta = c$.

TBP

Sol: $a \sec \theta - c \tan \theta = b = 0$

$$b \sec \theta + d \tan \theta - c = 0$$

Solving for $\sec \theta$ and $\tan \theta$.

$$\begin{matrix} -c & -b & a & -c \\ d & -c & b & d \end{matrix}$$

$$\frac{\sec \theta}{c^2 + bd} = \frac{\tan \theta}{ac - b^2} = \frac{1}{ad + bc}$$

$$\sec \theta = \frac{c^2 + bd}{ad + bc}, \tan \theta = \frac{ac - b^2}{ad + bc}$$

$$\sec^2 \theta = 1 + \tan^2 \theta$$

$$\frac{(c^2 + bd)^2}{(ad + bc)^2} = 1 + \frac{(ac - b^2)^2}{(ad + bc)^2}$$

$$(c^2 + bd)^2 = (ad + bc)^2 + (ac - b^2)^2$$

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7) If $x = \sum_{n=0}^{\infty} \cos^{2n} \theta$, $y = \sum_{n=0}^{\infty} \sin^{2n} \theta$, $z = \sum_{n=0}^{\infty} \cos^{2n} \theta \sin^{2n} \theta$. $0 < \theta < \pi/2$ then
 T.B.P S.T. $xyz = x + y + z$.

Sol: $\because 1 + x + x^2 + x^3 + \dots = \frac{1}{1-x}$ $|x| < 1$.

$$x = \frac{1}{1 - \cos^2 \theta}, \quad y = \frac{1}{1 - \sin^2 \theta}, \quad z = \frac{1}{1 - \sin^2 \theta \cos^2 \theta}$$

$$= \frac{1}{\sin^2 \theta}, \quad = \frac{1}{\cos^2 \theta}, \quad = \frac{1}{1 - \sin^2 \theta \cos^2 \theta}$$

$$xyz = \frac{1}{\sin^2 \theta} \cdot \frac{1}{\cos^2 \theta} \cdot \frac{1}{1 - \sin^2 \theta \cos^2 \theta} = \frac{1}{\sin^2 \theta \cos^2 \theta (1 - \sin^2 \theta \cos^2 \theta)} \quad \text{--- ①}$$

$$x + y + z = \frac{1}{\sin^2 \theta} + \frac{1}{\cos^2 \theta} + \frac{1}{1 - \sin^2 \theta \cos^2 \theta}$$

$$= \frac{\cos^2 \theta (1 - \sin^2 \theta \cos^2 \theta) + \sin^2 \theta (1 - \sin^2 \theta \cos^2 \theta) + \sin^2 \theta \cos^2 \theta}{\sin^2 \theta \cos^2 \theta (1 - \sin^2 \theta \cos^2 \theta)}$$

$$= \frac{\cos^2 \theta - \sin^2 \theta \cos^4 \theta + \sin^2 \theta - \sin^4 \theta \cos^2 \theta + \sin^2 \theta \cos^2 \theta}{\sin^2 \theta \cos^2 \theta (1 - \sin^2 \theta \cos^2 \theta)}$$

$$= \frac{1 - \sin^2 \theta \cos^2 \theta (\sin^2 \theta + \cos^2 \theta) + \sin^2 \theta \cos^2 \theta}{\sin^2 \theta \cos^2 \theta (1 - \sin^2 \theta \cos^2 \theta)}$$

$$= \frac{1}{\sin^2 \theta \cos^2 \theta (1 - \sin^2 \theta \cos^2 \theta)} \quad \text{--- ②}$$

From ① and ②

$$xyz = x + y + z$$

TBP 8) If $a \cos x - b \sin x = c$ S.T $a \sin x + b \cos x = \pm \sqrt{a^2 + b^2 - c^2}$

Sol: Consider.

$$(a \cos x - b \sin x)^2 + (a \sin x + b \cos x)^2 = a^2 \cos^2 x + b^2 \sin^2 x - 2ab \sin x \cos x + a^2 \sin^2 x + b^2 \cos^2 x + 2ab \sin x \cos x$$

$$= a^2 + b^2$$

$$\therefore (a \sin x + b \cos x)^2 = (a^2 + b^2) - (a \cos x - b \sin x)^2$$

$$= a^2 + b^2 - c^2$$

$$a \sin x + b \cos x = \pm \sqrt{a^2 + b^2 - c^2}$$

9) If $\sec x + \tan x = P$ obtain the values of $\sec x$, $\tan x$, $\sin x$ in terms of P .

Sol: We know that $\sec^2 x - \tan^2 x = 1$

$$(\sec x + \tan x)(\sec x - \tan x) = 1$$

$$P(\sec x - \tan x) = 1$$

$$\sec x - \tan x = \frac{1}{P}$$

$$(\sec x + \tan x) + (\sec x - \tan x) = P + \frac{1}{P}$$

$$(+)\quad 2\sec x = \frac{P^2 + 1}{P} \Rightarrow \sec x = \frac{P^2 + 1}{2P}$$

$$(-)\quad 2\tan x = P - \frac{1}{P} \\ = \frac{P^2 - 1}{P} \Rightarrow \tan x = \frac{P^2 - 1}{2P}$$

$$\sin x = \frac{\tan x}{\sec x} = \frac{P^2 - 1}{2P} \times \frac{2P}{P^2 + 1} = \frac{P^2 - 1}{P^2 + 1}$$

10) If $m = a \cos^3 x + 3a \cos x \sin^2 x$, $n = a \sin^3 x + 3a \cos^2 x \sin x$. P.T
 $(m+n)^{2/3} + (m-n)^{2/3} = 2a^{2/3}$

Sol: Procedure: Find $m+n$ and $m-n$. Raise the power to $2/3$ on both $m+n$ and $m-n$. Then add.

$$m+n = a \cos^3 x + 3a \cos x \sin^2 x + a \sin^3 x + 3a \cos^2 x \sin x \\ = a(\cos^3 x + \sin^3 x) + 3a \sin x \cos x (\sin x + \cos x)$$

$$= a(\cos x + \sin x)^3$$

$$(m+n)^{2/3} = a^{2/3} (\cos x + \sin x)^2 \quad \text{--- (1)}$$

$$(m-n)^{2/3} = a^{2/3} (\cos x - \sin x)^2 \quad \text{--- (2)}$$

$$\textcircled{1} + \textcircled{2} \quad (m+n)^{2/3} + (m-n)^{2/3} = a^{2/3} \left[\cos^2 x + \sin^2 x + 2\sin x \cos x \right. \\ \left. + \cos^2 x + \sin^2 x - 2\sin x \cos x \right] \\ = a^{2/3} [2(\cos^2 x + \sin^2 x)]$$

$$(m+n)^{2/3} + (m-n)^{2/3} = 2a^{2/3}$$

11. If $2\tan^2 \alpha \tan^2 \beta \tan^2 \gamma + \tan^2 \alpha \tan^2 \beta + \tan^2 \beta \tan^2 \gamma + \tan^2 \alpha \tan^2 \gamma = 1$.
 Then P.T $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 1$.

Sol: Procedure: $\div$ by $\tan^2 \alpha \tan^2 \beta \tan^2 \gamma$ and change the Trigonometric formulas and simplify $\cot^2 \alpha = \frac{1}{\tan^2 \alpha}$
 Second time $\div$ $\cot^2 \alpha \cot^2 \beta \cot^2 \gamma$.

$$\frac{2\tan^2\alpha\tan^2\beta\tan^2\gamma + \tan^2\alpha\tan^2\beta + \tan^2\beta\tan^2\gamma + \tan^2\alpha\tan^2\gamma}{\tan^2\alpha\tan^2\beta\tan^2\gamma} = \frac{1}{\tan^2\alpha\tan^2\beta\tan^2\gamma}$$

$$2 + \frac{1}{\tan^2\gamma} + \frac{1}{\tan^2\alpha} + \frac{1}{\tan^2\beta} = \frac{1}{\tan^2\alpha\tan^2\beta\tan^2\gamma}$$

$$2 + \cot^2\alpha + \cot^2\beta + \cot^2\gamma = \cot^2\alpha \cot^2\gamma \cdot \cot^2\beta$$

$$2 + (\csc^2\alpha - 1) + (\csc^2\beta - 1) + (\csc^2\gamma - 1) = (\csc^2\alpha - 1)(\csc^2\beta - 1)(\csc^2\gamma - 1)$$

$$\begin{aligned} \csc^2\alpha + \csc^2\beta + \csc^2\gamma - 3 &= (\csc^2\alpha \csc^2\beta - \csc^2\alpha - \csc^2\beta + 1)(\csc^2\gamma - 1) \\ &= \csc^2\alpha \csc^2\beta \csc^2\gamma - \csc^2\alpha \csc^2\beta - \csc^2\alpha \csc^2\gamma + \csc^2\alpha \\ &\quad - \csc^2\beta \csc^2\gamma + \csc^2\beta + \csc^2\gamma - 1 \end{aligned}$$

$$\csc^2\alpha \csc^2\beta + \csc^2\beta \csc^2\gamma + \csc^2\alpha \csc^2\gamma = \csc^2\alpha \csc^2\beta \csc^2\gamma$$

$$\div \csc^2\alpha \csc^2\beta \csc^2\gamma$$

$$\frac{1}{\csc^2\gamma} + \frac{1}{\csc^2\alpha} + \frac{1}{\csc^2\beta} = 1$$

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 1$$

12) If $m = \sin x + \cos x$, P.T $\sin^6 x + \cos^6 x = \frac{4 - 3(m^2 - 1)^2}{4}$

TBP.

Sol: P.T LHS = RHS. Procedure
use $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ and $a^2 + b^2 = (a+b)^2 - 2ab$.

For LHS $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ and $a^2 + b^2 = (a+b)^2 - 2ab$.

RHS: Sub the m value.

$$\begin{aligned} \text{LHS} &= \sin^6 x + \cos^6 x = (\sin^2 x)^3 + (\cos^2 x)^3 \\ &= (\sin^2 x + \cos^2 x) [(\sin^2 x)^2 - \sin^2 x \cos^2 x + (\cos^2 x)^2] \\ &= (\sin^2 x + \cos^2 x)^2 - \sin^2 x \cos^2 x \\ &= (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x - \sin^2 x \cos^2 x \\ &= 1 - 3\sin^2 x \cos^2 x \end{aligned}$$

$$\begin{aligned} \text{RHS: } \frac{4 - 3[(\sin^2 x + \cos^2 x)^2 - 1]^2}{4} &= \frac{4 - 3(\sin^2 x + \cos^2 x + 2\sin x \cos x - 1)^2}{4} \\ &= \frac{4 - 3 \cdot 4\sin^2 x \cos^2 x}{4} \\ &= 1 - 3\sin^2 x \cos^2 x \end{aligned}$$

$$\text{LHS} = \text{RHS}$$

13) If $a \cos x + b \sin x = m$ and $a \sin x - b \cos x = n$ P.T $a^2 + b^2 = m^2 + n^2$

Sol: Procedure: Squaring both m and n and adding.

$$m^2 = (a \cos x + b \sin x)^2 = a^2 \cos^2 x + b^2 \sin^2 x + 2ab \sin x \cos x.$$

$$n^2 = (a \sin x - b \cos x)^2 = a^2 \sin^2 x + b^2 \cos^2 x - 2ab \sin x \cos x.$$

$$\begin{aligned} m^2 + n^2 &= a^2 (\cos^2 x + \sin^2 x) + b^2 (\sin^2 x + \cos^2 x) \\ &= a^2 + b^2 \end{aligned}$$

14) If $\cos x + \sin x = \sqrt{2} \cos x$, P.T $\cos x - \sin x = \sqrt{2} \sin x$.

Sol: Procedure: Find $(\cos x + \sin x)^2 + (\cos x - \sin x)^2$ and simplify.

$$(\cos x + \sin x)^2 + (\cos x - \sin x)^2 = (\sqrt{2} \cos x)^2 + (\sqrt{2} \sin x)^2$$

$$(\sqrt{2} \cos x)^2 + (\cos x - \sin x)^2 = 2$$

$$\begin{aligned} (\cos x - \sin x)^2 &= 2 - 2 \cos^2 x \\ &= 2(1 - \cos^2 x) \end{aligned}$$

$$= 2 \sin^2 x.$$

$$\cos x - \sin x = \sqrt{2} \sin x.$$

15) P.T $\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{1 + \sin \theta}{\cos \theta}$

(BP.

Sol: $\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{(\tan \theta + \sec \theta) - (\sec^2 \theta - \tan^2 \theta)}{\tan \theta - \sec \theta + 1}$

$$= \frac{(\tan \theta + \sec \theta) - (\sec \theta + \tan \theta)(\sec \theta - \tan \theta)}{\tan \theta - \sec \theta + 1}$$

$$= \frac{(\tan \theta + \sec \theta)[1 - \sec \theta + \tan \theta]}{\tan \theta - \sec \theta + 1}$$

$$= \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta} \Rightarrow \frac{1 + \sin \theta}{\cos \theta}.$$

16. P.T $(\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A) = \tan A \sec A - \cot A \operatorname{cosec} A$.

Sol: LHS: $(\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A)$

$$= \left(\frac{1}{\cos A} - \frac{1}{\sin A} \right) \left(1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \right)$$

$$= \left(\frac{\sin A - \cos A}{\sin A \cos A} \right) \left(\frac{\sin^2 A \cos A + \sin^2 A + \cos^2 A}{\sin A \cos A} \right)$$

$$= \frac{\sin^3 A - \cos^3 A}{\sin^2 A \cos^2 A}.$$

RHS: $\frac{\sin A}{\cos A} \cdot \frac{1}{\cos A} - \frac{\cos A}{\sin A} \cdot \frac{1}{\sin A}.$

$$= \frac{\sin^3 A - \cos^3 A}{\sin^2 A \cos^2 A}$$

∴ LHS = RHS.

17) Identify the quadrant in which an angle of each given measure lies.

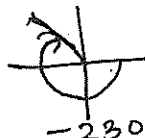
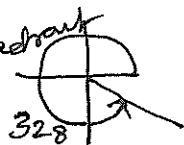
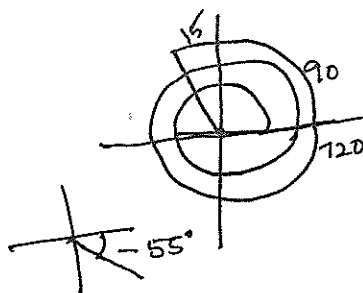
Q.B.P. 1) 25° - I quadrant

2) 8225° - II quadrant

3) -55° - IV quadrant

4) 328° - IV quadrant

5) -230° - II quadrant.



18) For each given angle, Find the co-terminal angle with measure

Q.B.P. of θ $0 \leq \theta < 360$

1) $(395^\circ, 35^\circ)$

$$\because 35 - 395 = -360$$

$$= -1(360) \therefore -1 \text{ rotation}$$

2) $(525, 885)$

$$\because 885 - 525 = 360$$

3)

$(1150, 70)$

$$2230 - 1150$$

$$70 - 1150 = -1080$$

$$= -3(360)$$

4) $(-270, 90)$

$$90 + 270 = 360$$

5) $(-450, 270)$

$$270 + 450 = 720 = 2(360)$$

19) If $a = \sec x - \tan x$, $b = \csc x + \cot x$ s.t. $ab + a - b + 1 = 0$

Sol: Procedure: Find ab , and $a - b$ separately and ab in $a - b$ we will get the answer.

$$ab = (\csc x + \cot x)(\sec x - \tan x)$$

$$= \sec x \csc x + \sec x \cot x - \tan x \csc x - \tan x \cot x$$

$$= \sec x \csc x + \frac{1}{\sin x} - \frac{1}{\cos x} - 1$$

$$= \sec x \csc x + \csc x - \sec x - 1$$

$$1 + ab = \sec x \csc x + \csc x - \sec x - 1 \quad \text{--- (1)}$$

$$a - b = \sec x - \tan x - \csc x - \cot x$$

$$= \sec x - \frac{\sin x}{\cos x} - \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right) = \sec x - \csc x - \left(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \right)$$

$$a - b = - (1 + ab) \Rightarrow ab + a - b + 1 = 0$$

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20) If $\tan x = \frac{b}{a}$ then find the value of $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$.

Sol: Procedure. Dividing both Nr and Dr by a and sub. $\tan x = \frac{b}{a}$ and simplify.

$$\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} = \sqrt{\frac{1+\frac{b}{a}}{1-\frac{b}{a}}} + \sqrt{\frac{1-\frac{b}{a}}{1+\frac{b}{a}}}$$

$$= \frac{\sqrt{1+\tan x}}{\sqrt{1-\tan x}} + \frac{\sqrt{1-\tan x}}{\sqrt{1+\tan x}}$$

$$= \frac{1+\tan x + 1-\tan x}{\sqrt{1-\tan x} \sqrt{1+\tan x}} = \frac{2}{\sqrt{1-\tan^2 x}}$$

$$= \frac{2}{\sqrt{1-\frac{\sin^2 x}{\cos^2 x}}}$$

$$= \frac{2 \cos x}{\sqrt{\cos^2 x - \sin^2 x}}$$

21) If $a \cos x + b \sin x = m$ and $a \sin x - b \cos x = n$ P.T $a^2 + b^2 = m^2 + n^2$

Sol: Procedure: Find m^2 and n^2 , add and simplify.

$$m^2 = (a \cos x + b \sin x)^2 = a^2 \cos^2 x + b^2 \sin^2 x + 2ab \sin x \cos x$$

$$n^2 = (a \sin x - b \cos x)^2 = a^2 \sin^2 x + b^2 \cos^2 x - 2ab \sin x \cos x$$

$$m^2 + n^2 = a^2 (\sin^2 x + \cos^2 x) + b^2 (\sin^2 x + \cos^2 x) \\ = a^2 + b^2$$

22) If $m = \tan x + \sin x$, $n = \tan x - \sin x$ S.T $m^2 - n^2 = 4\sqrt{mn}$.

Sol: Procedure: Find both $m^2 - n^2$ and $4\sqrt{mn}$ separately and show that they are equal.

$$m^2 - n^2 = (\tan x + \sin x)^2 - (\tan x - \sin x)^2 = 4 \tan x \sin x$$

$$\therefore (a+b)^2 - (a-b)^2 = 4ab$$

$$4\sqrt{mn} = 4\sqrt{(\tan x + \sin x)(\tan x - \sin x)}$$

$$1 + \tan^2 x = \sec^2 x$$

$$= 4\sqrt{\tan^2 x - \sin^2 x}$$

$$= 4\sqrt{\frac{\sin^2 x}{\cos^2 x} - \sin^2 x} = 4 \sin x \sqrt{\frac{1}{\cos^2 x} - 1}$$

$$= 4 \sin x \sqrt{\sec^2 x - 1}$$

$$= 4 \sin x \tan x$$

$$\therefore m^2 - n^2 = 4\sqrt{mn}$$

P.T
23) $\left| \sqrt{\frac{1-\sin x}{1+\sin x}} + \sqrt{\frac{1+\sin x}{1-\sin x}} \right| = -\frac{2}{\cos x} \quad \frac{\pi}{2} < x < \pi$

Sol: Just simplify and put the sign for II Quadrant.

$$\therefore \frac{\pi}{2} < x < \pi.$$

$$\frac{\sqrt{1-\sin x}}{\sqrt{1+\sin x}} + \frac{\sqrt{1+\sin x}}{\sqrt{1-\sin x}} = \frac{1-\sin x + 1+\sin x}{\sqrt{1+\sin x}\sqrt{1-\sin x}}$$

$$= \frac{2}{\sqrt{1-\sin^2 x}}$$

$$= \frac{2}{\cos x} \text{ or } 2 \sec x.$$

$$= -\frac{2}{\cos x} \quad \because \sec x = -ve \text{ in II quadrant}$$

24) Find the degree measure of the angle subtended at a centre of a circle of radius 100 cm by an arc of length 22 cm as shown in fig. ($\pi = \frac{22}{7}$)

Sol: $l = r\theta$

$$\theta = \frac{l}{r} = \frac{22}{100} = 0.22^\circ$$

$$= 0.22 \times \frac{180}{\pi} \text{ deg.} \Rightarrow 0.22 \times \frac{180}{22} \times 7 = 12.60$$

$$= 12 \frac{6}{10} \times \frac{60}{100} = 12.36'$$



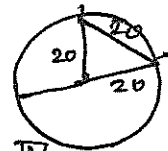
25) In a circular diameter 40 cm a chord of length 20 cm. Find the length of the minor arc of the chord.

Sol: Since radius = 20

and length of the chord = 20.

$\therefore$ The Δ is equilateral Δ

$$\therefore \theta = 60^\circ = 60 \times \frac{\pi}{180} = \frac{\pi}{3}$$



length of the arc = $r\theta$

$$= 20 \times \frac{\pi}{3} = 20 \times \frac{22}{7 \times 3} = \frac{440}{21} \text{ cm.}$$

Relation between degree and Radian measure

26) Convert i) 18° ii) -108° into radians.

TBP:

Sol: $180^\circ = \pi^c$

$$1^\circ = \frac{\pi^c}{180}$$

$$1) 18^\circ = \frac{\pi}{180} \times 18 = \frac{\pi}{10} \text{ radian.}$$

$$2) -108^\circ = \frac{\pi}{180} \times (-108) = -\frac{3\pi}{5}$$

27) Convert i) 30° 2) 135° 3) -205° 4) 150° 5) 330° into radian.

TBP

Sol: $180^\circ = \pi^c \Rightarrow 1^\circ = \frac{\pi^c}{180}$

$$1) 30^\circ = \frac{\pi}{180} \times 30 = \frac{\pi}{6}$$

$$2) 135^\circ = \frac{\pi}{180} \times 135 = \frac{3\pi}{4}$$

$$3) -205^\circ = \frac{\pi}{180} \times (-205) = -\frac{5\pi}{4}$$

$$4) 150^\circ = \frac{\pi}{180} \times 150 = \frac{5\pi}{6}$$

$$5) 330^\circ = \frac{\pi}{180} \times 330 = \frac{11\pi}{6}$$

28) change into degree measure.

TBP: 1) $\frac{\pi}{5}$ 2) 6 radians.

Sol: 1) $\pi^c = 180^\circ$

$$1^c = \frac{180}{\pi}$$

$$\frac{\pi}{5} = \frac{180 \times \pi}{\pi \times 5} = 36^\circ$$

$$2) 1^c = \frac{180}{\pi} = \frac{180 \times 7}{22}$$

$$6^c = \left(\frac{180 \times 7 \times 6}{22 \times 11} \right)^\circ = \left(343 \frac{7}{11} \right)^\circ$$

29) Change into degree measure

TBP 1) $\frac{\pi}{3}$ 2) $\frac{\pi}{9}$ 3) $\frac{2\pi}{5}$ 4) $\frac{7\pi}{3}$ 5) $\frac{10\pi}{9}$.

Sol: $1^c = \frac{\pi}{180} \cdot \frac{180}{\pi}$

$$1) \frac{\pi}{3} = \frac{180}{\pi} \times \frac{\pi}{3} = 60^\circ$$

$$2) \frac{\pi}{9} = \frac{180}{\pi} \times \frac{\pi}{9} = 20^\circ$$

$$3) \frac{2\pi}{5} = \frac{180}{\pi} \times \frac{2\pi}{5} = 72^\circ$$

$$4) \frac{10\pi}{9} = \frac{180}{\pi} \times \frac{10\pi}{9} = 200^\circ$$

30) what is the length of the arc intercepted by a central angle of 41° in a circle of radius 10 ft?

$$\text{Sol: } \theta = 41^\circ = \frac{\pi}{180} \times 41^\circ$$

$$r = 10$$

$$\therefore l = r\theta = \frac{\pi}{180} \times 41 = \frac{22 \times 41}{7 \times 180} = \frac{902}{1260} \text{ ft.} = \frac{82}{126} \text{ ft.}$$

31) If the arc of the same length in two circles subtend angles 60° and 75° at the centre. find the ratio of their radii.

$$\text{Sol: } 60^\circ = \frac{\pi}{180} \times 60 = \pi/3$$

$$75^\circ = \frac{\pi}{180} \times 75 = \frac{5\pi}{12}$$

$$\theta_1 = \frac{l}{r_1} = l = r_1 \theta_1, \quad l = r_2 \theta_2$$

$$\therefore r_1 \theta_1 = r_2 \theta_2$$

$$\frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{\frac{5\pi}{12}}{\frac{\pi}{3}} = \frac{5\pi \times 3}{12 \times \pi} = \frac{5}{4}$$

$$r_1 : r_2 = 5 : 4$$

32) If the arc of the same length in two circles subtended angles 65° and 110° at the centre. find their ratio of their radii.

33) The perimeter of a certain sector of a circle is equal to the length of the arc of a semi circle having the same radius. Express the angle of the sector in degrees and seconds.

$$\text{Sol: } \text{Perimeter of the sector} = 2r + r\theta$$

$$\text{length of the semi circle} = \pi r$$

$$\text{Given } 2r + r\theta = \pi r$$

$$r(2 + \theta) = \pi r$$

$$2 + \theta = \pi$$

$$\therefore \theta = \pi - 2 \text{ rad.}$$

$$= \frac{180}{\pi} \times (\pi - 2)$$

$$= \frac{180 \times \pi}{\pi} - \frac{180 \times 2}{\pi}$$

$$\theta = 180 - \frac{360}{\pi} = 180 - 114^\circ 32' 44'' = 65^\circ 27' 16''$$

34) what must be the radius of circular running path around which an athlete must run 5 times in order to describe 5 km.

Sol: For 5 rounds = 5 km
= 5000 m.

1 round = 1000 m.

$$2\pi r = 1000 \text{ m.}$$

$$r = \frac{1000 \times 7}{2 \times 22} = \frac{1750}{11} = 158^{\circ} 18'$$

35) Find the length of an arc of a circle of radius 5 cm. subtending a central angle 15° .

Sol: $l = r\theta$.

$$= 5 \times \frac{\pi}{12} \text{ cm.}$$

$$\theta = 15^{\circ} = 15 \times \frac{\pi}{180}$$

$$r = 5 \text{ cm.}$$

36) If the arc of the same length in two circles subtend central angles 30° and 80° find the ratio of their radii.

$$\text{Sol: } \theta_1 = 30^{\circ} = \frac{\pi}{6}$$

$$\theta_2 = 80^{\circ} = \frac{\pi}{9} \times \frac{4}{3} = \frac{4\pi}{9}$$

$$l = r_1\theta_1 = r_2\theta_2$$

$$\frac{r_1}{r_2} = \frac{\theta_2}{\theta_1}$$

$$= \frac{4\pi \times \frac{62}{\pi}}{9 \times \frac{1}{6}}$$

$$= \frac{8}{3}$$

$$\therefore r_1 : r_2 = 8 : 3.$$

37) what must be the radius of a circular running path around which an athlete must run 5 times in order to describe 1 km?

Sol: For 5 rounds = 1 km 1000 m.

For 1 round = 200 m.

$$2\pi r = 200 \text{ m}$$

$$r = \frac{100 \times 7}{22} = \frac{700}{22} = 31^{\circ} 49' 5''.$$

$$\frac{18 \times 60}{22} = \frac{520}{11} = 47 \frac{60}{11}$$

40) A Train is moving on a circular track of 1500 m radius
at the rate of 66 km/hr. What angle will it turn in
20 seconds.

$$r = 1500 \text{ m.}$$

$$\text{in 20 seconds} = \frac{66000}{3600} \times 20 = \frac{3300}{9} \checkmark$$

$$\text{min length } l = \frac{3300}{9}$$

$$\theta = \frac{l}{r} = \frac{\frac{3300}{9}}{1500} = \frac{11}{45}^\circ$$

41) If $\sec \theta + \tan \theta = P$ obtain the values of $\sec \theta$, $\tan \theta$ and $\sin \theta$
in terms of P .

Sol: Given $\sec \theta + \tan \theta = P \rightarrow \text{①}$

$$\text{we know } \sec^2 \theta - \tan^2 \theta = 1$$

$$(\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = 1$$

$$P(\sec \theta - \tan \theta) = 1$$

$$\sec \theta - \tan \theta = \frac{1}{P} \rightarrow \text{②}$$

$$\text{①} + \text{②} \quad 2\sec \theta = P + \frac{1}{P} \quad \text{①} - \text{②}$$

$$= \frac{P^2 + 1}{P}$$

$$\sec \theta = \frac{P^2 + 1}{2P}$$

$$2\tan \theta = P - \frac{1}{P}$$

$$= \frac{P^2 - 1}{P}$$

$$\tan \theta = \frac{P^2 - 1}{2P}$$

$$\frac{\tan \theta}{\sec \theta} = \frac{\frac{P^2 - 1}{2P}}{\frac{P^2 + 1}{2P}}$$

$$\frac{\sin \theta}{\cos \theta \cdot \sec \theta} = \frac{P^2 - 1}{P^2 + 1} \Rightarrow \sin \theta = \frac{P^2 - 1}{P^2 + 1}$$

P.T. $3(\sin^4 x - \cos^4 x) + 6(\sin^2 x + \cos^2 x) + 4(\sin^6 x + \cos^6 x) = 13.$

Sol: $(\sin^4 x - \cos^4 x)^2 = [(\sin^2 x - \cos^2 x)^2]^2$

1)
$$= (\sin^2 x + \cos^2 x - 2\sin x \cos x)^2$$

$$= (1 - 2\sin x \cos x)^2$$

$$= 1 + 4\sin^2 x \cos^2 x - 4\sin x \cos x.$$

$$(\sin^2 x + \cos^2 x)^2 = \sin^2 x + \cos^2 x + 2\sin x \cos x$$

$$= 1 + 2\sin x \cos x.$$

$$\sin^6 x + \cos^6 x = (\sin^2 x)^3 + (\cos^2 x)^3$$

$$= (\sin^2 x + \cos^2 x)^3 - 3\sin^2 x \cos^2 x (\sin^2 x + \cos^2 x)$$

$$= 1 - 3\sin^2 x \cos^2 x.$$

∴ L.H.S = $3(1 + 4\sin^2 x \cos^2 x - 4\sin x \cos x) + 6(1 + 2\sin x \cos x)$

$$+ 4(1 - 3\sin^2 x \cos^2 x)$$

$$= 3 + 12\sin^2 x \cos^2 x - 12\sin x \cos x + 6 + 12\sin x \cos x$$

$$+ 4 - 12\sin^2 x \cos^2 x$$

$$= 3 + 6 + 4$$

$$= 13$$

2) If $\tan \theta + \sin \theta = p$, $\tan \theta - \sin \theta = q$, $p > q$ P.T $p^2 - q^2 = 4\sqrt{pq}$

Sol: $p^2 = (\tan \theta + \sin \theta)^2$

$$= \tan^2 \theta + \sin^2 \theta + 2\tan \theta \sin \theta$$

$$q^2 = (\tan \theta - \sin \theta)^2$$

$$= \tan^2 \theta + \sin^2 \theta - 2\sin \theta \tan \theta.$$

$$p^2 - q^2 = 4\tan \theta \sin \theta \quad \text{--- (1)}$$

$$pq = (\tan \theta + \sin \theta)(\tan \theta - \sin \theta)$$

$$= \tan^2 \theta - \sin^2 \theta$$

$$= \frac{\sin^2 \theta}{\cos^2 \theta} - \sin^2 \theta = \sin^2 \theta \left(\frac{1}{\cos^2 \theta} - 1 \right)$$

$$= \sin^2 \theta (\sec^2 \theta - 1)$$

$$\sqrt{pq} = \sin \theta \tan \theta$$

$$4\sqrt{pq} = 4\sin \theta \tan \theta \quad \text{--- (2)}$$

From (1) and (2)

$$p^2 - q^2 = 4\sqrt{pq}.$$

$$\sqrt{pq} = \sin \theta \tan \theta$$

$$4\sqrt{pq} = 4 \tan \theta \sin \theta \quad \text{--- (2)}$$

$$\therefore p^2 - q^2 = 4\sqrt{pq}$$

$$3) \text{ P.T } \frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{1 + \sin \theta}{\cos \theta}$$

$$\begin{aligned} \text{L.H.S. } \frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} &= \frac{(\tan \theta + \sec \theta) - (\sec^2 \theta - \tan^2 \theta)}{\tan \theta - \sec \theta + 1} \\ &= \frac{(\tan \theta + \sec \theta) - (\sec \theta + \tan \theta)(\sec \theta - \tan \theta)}{\tan \theta - \sec \theta + 1} \\ &= \frac{(\tan \theta + \sec \theta)(1 - \sec \theta + \tan \theta)}{(\tan \theta - \sec \theta + 1)} \\ &= \tan \theta + \sec \theta \\ &= \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta} \\ &= \frac{1 + \sin \theta}{\cos \theta} \end{aligned}$$

$$4) \text{ S.T } \sin^2 A \tan A + \cos^2 A \cot A + 2 \sin A \cos A = \tan A + \cot A.$$

$$\text{L.H.S. } \sin^2 A \tan A + \cos^2 A \cot A + 2 \sin A \cos A.$$

$$= \sin^2 A \frac{\sin A}{\cos A} + \cos^2 A \frac{\cos A}{\sin A} + 2 \sin A \cos A.$$

$$= \frac{\sin^4 A + \cos^4 A + 2 \sin^2 A \cos^2 A}{\sin A \cos A}$$

$$= \frac{(\sin^2 A + \cos^2 A)^2}{\sin A \cos A} = \frac{1}{\sin A \cos A} = \frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$$

$$= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= \tan A + \cot A$$

5. If $\tan \theta + \sec \theta = x$: $\therefore$ P.T $2 \tan \theta = x - \frac{1}{x}$, $2 \sec \theta = x + \frac{1}{x}$

$$\sin \theta = \frac{x^2 - 1}{x^2 + 1}$$

Sol: $x = \tan \theta + \sec \theta$

$$\frac{1}{x} = \frac{1}{\tan \theta + \sec \theta} = \frac{1}{\tan \theta + \sec \theta} \times \frac{\tan \theta - \sec \theta}{\tan \theta - \sec \theta}$$

$$= \frac{\tan \theta - \sec \theta}{\tan^2 \theta - \sec^2 \theta}$$

$$= \frac{\tan \theta - \sec \theta}{-1}$$

$$= \sec \theta - \tan \theta$$

$$\therefore x - \frac{1}{x} = \tan \theta + \sec \theta - \sec \theta + \tan \theta$$

$$= 2 \tan \theta$$

$$x + \frac{1}{x} = \tan \theta + \sec \theta + \sec \theta - \tan \theta$$

$$= 2 \sec \theta$$

$$\frac{2 \tan \theta}{2 \sec \theta} = \frac{x - \frac{1}{x}}{x + \frac{1}{x}} = \frac{\frac{x^2 - 1}{x}}{\frac{x^2 + 1}{x}}$$

$$\frac{\sin \theta}{\cos \theta} \times \cos \theta = \frac{x^2 - 1}{x^2 + 1}$$

$$\sin \theta = \frac{x^2 - 1}{x^2 + 1}$$

6) P.T $(1 + \tan A + \sec A)(1 + \cot A - \operatorname{cosec} A) = 2$

L.H.S. $(1 + \tan A + \sec A)(1 + \cot A - \operatorname{cosec} A)$

$$= \left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right) \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)$$

$$= (\cos A + \sin A + 1) / (\sin A + \cos A - 1)$$

$$\begin{aligned}
 & \frac{(\cos A + \sin A + 1)(\cos A + \sin A - 1)}{\sin A \cos A} \\
 &= \frac{(\cos A + \sin A)^2 - 1}{\sin A \cos A} = \frac{\cos^2 A + \sin^2 A + 2 \sin A \cos A - 1}{\sin A \cos A} \\
 &= \frac{2 \sin A \cos A}{\sin A \cos A} \\
 &= 2
 \end{aligned}$$

$$7) \text{ P.T } (1 + \cot A + \tan A)(\sin A - \cos A) = \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A}.$$

$$\text{LHS: } (1 + \cot A + \tan A)(\sin A - \cos A)$$

$$= \left(1 + \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}\right)(\sin A - \cos A)$$

$$= \frac{(\sin A \cos A + \cos^2 A + \sin^2 A)(\sin A - \cos A)}{\sin A \cos A}$$

$$= \frac{\sin^2 A \cos A + \sin A \cos^2 A + \sin^3 A - \sin A \cos^2 A - \cos^3 A - \sin^2 A \cos A}{\sin A \cos A}$$

$$= \frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A}$$

$$= \frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A}$$

$$= \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A}.$$

8) Find the value of $\sin 18^\circ$.

TBP let $A = 18^\circ \Rightarrow 5A = 90^\circ$

$$2A + 3A = 90^\circ$$

$$2A = 90 - 3A$$

$$\sin 2A = \sin(90 - 3A) \\ = \cos 3A$$

$$2 \sin A \cos A = 4 \cos^3 A - 3 \cos A$$

$$\begin{aligned}
 2 \sin A &= 4(1 - \sin^2 A) - 3 \\
 &= 4 - 4 \sin^2 A - 3 \\
 &= 1 - 4 \sin^2 A
 \end{aligned}$$

$$4 \sin^2 A + 2 \sin A - 1 = 0 \quad a=4, b=2, c=-1$$

$$\sin A = \frac{-2 \pm \sqrt{4+16}}{8}$$

$$= \frac{-2 \pm 2\sqrt{5}}{8} = \frac{-1 \pm \sqrt{5}}{4}$$

Extras Either one mark or Two marks.

1. change 150° into radian measure

$$180^\circ = \pi^c$$

$$150^\circ = \frac{\pi}{180} \times 150 = \frac{5\pi}{6}$$

2. change $\frac{3\pi}{4}$ into degree measure

$$\pi \text{ radian} = 180^\circ$$

$$\frac{3\pi}{4} = \frac{45}{180} \times \frac{3\pi}{4} = 135^\circ$$

3. change $\frac{1}{4}$ into degree measure.

$$= \frac{1}{4} \times \frac{180}{\pi} = \frac{1}{4} \times \frac{180}{22} \times 7 = 14^\circ 19' 5''$$

4. Express in which quadrant it appears.

$$1. 380 = 360 + 20, \text{ I quadrant}$$

$$2. -140 = -(90 + 50), \text{ II quadrant}$$

$$3. 1100 = 3 \times 360 + 20, \text{ I quadrant}$$

5) 1. $\tan 735^\circ = \tan (2 \times 360 + 15) = \tan 15^\circ$

$$2. \cos 980 = \cos (2 \times 360 + 260) = \cos 260 = \cos (270 - 10) = -\sin 10^\circ$$

$$3. \sin (2460) = \sin (6 \times 360 + 300) = \sin 300 = \sin (360 - 60) = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$$

$$4) \cos (-870) = \cos 870 = \cos (2 \times 360 + 150) = \cos 150 = \cos (180 - 30) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

$$5. \sin (-780) = -\sin (780)$$

$$\begin{aligned}
 6. \cot(-855) &= -\cot 855 \\
 &= -\cot(2 \times 360 + 135) \\
 &= -\cot 135^\circ \\
 &= -\cot(180 - 45) \\
 &= \cot 45 \\
 &= 1.
 \end{aligned}$$

$$\begin{aligned}
 7. \operatorname{cosec}(2040) &= \operatorname{cosec}(5 \times 360 + 240) \\
 &= \operatorname{cosec} 240 \\
 &= \operatorname{cosec}(180 + 60) \\
 &= -\operatorname{cosec} 60^\circ \\
 &= -2/\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 8. \sec(-1305) &= \sec(1305) \\
 &= \sec(3 \times 360 + 225) \\
 &= \sec(225) \\
 &= \sec(180 + 45) \\
 &= -\sec 45^\circ \\
 &= -\sqrt{2}
 \end{aligned}$$

$$6. \text{ p.T } \frac{\sin 300 \tan 330 \sec 420}{\cot 135^\circ \cos 210^\circ \operatorname{cosec} 315^\circ} = -\sqrt{\frac{2}{3}}.$$

$$\text{LHS } \frac{\sin 300 \tan 330 \sec 420}{\cot 135^\circ \cos 210^\circ \operatorname{cosec} 315^\circ}$$

$$= \frac{\left(-\frac{\sqrt{3}}{2}\right) \left(-\frac{1}{\sqrt{3}}\right) \left(\frac{\sqrt{2}}{2}\right)}{(-1) \left(-\frac{\sqrt{3}}{2}\right) (-\sqrt{2})}$$

$$= -\frac{\sqrt{2}}{\sqrt{3}}.$$

$$\begin{aligned}
 \sin 300 &= \sin(360 - 60) \\
 &= -\sin 60 \\
 &= -\frac{\sqrt{3}}{2}
 \end{aligned}$$

$$\begin{aligned}
 \tan 330 &= \tan(360 - 30) \\
 &= -\tan 30 \\
 &= -\frac{1}{\sqrt{3}}
 \end{aligned}$$

$$\begin{aligned}
 \sec 420 &= \sec(360 + 60) \\
 &= \sec 60^\circ \\
 &= 2
 \end{aligned}$$

$$\begin{aligned}
 \cot 135^\circ &= \cot(180 - 45) \\
 &= -\cot 45 \\
 &= -1
 \end{aligned}$$

$$\begin{aligned}
 \cos 210^\circ &= \cos(180 + 30) \\
 &= -\cos 30^\circ \\
 &= -\frac{\sqrt{3}}{2}
 \end{aligned}$$

7) Express as a function of A $\sec\left(A - \frac{3\pi}{2}\right)$

$$\sec\left(-\left(\frac{3\pi}{2} - A\right)\right) = \sec\left(\frac{3\pi}{2} - A\right)$$

..... A

..... $(2\pi - A)$

8) T.B.P. P.T $\frac{\sin(180+A) \cos(90-A) \tan(270-A)}{\sec(540-A) \cos(360+A) \operatorname{cosec}(270+A)} = -\sin A \cos^2 A.$

$\sin(180+A) = -\sin A, \cos(90-A) = \sin A, \tan(270-A) = -\cot A.$

$\sec(540-A) = \sec(180-A) \cos(360+A) \operatorname{cosec}(270+A)$
 $= -\sec A / \quad = \cos A \quad = -\sec A.$

L.H.S: $\frac{\sin(180+A) \cos(90-A) \tan(270-A)}{\sec(540-A) \cos(360+A) \operatorname{cosec}(270+A)}$

$= \frac{(-\sin A)(\sin A)(-\cot A)}{(-\sec A)(\cos A)(-\sec A)}$

$= \frac{(-\sin A)(\sin A)(\cos A)}{(-\frac{1}{\cos A})(\cos A)(-\frac{1}{\cos A})(\sin A)}$

$= -\sin A \cos^2 A$

9) Find the value of $\frac{1}{2} \sin^2 60 - \frac{1}{2} \sec 60 \tan^2 30 + \frac{4}{5} \sin^2 45 \tan^2 60$

Sol: $\frac{1}{2} \sin^2 60 - \frac{1}{2} \sec 60 \tan^2 30 + \frac{4}{5} \sin^2 45 \tan^2 60$

$= \frac{1}{2} \left(\frac{\sqrt{3}}{2}\right)^2 - \frac{1}{2} \cdot 2 \left(\frac{1}{\sqrt{3}}\right)^2 + \frac{4}{5} \cdot \frac{1}{2} \cdot 3$

$= \frac{3}{8} - \frac{1}{3} + \frac{12}{10} = \frac{45 - 40 + 144}{120}$

$= \frac{149}{120}$

10) P.T $\sin^4 A - \cos^4 A = 1 - 2\cos^2 A$

L.H.S: $\sin^4 A - \cos^4 A$

$= (\sin^2 A)^2 - (\cos^2 A)^2$
 $= (\sin^2 A + \cos^2 A) (\sin^2 A - \cos^2 A)$

$= \sin^2 A - \cos^2 A$

$= 1 - \cos^2 A - \cos^2 A$

$$1) \sin^3 A - \cos^3 A = (\sin A - \cos A)(1 + \sin A \cos A)$$

$$\begin{aligned} \text{LHS: } \sin^3 A - \cos^3 A &= (\sin A - \cos A)(\sin^2 A + \sin A \cos A + \cos^2 A) \quad a^3 - b^3 = (a-b)(a^2 + ab + b^2) \\ &= (\sin A - \cos A)(1 + \sin A \cos A). \end{aligned}$$

$$12) (\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$$

$$\begin{aligned} \text{LHS: } (\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 \\ &= \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta + \sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta \\ &= 2(\sin^2 \theta + \cos^2 \theta) \\ &= 2 \end{aligned}$$

$$13) (\tan \theta + \cot \theta)^2 = \sec^2 \theta + \operatorname{cosec}^2 \theta$$

$$\begin{aligned} \text{LHS: } (\tan \theta + \cot \theta)^2 \\ &= \tan^2 \theta + \cot^2 \theta + 2 \tan \theta \cot \theta \\ &= \tan^2 \theta + \cot^2 \theta + 2 \\ &= 1 + \tan^2 \theta + 1 + \cot^2 \theta \\ &= \sec^2 \theta + \operatorname{cosec}^2 \theta. \end{aligned}$$

$$14) \frac{1}{1+\sin \theta} + \frac{1}{1-\sin \theta} = 2 \sec^2 \theta.$$

$$\begin{aligned} \text{LHS: } \frac{1}{1+\sin \theta} + \frac{1}{1-\sin \theta} \\ &= \frac{1-\sin \theta + 1+\sin \theta}{(1+\sin \theta)(1-\sin \theta)} = \frac{2}{1-\sin^2 \theta} \\ &= \frac{2}{\cos^2 \theta} = 2 \sec^2 \theta. \end{aligned}$$

$$15) \frac{\sec x + \tan x}{\sec x - \tan x} = (\sec x + \tan x)^2$$

$$\begin{aligned} \text{LHS: } \frac{\sec x + \tan x}{\sec x - \tan x} \times \frac{\sec x + \tan x}{\sec x + \tan x} \\ &= \frac{(\sec x + \tan x)^2}{\sec^2 x - \tan^2 x} \end{aligned}$$

$$\therefore \sec^2 x - \tan^2 x = 1$$

$$(6) \frac{1}{\tan \theta + \sec \theta} = \sec \theta - \tan \theta.$$

$$\text{LHS: } \frac{1}{\tan \theta + \sec \theta} \times \frac{\tan \theta - \sec \theta}{\tan \theta - \sec \theta}$$

$$= \frac{\tan \theta - \sec \theta}{\tan^2 \theta - \sec^2 \theta} = \frac{\tan \theta - \sec \theta}{-1} = \sec \theta - \tan \theta.$$

$$17) \frac{\operatorname{cosec} \theta}{\cot \theta + \tan \theta} = \cos \theta$$

$$\begin{aligned} \text{LHS: } \frac{\operatorname{cosec} \theta}{\cot \theta + \tan \theta} &= \frac{\operatorname{cosec} \theta}{\frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta}} \\ &= \frac{\operatorname{cosec} \theta}{\frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta}} \end{aligned}$$

$$= \operatorname{cosec} \theta \times \sin \theta \cdot \cos \theta$$

$$= \cos \theta.$$

$$18. \frac{1}{\operatorname{cosec} \theta - \cot \theta} = \frac{1 + \cos \theta}{\sin \theta}.$$

$$\text{LHS: } \frac{1}{\operatorname{cosec} \theta - \cot \theta} =$$

$$= \frac{1}{\operatorname{cosec} \theta - \cot \theta} \times \frac{\operatorname{cosec} \theta + \cot \theta}{\operatorname{cosec} \theta + \cot \theta}$$

$$= \frac{\operatorname{cosec} \theta + \cot \theta}{\operatorname{cosec}^2 \theta - \cot^2 \theta} = \frac{\operatorname{cosec} \theta + \cot \theta}{\frac{1}{\sin^2 \theta} - \frac{\cos^2 \theta}{\sin^2 \theta}}$$

$$= \frac{1 + \cos \theta}{\sin \theta}.$$

$$19. (\sec \theta + \cos \theta)(\sec \theta - \cos \theta) = \tan^2 \theta + \sin^2 \theta.$$

$$\text{LHS: } (\sec \theta + \cos \theta)(\sec \theta - \cos \theta)$$

$$= \sec^2 \theta - \cos^2 \theta$$

$$= (1 + \tan^2 \theta) - (1 - \sin^2 \theta) = \tan^2 \theta + \sin^2 \theta.$$

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தமிழ்நாடு அரசுப் பல்கலைக் கழகம்

Trigonometric functions and their properties.

1) The terminal side of an angle θ in standard position passes through the point $(3, -4)$. Find the trigonometric functions.

Sol: $r = \sqrt{9+16} = 5$

$$\sin \theta = \frac{y}{r} = \frac{-4}{5} \quad \therefore \theta \text{ is in IV}$$

$$\sin \theta = -4/5$$

$$\cos \theta = \frac{x}{r} = \frac{3}{5}$$

$$\cos \theta = 3/5$$

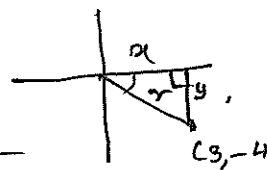
$$\tan \theta = \frac{y}{x} = \frac{-4}{3}$$

$$\tan \theta = -4/3$$

$$\therefore \csc \theta = -5/4$$

$$\sec \theta = 5/3$$

$$\cot \theta = -3/4$$



2) If $\sin \theta = 3/5$ and the angle θ is in II quadrant find the value of other functions.

$$\cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$$

$$\tan \theta = 3/4$$

$\therefore \theta$ lies in second quadrant $\sin \theta = 3/5, \csc \theta = 5/3$

$$\cos \theta = -4/5, \sec \theta = -5/4$$

$$\tan \theta = -3/4, \cot \theta = -4/3$$

3) Find the value of 1) $\sin(-45^\circ)$ 2) $\cos(-45^\circ)$ 3) $\cot(-45^\circ)$

1) $\sin(-45^\circ) = -\sin 45^\circ = -1/\sqrt{2}$

$$\sin(-\theta) = -\sin \theta$$

2) $\cos(-45^\circ) = \cos 45^\circ = 1/\sqrt{2}$

$$\cos(-\theta) = \cos \theta$$

3) $\cot(-45^\circ) = -\cot 45^\circ = -1$

$$\cot(-\theta) = -\cot \theta$$

4) Find the value of 1) $\sin 150^\circ$ 2) $\cos 135^\circ$ 3) $\tan 120^\circ$

1) $\sin 150^\circ = \sin(180^\circ - 30^\circ) = \sin 30^\circ = 1/2$

2) $\cos 135^\circ = \cos(180^\circ - 45^\circ) = -\cos 45^\circ = -1/\sqrt{2}$

3) $\tan 120^\circ = \tan(180^\circ - 60^\circ) = -\tan 60^\circ = -\sqrt{3}$

5) Find the value of 1) $\sin 765^\circ$ 2) $\operatorname{cosec}(-1410)$, $\cot\left(-\frac{15\pi}{4}\right)$

1) $\sin 765 = \sin(360 + 405) = \sin 405 = \frac{1}{\sqrt{2}}$

2) $\operatorname{cosec}(-1410) = -\operatorname{cosec}(1410)$
 $= -\operatorname{cosec}(360 + 330) = -\operatorname{cosec} 330$
 $= -\operatorname{cosec}(360 - 30) = -\operatorname{cosec} 30^\circ$
 $= -2$

3) $\cot\left(-\frac{15\pi}{4}\right) = -\cot\left(4\pi - \frac{\pi}{4}\right) = +\cot \frac{\pi}{4} = +1$

6) B.P. P.T $\tan 315^\circ \cot(-405^\circ) + \cot 495^\circ \tan(-585^\circ) = 2$.

$\tan 315^\circ = \tan(360 - 45) = -\tan 45 = -1$

$\cot(-405) = -\cot 405 = -\cot(360 + 45) = -\cot 45 = -1$

$\cot 495 = \cot(360 + 135) = \cot 135 = \cot(180 - 45) = -\cot 45 = -1$

$\tan(-585) = -\tan(585) = -\tan(180 + 405)$
 $= -\tan(180 + 45) = -\tan 45 = -1$

$\therefore \tan 315^\circ \cot(-405) + \cot 495^\circ \tan(-585) = (-1)(-1) + (-1)(-1)$
 $= 1 + 1$
 $= 2$

7) Determine the functions are even or odd or neither.

1) $\sin^2 x - 2\cos^2 x - \cos x$

2) $f(-x) = \sin^2 x - 2\cos^2 x - \cos x$
 $= f(x) \therefore$ even function

2) $\sin(\cos x) : f(-x) = \sin(\cos(-x))$
 $= \sin(\cos x) = \text{even fn.}$
 $= f(x)$

3) $f(x) = \cos(\sin x) : f(-x) = \cos(\sin(-x)) = \cos(-\sin x)$
 $= \cos(\sin x) = f(x) \text{ even.}$

4. $f(x) = \sin x + \cos x$
 $f(-x) = -\sin x + \cos x$
 $\therefore f(x) \neq f(-x) \therefore$ neither even nor odd.

Q) Find the values of 1) $\sin 480^\circ$ 2) $\sin(-1110)$ 3) $\cos(300)$ 4) $\tan(1050)$
 5) $\cot 660^\circ$ 6) $\tan \frac{19\pi}{3}$ 7) $\sin(-\frac{11\pi}{3})$

Sol: 1. $\sin 480 = \sin(360 + 120) = \sin 120 = \sin(180 - 60)$
 $= \sin 60 = \frac{\sqrt{3}}{2}$

2. $\sin(-1110) = -\sin(1110) = -\sin(3 \times 360 + 30)$
 $= -\sin 30 = -\frac{1}{2}$

3. $\cos(300) = \cos(360 - 60) = \cos 60 = \frac{1}{2}$

4. $\tan(1050) = \tan(3 \times 360 - 30) = -\tan 30 = -\frac{1}{\sqrt{3}}$

5. $\cot(660^\circ) = \cot(2 \times 360 - 60) = -\cot 60 = -\frac{1}{\sqrt{3}}$

6. $\tan\left(\frac{19\pi}{3}\right) = \tan\left(6\pi + \frac{2\pi}{3}\right) = \tan \frac{\pi}{3} = \sqrt{3}$

7. $\sin\left(-\frac{11\pi}{3}\right) = \sin\left(4\pi - \frac{\pi}{3}\right) = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

Q) PT $\frac{\cot(180+\theta) \sin(90-\theta) \cos(-\theta)}{\sin(270+\theta) \tan(-\theta) \operatorname{cosec}(360+\theta)} = \cos^2 \theta \tan \theta$

Sol: $\cot(180+\theta) = \cot \theta$ $\therefore \frac{\cot(180+\theta) \sin(90-\theta) \cos(-\theta)}{\sin(270+\theta) \tan(-\theta) \operatorname{cosec}(360+\theta)}$
 $\sin(90-\theta) = \cos \theta$ $\frac{\sin(90-\theta) \cos(-\theta)}{\sin(270+\theta) \tan(-\theta) \operatorname{cosec}(360+\theta)}$
 $\cos(-\theta) = \cos \theta$ $= \frac{(\cot \theta) (\cos \theta) (\cos \theta)}{(-\cos \theta) (-\tan \theta) \cdot \operatorname{cosec} \theta}$
 $\sin(270+\theta) = -\cos \theta$ $= \frac{\cot \theta \cdot \cos^2 \theta}{(-\cos \theta) (-\tan \theta) \cdot \operatorname{cosec} \theta}$
 $\tan(-\theta) = -\tan \theta$ $= \frac{\cot \theta \cdot \cos^2 \theta}{\cos \theta \operatorname{cosec} \theta \cdot \tan \theta}$
 $\operatorname{cosec}(360+\theta) = \operatorname{cosec} \theta$ $= \cos \theta \cdot \cot \theta$

Q) Find the values of other five trigonometric functions for the following

1) $\cos \theta = -\frac{1}{2}$ & lies in the III quadrant.
 $\sin \theta = -\frac{\sqrt{3}}{2}$ $\operatorname{cosec} \theta = -\frac{2}{\sqrt{3}}$
 $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3}$ $\sec \theta = -2$
 $\cot \theta = \frac{1}{\sqrt{3}}$

2) $\cos \theta = \frac{2}{3}$ & lies in I quadrant.

$\sin \theta = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$
 $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{\sqrt{5}}{3}}{\frac{2}{3}} = \frac{\sqrt{5}}{2}$ $\sec \theta = \frac{3}{2}$
 $\operatorname{cosec} \theta = \frac{3}{\sqrt{5}}$ $\cot \theta = \frac{2}{\sqrt{5}}$

3) $\sin \theta = -\frac{2}{3}$ & lies in the IV quadrant.

$$\cos \theta = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$$

$$\tan \theta = -\frac{2}{\sqrt{5}} \quad \sec \theta = \frac{3}{\sqrt{5}}$$

$$\csc \theta = -\frac{3}{2} \quad \cot \theta = -\frac{\sqrt{5}}{2}$$

4) $\tan \theta = -\frac{2}{1}$ & lies in II quadrant.

$$\sin \theta = \frac{2}{\sqrt{5}}, \csc \theta = \frac{\sqrt{5}}{2}$$

$$\cos \theta = -\frac{1}{\sqrt{5}} \quad \sec \theta = -\sqrt{5}$$

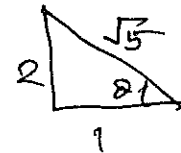
$$\cot \theta = -\frac{1}{2}$$

5) $\sec \theta = \frac{13}{5}$ & is in IV quadrant

$$\cos \theta = \frac{5}{13}, \tan \theta = -\frac{12}{5}$$

$$\sin \theta = \sqrt{1 - \frac{25}{169}} = -\frac{12}{13}$$

$$\csc \theta = -\frac{13}{12}$$



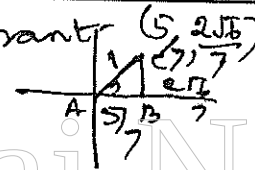
11) $\left(\frac{7}{5}, \frac{2\sqrt{6}}{7}\right)$ is a point on the terminal side of an angle θ in standard position. Determine the trigonometric values of angle θ .

Sol: $AC = \sqrt{\frac{25}{49} + \frac{24}{49}} = 1$ & lies in I quadrant

$$\sin \theta = \frac{2\sqrt{6}}{7}, \csc \theta = \frac{7}{2\sqrt{6}}$$

$$\cos \theta = \frac{5}{7}, \sec \theta = \frac{7}{5}$$

$$\tan \theta = \frac{2}{\sqrt{6}} \times \frac{7}{5} = \frac{14}{5\sqrt{6}}, \cot \theta = \frac{5\sqrt{6}}{14}$$



12) Find all the angles between 0 and 360° which satisfies the equation $\sin^2 \theta = \frac{3}{4}$

$$\sin^2 \theta = \frac{3}{4} \Rightarrow \sin \theta = \pm \frac{\sqrt{3}}{2}$$

If $\sin \theta = \frac{\sqrt{3}}{2}$ & lies in I or II quadrant.

If θ is in I quadrant

$$\cos \theta = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}, \sec \theta = 2$$

$$\tan \theta = \sqrt{3}, \cot \theta = \frac{1}{\sqrt{3}}$$

If θ is in II quadrant

$$\sin \theta = \frac{\sqrt{3}}{2}, \csc \theta = \frac{2}{\sqrt{3}}$$

$$\cos \theta = -\frac{1}{2}, \sec \theta = -2$$

If $\sin \theta = -\frac{\sqrt{3}}{2}$ & may be in III or IV quadrant

If θ is in III

$$\sin \theta = -\frac{\sqrt{3}}{2}, \csc \theta = -\frac{2}{\sqrt{3}}$$

$$\cos \theta = -\frac{1}{2}, \sec \theta = -2$$

$$\tan \theta = \sqrt{3}, \cot \theta = \frac{1}{\sqrt{3}}$$

If θ is in IV

$$\sin \theta = -\frac{\sqrt{3}}{2}, \csc \theta = -\frac{2}{\sqrt{3}}$$

$$\cos \theta = \frac{1}{2}, \sec \theta = 2$$

$$\tan \theta = -\frac{1}{\sqrt{3}}$$

13) P.T $\frac{\cos(90+\theta) \sec(-\theta) \tan(180-\theta)}{\sec(360-\theta) \sin(180+\theta) \cot(90-\theta)} = -1.$

Sol: $\cos(90+\theta) = -\sin\theta$
 $\sec(-\theta) = \sec\theta$
 $\tan(180-\theta) = -\tan\theta$

$\sec(360-\theta) = \sec\theta$
 $\sin(180+\theta) = -\sin\theta$
 $\cot(90-\theta) = \tan\theta$

$\therefore \frac{\cos(90+\theta) \sec(-\theta) \tan(180-\theta)}{\sec(360-\theta) \sin(180+\theta) \cot(90-\theta)} = \frac{(-\sin\theta)(\sec\theta)(-\tan\theta)}{(\sec\theta)(-\sin\theta)(\tan\theta)} = -1.$

14) P.T $\sin^2 \frac{\pi}{18} + \sin^2 \frac{\pi}{9} + \sin^2 \frac{7\pi}{18} + \sin^2 \frac{4\pi}{9} = 2.$

KBP Sol: $\sin \frac{7\pi}{18} = \sin\left(\frac{9\pi-2\pi}{18}\right) = \sin\left(\frac{\pi}{2} - \frac{\pi}{9}\right) = \cos \frac{\pi}{9}$

$\sin \frac{4\pi}{9} = \sin \frac{8\pi}{18} = \sin\left(\frac{9\pi-\pi}{18}\right) = \sin\left(\frac{\pi}{2} - \frac{\pi}{18}\right) = \cos \frac{\pi}{18}$

$\therefore \text{LHS} = \sin^2 \frac{\pi}{18} + \sin^2 \frac{\pi}{9} + \sin^2 \frac{7\pi}{18} + \sin^2 \frac{4\pi}{9}$

$= \sin^2 \frac{\pi}{18} + \sin^2 \frac{\pi}{9} + \cos^2 \frac{\pi}{18} + \cos^2 \frac{\pi}{9}$

$= \left(\sin^2 \frac{\pi}{18} + \cos^2 \frac{\pi}{18}\right) + \left(\sin^2 \frac{\pi}{9} + \cos^2 \frac{\pi}{9}\right)$

$\uparrow = 1+1 = 2.$

Procedure: Change $\frac{7\pi}{18}$ as $\frac{9\pi-2\pi}{18}$

and $\frac{4\pi}{9}$ as $\frac{8\pi}{18}$ and $\frac{9\pi-\pi}{18}$

15) P.T $\sin^2 \frac{\pi}{8} + \sin^2 \frac{3\pi}{8} + \sin^2 \frac{5\pi}{8} + \sin^2 \frac{7\pi}{8} = 2.$

Sol: Procedure: $\frac{7\pi}{8} = \frac{4\pi-3\pi}{8} = \frac{\pi}{2} - \frac{3\pi}{8}$ and $\frac{5\pi}{8} = \frac{4\pi-\pi}{8} = \left(\frac{\pi}{2} - \frac{\pi}{8}\right)$

$\therefore \sin \frac{7\pi}{8} = \sin\left(\frac{\pi}{2} - \frac{3\pi}{8}\right) = \cos\left(\frac{3\pi}{8}\right)$

$\sin\left(\frac{5\pi}{8}\right) = \sin\left(\frac{4\pi-\pi}{8}\right) = \sin\left(\frac{\pi}{2} - \frac{\pi}{8}\right) = \cos \frac{\pi}{8}$

$\therefore \text{LHS} = \sin^2 \frac{\pi}{8} + \sin^2 \frac{3\pi}{8} + \sin^2 \frac{5\pi}{8} + \sin^2 \frac{7\pi}{8}$

$= \sin^2 \frac{\pi}{8} + \sin^2 \frac{3\pi}{8} + \cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8}$

$= (\sin^2 \frac{\pi}{8} + \cos^2 \frac{\pi}{8}) + (\sin^2 \frac{3\pi}{8} + \cos^2 \frac{3\pi}{8})$

$$16) P.T \cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} + \cos^2 \frac{7\pi}{8} + \cos^2 \frac{5\pi}{8} = 2.$$

Same as problem (15)

$$17) P.T [1 + \cot \alpha - \sec(\alpha + \pi/2)] [1 + \cot \alpha + \sec(\alpha + \pi/2)] = 2 \cot \alpha$$

Sol: Procedure -
Find $\sec(\alpha + \pi/2)$ and use $(a+b)(a-b) = a^2 - b^2$
and also $1 + \cot^2 \alpha = \csc^2 \alpha$.

$$\sec(\alpha + \pi/2) = \csc \alpha.$$

$$\begin{aligned} \therefore L.H.S: [1 + \cot \alpha - \csc \alpha] [1 + \cot \alpha + \csc \alpha] \\ = (1 + \cot \alpha)^2 - \csc^2 \alpha \\ = 1 + \cot^2 \alpha + 2 \cot \alpha - \csc^2 \alpha \\ = \cancel{\csc^2 \alpha} + 2 \cot \alpha - \cancel{\csc^2 \alpha} \\ = 2 \cot \alpha. \end{aligned}$$

$$18) P.T \cos(\frac{3\pi}{2} + \theta) \cos(2\pi + \theta) [\cot(\frac{3\pi}{2} - \theta) + \cot(2\pi + \theta)] = 1$$

Sol: Procedure: $\frac{3\pi}{2} = 270^\circ$, Find all given trigonometric values and substitute.

$$\begin{array}{l|l} \cos(270 + \theta) = -\sin \theta & \cot(270 - \theta) = \tan \theta \\ \cos(2\pi + \theta) = \cos \theta & \cot(2\pi + \theta) = \cot \theta \end{array}$$

$$\begin{aligned} L.H.S: \cos(\frac{3\pi}{2} + \theta) \cos(2\pi + \theta) [\cot(\frac{3\pi}{2} - \theta) + \cot(2\pi + \theta)] \\ = (-\sin \theta \cdot \cos \theta) (\tan \theta + \cot \theta) \\ = -\sin \theta \cdot \cos \theta \cdot \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right) \\ = -\cancel{\sin \theta} \cos \theta \left(\frac{\sin^2 \theta + \cos^2 \theta}{\cancel{\sin \theta} \cos \theta} \right) = 1. \end{aligned}$$

$$19) P.T \sec(\frac{3\pi}{2} - \theta) \sec(\theta - \frac{5\pi}{2}) + \tan(\frac{5\pi}{2} + \theta) \tan(\theta - \frac{3\pi}{2}) = -1.$$

Sol: Procedure: $\frac{3\pi}{2} = 270^\circ$, $\sec(-\theta) = \sec \theta$, $\tan(-\theta) = -\tan \theta$.
 $\frac{5\pi}{2} = 450 = 360 + 90$. Use these to find the values and find L.H.S.

$$\begin{array}{l|l} \sec(270 - \theta) = -\csc \theta & = \sec(450 - \theta) \\ & = \sec(360 + 90 - \theta) = \sec(90 - \theta) \end{array}$$

$$\begin{aligned} \text{and } \tan\left(\theta - \frac{3\pi}{2}\right) &= \tan\left(-\left(\frac{3\pi}{2} - \theta\right)\right) \\ &= -\tan\left(270 - \theta\right) \\ &= -\cot\theta. \end{aligned}$$

$$\begin{aligned} \therefore \text{LHS} &= (-\csc\theta)(\sec\theta) + (-\cot\theta)(-\cot\theta) \\ &= (\cot^2\theta - \csc^2\theta) = -1. \end{aligned}$$

$$20) \text{ Simplify } \frac{\cos(90+\theta) \sec(-\theta) \tan(180-\theta)}{\sec(360-\theta) \sin(180+\theta) \cot(90+\theta)} = 1$$

Already worked out same model.

$$21) \text{ Simplify } \frac{\tan(90-\theta) \sec(180-\theta) \sin(-\theta)}{\sin(180+\theta) \cot(360-\theta) \csc(90+\theta)} \quad \text{Ans: } 1$$

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Trigonometric identities (Sum and difference of two angles)

1) Find the values of $\cos 15^\circ$ 2) $\tan 165^\circ$.

$\times$ BP 1) $\cos 15^\circ = \cos (45 - 30) = \cos 45 \cos 30 + \sin 45 \sin 30$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

Also $\sin 75^\circ = \frac{\sqrt{3} + 1}{2\sqrt{2}}$

2) $\tan (165) = \tan (120 + 45)$

$$= \frac{\tan 120 + \tan 45}{1 - \tan 120 \tan 45}$$

$$= \frac{-\sqrt{3} + 1}{1 + \sqrt{3}} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$$

But $\tan 120$

$$= \tan (90 + 30)$$

$$= -\cot 30^\circ$$

$$= -\sqrt{3}$$

2) If $\sin x = \frac{4}{5}$ (I quadrant) and $\cos y = -\frac{12}{13}$ (second quadrant)

$\times$ BP

Find 1) $\sin (x - y)$ 2) $\cos (x - y)$

Sol: $\sin x = \frac{4}{5}$ $\cos x = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$ $\therefore x$ is in I quadrant

$\cos y = -\frac{12}{13}$ $\sin y = \sqrt{1 - \frac{144}{169}} = \frac{5}{13}$ $\therefore y$ is second quadrant

1) $\sin (x - y) = \sin x \cos y - \cos x \sin y$

$$= \frac{4}{5} \left(-\frac{12}{13}\right) - \frac{3}{5} \cdot \frac{5}{13} = \frac{-48 - 15}{65} = -\frac{63}{65}$$

2. $\cos (x - y) = \cos x \cos y + \sin x \sin y$

$$= \frac{3}{5} \left(-\frac{12}{13}\right) + \frac{4}{5} \cdot \frac{5}{13} = \frac{-36 + 20}{65} = -\frac{16}{65}$$

3) PT $\cos \left(\frac{3\pi}{4} + x\right) - \cos \left(\frac{3\pi}{4} - x\right) = -\sqrt{2} \sin x$

$\times$ BP

$$\underline{\underline{LHS}} \left(\cos \frac{3\pi}{4} \cos x - \sin \frac{3\pi}{4} \sin x \right) - \left(\cos \frac{3\pi}{4} \cos x + \sin \frac{3\pi}{4} \sin x \right)$$

$$= -2 \sin \frac{3\pi}{4} \sin x$$

$$\sin \frac{3\pi}{4} = \sin \left(\pi - \frac{\pi}{4}\right)$$

$$= -2 \cdot \frac{1}{\sqrt{2}} \sin x = -\sqrt{2} \sin x = \sin \frac{\pi}{4}$$

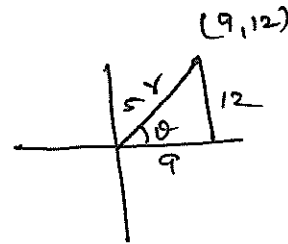
Note: $\cos (A + B) - \cos (A - B) = -2 \sin A \sin B$ 

- 4) Point A (9,12) rotates around the origin O in the plane through 60° in the anticlockwise direction to the new position B. Find the Co-ordinates of B.

Sol: Here $r \cos \theta = 9$, $r \sin \theta = 12$

$$r = \sqrt{81+144} = \sqrt{225} = 15$$

$$\therefore 15 \cos \theta = 9, 15 \sin \theta = 12$$



$$15 \cos(\theta + 60) = \cos \theta \cos 60 - \sin \theta \sin 60$$

$$= 15 \left(\frac{9}{15} \cdot \frac{1}{2} - \frac{12}{15} \cdot \frac{\sqrt{3}}{2} \right) = 15 \left(\frac{9 - 12\sqrt{3}}{30} \right)$$

$$\text{Hence } 15 \sin(\theta + 60) = \frac{3}{2} (4 + 3\sqrt{3})$$

$$\text{Hence } B = \left(\frac{3}{2} (3 - 4\sqrt{3}), \frac{3}{2} (4 + 3\sqrt{3}) \right)$$

- 5) Expand) $\sin(A+B+C)$, 2) $\cos(A+B+C)$

BP:

Sol: 1) $\sin(A+B+C) = \sin A \cos(B+C) + \cos A (\sin(B+C))$

$$= \sin A (\cos B \cos C - \sin B \sin C) + \cos A (\sin B \cos C + \cos B \sin C)$$

$$= \sin A \cos B \cos C - \sin A \sin B \sin C + \cos A \sin B \cos C + \cos A \cos B \sin C$$

2) $\tan(A+B+C) = \tan[A+(B+C)]$

$$= \frac{\tan A + \tan(B+C)}{1 - \tan A \tan(B+C)}$$

$$= \frac{\tan A + \tan B + \tan C}{1 - \tan B \tan C}$$

$$\tan(A+B+C) = \frac{\tan A + \tan B + \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan A \tan C}$$

$$= \frac{\tan A (1 - \tan B \tan C) + \tan B + \tan C}{1 - \tan B \tan C - \tan A \tan B - \tan A \tan C}$$

$$\tan(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan A \tan C}$$

— (X)

Note: If $A+B+C = 0$ or π

From (2). $0 = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$

$$\Rightarrow \tan A \tan B \tan C = \tan A + \tan B + \tan C.$$

If $A+B+C = \pi/2$ $1 = \tan A \tan B + \tan B \tan C + \tan C \tan A.$

From (3) $D = 0$

6) Expand $\cos(A+B+C)$ Hence p.T

KBP $\cos A \cos B \cos C = \sin A \sin B \cos C + \sin B \sin C \cos A + \sin C \sin A \cos B$
If $A+B+C = \pi/2$.

Sol: $\cos(A+B+C) = \cos[A+(B+C)]$
 $= \cos A \cos(B+C) - \sin A \sin(B+C)$
 $= \cos A [\cos B \cos C - \sin B \sin C] - \sin A [\sin B \cos C + \cos B \sin C]$
 $= \cos A \cos B \cos C - \cos A \sin B \sin C - \sin A \sin B \cos C - \sin A \cos B \sin C.$

If $A+B+C = \pi/2$ $\cos(\pi/2) = 0$

$\therefore \cos A \cos B \cos C = \sin A \sin B \cos C + \sin B \sin C \cos A + \sin A \sin C \cos B.$

7) If $A+B+C = \pi/2$ then $\tan A \tan B + \tan B \tan C + \tan C \tan A = 1.$

KBP Sol: $A+B+C = \pi/2$
 $A+B = \frac{\pi}{2} - C \Rightarrow \tan(A+B) = \tan(\frac{\pi}{2} - C)$
 $\frac{\tan A + \tan B}{1 - \tan A \tan B} = \cot C = \frac{1}{\tan C}.$

$$\tan A \tan C + \tan B \tan C = 1 - \tan A \tan B$$

$$\tan A \tan C + \tan B \tan C + \tan A \tan B = 1$$

8) P.T $\sin(45^\circ + \theta) - \sin(45^\circ - \theta) = \sqrt{2} \sin \theta.$

KBP $\sin(30^\circ + \theta) + \cos(60^\circ + \theta) = \cos \theta.$

1) $\sin(A+B) - \sin(A-B) = 2 \cos A \sin B.$

$$\therefore \sin(45^\circ + \theta) - \sin(45^\circ - \theta) = 2 \cos 45^\circ \sin \theta = 2 \cdot \frac{1}{\sqrt{2}} \sin \theta = \sqrt{2} \sin \theta.$$

2) $\sin(30^\circ + \theta) + \cos(60^\circ + \theta)$

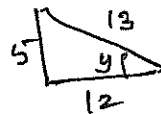
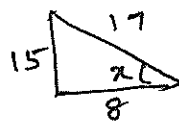
$$= \sin 30^\circ \cos \theta + \cos 30^\circ \sin \theta + \cos 60^\circ \cos \theta - \sin 60^\circ \sin \theta$$

$$= \frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta + \frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta = \cos \theta.$$

Q) If $\sin x = \frac{15}{17}$ $\cos y = \frac{12}{13}$ $0 < x < \frac{\pi}{2}$ and $0 < y < \frac{\pi}{2}$
 (BP) Find the value of 1) $\sin(x+y)$ 2) $\cos(x-y)$ 3) $\tan(x+y)$.

Sol: x and y are in I Quadrant.

$$\sin x = \frac{15}{17} \quad \cos x = \frac{8}{17} \quad \tan x = \frac{15}{8}$$



$$\sin y = \frac{5}{13} \quad \cos y = \frac{12}{13} \quad \tan y = \frac{5}{12}$$

$$1) \sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$= \frac{15}{17} \cdot \frac{12}{13} + \frac{8}{17} \cdot \frac{5}{13} = \frac{180 + 40}{221} = \frac{220}{221}$$

$$2) \cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$= \frac{8}{17} \cdot \frac{12}{13} - \frac{15}{17} \cdot \frac{5}{13} = \frac{96 - 75}{221} = \frac{21}{221}$$

Q) If $\sin A = \frac{3}{5}$ and $\cos B = \frac{9}{41}$ $0 < A < \frac{\pi}{2}$ $0 < B < \frac{\pi}{2}$
 (BP) Find 1) $\sin(A+B)$ 2) $\cos(A-B)$

Sol: A lies in I Quadrant.

$$\sin A = \frac{3}{5} \quad \cos A = \frac{4}{5} \quad \tan A = \frac{3}{4}$$



$$1) \sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$= \frac{3}{5} \cdot \frac{9}{41} + \frac{4}{5} \cdot \frac{40}{41}$$

$$= \frac{27 + 160}{205} = \frac{187}{205}$$

$$\sin B = \frac{9}{41}$$

$$\cos B = \frac{40}{41}$$

$$\tan B = \frac{9}{40}$$

$$2) \cos(A-B) = \cos A \cos B + \sin A \sin B$$

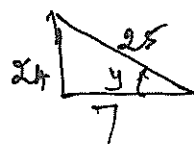
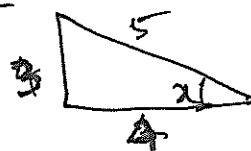
$$= \frac{4}{5} \cdot \frac{9}{41} + \frac{3}{5} \cdot \frac{40}{41} = \frac{36 + 120}{205} = \frac{156}{205}$$

Q) Find $\cos(x-y)$, given that $\cos x = -\frac{4}{5}$ $\pi < x < \frac{3\pi}{2}$, $\pi < y < \frac{3\pi}{2}$
 (B.P) $\sin y = -\frac{24}{25}$

x, y lies in III Quadrant.

$$\sin x = -\frac{3}{5} \quad \cos x = -\frac{4}{5}$$

$$\sin y = -\frac{24}{25} \quad \cos y = -\frac{7}{25}$$



$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$= \left(-\frac{4}{5}\right)\left(-\frac{7}{25}\right) + \left(-\frac{3}{5}\right)\left(-\frac{24}{25}\right) = \frac{28 + 72}{125} = \frac{100}{125} = \frac{4}{5}$$

12) Find $\sin(x-y)$ given that $\sin x = \frac{8}{17}$ with $0 < x < \frac{\pi}{2}$ and
 (BP) $\cos y = -\frac{24}{25}$ with $\pi < y < \frac{3\pi}{2}$

Sol: x lies in the I quadrant, y lies in the III quadrant.

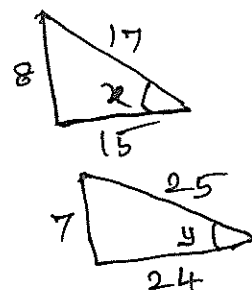
$$\sin x = \frac{8}{17}, \cos x = \frac{15}{17}$$

$$\cos y = -\frac{24}{25}, \sin y = -\frac{7}{25}$$

$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$

$$= \frac{8}{17} \left(-\frac{24}{25}\right) - \frac{15}{17} \left(-\frac{7}{25}\right)$$

$$= \frac{-192 + 105}{425} = \frac{-87}{425}$$



13) Find the value of i) $\cos 105^\circ$ 2) $\sin 105^\circ$ 3) $\tan \frac{7\pi}{12}$
 (BP)

Sol: 1) $\cos 105 = \cos(60 + 45)$

$$= \cos 60 \cos 45 - \sin 60 \sin 45$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} = \frac{1-\sqrt{3}}{2\sqrt{2}}$$

2) $\sin 105 = \sin(60 + 45)$

$$= \sin 60 \cos 45 + \cos 60 \sin 45$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{2} \cdot \frac{1}{\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

3) $\tan \frac{7\pi}{12} = \tan \frac{7 \times 15^\circ}{12} = \tan 105$

$$= \tan(60 + 45) = \frac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45}$$

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}} = \frac{(1+\sqrt{3})(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})}$$

$$= \frac{1+3+2\sqrt{3}}{1-3}$$

$$= \frac{2(2+\sqrt{3})}{-2}$$

$$= -(2+\sqrt{3})$$

12) P.T $\sin(n+1)\theta \sin(n-1)\theta + \cos(n+1)\theta \cos(n-1)\theta = \cos 2\theta \quad n \in \mathbb{Z}$.

B.P

Sol: Procedure Use $\cos A \cos B + \sin A \sin B = \cos(A-B)$

$$\begin{aligned} \text{LHS: } \cos(n+1)\theta \cos(n-1)\theta + \sin(n+1)\theta \sin(n-1)\theta \\ = \cos((n+1)\theta - (n-1)\theta) \\ = \cos 2\theta \end{aligned}$$

13) If $x \cos \theta = y \cos(\theta + \frac{2\pi}{3}) = z \cos(\theta + \frac{4\pi}{3})$ Find the value of $xy + yz + zx$

B.P

Sol: Procedure: Find the values of each separately and put them equal to k . Then find x, y, z . Then calculate $xy + yz + zx$.

$$x \cos \theta = y \cos(\theta + \frac{\pi}{3}) = z \cos(\theta + \frac{2\pi}{3})$$

$$x \cos \theta = y [\cos \theta \cos \frac{\pi}{3} + \sin \theta \sin \frac{\pi}{3}] = z [\cos \theta \cos \frac{2\pi}{3} - \sin \theta \sin \frac{2\pi}{3}]$$

$$x \cos \theta = y [\cos \theta \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \sin \theta] = z [\frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta] = K.$$

$$x = \frac{K}{\cos \theta}, \quad y = \frac{K}{\frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta}, \quad z = \frac{K}{\frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta}.$$

$$\therefore xy + yz + zx = \frac{K^2}{\frac{1}{2} \cos^2 \theta + \frac{\sqrt{3}}{2} \sin \theta \cos \theta} + \frac{K^2}{\frac{1}{4} \cos^2 \theta - \frac{3}{4} \sin^2 \theta} + \frac{K^2}{\frac{1}{2} \cos^2 \theta - \frac{\sqrt{3}}{2} \sin \theta \cos \theta}$$

$$= \frac{2K^2}{\frac{1}{4} \cos^4 \theta - \frac{3}{4} \sin^2 \theta \cos^2 \theta} +$$

$$x \cos \theta = y \cos(\frac{\pi}{3} - \theta) = z \cos(\frac{\pi}{3} + \theta) = K.$$

$$\frac{K^2}{\cos \theta (\cos \frac{\pi}{3} - \theta)} + \frac{K^2}{\cos(\frac{\pi}{3} - \theta) \cos(\frac{\pi}{3} + \theta)} + \frac{K^2}{-\cos \theta (\cos(\frac{\pi}{3} + \theta))}$$

$$= -K^2 \left[\frac{\cos(\frac{\pi}{3} + \theta) \cos \theta + \cos(\frac{\pi}{3} - \theta) \cos \theta}{-\cos \theta (\cos \frac{\pi}{3} - \theta) \cos(\frac{\pi}{3} + \theta)} \right]$$

$$= -K^2 (\cos \frac{\pi}{3} \cos \theta - \sin \frac{\pi}{3} \sin \theta) - K^2 \cos \theta + K^2 (\cos \frac{\pi}{3} \cos \theta + \sin \frac{\pi}{3} \sin \theta)$$

$$= +K^2 \frac{\sqrt{3}}{2} \sin \theta - K^2 \cos \theta + \frac{\sqrt{3}}{2} K^2 \sin \theta.$$

13 P.T $\sin(n+1)\theta \sin(n-1)\theta + \cos(n+1)\theta \cos(n-1)\theta = \cos 2\theta \quad n \in \mathbb{Z}$.

ABP

Sol: Procedure: Use $\cos A \cos B + \sin A \sin B = \cos(A-B)$

$$\begin{aligned} & \cos(n+1)\theta \cos(n-1)\theta + \sin(n+1)\theta \sin(n-1)\theta \\ &= \cos((n+1)-(n-1))\theta \\ &= \cos 2\theta. \end{aligned}$$

14) If $x \cos \theta = y \cos(\theta + \frac{2\pi}{3}) = z \cos(\theta + \frac{4\pi}{3})$. Find $xy + yz + zx$.

ABP

Sol: Procedure: Simplify each one and put they are equal to k . Then find x, y, z then sub. in $xy + yz + zx$.

$$\begin{aligned} y \cos(\pi - \frac{\pi}{3} + \theta) &= y \cos(\pi - (\frac{\pi}{3} - \theta)) = -y \cos(\frac{\pi}{3} - \theta) \\ &= -y [\cos \frac{\pi}{3} \cos \theta + \sin \frac{\pi}{3} \sin \theta] \\ &= -y [\frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta] \\ z \cos(\pi + \frac{\pi}{3} + \theta) &= -z \cos(\frac{\pi}{3} + \theta) \\ &= -z (\cos \frac{\pi}{3} \cos \theta - \sin \frac{\pi}{3} \sin \theta) \\ &= -z (\frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta) \end{aligned}$$

Now $x \cos \theta = -y [\frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta] = -z (\frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta) = k$.

$$x = \frac{k}{\cos \theta}, \quad y = \frac{k}{-(\frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta)}, \quad z = \frac{k}{\frac{\sqrt{3}}{2} \sin \theta - \frac{1}{2} \cos \theta}.$$

$$\begin{aligned} \therefore xy + yz + zx &= \frac{k^2}{-\cos \theta (\frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta)} + \frac{k^2}{-(\frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta) (\frac{\sqrt{3}}{2} \sin \theta - \frac{1}{2} \cos \theta)} \\ &\quad + \frac{k^2}{\cos \theta (\frac{\sqrt{3}}{2} \sin \theta - \frac{1}{2} \cos \theta)} \\ &= \frac{-k^2 (\frac{\sqrt{3}}{2} \sin \theta - \frac{1}{2} \cos \theta) - k^2 (\cos \theta) + k^2 (\frac{\sqrt{3}}{2} \sin \theta + \frac{1}{2} \cos \theta)}{\cos \theta (\frac{\sqrt{3}}{2} \sin \theta + \frac{1}{2} \cos \theta) (\frac{\sqrt{3}}{2} \sin \theta - \frac{1}{2} \cos \theta)} \\ &= \frac{-k^2 \frac{\sqrt{3}}{2} \sin \theta + \frac{k^2}{2} \cos \theta - k^2 \cos \theta + k^2 \frac{\sqrt{3}}{2} \sin \theta + \frac{k^2}{2} \cos \theta}{k^2 \cos \theta - k^2 \cos \theta} \\ &= 0. \end{aligned}$$

15) If $\cos(\alpha - \beta) + \cos(\beta - \gamma) + \cos(\gamma - \alpha) = -\frac{3}{2}$ P.T $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 0$ $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 0$

Sol: Expand LHS and crossmultiply with 2. Take -3 to LHS as $1+1+1$. Put $1 = \sin^2 \alpha + \cos^2 \alpha$, $1 = \sin^2 \beta + \cos^2 \beta$, $1 = \sin^2 \gamma + \cos^2 \gamma$. Simplify. use $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$$\text{LHS: } (\cos \alpha \cos \beta + \sin \alpha \sin \beta) + (\cos \beta \cos \gamma + \sin \beta \sin \gamma) + (\cos \gamma \cos \alpha + \sin \gamma \sin \alpha) = -\frac{3}{2}$$

$$2 \cos \alpha \cos \beta + 2 \sin \alpha \sin \beta + 2 \cos \beta \cos \gamma + 2 \sin \beta \sin \gamma + 2 \cos \gamma \cos \alpha + 2 \sin \gamma \sin \alpha = -3$$

$$\therefore 2 \cos \alpha \cos \beta + 2 \sin \alpha \sin \beta + 2 \cos \beta \cos \gamma + 2 \sin \beta \sin \gamma + 2 \cos \gamma \cos \alpha + 2 \sin \gamma \sin \alpha + 1 + 1 + 1 = 0$$

$$2 \cos \alpha \cos \beta + 2 \sin \alpha \sin \beta + 2 \cos \beta \cos \gamma + 2 \sin \beta \sin \gamma + 2 \cos \gamma \cos \alpha + 2 \sin \gamma \sin \alpha + \sin^2 \alpha + \cos^2 \alpha + \sin^2 \beta + \cos^2 \beta + \sin^2 \gamma + \cos^2 \gamma = 0$$

$$\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma + 2 \sin \alpha \sin \beta + 2 \sin \beta \sin \gamma + 2 \sin \gamma \sin \alpha + \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + 2 \cos \alpha \cos \beta + 2 \cos \beta \cos \gamma + 2 \cos \gamma \cos \alpha = 0$$

$$(\sin \alpha + \sin \beta + \sin \gamma)^2 + (\cos \alpha + \cos \beta + \cos \gamma)^2 = 0$$

$$\Rightarrow \sin \alpha + \sin \beta + \sin \gamma = 0$$

$$\cos \alpha + \cos \beta + \cos \gamma = 0$$

16) Find the ^{quadratic} equation whose roots are $\sin 15^\circ$ and $\cos 15^\circ$.
Sol: Procedure: write down the values of $\sin 15^\circ$ and $\cos 15^\circ$. Find the sum of the roots and product of the roots. Then form the equation $x^2 - (SR)x + PR = 0$.

$$\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}, \quad \cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$SR = \frac{\sqrt{3}-1}{2\sqrt{2}} + \frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{2\sqrt{3}}{2\sqrt{2}}$$

$$PR = \left(\frac{\sqrt{3}-1}{2\sqrt{2}} \right) \left(\frac{\sqrt{3}+1}{2\sqrt{2}} \right) = \frac{3-1}{8} = \frac{2}{8} = \frac{1}{4}$$

$$\text{Rem } x^2 - \frac{\sqrt{3}}{\sqrt{2}}x + \frac{1}{4} = 0$$

$$x^2 - \frac{\sqrt{3}\sqrt{2}}{2}x + \frac{1}{4} = 0 \Rightarrow 4x^2 - 2\sqrt{6}x + 1 = 0$$

*7) P.T. $\sin(45+\theta) - \sin(45-\theta) = \sqrt{2} \sin \theta$

TBP. 2. $\sin(30+\theta) + \cos(60+\theta) = \cos \theta$.

Sol: Procedure: Use $\sin(A+B)$, $\sin(A-B)$, $\cos(A+B)$.

1) LHS: $\sin(45+\theta) - \sin(45-\theta)$

$$= (\sin 45 \cos \theta + \cos 45 \sin \theta) - (\sin 45 \cos \theta - \cos 45 \sin \theta)$$

$$= 2 \cos 45 \sin \theta$$

$$= 2 \cdot \frac{1}{\sqrt{2}} \sin \theta = \sqrt{2} \sin \theta.$$

2) LHS $\sin(30+\theta) + \cos(60+\theta)$

$$= (\sin 30 \cos \theta + \cos 30 \sin \theta) + (\cos 60 \cos \theta - \sin 60 \sin \theta)$$

$$= \frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta + \frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta$$

$$= \cos \theta.$$

18) P.T 1) $\cos(30^\circ + x) = \frac{\sqrt{3} \cos x - \sin x}{2}$

TBP.

2) $\cos(\pi + \theta) = -\cos \theta$

3) $\sin(\pi + \theta) = -\sin \theta$.

Sol: 1) LHS: $\cos(30+x) = \cos 30 \cos x - \sin 30 \sin x$,

$$= \frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x.$$

$$= \frac{\sqrt{3} \cos x - \sin x}{2}.$$

2. $\cos(\pi + \theta) = \cos \pi \cos \theta - \sin \pi \sin \theta$

$$= -1 \cdot \cos \theta - 0$$

$$= -\cos \theta.$$

3) $\sin(\pi + \theta) = \sin \pi \cos \theta + \cos \pi \sin \theta$

$$= 0 - 1 \cdot \sin \theta$$

$$= -\sin \theta.$$

19) RT $\sin 105 + \cos 105 = \cos 45^\circ$.

TBP.

Sol: Procedure write 105 as 60+45. and Expand.

LHS: $\sin(60+45) + \cos(60+45)$

$$(\sin 60 \cos 45 + \cos 60 \sin 45) + (\cos 60 \cos 45 - \sin 60 \sin 45)$$

$$\frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$= \cos 45^\circ.$$

20) P.T $\sin 75^\circ - \sin 15^\circ = \cos 105^\circ + \cos 15^\circ$

Sol: Use $\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$

Procedure $\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$

LHS $\sin 75^\circ - \sin 15^\circ = 2 \cos \frac{90^\circ}{2} \sin \frac{60^\circ}{2}$

$= 2 \cos 45^\circ \sin 30^\circ$

$= 2 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{1}{\sqrt{2}}$

RHS $\cos 105^\circ + \cos 15^\circ = 2 \cos \frac{120^\circ}{2} \cos \frac{90^\circ}{2}$

$= 2 \cos 60^\circ \cos 45^\circ$

$\therefore \text{LHS} = \text{RHS}$

$= 2 \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

21) P.T $\tan 75^\circ + \cot 75^\circ = 4$

K.B.P. Sol: Procedure: Use $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\frac{(a+b)^2 + (a-b)^2}{2(a^2+b^2)}$ $\cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$

LHS $\tan 75^\circ + \cot 75^\circ = \tan(45^\circ + 30^\circ) + \cot(45^\circ + 30^\circ)$
 $= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} + \frac{\cot 45^\circ \cot 30^\circ - 1}{\cot 30^\circ + \cot 45^\circ}$

$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} + \frac{1 \cdot \sqrt{3} - 1}{\sqrt{3} + 1}$

$= \frac{\sqrt{3} + 1}{\sqrt{3} - 1} + \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{(\sqrt{3} + 1)^2 + (\sqrt{3} - 1)^2}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$

$= \frac{2(3+1)}{3-1} = 4$

22) 1) S.T $\tan(45^\circ + A) = \frac{1 + \tan A}{1 - \tan A}$ 2) $\tan(45^\circ - A) = \frac{1 - \tan A}{1 + \tan A}$

K.B.P.

1) $\tan(45^\circ + A) = \frac{\tan 45^\circ + \tan A}{1 - \tan 45^\circ \tan A}$
 $= \frac{1 + \tan A}{1 - \tan A}$

2) $\tan(45^\circ - A) = \frac{\tan 45^\circ - \tan A}{1 + \tan 45^\circ \tan A}$
 $= \frac{1 - \tan A}{1 + \tan A}$

23) P.T $\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$
 KBP

Sol: $\cot(A+B) = \frac{1}{\tan(A+B)} = \frac{1}{\frac{\tan A + \tan B}{1 - \tan A \tan B}}$
 $= \frac{1 - \tan A \tan B}{\tan A + \tan B}$
 $\therefore$ both Nr and Dr by $\tan A \tan B$
 $= \frac{1}{\tan A \tan B} - 1$
 $= \frac{\frac{\tan A}{\tan A \cdot \tan B} + \frac{\tan B}{\tan A \tan B}}{\frac{\tan A}{\tan A \cdot \tan B} + \frac{\tan B}{\tan A \tan B}}$
 $= \frac{\cot A \cot B - 1}{\cot B + \cot A}$

24) If $\tan x = \frac{n}{n+1}$ and $\tan y = \frac{1}{2n+1}$ find $\tan(x+y)$
 KBP

Procedure: Use $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ and Sub. values.

Sol: $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{\frac{n}{n+1} + \frac{1}{2n+1}}{1 - \left(\frac{n}{n+1}\right)\left(\frac{1}{2n+1}\right)}$
 $= \frac{n(2n+1) + (n+1)}{(n+1)(2n+1) - n}$
 $= \frac{2n^2 + n + n + 1}{2n^2 + 3n + 1 - n} = \frac{2n^2 + 2n + 1}{2n^2 + 2n + 1} = 1$
 $x+y = \tan^{-1}(1) = \frac{\pi}{4}$

25) P.T $\tan\left(\frac{\pi}{4} + \theta\right) \tan\left(\frac{3\pi}{4} + \theta\right) = -1$.

Sol: write $\frac{3\pi}{4}$ as $\pi - \frac{\pi}{4}$ and $\tan(\pi - \theta) = -\tan \theta$.

$\tan\left(\frac{3\pi}{4} + \theta\right) = \tan\left(\pi - \left(\frac{\pi}{4} - \theta\right)\right) = -\tan\left(\frac{\pi}{4} - \theta\right)$
 $\therefore \left[\tan\left(\frac{\pi}{4} + \theta\right)\right] \left[-\tan\left(\frac{\pi}{4} - \theta\right)\right] = -\frac{2 \tan 45^\circ}{1 - \tan^2 45^\circ \tan^2 \theta} = \frac{2}{1 - \tan^2 \theta}$
 $= \left(\frac{\tan \frac{\pi}{4} + \tan \theta}{1 - \tan \frac{\pi}{4} \tan \theta}\right) \left(\frac{\tan \frac{\pi}{4} - \tan \theta}{1 + \tan \frac{\pi}{4} \tan \theta}\right) =$

$$= - \left(\frac{1+\tan \theta}{1-\tan \theta} \right) \left(\frac{1-\tan \theta}{1+\tan \theta} \right)$$

$$= -1$$

26 If $\theta + \phi = \alpha$ and $\tan \theta = k \tan \phi$ P.T $\sin(\theta - \phi) = \frac{k-1}{k+1} \sin \alpha$.
 (BP)

Sol: by using componendo and dividendo method.

Given $\tan \theta = k \tan \phi$

$$\frac{\tan \theta}{\tan \phi} = \frac{k}{1} \Rightarrow \frac{k+1}{k-1} = \frac{\tan \theta + \tan \phi}{\tan \theta - \tan \phi}$$

$$\therefore \frac{k+1}{k-1} = \frac{\sin(\theta + \phi)}{\sin(\theta - \phi)}$$

$$\therefore \frac{\sin(\theta - \phi)}{\sin(\theta + \phi)} = \frac{k-1}{k+1}$$

$$\frac{\sin(\theta - \phi)}{\sin \alpha} = \frac{k-1}{k+1}$$

$$\sin(\theta - \phi) = \frac{k-1}{k+1} \sin \alpha$$

$$= \frac{\sin \theta}{\cos \theta} + \frac{\sin \phi}{\cos \phi}$$

$$\frac{\sin \theta}{\cos \theta} - \frac{\sin \phi}{\cos \phi}$$

$$= \frac{\sin \theta \cos \phi + \cos \theta \sin \phi}{\cos \theta \cos \phi}$$

$$\frac{\sin \theta \cos \phi - \cos \theta \sin \phi}{\cos \theta \cos \phi}$$

27) Find the value of $\tan(\alpha + \beta)$ given that $\cot \alpha = \frac{1}{2}$ $\alpha \in (\pi, \frac{3\pi}{2})$
 (BP) and $\sec \beta = -\frac{5}{3}$ $\beta \in (\frac{\pi}{2}, \pi)$

Sol: Procedure: Identify α, β belongs to which quadrant.

$$\alpha \in (\pi, \frac{3\pi}{2}) = \text{III} \quad \beta \in (\frac{\pi}{2}, \pi) = \text{II}$$

$$\cot \alpha = \frac{1}{2} \Rightarrow \tan \alpha = 2 \quad \because \alpha \in \text{III} \text{ it is +ve}$$

$$\tan^2 \beta = \sec^2 - 1 = \frac{25}{9} - 1 = \frac{16}{9}$$

$$\tan \beta = -\frac{4}{3} \quad \because \beta \text{ lies in II quadrant.}$$

$$\therefore \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{\frac{1}{2} - \frac{4}{3}}{1 - \frac{1}{2}(-\frac{4}{3})} = \frac{\frac{3-8}{6}}{\frac{6+4}{6}} = -\frac{5}{10} = -\frac{1}{2}$$

28) P.T 1. $\sin(A+B) \sin(A-B) = \sin^2 A - \sin^2 B$
 (BP) 2. $\cos(A+B) \cos(A-B) = \cos^2 A - \sin^2 B$

Procedure: Apply $\sin(A+B)$, $\sin(A-B)$, $\cos(A+B)$ $\cos(A-B)$.
 and For ① change all into sin For ② change angle A to cos and B to sin.

LHS
Sol: 1) $\sin(A+B) \cdot \sin(A-B)$

$$= (\sin A \cos B + \cos A \sin B) (\sin A \cos B - \cos A \sin B)$$

$$= \sin^2 A \cos^2 B - \cos^2 A \sin^2 B$$

$$= \sin^2 A (1 - \sin^2 B) - (1 - \sin^2 A) \sin^2 B$$

$$= \sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B$$

$$= \sin^2 A - \sin^2 B$$

2) $\cos(A+B) \cdot \cos(A-B)$

$$= (\cos A \cos B - \sin A \sin B) (\cos A \cos B + \sin A \sin B)$$

$$= \cos^2 A \cos^2 B - \sin^2 A \sin^2 B$$

$$= \cos^2 A (1 - \sin^2 B) - (1 - \cos^2 A) \sin^2 B$$

$$= \cos^2 A - \cos^2 A \sin^2 B - \sin^2 B + \cos^2 A \sin^2 B$$

$$= \cos^2 A - \sin^2 B$$

29) 1) $\sin^2(A+B) - \sin^2(A-B) = \sin 2A \sin 2B$

ABP 2) $\cos 80^\circ \cos 20^\circ = \cos^2 50^\circ - \sin^2 30^\circ$

Sol: Procedure: Use $\sin(A+B) \cdot \sin(A-B) = \sin^2 A - \sin^2 B$
 $\cos(A+B) \cdot \cos(A-B) = \cos^2 A - \sin^2 B$

1) LHS: $\sin(A+B) \cdot \sin(A-B) = \sin(A+B+A-B) \cdot \sin(A+B-A-B)$
 $\quad \quad \quad A \quad \quad \quad B \quad \quad \quad = \sin 2A \cdot \sin 2B$

2) RHS: $\cos^2 50^\circ - \sin^2 30^\circ = \cos(80^\circ + 30^\circ) \cdot \cos(80^\circ - 30^\circ)$
 $\quad \quad \quad = \cos 110^\circ \cdot \cos 50^\circ$

30) If $a \cos(x+y) = b(\cos(x-y))$ then P.T. $(a+b) \tan x = (a-b) \tan y$.

ABP: by componendo dividendo method.

Sol:

$$a \cos(x+y) = b \cos(x-y)$$

$$\frac{a}{b} = \frac{\cos(x-y)}{\cos(x+y)}$$

Now $\frac{a+b}{a-b} = \frac{\cos(x-y) + \cos(x+y)}{\cos(x-y) - \cos(x+y)}$

$$= \frac{\cos x \cos y + \sin x \sin y + \cos x \cos y - \sin x \sin y}{\cos x \cos y + \sin x \sin y - \cos x \cos y + \sin x \sin y}$$

$$\frac{a+b}{a-b} = \frac{2 \cos x \cos y}{2 \sin x \sin y} = \cot x \cot y$$

$$\frac{a+b}{a-b} = \frac{\cot y}{\tan x}$$

$$(a+b) \tan x = (a-b) \cot y.$$

$$31) \text{ P.T } \sin(n+1)x \sin(n+2)x + \cos(n+1)x \cos(n+2)x = \cos x.$$

Sol: Procedure: Use $\cos A \cos B + \sin A \sin B = \cos(A-B)$

$$\begin{aligned} \text{LHS: } & \cos(n+1)x \cos(n+2)x + \sin(n+1)x \sin(n+2)x \\ &= \cos[(n+1)-(n+2)]x \\ &= \cos(-x) = \cos x. \end{aligned}$$

$$32) \text{ S.T } \cos\left(\frac{\pi}{4}-\theta\right) \cos\left(\frac{\pi}{4}-\phi\right) - \sin\left(\frac{\pi}{4}-\theta\right) \sin\left(\frac{\pi}{4}-\phi\right) = \sin(\theta+\phi)$$

Sol: LHS: Procedure: Use $\cos A \cos B - \sin A \sin B = \cos(A+B)$

$$\begin{aligned} & \cos\left(\frac{\pi}{4}-\theta\right) \cos\left(\frac{\pi}{4}-\phi\right) - \sin\left(\frac{\pi}{4}-\theta\right) \sin\left(\frac{\pi}{4}-\phi\right) \\ &= \cos\left(\frac{\pi}{4}-\theta + \frac{\pi}{4}-\phi\right) \\ &= \cos(90 - (\theta+\phi)) \\ &= \cos(\theta+\phi) \end{aligned}$$

$$33) \text{ If } \cos(\theta+\phi) = m \cos(\theta-\phi) \text{ then find the value of } \frac{1-m}{1+m} \cot \phi.$$

$$\frac{1}{m} = \frac{\cos(\theta-\phi)}{\cos(\theta+\phi)}$$

$$\begin{aligned} \cos(A-B) - \cos(A+B) &= 2 \sin A \sin B \\ \cos(A-B) + \cos(A+B) &= 2 \cos A \cos B \end{aligned}$$

$$\begin{aligned} \text{Now } \frac{1-m}{1+m} &= \frac{\cos(\theta-\phi) - \cos(\theta+\phi)}{\cos(\theta-\phi) + \cos(\theta+\phi)} \\ &= \frac{2 \sin \theta \sin \phi}{2 \cos \theta \cos \phi} \end{aligned}$$

$$\begin{aligned} \frac{1-m}{1+m} &= \tan \theta \tan \phi. \\ &= \frac{\tan \theta}{\cot \phi} \end{aligned}$$

$$\left(\frac{1-m}{1+m}\right) \cot \phi = \tan \theta.$$

$$34) \text{ S.T } \sin(40^\circ+\theta) \cos(10^\circ+\theta) - \cos(40^\circ+\theta) \sin(10^\circ+\theta) = \frac{1}{2}$$

Procedure: Use $\sin A \cos B - \cos A \sin B = \sin(A-B)$

$$\begin{aligned} \text{LHS: } & \sin(40^\circ+\theta) \cos(10^\circ+\theta) - \cos(40^\circ+\theta) \sin(10^\circ+\theta) \\ &= \sin(40^\circ+\theta - 10^\circ-\theta) = \sin 30^\circ = \frac{1}{2} \end{aligned}$$

35) If $\cos(\alpha + \beta) = \frac{4}{5}$, $\sin(\alpha - \beta) = \frac{5}{13}$ and $0 < \alpha, \beta < \frac{\pi}{4}$, then
P.T $\tan(2\alpha) = \frac{56}{33}$.

Sol: Procedure: Find $\sin(\alpha + \beta)$ and $\cos(\alpha + \beta)$ then $\tan(\alpha + \beta)$ and $\tan(\alpha - \beta)$ from the given values.
 $\tan 2\alpha = \tan(\alpha + \beta + \alpha - \beta) = \tan(A+B)$ type.

Since $0 < \alpha < \frac{\pi}{4} \Rightarrow \frac{\pi}{4} - \beta < \frac{\pi}{4}$ and $0 < \alpha + \beta < \frac{\pi}{2}$.

$$\cos(\alpha + \beta) = \sqrt{1 - \sin^2(\alpha + \beta)} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5} = \sqrt{1 - \frac{25}{169} - \frac{12}{13}}$$

$$\sin(\alpha + \beta) = \sqrt{1 - \cos^2(\alpha + \beta)} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5} = \sqrt{1 - \frac{16}{25} - \frac{12}{13}}$$

$$\tan(\alpha + \beta) = \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$$

$$\tan(\alpha - \beta) = \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)} = \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{12}$$

$$\begin{aligned} \tan 2\alpha &= \tan(\alpha + \beta + \alpha - \beta) = \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta) \cdot \tan(\alpha - \beta)} \\ &= \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \cdot \frac{5}{12}} = \frac{\frac{36 + 20}{48}}{\frac{48 - 20}{48}} = \frac{56}{33} \end{aligned}$$

36) If $\tan(\pi \cos \theta) = \cot(\pi \sin \theta)$ then P.T $\cos(\theta - \frac{\pi}{4}) = \pm \frac{1}{2\sqrt{2}}$

Sol: Procedure: Change LHS and RHS in terms of $\sin$ and $\cos$. then cross multiply. Use $\cos A \cos B - \sin A \sin B = \cos(A+B)$ and multiply on both sides by $\frac{1}{\sqrt{2}}$ (ie) $\cos \frac{\pi}{4}$ and $\sin \frac{\pi}{4}$.

$$\text{Given } \tan(\pi \cos \theta) = \cot(\pi \sin \theta)$$

$$\frac{\sin(\pi \cos \theta)}{\cos(\pi \cos \theta)} = \frac{\cos(\pi \sin \theta)}{\sin(\pi \sin \theta)}$$

$$\sin(\pi \cos \theta) \sin(\pi \sin \theta) = \cos(\pi \sin \theta) \cos(\pi \cos \theta)$$

$$\cos(\pi \sin \theta) \cdot \cos(\pi \cos \theta) - \sin(\pi \sin \theta) \sin(\pi \cos \theta) = 0$$

$$\cos(\pi \sin \theta + \pi \cos \theta) = 0$$

$$\pi \sin \theta + \pi \cos \theta = \cos^{-1}(0) = \pm \frac{\pi}{2}$$

$$\pi(\sin \theta + \cos \theta) = \pm \frac{\pi}{2}$$

36) If $\tan \alpha = \frac{1}{\sqrt{x(x^2+x+1)}}$, $\tan \beta = \frac{\sqrt{x}}{\sqrt{x^2+x+1}}$, $\tan \gamma = \sqrt{\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}}$

Then P.T $\alpha + \beta = \gamma$.

Sol: Procedure: Find $\tan(\alpha + \beta)$ & it is equal to $\tan \gamma$.

$$\begin{aligned} \tan(\alpha + \beta) &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{\frac{1}{\sqrt{x(x^2+x+1)}} + \frac{\sqrt{x}}{\sqrt{x^2+x+1}}}{1 - \left(\frac{1}{\sqrt{x(x^2+x+1)}}\right)\left(\frac{\sqrt{x}}{\sqrt{x^2+x+1}}\right)} \\ &= \frac{(1+x)(x^2+x+1)}{\sqrt{x}(x(x^2+x+1))\sqrt{x^2+x+1}} \\ &= \frac{\sqrt{x}\sqrt{x^2+x+1}}{\sqrt{x} \cdot \sqrt{x} \cdot x} \\ &= \frac{\sqrt{x^3+x^2+x}}{x^2} \\ &= \sqrt{\frac{x^3+x^2+x}{x^4}} = \sqrt{\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}} \\ &= \tan \gamma. \end{aligned}$$

37) P.T $\frac{\tan(\frac{\pi}{4} + x)}{\tan(\frac{\pi}{4} - x)} = \frac{(1 + \tan x)^2}{(1 - \tan x)^2}$ Procedure: Just expand LHS and simplify.

$$\begin{aligned} \text{Sol: } \frac{\tan(\frac{\pi}{4} + x)}{\tan(\frac{\pi}{4} - x)} &= \frac{\tan \frac{\pi}{4} + \tan x}{1 - \tan \frac{\pi}{4} \tan x} \\ &= \frac{\tan \frac{\pi}{4} - \tan x}{1 + \tan \frac{\pi}{4} \tan x} \\ &= \frac{1 + \tan x}{1 - \tan x} = \frac{(1 + \tan x)^2}{(1 - \tan x)^2} \\ &= \frac{1 - \tan x}{1 + \tan x} \end{aligned}$$

38) If $A + B + C = \pi$ or P.T $\tan A + \tan B + \tan C$.

BP. If $A + B + C = \pi$
 $A + B = \pi - C$
 $\tan(A + B) = \tan(\pi - C)$
 $\tan A + \tan B = -\tan C$.

$\tan A + \tan B = -\tan C + \tan A \tan B \tan C$
 $\tan A + \tan B + \tan C = \tan A \tan B \tan C$
 This result is also true in the case of oblique triangles.

39) P.T) $\tan\left(\frac{\pi}{4} + \theta\right) \tan\left(\frac{\pi}{4} - \theta\right) = 1$

2) $\tan\left(\frac{\pi}{4} + \theta\right) \tan\left(\frac{3\pi}{4} + \theta\right) = -1$

3) $\cot\left(\frac{\pi}{4} + \theta\right) \cot\left(\frac{\pi}{4} - \theta\right) = 1$

Sol: Procedure ^{use} $\tan\left(\frac{\pi}{4} + \theta\right) = \frac{1 + \tan\theta}{1 - \tan\theta}$
 and $\tan\left(\frac{\pi}{4} - \theta\right) = \frac{1 - \tan\theta}{1 + \tan\theta}$.

1) $\tan\left(\frac{\pi}{4} + \theta\right) \tan\left(\frac{\pi}{4} - \theta\right) = \frac{1 + \tan\theta}{1 - \tan\theta} \times \frac{1 - \tan\theta}{1 + \tan\theta} = 1$

2) $\tan\left(\frac{3\pi}{4} + \theta\right) = \tan\left(\pi - \left(\frac{\pi}{4} - \theta\right)\right) = -\tan\left(\frac{\pi}{4} - \theta\right)$

LHS: $\tan\left(\frac{\pi}{4} + \theta\right) \left(-\tan\left(\frac{\pi}{4} - \theta\right)\right) = \left(\frac{1 + \tan\theta}{1 - \tan\theta}\right) \left(-\frac{1 - \tan\theta}{1 + \tan\theta}\right)$

$= \underline{\underline{-1}}$.

4) $\cot\left(\frac{\pi}{4} + \theta\right) \cdot \cot\left(\frac{\pi}{4} - \theta\right) = \frac{1}{\tan\left(\frac{\pi}{4} + \theta\right) \tan\left(\frac{\pi}{4} - \theta\right)}$

$= \frac{1}{\frac{1 + \tan\theta}{1 - \tan\theta} \times \frac{1 - \tan\theta}{1 + \tan\theta}}$

$= \frac{1}{1} = 1$.

40) If $A + B = 45^\circ$ P.T) $(1 + \tan A)(1 + \tan B) = 2$

2) $(\cot A - 1)(\cot B - 1) = 2$.

Sol: Procedure: ^{use} $\tan\left(\frac{\pi}{4} - \theta\right) = \frac{1 - \tan\theta}{1 + \tan\theta}$

If $A + B = 45^\circ$

$B = 45^\circ - A$

$\tan B = \tan\left(\frac{\pi}{4} - A\right) = \frac{1 - \tan A}{1 + \tan A}$.

∴ 1) LHS $(1 + \tan A)(1 + \tan B) = (1 + \tan A) \left(1 + \frac{1 - \tan A}{1 + \tan A}\right)$
 $= (1 + \tan A) \left(\frac{1 + \tan A + 1 - \tan A}{1 + \tan A}\right)$
 $= \frac{2(1 + \tan A)}{1 + \tan A} = 2$

2) $(\cot A - 1)(\cot B - 1)$
 $= \left(\frac{1}{\tan A} - 1\right) \left(\frac{1}{\tan B} - 1\right) = \frac{(1 - \tan A)(1 - \tan B)}{\tan A \tan B}$

$$\begin{aligned}
 & \frac{(1 - \tan A) \left(1 - \frac{1 - \tan A}{1 + \tan A}\right)}{\tan A \cdot \frac{1 - \tan A}{1 + \tan A}} \\
 & = \frac{(1 - \tan A) \left(\frac{1 + \tan A - 1 + \tan A}{1 + \tan A}\right)}{\frac{\tan A (1 - \tan A)}{1 + \tan A}} = \frac{(1 - \tan A) \cdot 2 \tan A}{\tan A (1 - \tan A)} \\
 & = 2.
 \end{aligned}$$

41) If $A + B = 225^\circ$ P.T $\frac{\cot A}{1 + \cot A} \cdot \frac{\cot B}{1 + \cot B} = \frac{1}{2}$.

Sol: Procedure: $A + B = 225^\circ$
 $B = 225^\circ - A = (180^\circ + 45^\circ - A)$

$$\tan B = \tan(180^\circ + 45^\circ - A) = \tan(45^\circ - A)$$

$$= \frac{1 - \tan A}{1 + \tan A}$$

LHS: $\frac{1}{1 + \frac{1}{\tan A}} \cdot \frac{1}{1 + \frac{1}{\tan B}} = \frac{1 \times \tan A}{\tan A (1 + \tan A)} \cdot \frac{\tan B}{\tan B (1 + \tan B)}$

$$= \frac{1}{1 + \tan A} \times \frac{1}{1 + \tan B}$$

$$= \frac{1}{1 + \tan A} \times \frac{1}{1 + \frac{1 - \tan A}{1 + \tan A}}$$

$$= \frac{1}{1 + \tan A} \times \frac{1 + \tan A}{1 + \tan A + 1 - \tan A}$$

$$= \frac{1}{2}$$

42) P.T $\tan(A-B) + \tan(B-C) + \tan(C-A) = \tan(A-B) \tan(B-C) \tan(C-A)$

Sol: Procedure: Consider $\alpha = A - B$, $\beta = B - C$, $\gamma = C - A$.
 and apply $\tan(A+B)$ formula.

$$\alpha + \beta + \gamma = A - B + B - C + C - A = 0$$

$$\alpha + \beta = -\gamma$$

$$\tan(\alpha + \beta) = \tan(-\gamma)$$

$$\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = -\tan \gamma$$

$$1 - \tan \alpha \tan \beta$$

$$\Rightarrow \tan \alpha + \tan \beta = -\tan \gamma (1 - \tan \alpha \tan \beta)$$

$$\tan \alpha + \tan \beta = -\tan \gamma + \tan \alpha \tan \beta \tan \gamma$$

$$\tan \alpha + \tan \beta + \tan \gamma = \tan \alpha \tan \beta \tan \gamma$$

($\therefore \tan(A-B) + \tan(B-C) + \tan(C-A) = \tan(A-B) \tan(B-C) \tan(C-A)$)

Multiple angle identities and Sub Multiple angle identities

1) Find the value of $\sin 22\frac{1}{2}^\circ$

ABP Sol: Let $\theta = 45^\circ$

$$\cos \theta = 1 - 2\sin^2 \frac{\theta}{2} \Rightarrow 2\sin^2 \frac{\theta}{2} = 1 - \cos \theta$$

$$\sin^2 \frac{\theta}{2} = \frac{1 - \cos \theta}{2}$$

$$\therefore \sin 22\frac{1}{2}^\circ = \pm \sqrt{\frac{1 - \cos 45^\circ}{2}}$$

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$= \pm \sqrt{\frac{1 - \frac{1}{\sqrt{2}}}{2}} = \pm \sqrt{\frac{\sqrt{2} - 1}{2}} = \frac{\sqrt{1 - \sqrt{2}}}{2} \quad \because 22\frac{1}{2}^\circ \text{ is acute.}$$

2) Find the value of $\sin 2\theta$ when $\sin \theta = \frac{12}{13}$, θ lies in I Quadrant

ABP

Sol: $\cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - \frac{144}{169}} = \frac{5}{13}$, $\because \theta$ lies in I Quadrant

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cdot \frac{12}{13} \cdot \frac{5}{13} = \frac{120}{169}$$

3) P.T $\sin 4A = 4 \sin A \cos^3 A - 4 \cos A \sin^3 A$

ABP Sol: R.H.S: $4 \sin A \cos^3 A - 4 \cos A \sin^3 A$

$$= 4 \sin A \cos A (\cos^2 A - \sin^2 A)$$

$$= 2 \cdot 2 \sin A \cos A \cdot \cos 2A$$

$$= 2 \sin 2A \cdot \cos 2A$$

$$= \sin 4A$$

4) P.T $\sin x = 2^{10} \sin \left(\frac{x}{2^{10}}\right) \cos \left(\frac{x}{2}\right) \cos \left(\frac{x}{2^2}\right) \dots \cos \left(\frac{x}{2^{10}}\right)$

ABP

Sol: L.H.S: $\sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2}$

$$= 2 \cdot 2 \sin \frac{x}{4} \cos \frac{x}{4} \cdot \cos \frac{x}{2}$$

$$= 4 \cdot 2 \cdot \sin \frac{x}{8} \cos \frac{x}{8} \cdot \cos \frac{x}{4} \cdot \cos \frac{x}{2}$$

$$= 2^4 \sin \frac{x}{16} \cos \frac{x}{16} \cdot \cos \frac{x}{8} \cdot \cos \frac{x}{4} \cdot \cos \frac{x}{2}$$

we can proceed the same way

$$\sin x = 2^{10} \left(\sin \frac{x}{2^{10}} \right) \cdot \cos \frac{x}{2^{10}} \cdot \cos \frac{x}{2^9} \dots \cos \frac{x}{2^3} \cdot \cos \frac{x}{2^2} \cdot \cos \frac{x}{2}$$

5) P.T $\frac{\sin \theta + \sin 2\theta}{1 + \cos \theta + \cos 2\theta} = \tan \theta$

ABP

Sol: L.H.S: $\frac{\sin \theta + \sin 2\theta}{1 + \cos \theta + \cos 2\theta} = \frac{\sin \theta + 2 \sin \theta \cos \theta}{\cos \theta + 1 + \cos 2\theta} = \frac{\sin \theta (1 + 2 \cos \theta)}{\cos \theta + 1 + \cos 2\theta} = \frac{\sin \theta (1 + 2 \cos \theta)}{\cos \theta (1 + 2 \cos \theta)}$

$$= \tan \theta$$

6) P.T / ABP $1 - \frac{1}{2} \sin^2 x = \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x}$

Sol: Procedure: apply $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$$\begin{aligned} \text{RHS: } \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} &= \frac{(\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)}{(\sin x + \cos x)} \\ &= 1 - \sin x \cos x \\ &= 1 - \frac{1}{2} \sin 2x \\ &= 1 - \frac{1}{2} \sin 2x. \end{aligned}$$

7) Find x s.t. $-\pi \leq x \leq \pi$ and $\cos 2x = \sin x$.

Sol: Given. $\cos 2x = \sin x$

$$1 - 2\sin^2 x = \sin x$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$\sin x = \frac{-1 \pm 3}{4} = -1 \text{ or } \frac{1}{2} \quad \because -\pi < x < \pi$$

$$\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$= \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\sin x = -1 \Rightarrow x = -\frac{\pi}{2}$$

$$\therefore x = -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$$

8) Find the values of 1) $\sin 18^\circ$ 2) $\cos 18^\circ$ 3) $\sin 72^\circ$ 4) $\cos 36^\circ$ 5) $\sin 54^\circ$.

Sol: Let $\theta = 18^\circ$

$$1) \sin 18^\circ \quad 5\theta = 90^\circ$$

$$3\theta + 2\theta = 90$$

$$2\theta = 90 - 3\theta$$

$$\sin 2\theta = \sin(90 - 3\theta)$$

$$= \cos 3\theta$$

$$2 \sin \theta \cos \theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$\div \cos \theta \quad 2 \sin \theta = 4 \cos^2 \theta - 3$$

$$= 4(1 - \sin^2 \theta) - 3$$

$$= 4 - 4 \sin^2 \theta - 3$$

$$= 1 - 4 \sin^2 \theta$$

$$\therefore 4 \sin^2 \theta + 2 \sin \theta - 1 = 0 \quad a = 4$$

$$\sin \theta = \frac{-2 \pm \sqrt{4 + 16}}{8} \quad b = 2$$

$$= \frac{-2 \pm \sqrt{20}}{8} = \frac{-1 \pm \sqrt{5}}{4} \quad c = 1$$

$$\therefore \sin 18^\circ = \frac{-1 + \sqrt{5}}{4}$$

$$2) \cos 18^\circ = \sqrt{1 - \sin^2 18^\circ}$$

$$= \sqrt{1 - \left(\frac{\sqrt{5}-1}{4}\right)^2}$$

$$= \sqrt{\frac{16 - (5 + 1 - 2\sqrt{5})}{16}}$$

$$= \sqrt{\frac{10 + 2\sqrt{5}}{4}}$$

$$3) \sin 72^\circ = \sin(90 - 18^\circ)$$

$$= \cos 18^\circ$$

$$= \frac{\sqrt{10 + 2\sqrt{5}}}{4}$$

$$4) \cos 36^\circ = 1 - 2 \sin^2 18^\circ$$

$$= 1 - 2 \left(\frac{\sqrt{5}-1}{4}\right)^2$$

$$= \frac{\sqrt{5} + 1}{4}$$

$$5) \sin 54^\circ = \sin(90 - 36^\circ)$$

$$= \cos 36^\circ = \frac{\sqrt{5} + 1}{4}$$

9) If $\tan \frac{\theta}{2} = \sqrt{\frac{1-a}{1+a}} \tan \frac{\phi}{2}$ then P.T $\cos \phi = \frac{\cos \theta - a}{1 - a \cos \theta}$.
TBP

$$\text{LHS: } \cos \phi = \frac{1 - \tan^2 \frac{\phi}{2}}{1 + \tan^2 \frac{\phi}{2}} \quad (\text{half angle formula})$$

$$= \frac{1 - \left(\frac{1+a}{1-a}\right) \tan^2 \frac{\theta}{2}}{1 + \left(\frac{1+a}{1-a}\right) \tan^2 \frac{\theta}{2}} =$$

$$= \frac{(1-a) - (1+a) \tan^2 \frac{\theta}{2}}{(1-a) + (1+a) \tan^2 \frac{\theta}{2}} = \frac{(1 - \tan^2 \frac{\theta}{2}) - a(1 + \tan^2 \frac{\theta}{2})}{(1 + \tan^2 \frac{\theta}{2}) + a(1 - \tan^2 \frac{\theta}{2})}$$

$$= \frac{1 - \tan^2 \frac{\theta}{2} - a}{1 + \tan^2 \frac{\theta}{2} + a}$$

$$= \frac{\cos \theta - a}{1 - a \cos \theta}$$

10) Find the value of $\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ$

TBP Sol: $\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ$

$$= \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ}$$

$$= \frac{\sqrt{3} \cos 20^\circ - \sin 20^\circ}{\sin 20^\circ \cos 20^\circ}$$

$$= \frac{\sqrt{3}/2 \cos 20^\circ - \frac{1}{2} \sin 20^\circ}{2 \sin 20^\circ \cos 20^\circ}$$

$\div$ Nr by 2
Multipln Dr by 2.

$$= 4 \left[\frac{\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ}{\sin 40^\circ} \right]$$

$$= 4 \left[\frac{\sin(60^\circ - 20^\circ)}{\sin 40^\circ} \right] = 4.$$

11) P.T $\cos A \cdot \cos 2A \cos 2^2 A \cos 2^3 A \dots \cos 2^{n-1} A = \frac{\sin 2^n A}{2^n \sin A}$.
TBP

12) P-T $\cos A \cos 2A \cos 2^2 A \cos 2^3 A \dots \cos 2^{n-1} A = \frac{\sin 2^n A}{2^n \sin A}$
 TBQ.

Sol: Procedure: To use $\sin 2A$ formula dividing and Multiplying
 by $2 \sin A$, similarly till n^{th} step.

LHS $\cos A \cos 2A \cos 2^2 A \cos 2^3 A \dots \cos 2^{n-1} A$

$$= \frac{1}{2 \sin A} \cdot 2 \sin A \cos A \cdot \cos 2A \cos 2^2 A \dots \cos 2^{n-1} A$$

$$= \frac{1}{2 \sin A} \cdot \sin 2A \cos 2A \cos 2^2 A \cos 2^3 A \dots \cos 2^{n-1} A$$

$$= \frac{1}{2^2 \sin A} \cdot 2 \sin 2A \cos 2A \cdot \cos 2^2 A \cos 2^3 A \dots \cos 2^{n-1} A$$

$$= \frac{1}{2^2 \sin A} \cdot \sin 2^2 A \cdot \cos 2^2 A \cos 2^3 A \dots \cos 2^{n-1} A$$

$$= \frac{1}{2^3 \sin A} \cdot 2 \sin 2^2 A \cos 2^2 A \cdot \cos 2^3 A \dots \cos 2^{n-1} A$$

$$= \frac{1}{2^3 \sin A} \cdot \sin 2^3 A \cos 2^3 A \dots \cos 2^{n-1} A$$

Proceed as follows for n steps -

$$= \frac{1}{2^n \sin A} \cdot \sin 2^n A$$

13) Find the value of $\cos 2A$, A lies in the first quadrant when
 TBP 1) $\cos A = \frac{15}{17}$ 2) $\sin A = \frac{4}{5}$ 3) $\tan A = \frac{16}{63}$

When 1) $\cos A = \frac{15}{17}$ $\sin A = \sqrt{1 - \frac{225}{289}} = \frac{8}{17}$

$$\therefore \cos 2A = 1 - 2 \sin^2 A = 1 - 2 \cdot \frac{64}{289} = \frac{161}{289}$$

$$\frac{289}{128} - \frac{161}{161}$$

When 2) $\sin A = \frac{4}{5}$ $\cos A = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

$$\cos 2A = 2 \cos^2 A - 1 = 2 \cdot \frac{9}{25} - 1 = -\frac{7}{25}$$

14) P.T $\cos 5x = 16\cos^5 x - 20\cos^3 x + 5\cos x$

TBP Sol: Procedure: Take $\cos 5x = \cos(3x+2x)$. Expand. Then use $\cos 3x, \sin 3x, \cos 2x, \sin 2x$ formulae and simplify. and change all $\sin$ into $\cos$.

$$\begin{aligned}\cos 5x &= \cos(3x+2x) = \cos 3x \cos 2x - \sin 3x \sin 2x \\ &= (4\cos^3 x - 3\cos x)(2\cos^2 x - 1) - (3\sin x - 4\sin^3 x) \cdot 2\sin x \cos x \\ &= (4\cos^3 x - 3\cos x)(2\cos^2 x - 1) - (3 - 4\sin^2 x) \cdot 2\sin^2 x \cos x \\ &= (4\cos^3 x - 3\cos x)(2\cos^2 x - 1) - (3 - 4(1 - \cos^2 x)) \cdot 2(1 - \cos^2 x) \cdot \cos x \\ &= (8\cos^5 x - 4\cos^3 x - 6\cos^3 x + 3\cos x) - (3 - 4 + 4\cos^2 x)(2\cos x - 2\cos^3 x) \\ &= (8\cos^5 x - 10\cos^3 x + 3\cos x) - (-2\cos x + 2\cos^3 x + 8\cos^3 x - 8\cos^5 x) \\ &= 16\cos^5 x - 20\cos^3 x + 5\cos x.\end{aligned}$$

15) P.T $\cot 7\frac{1}{2}^\circ = \tan 82\frac{1}{2}^\circ = (\sqrt{3} + \sqrt{2})(\sqrt{2} + 1)$ (or) $\sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$.

T.B.P

$$\begin{aligned}\text{L.H.S} &= \tan 82\frac{1}{2}^\circ = \tan(90 - 7\frac{1}{2}^\circ) \\ &= \cot 7\frac{1}{2}^\circ = \cot A \text{ (say)}\end{aligned}$$

$$\cot A = \frac{\cos A}{\sin A} = \frac{\cos A (2\cos A)}{\sin A \cdot 2(\cos A)} = \frac{1 + \cos 2A}{\sin 2A}$$

$$\cot 7\frac{1}{2}^\circ = \frac{1 + \cos 15^\circ}{\sin 15^\circ} = \frac{1 + \cos(45^\circ - 30^\circ)}{\sin(45^\circ - 30^\circ)}$$

$$= \frac{1 + (\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ)}{\sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ}$$

Procedure

Consider $\cot A = \frac{\cos A}{\sin A}$

make it as $\frac{1 + \cos 2A}{\sin 2A}$

by multiplying and dividing both Nr and Dr by $2\cos A$.

Apply $A = 7\frac{1}{2}^\circ$ (ie) $2A = 15^\circ$

Make 15° as $(45^\circ - 30^\circ)$ then simplify.

$$= 1 + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{2\sqrt{2} + \sqrt{3} + 1}{2\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$\frac{\sqrt{3} - 1}{2\sqrt{2}}$$

$$= \frac{(2\sqrt{2} + \sqrt{3} + 1)(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

$$= \frac{2\sqrt{6} + 2\sqrt{2} + 3 + \sqrt{3} + \sqrt{3} + 1}{2}$$

$$= \sqrt{6} + \sqrt{2} + \sqrt{3} + 2$$

$$= \sqrt{6} + \sqrt{2} + \sqrt{3} + \sqrt{4}$$

$$= (\sqrt{3} + \sqrt{2})(\sqrt{2} + 1)$$

16) P.T $32\sqrt{3} \sin \frac{\pi}{48} \cos \frac{\pi}{48} \cos \frac{\pi}{24} \cos \frac{\pi}{12} \cos \frac{\pi}{6} = 3$

TBP Sol: Procedure: in order to use $2\sin A \cos A = \sin 2A$ split 32 into 5, 16 and join with all functions. and simplify.

16) If $A+B=45^\circ$ P.T $(1+\tan A)(1+\tan B)=2$

TBP Sol: Procedure: Apply tan on both sides and add 1 on both sides to simplify factorise we will get answer.

Given $A+B=45^\circ$

$$\tan(A+B) = \tan 45^\circ$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$$\tan A + \tan B = 1 - \tan A \tan B$$

$$\tan A + \tan B + \tan A \tan B = 1$$

$$(1 + \tan A) + \tan B(1 + \tan B) = 1 + 1$$

$$(1 + \tan A)(1 + \tan B) = 2$$

19) P.T $\tan\left(\frac{\pi}{4} + \theta\right) - \tan\left(\frac{\pi}{4} - \theta\right) = 2 \tan 2\theta$

TBP

Sol: Procedure: Apply $\tan\left(\frac{\pi}{4} + \theta\right) = \frac{1 + \tan \theta}{1 - \tan \theta}$

$$\tan\left(\frac{\pi}{4} - \theta\right) = \frac{1 - \tan \theta}{1 + \tan \theta}$$

and simplify.

$$\text{LHS} = \tan\left(\frac{\pi}{4} + \theta\right) - \tan\left(\frac{\pi}{4} - \theta\right) = \frac{1 + \tan \theta}{1 - \tan \theta} - \frac{1 - \tan \theta}{1 + \tan \theta}$$

$$= \frac{(1 + \tan \theta)^2 - (1 - \tan \theta)^2}{1 - \tan^2 \theta}$$

$$= \frac{4 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \cdot 2 \cdot \tan \theta}{1 - \tan^2 \theta}$$

$$= 2 \tan 2\theta$$

20)

LHS: $32\sqrt{3} \sin \frac{\pi}{48} \cos \frac{\pi}{48} \cos \frac{\pi}{24} \cos \frac{\pi}{12} \cos \frac{\pi}{6}$ 16th continuation

$$= \sqrt{3} \cdot 2 \cdot 2 \sin \frac{\pi}{48} \cos \frac{\pi}{48} \cdot 2 \cos \frac{\pi}{24} \cdot 2 \cos \frac{\pi}{12} \cdot 2 \cos \frac{\pi}{6}$$

$$= 2\sqrt{3} \cdot \sin \frac{2\pi}{48} \cdot 2 \cos \frac{\pi}{24} \cdot 2 \cos \frac{\pi}{12} \cdot 2 \cos \frac{\pi}{6}$$

$$= 2\sqrt{3} \cdot \sin \frac{2\pi}{24} \cdot 2 \cos \frac{\pi}{12} \cdot 2 \cos \frac{\pi}{6}$$

$$= 2\sqrt{3} \sin \frac{2\pi}{12} \cdot 2 \cos \frac{\pi}{6}$$

$$= 2\sqrt{3} \sin \frac{2\pi}{6}$$

$$= 2\sqrt{3} \cdot \frac{\sqrt{3}}{2} = 3 = \text{RHS}$$

17) P.T $\cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \cos \frac{16\pi}{15} = \frac{1}{16}$

Sol: Procedure: Multiplying and dividing the LHS by $16 \sin \frac{16\pi}{15}$ and this 16 is also split into 4, two and join with all f's and rearrange into descending order.

$$\begin{aligned} \text{LHS} &= \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \cos \frac{16\pi}{15} \\ &= \frac{1}{16 \sin \frac{2\pi}{15}} \cdot 2 \sin \frac{2\pi}{15} \cos \frac{2\pi}{15} \cdot 2 \cos \frac{4\pi}{15} \cdot 2 \cos \frac{8\pi}{15} \cdot 2 \cos \frac{16\pi}{15} \\ &= \frac{1}{16 \sin \frac{2\pi}{15}} \cdot 2 \sin \frac{2\pi}{15} \cos \frac{2\pi}{15} \cdot 2 \cos \frac{4\pi}{15} \cdot 2 \cos \frac{8\pi}{15} \cdot 2 \cos \frac{16\pi}{15} \\ &= \frac{1}{16 \sin \frac{2\pi}{15}} \cdot \sin \frac{4\pi}{15} \cdot 2 \cos \frac{4\pi}{15} \cdot 2 \cos \frac{8\pi}{15} \cdot 2 \cos \frac{16\pi}{15} \\ &= \frac{1}{16 \sin \frac{2\pi}{15}} \cdot \sin \frac{8\pi}{15} \cdot 2 \cos \frac{8\pi}{15} \cdot 2 \cos \frac{16\pi}{15} \\ &= \frac{1}{16 \sin \frac{2\pi}{15}} \cdot \sin \frac{16\pi}{15} \cdot 2 \cos \frac{16\pi}{15} \\ &= \frac{1}{16 \sin \frac{2\pi}{15}} \cdot \sin \frac{32\pi}{15} \\ &= \frac{1}{16 \sin \frac{2\pi}{15}} \cdot \sin \frac{2\pi}{15} \\ &= \frac{1}{16} = \text{RHS} \end{aligned}$$

$\left(\sin \frac{32\pi}{15} = \sin \left(2\pi + \frac{2\pi}{15} \right) \right)$
 $= \sin \frac{2\pi}{15}$

2.1) If $\tan \alpha = \frac{1}{7}$, $\sin \beta = \frac{1}{\sqrt{10}}$ P.T $\alpha + 2\beta = \frac{\pi}{4}$ $0 < \alpha < \frac{\pi}{2}$, $0 < \beta < \frac{\pi}{2}$

Sol: Procedure: α, β lies in the first quadrant. $\therefore$ All ratios are +ve. From $\sin \beta$ find $\cos \beta$ and then $\tan \beta$.

By using $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$ find $\tan 2\beta$ then by using

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ find $\tan(\alpha + 2\beta)$ then we get the result.

$$\cos \beta = \sqrt{1 - \sin^2 \beta} = \sqrt{1 - \frac{1}{10}} = \frac{3}{\sqrt{10}} \quad \therefore \tan \beta = \frac{\sin \beta}{\cos \beta} = \frac{\frac{1}{\sqrt{10}}}{\frac{3}{\sqrt{10}}} = \frac{1}{3}$$

$$\tan 2\beta = \frac{2 \tan \beta}{1 - \tan^2 \beta} = \frac{2 \cdot \frac{1}{3}}{1 - \frac{1}{9}} = \frac{\frac{2}{3} \times \frac{3}{8}}{\frac{8}{9}} = \frac{3}{4}$$

$$\tan(\alpha + 2\beta) = \frac{\tan \alpha + \tan 2\beta}{1 - \tan \alpha \cdot \tan 2\beta} = \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7} \cdot \frac{3}{4}} = \frac{\frac{4+21}{28}}{\frac{28-25}{28}} = \frac{25/28}{25/28} = 1$$

$$\tan(\alpha + 2\beta) = 1$$

$$\alpha + 2\beta = \frac{\pi}{4}$$

2.2) If $\cos \theta = \frac{1}{2} \left(a + \frac{1}{a} \right)$ S.T $\cos 3\theta = \frac{1}{2} \left(a^3 + \frac{1}{a^3} \right)$

TBP

Sol: Procedure: Take LHS as $2 \cos 3\theta$ and use $\cos 3\theta$ formula. and sub. the value of $2 \cos \theta$.
use $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

$$\text{LHS} = 2 \cos 3\theta = 2 [4 \cos^3 \theta - 3 \cos \theta]$$

$$= 8 \cos^3 \theta - 6 \cos \theta$$

$$= (2 \cos \theta)^3 - 3(2 \cos \theta)$$

Given. sub.

$$\therefore 2 \cos \theta = a + \frac{1}{a}$$

$$= \left(a + \frac{1}{a} \right)^3 - 3 \left(a + \frac{1}{a} \right)$$

$$= a^3 + \frac{1}{a^3} + 3a \cdot \frac{1}{a} \left(a + \frac{1}{a} \right) - 3 \left(a + \frac{1}{a} \right)$$

$$= a^3 + \frac{1}{a^3}$$

2.3) P.T $(1 + \tan^2 1^\circ)(1 + \tan^2 2^\circ) \dots (1 + \tan^2 44^\circ)$ is multiple of 4

TBP. Sol: we know that if $A+B=45^\circ$ $(1 + \tan A)(1 + \tan B) = 2$

$$1+44=45 \quad \therefore (1 + \tan 1^\circ)(1 + \tan 44^\circ) = 2$$

$$2+43=45 \quad (1 + \tan 2^\circ)(1 + \tan 43^\circ) = 2$$

||| we can get 22 sets of 2s $\therefore (1 + \tan 1^\circ)(1 + \tan 2^\circ) \dots (1 + \tan 44^\circ) = 2^{22} = 4^{11}$

24) P.T $\sin 4x = 4 \tan x \left(\frac{1 - \tan^2 x}{1 + \tan^2 x} \right)^2$
TBP

Sol: Procedure: Use $\sin 2x = \frac{2 \tan x}{1 + \tan^2 x}$, $\cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$

LHS: $\sin 4x = \sin 2(2x)$

$= 2 \sin 2x \cos 2x$

$= 2 \cdot \frac{2 \tan x}{1 + \tan^2 x} \times \frac{1 - \tan^2 x}{1 + \tan^2 x}$

$= 4 \tan x \frac{(1 - \tan^2 x)}{(1 + \tan^2 x)^2}$

25) P.T $(1 + \sec 2x)(1 + \sec 4x)(1 + \sec 8x) \dots (1 + \sec 2^n x) = \tan 2^n x \cot x$
TBP

Sol: Procedure: Use $\sec 2x = \frac{1}{\cos 2x}$, $1 + \cos 2x = 2 \cos^2 x$

and $\cos x \cos 2x \cos 2^2 x \dots \cos 2^{n-1} x = \frac{\sin 2^n x}{2^n \sin x}$

LHS: $(1 + \sec 2x)(1 + \sec 4x)(1 + \sec 8x) \dots (1 + \sec 2^n x)$

$= \left(1 + \frac{1}{\cos 2x}\right) \left(1 + \frac{1}{\cos 4x}\right) \left(1 + \frac{1}{\cos 8x}\right) \dots \left(1 + \frac{1}{\cos 2^n x}\right)$

$= \frac{(1 + \cos 2x)(1 + \cos 4x)(1 + \cos 8x) \dots (1 + \cos 2^n x)}{\cos 2x \cdot \cos 4x \cdot \cos 8x \dots \cos 2^n x}$

$= \frac{2 \cos^2 x \cdot 2 \cos^2 2x \cdot 2 \cos^2 2^2 x \dots 2 \cos^2 2^{n-1} x}{\cos 2x \cos 2^2 x \cos 2^3 x \dots \cos 2^n x}$

$= \frac{2^n \cos x [\cos x \cdot \cos 2x \cdot \cos 2^2 x \dots \cos 2^{n-1} x]}{\cos 2^n x}$

$= \frac{2^n \cos x}{\cos 2^n x} \left[\frac{\sin 2^n x}{2^n \sin x} \right] = \tan 2^n x \cdot \cot x$

26) P.T $\sqrt{2 + \sqrt{2 + \sqrt{2 + 2 \cos 8x}}} = 2 \cos x$, $0 < x < \pi/8$

Sol: Procedure Use $1 + \cos x = 2 \cos^2 x/2$

LHS: $\sqrt{2 + \sqrt{2 + \sqrt{2 + 2 \cos 8x}}} = \sqrt{2 + \sqrt{2 + \sqrt{2(1 + \cos 8x)}}} = \sqrt{2 + \sqrt{2 + \sqrt{2 \cdot 2 \cos^2 4x}}} = \sqrt{2 + \sqrt{2 + \sqrt{4 \cos^2 4x}}} = \sqrt{2 + \sqrt{2 + 2 \cos 4x}} = \sqrt{2 + \sqrt{2(1 + \cos 4x)}} = \sqrt{2 + \sqrt{2 \cdot 2 \cos^2 2x}} = \sqrt{2 + 2 \cos 2x} = 2 \cos x$

$$27) P.T \tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ = 1.$$

$$\text{Sol: Change } \tan 89^\circ = \tan(90-1) = \cot 1$$

$$\tan 88^\circ = \tan(90-2) = \cot 2$$

$$\dots \dots \dots \tan 46^\circ = \tan(90-44) = \cot 44$$

$$\therefore \text{LHS: } \tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ$$

$$= (\tan 1^\circ \cdot \tan 89^\circ) (\tan 2^\circ \tan 88^\circ) \dots (\tan 44^\circ \tan 46^\circ) \cdot \tan 45^\circ$$

$$= (\tan 1^\circ \cot 1^\circ) (\tan 2^\circ \cot 2^\circ) \dots (\tan 44^\circ \cot 44^\circ) \cdot \tan 45^\circ$$

$$= 1 \cdot 1 \cdot 1 \cdot \dots \cdot 1 \cdot 1 = \underline{\underline{1}}$$

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Product to sum and Sum to product identities

1) Express each of the following product as a sum (or) difference.

TBP 1. $\sin 40^\circ \cos 30^\circ = \frac{1}{2} 2 \sin 40^\circ \cos 30^\circ$

$$= \frac{1}{2} [\sin 70^\circ + \sin 10^\circ]$$

2. $\cos 110^\circ \sin 55^\circ = \frac{1}{2} \cdot 2 \cos 110^\circ \sin 55^\circ$

$$= \frac{1}{2} [\sin 165^\circ - \sin 55^\circ]$$

3. $\sin \frac{x}{2} \cos \frac{3x}{2} = \frac{1}{2} [2 \cos \frac{3x}{2} \sin \frac{x}{2}]$

$$= \frac{1}{2} [\sin \frac{4x}{2} - \sin \frac{2x}{2}]$$

$$= \frac{1}{2} [\sin 2x - \sin x]$$

2) Express each of the following sum or difference as a product.

TBP

1) $\sin 50^\circ + \sin 20^\circ = 2 \sin \frac{70^\circ}{2} \cos \frac{30^\circ}{2}$

$$= 2 \sin 35^\circ \cos 15^\circ$$

2) $\cos 60^\circ + \cos 20^\circ = 2 \cos \frac{80^\circ}{2} \cos \frac{40^\circ}{2}$

$$= 2 \cos 40^\circ \cos 20^\circ$$

3) $\cos \frac{3x}{2} - \cos \frac{9x}{2} = 2 \sin \frac{12x}{2} \sin \frac{6x}{2}$

$$= 2 \sin \frac{12x}{4} \sin \frac{6x}{4}$$

$$= 2 \sin 3x \sin \frac{3x}{2}$$

3) Find the value of $\sin 34^\circ + \cos 54^\circ - \cos 4^\circ$

TBP

Sol: $\sin 34^\circ + (-2 \sin 34^\circ \sin 30^\circ)$

$$= \sin 34^\circ - \sin 34^\circ$$

$$= 0$$

4) P-T $\cos 36^\circ \cos 72^\circ \cos 108^\circ \cos 144^\circ = \frac{1}{16}$

TBP
Sol:

$$\text{LHS} = \cos 36^\circ \cos 72^\circ \cos 108^\circ \cos 144^\circ$$

$$= \cos 36^\circ \sin (90^\circ - 18^\circ) \cos (90^\circ + 18^\circ) \cos (180^\circ - 36^\circ)$$

$$= \cos 36^\circ \sin 18^\circ (-\sin 18^\circ) (-\cos 36^\circ)$$

$$= \sin^2 18^\circ \cdot \cos^2 36^\circ$$

Procedure.

$$\sin 18^\circ = \frac{\sqrt{5}-1}{4}$$

$$\cos 36^\circ = \frac{\sqrt{5}+1}{4}$$

This type is used only when given angles are related with $90^\circ, 180^\circ, 270^\circ, 360^\circ$.

5) Simplify $\frac{\sin 75^\circ - \sin 15^\circ}{\cos 75^\circ + \cos 15^\circ}$ Just apply formula.
TBP

$$\begin{aligned} \text{Sol: } \frac{\sin 75^\circ - \sin 15^\circ}{\cos 75^\circ + \cos 15^\circ} &= \frac{2 \cos \frac{90^\circ}{2} \sin \frac{60^\circ}{2}}{2 \cos \frac{90^\circ}{2} \cos \frac{60^\circ}{2}} \\ &= \frac{\sin 30^\circ}{\cos 30^\circ} = \tan 30^\circ = \frac{1}{\sqrt{3}}. \end{aligned}$$

6) S.T $\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ = \frac{3}{16}$.
TBP

Sol: Procedure: Use $\cos(60-A) \cos A \cos(60+A) = \frac{1}{4} \cos 3A$.
This type is used only when the angles are related
with 60° (ie) write it as $60+\theta$, $60-\theta$, & .

$$\begin{aligned} \cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ &= \cos 30^\circ \cdot \cos(60-10) \cos 10^\circ \cos(60+10) \\ &= \frac{\sqrt{3}}{2} \cdot \frac{1}{4} \cos 30^\circ \\ &= \frac{\sqrt{3}}{8} \cdot \frac{\sqrt{3}}{2} = \frac{3}{16}. \end{aligned}$$

7) P.T $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$. Same type as problem 6

LHS $\sin 30^\circ \sin(60-20) \sin 10^\circ \sin(60+10)$

$$\begin{aligned} &= \frac{1}{2} \cdot \frac{1}{4} \sin 30^\circ \\ &= \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{16}. \end{aligned}$$

8) P.T $4 \cos 12^\circ \cos 48^\circ \cos 72^\circ = \cos 36^\circ$ Same type as Problem 6

Sol: LHS $= 4 \cos 12^\circ \cos 48^\circ \cos 72^\circ$

$$\begin{aligned} &= 4 \cos(60-12) \cos 12^\circ \cos(60+12) \\ &= 4 \cdot \frac{1}{4} \cos 36^\circ \\ &= \cos 36^\circ. \end{aligned}$$

9) P.T $\cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \cos \frac{16\pi}{15} = \frac{1}{16}$. already done

TBP
9) P.T $\cos \frac{\pi}{15} \cos \frac{2\pi}{15} \cos \frac{3\pi}{15} \cos \frac{4\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15} \cos \frac{7\pi}{15} = \frac{1}{128}$

Sol: Procedure: Convert all radians into degree measure. Substitute the known values directly including $\sin 18^\circ$, $\cos 72^\circ$, $\sin 56^\circ$, $\cos 36^\circ$, - - . Then arrange the remaining to use $\sin 2A$ formula.

$$1) LHS \cos 12^\circ \cos 24^\circ \cos 36^\circ \cos 48^\circ \cos 60^\circ \cos 72^\circ \cos 84^\circ.$$

$$= \cos 12^\circ \cos 24^\circ \cos 48^\circ \cos 84^\circ \cdot \cos 36^\circ \cdot \cos 72^\circ \cdot \cos 60^\circ$$

$$= \frac{1}{8 \sin 12^\circ} \cdot 2 \sin 12^\circ \cos 12^\circ \cdot 2 \cos 24^\circ \cdot 2 \cos 48^\circ \cdot 2 \cos 84^\circ \left(\frac{\sqrt{5}-1}{4} \right) \left(\frac{\sqrt{5}+1}{4} \right) \cdot \frac{1}{2}$$

$$= \frac{1}{8 \sin 12^\circ} \sin 24^\circ \cdot 2 \cos 24^\circ \cdot 2 \cos 48^\circ \cdot \cos 84^\circ \cdot \frac{5-1}{4 \times 4 \times 2}$$

$$= \frac{1}{8 \sin 12^\circ} \sin 48^\circ \cdot 2 \cos 48^\circ \cos 84^\circ \cdot \frac{1}{8}$$

$$= \frac{1}{64 \sin 12^\circ} \cdot \sin 96^\circ \cdot \cos 84^\circ$$

$$= \frac{1}{64 \sin 12^\circ} \cdot \cos 6^\circ \sin 6^\circ$$

$$= \frac{1}{128 \cdot \sin 12^\circ} \cdot 2 \sin 6^\circ \cos 6^\circ$$

$$= \frac{1}{128 \sin 12^\circ} \cdot \sin 12^\circ = \frac{1}{128}$$

$$10) P.T \sin 12^\circ \sin 48^\circ \sin 54^\circ = \frac{1}{8}$$

TBP Sol: Procedure: Set the known value directly and use $2 \sin A \sin B = \cos(B-A) - \cos(A+B)$

$$LHS \sin 12^\circ \sin 48^\circ \sin 54^\circ$$

$$= \frac{1}{2} 2 \sin 12^\circ \sin 48^\circ \cdot \sin 54^\circ$$

$$= \frac{1}{2} [\cos(-36^\circ) - \cos 60^\circ] \sin 54^\circ$$

$$= \frac{1}{2} [\cos 36^\circ - \cos 60^\circ] \sin 54^\circ$$

$$= \frac{1}{2} \left[\frac{\sqrt{5}+1}{4} - \frac{1}{2} \right] \frac{\sqrt{5}+1}{4}$$

$$= \frac{1}{2} \frac{(\sqrt{5}+1)^2}{16} - \frac{1}{4} \cdot \frac{\sqrt{5}+1}{4}$$

$$= \frac{5+1+2\sqrt{5}}{32} - \frac{2\sqrt{5}-2}{32} = \frac{4}{32} = \frac{1}{8}$$

11) S.T $\sin 8x \cos 2x - \sin 6x \cos 3x$

TBP. $\cos 2x \cos x - \sin 3x \sin 4x$.

Sol: Procedure apply $2 \sin A \sin B$, $2 \cos A \cos B$ formula.

LHS: $\sin 8x \cos 2x - \sin 6x \cos 3x$

$2 \sin A \cos B$:

$\cos 2x \cos x - \sin 3x \sin 4x$.

$= \frac{2 \sin 8x \cos 2x - 2 \sin 6x \cos 3x}{2 \cos 2x \cos x - 2 \sin 3x \sin 4x}$

$= \frac{(\sin 9x + \sin 7x) - (\sin 9x + \sin 3x)}{(\cos 7x + \cos 3x) + (\cos 7x + \cos 3x)}$

$= \frac{\sin 7x - \sin 3x}{\cos 7x + \cos 3x} = \frac{2 \cos 5x \sin 2x}{2 \cos 5x \cos 2x}$

$= \tan 2x$.

12. P.T $(\cos 80^\circ - \cos 30^\circ)(\sin 80^\circ + \sin 20^\circ)$

TBP. $(\sin 50^\circ - \sin 10^\circ)(\cos 40^\circ - \cos 60^\circ) = 1$.

Sol: Procedure: Use $\sin C + \sin D$, $\sin C - \sin D$ formulae and soon.

LHS: $(\cos 80^\circ - \cos 30^\circ)(\sin 80^\circ + \sin 20^\circ)$

$(\sin 50^\circ - \sin 10^\circ)(\cos 40^\circ - \cos 60^\circ)$

$= \frac{2 \sin 20^\circ \sin 10^\circ \cdot 2 \sin 50^\circ \cos 30^\circ}{2 \cos 30^\circ \sin 20^\circ \cdot 2 \sin 10^\circ \sin 50^\circ}$

$= 1$

13. P.T $\frac{\sin 4x + \sin 2x}{\cos 4x + \cos 2x} = \tan 3x$.

TBP. $\cos 4x + \cos 2x$

Sol: Procedure: Use $\sin C + \sin D$, $\cos C + \cos D$ formula.

LHS: $\frac{\sin 4x + \sin 2x}{\cos 4x + \cos 2x} = \frac{2 \sin 3x \cos x}{2 \cos 3x \cos x}$

$= \tan 3x$.

14) P.T $\sin \frac{80^\circ}{2} \sin \frac{70^\circ}{2} + \sin \frac{30^\circ}{2} \sin \frac{10^\circ}{2} = \sin 20^\circ \sin 50^\circ$.

TBP

Sol: Procedure: Use $2 \sin A \sin B$ Formula.

LHS $= \frac{1}{2} [2 \sin \frac{80^\circ}{2} \sin \frac{70^\circ}{2} + 2 \sin \frac{30^\circ}{2} \sin \frac{10^\circ}{2}]$
 $= \frac{1}{2} [(\cos \frac{10^\circ}{2} - \cos \frac{40^\circ}{2}) + (\cos \frac{40^\circ}{2} - \cos \frac{70^\circ}{2})]$
 $= \frac{1}{2} [-\cos \frac{70^\circ}{2} + \cos \frac{30^\circ}{2}]$
 $= \frac{1}{2} [2 \sin 50^\circ \sin 20^\circ]$

15 P.T $\cos(30-A)\cos(30+A) + \cos(45-A)\cos(45+A) = \cos 2A + \frac{1}{4}$

TBP. Sol: ^{Apply.} Procedure: $\cos(A+B) \cdot \cos(A-B) = \cos^2 A - \sin^2 B$

LHS: $\cos(30-A)\cos(30+A) + \cos(45-A)\cos(45+A)$
 $= \cos^2 30 - \sin^2 A + \cos^2 45 - \sin^2 A$
 $= \frac{3}{4} - \sin^2 A + \frac{1}{2} - \sin^2 A$
 $= \frac{1}{4} + 1 - 2\sin^2 A$
 $= \frac{1}{4} + \cos 2A.$

18) P.T $\frac{\sin x + \sin 3x + \sin 5x + \sin 7x}{\cos x + \cos 3x + \cos 5x + \cos 7x} = \tan 4x.$

TBP. $\cos x + \cos 3x + \cos 5x + \cos 7x$

Sol: Procedure: Use $\sin C + \sin D$, $\cos C + \cos D$ formula two times

LHS: $\frac{\sin x + \sin 3x + \sin 5x + \sin 7x}{\cos x + \cos 3x + \cos 5x + \cos 7x}$
 $= \frac{2\sin 2x \cos 2x + 2\sin 6x \cos 2x}{2\cos 2x \cos 2x + 2\cos 6x \cos 2x}$
 $= \frac{2\cos 2x (\sin 2x + \sin 6x)}{2\cos 2x (\cos 2x + \cos 6x)} = \frac{2\sin 4x \cos 2x}{2\cos 4x \cos 2x}$
 $= \tan 4x.$

19) P.T $\frac{\sin(4A-2B) + \sin(4B-2A)}{\cos(4A-2B) + \cos(4B-2A)} = \tan(A+B)$

TBP. $\cos(4A-2B) + \cos(4B-2A)$

Sol: Procedure: Use $\sin C + \sin D$, $\cos C + \cos D$ formula.

LHS: $\frac{\sin(4A-2B) + \sin(4B-2A)}{\cos(4A-2B) + \cos(4B-2A)}$
 $= \frac{2\sin\left(\frac{4A-2B+4B-2A}{2}\right)\cos\left(\frac{4A-2B-4B+2A}{2}\right)}{2\cos\left(\frac{4A-2B+4B-2A}{2}\right)\cos\left(\frac{4A-2B-4B+2A}{2}\right)}$
 $= \frac{2\sin\left(\frac{2(A+B)}{2}\right)\cos\left(\frac{2(A-B)}{2}\right)}{2\cos\left(\frac{2(A+B)}{2}\right)\cos\left(\frac{2(A-B)}{2}\right)} = \tan(A+B)$

20) S.T $\cot(A+45) - \tan(A-45) = \frac{4 \cos 2A}{1+2 \sin 2A}$

18) TBP.

Sol:

18) P.T $\sin x + \sin 2x + \sin 3x = \sin 2x (1 + 2 \cos x)$

TBP Sol: Procedure use $\sin C + \sin D$ and arrange as $\sin 3x + \sin x + \sin 2x$ in order to get the $\sin$ in angle.

$$\begin{aligned} \text{LHS } \sin 3x + \sin x + \sin 2x &= 2 \sin 2x \cos x + 2 \sin x \cos x \\ &= 2 \cos x (\sin 2x + \sin x) \\ &= 2 \cos x (2 \sin x \cos x + \sin x) \\ &= 2 \sin x \cos x (2 \cos x + 1) \\ &= \sin 2x (1 + 2 \cos x) \end{aligned}$$

19) P.T $1 + \cos 2x + \cos 4x + \cos 6x = 4 \cos x \cos 2x \cos 3x$.

TBP Sol: Same as above problem. Re arrange the LHS

$$\begin{aligned} \text{LHS} &= 1 + \cos 4x + \cos 6x + \cos 2x \\ &= 1 + \cos 4x + 2 \cos 4x \cos 2x \\ &= 1 + \cos 2(2x) + 2 \cos 4x \cos 2x \\ &= 2 \cos^2 2x + 2 \cos 4x \cos 2x \\ &= 2 \cos 2x (\cos 2x + \cos 4x) \\ &= 2 \cos 2x \cdot 2 \cos 3x \cdot \cos x \\ &= 4 \cos x \cos 2x \cos 3x. \end{aligned}$$

Conditional Trigonometric Identities -

- 1) If $A+B+C = \pi$ P.T 1) $\cos A + \cos B + \cos C = 1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$
 TBP.
 2) $\sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \leq \frac{1}{8}$
 3) $1 < \cos A + \cos B + \cos C \leq \frac{3}{2}$

Sol: Procedure: Apply $\cos C + \cos D$ formula for first $\cos A + \cos B$ and write $\cos C$ as $1 - 2 \sin^2 \frac{C}{2}$. Then $\frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}$
 and $\sin \frac{C}{2}$ as $\sin \left(\frac{\pi}{2} - \frac{A+B}{2} \right)$
 Lastly Apply $\cos D - \cos C = 2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$.

$$\begin{aligned}
 \text{LHS: 1) } \cos A + \cos B + \cos C &= 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} + \cos C \\
 &= 2 \cos \left(\frac{\pi}{2} - \frac{C}{2} \right) \cdot \cos \frac{A-B}{2} + 1 - 2 \sin^2 \frac{C}{2} \\
 &= 1 + 2 \sin \frac{C}{2} \cdot \cos \frac{A-B}{2} - 2 \sin^2 \frac{C}{2} \\
 &= 1 + 2 \sin \frac{C}{2} \left[\frac{\cos A-B}{2} - \sin \left(\frac{\pi}{2} - \frac{A+B}{2} \right) \right] \\
 &= 1 + 2 \sin \frac{C}{2} \left[\frac{\cos A-B}{2} - \cos \frac{A+B}{2} \right] \\
 &= 1 + 2 \sin \frac{C}{2} \left[\frac{2 \cos \frac{A+B}{2} \sin \frac{A+B}{2} - 2 \cos \frac{A+B}{2} \sin \frac{A+B}{2}}{2 \times 2} \right] \\
 &= 1 + 4 \sin \frac{C}{2} \sin \frac{A}{2} \sin \frac{B}{2} \\
 &= 1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}.
 \end{aligned}$$

$$\begin{aligned}
 2) \text{ Let } u &= \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \\
 &= -\frac{1}{2} \left[\cos \frac{A+B}{2} - \cos \frac{A-B}{2} \right] \sin \frac{C}{2} \\
 &= -\frac{1}{2} \left[\cos \frac{A+B}{2} - \cos \frac{A-B}{2} \right] \sin \left(\frac{\pi}{2} - \frac{A+B}{2} \right) \\
 &= -\frac{1}{2} \left[\cos \frac{A+B}{2} - \cos \frac{A-B}{2} \right] \cos \frac{A+B}{2} \\
 2u &= -\cos^2 \frac{A+B}{2} + \cos \frac{A+B}{2} \cos \frac{A-B}{2} \\
 \cos^2 \frac{A+B}{2} - \cos \frac{A+B}{2} \cos \frac{A-B}{2} + 2u &= 0 \quad \text{This is quadratic Eqn in } \cos \frac{A+B}{2}
 \end{aligned}$$

$$b^2 - 4ac \geq 0$$

$$\cos^2 \frac{A-B}{2} - 8u \geq 0$$

$$\cos^2 \frac{A-B}{2} \geq 8u$$

$$\Rightarrow 8u \leq \cos^2 \frac{A-B}{2} \leq 1$$

$$u \leq \frac{1}{8} \cos^2 \frac{A-B}{2} \leq \frac{1}{8}$$

$$\sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \leq \frac{1}{8} (?)$$

$$a=1$$

$$b = -\cos \frac{A-B}{2}$$

$$c = 2u.$$

$$\sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \leq \frac{1}{8} \quad \text{Proved}$$

3) $\cos A + \cos B + \cos C > 1$ and
 $\cos A + \cos B + \cos C \leq 1 + 4 \cdot \frac{1}{8}$
 $1 < \cos A + \cos B + \cos C \leq \frac{9}{8}$

2) P.T $\sin \frac{A}{2} + \sin \frac{B}{2} + \sin \frac{C}{2} = 1 + 4 \sin \left(\frac{\pi-A}{4} \right) \sin \left(\frac{\pi-B}{4} \right) \sin \left(\frac{\pi-C}{4} \right)$
 TBP if $A+B+C = \pi$

Sol: Procedure: change $\sin$ terms into $\cos$ term first as

Express the sum of the first two terms as a product (by using cond D) and simplify the first factor from the relation connecting the angles.

Express the third term in terms of simplified factor. Take out common factor and express the sum with in brackets, first in terms of the first two angles and express it as a product (C, D formula)

$$\begin{aligned} \text{LHS: } \sin \frac{A}{2} + \sin \frac{B}{2} + \sin \frac{C}{2} &= \cos \left(\frac{\pi}{2} - \frac{A}{2} \right) + \cos \left(\frac{\pi}{2} - \frac{B}{2} \right) + \cos \left(\frac{\pi}{2} - \frac{C}{2} \right) \\ &= 2 \cos \left(\frac{\pi}{2} - \frac{A+B}{4} \right) \cos \left(\frac{B-A}{4} \right) + \left[1 - 2 \sin^2 \left(\frac{\pi}{4} - \frac{C}{4} \right) \right] \\ &= 1 + 2 \sin \frac{A+B}{4} \cos \frac{B-A}{4} - 2 \sin^2 \left(\frac{\pi}{4} - \frac{C}{4} \right) \\ &= 1 + 2 \sin \left(\frac{\pi}{4} - \frac{C}{4} \right) \cos \frac{B-A}{4} - 2 \sin^2 \left(\frac{\pi}{4} - \frac{C}{4} \right) \\ &= 1 - 2 \sin \left(\frac{\pi}{4} - \frac{C}{4} \right) \left[\cos \frac{B-A}{4} - \sin \left(\frac{\pi}{4} - \frac{C}{4} \right) \right] \\ &= 1 - 2 \sin \left(\frac{\pi}{4} - \frac{C}{4} \right) \left[\cos \frac{B-A}{4} - \sin \frac{\pi}{4} + \left(\frac{A+B}{4} + \frac{\pi}{4} \right) \right] \\ &= 1 - 2 \sin \left(\frac{\pi}{4} - \frac{C}{4} \right) \left[\cos \frac{B-A}{4} - \sin \frac{A+B}{4} \right] \\ &= 1 - 2 \sin \left(\frac{\pi}{4} - \frac{C}{4} \right) \left[\cos \frac{B-A}{4} - \cos \left(\frac{\pi}{2} - \frac{A+B}{4} \right) \right] \\ &= 1 - 2 \sin \left(\frac{\pi}{4} - \frac{C}{4} \right) \left[2 \sin \left(\frac{\frac{\pi}{2} - \frac{A+B}{4} + \frac{B-A}{4}}{2} \right) \sin \left(\frac{\frac{\pi}{2} - \frac{A+B}{4} - \frac{B-A}{4}}{2} \right) \right] \\ &= 1 - 2 \sin \left(\frac{\pi}{4} - \frac{C}{4} \right) \left[2 \sin \frac{\frac{\pi}{2} - \frac{A}{2}}{2} \sin \frac{\frac{\pi}{2} - \frac{B}{2}}{2} \right] \\ &= 1 - 4 \sin \left(\frac{\pi}{4} - \frac{C}{4} \right) \sin \left(\frac{\pi}{4} - \frac{A}{2} \right) \sin \left(\frac{\pi}{4} - \frac{B}{2} \right) \end{aligned}$$

3) If $A+B+C = 180$ P.T $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$.

TBP: Procedure: Same as above.

$$\begin{aligned} \text{LHS: } \sin 2A + \sin 2B + \sin 2C &= 2 \sin (A+B) \cos (A-B) + \sin 2C \\ &= 2 \sin (\pi - C) \cos (A-B) + \sin 2C \\ &= 2 \sin C \cos (A-B) + 2 \sin C \cos C \\ &= 2 \sin C [\cos (A-B) + \cos C] \\ &= 2 \sin C [\cos (A-B) + \cos (\pi - A+B)] \end{aligned}$$

$$= 2 \sin c [\cos (A-B) - \cos (A+B)]$$

$$= 2 \sin c \cdot 2 \sin A \sin B$$

$$= 4 \sin A \sin B \sin c.$$

4) If $A+B+C=0$ P.T $\cos A + \cos B - \cos C = -1 + 4 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$
T.B.P.

Sol: LHS: $\cos A + \cos B - \cos C = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} - \cos C$

T.B.P.

Procedure: Proceed as above problem.

$$= 2 \cos \left(\frac{\pi}{2} - \frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right) - \cos C$$

$$= 2 \sin \frac{C}{2} \cos \left(\frac{A-B}{2} \right) - \left(1 - 2 \sin^2 \frac{C}{2} \right)$$

$$= 2 \sin \frac{C}{2} \left[\cos \frac{A-B}{2} + \sin \frac{C}{2} \right] - 1$$

$$= 2 \sin \frac{C}{2} \left[\cos \frac{A-B}{2} + \sin \left(\frac{\pi}{2} - \frac{A+B}{2} \right) \right] - 1$$

$$= 2 \sin \frac{C}{2} \left[\cos \frac{A-B}{2} + \cos \frac{A+B}{2} \right]$$

$$= 2 \cdot \sin \frac{C}{2} \cdot 2 \cos \frac{A}{2} \cos \frac{B}{2} - 1$$

$$= -1 + 4 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$$

5) If $A+B+C=180^\circ$ P.T $\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$

Sol: Procedure: Proceed as above problem.

LHS: $\sin A + \sin B + \sin C = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} + \sin C$

$$= 2 \sin \left(\frac{\pi}{2} - \frac{C}{2} \right) \cos \left(\frac{A-B}{2} \right) + \sin C$$

$$= 2 \cos \left(\frac{C}{2} \right) \cos \left(\frac{A-B}{2} \right) + 2 \sin \frac{C}{2} \cos \frac{C}{2}$$

$$= 2 \cos \frac{C}{2} \left[\cos \frac{A-B}{2} + \sin \left(\frac{\pi}{2} \right) \right]$$

$$= 2 \cos \frac{C}{2} \left[\cos \frac{A-B}{2} + \sin \left(\frac{\pi}{2} - \frac{A+B}{2} \right) \right]$$

$$= 2 \cos \frac{C}{2} \left[\cos \frac{A-B}{2} + \cos \frac{A+B}{2} \right]$$

$$= 2 \cos \frac{C}{2} \cdot 2 \cos \frac{A}{2} \cos \frac{B}{2}$$

$$= 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$$

6) If $A+B+C=180^\circ$ P.T $\sin^2 A + \sin^2 B - \sin^2 C = 4 \cos A \cos B \sin C$.

Sol: Procedure: Same as above problem.

LHS: $\sin^2 A + \sin^2 B - \sin^2 C = 2 \sin (A+B) \cos (A-B) - \sin^2 C$

$$= 2 \sin (\pi - C) \cos (A-B) - \sin^2 C$$

$$= 2 \sin C \cos (A-B) - 2 \sin C \cos C$$

$$= 2 \sin C (\cos (A-B) - \cos C)$$

$$= 2 \sin C (\cos (A-B) - \cos (\pi - A+B))$$

$$= 2 \sin C (\cos (A-B) + \cos (A+B))$$

$$= 2 \sin C \cdot 2 \cos A \cos B$$

$$= 4 \cos A \cos B \sin C.$$

7) P.T of $A+B+C=\pi$ P.T $\cos^2 A + \cos^2 B + \cos^2 C \geq 1 - 2\cos A \cos B \cos C$.

TBP. sol: Procedure: change the squares of Sines or cosines into cosines of double the angles by using $\cos^2 A = \frac{1+\cos 2A}{2}$

and prove it as in the previous type. $\sin^2 A = \frac{1-\cos 2A}{2}$

$$\text{LHS: } \cos^2 A + \cos^2 B + \cos^2 C = \frac{1+\cos 2A}{2} + \frac{1+\cos 2B}{2} + \frac{1+\cos 2C}{2}$$

$$= \frac{3}{2} + \frac{1}{2} [\cos 2A + \cos 2B + \cos 2C] \quad \text{--- (1)}$$

$$\text{Now let } \cos 2A + \cos 2B + \cos 2C = 2\cos(A+B)\cos(A-B) + \cos 2C.$$

$$= 2\cos(\pi-C)\cos(A-B) + \cos 2C$$

$$= -2\cos C \cos(A-B) + 2\cos^2 C - 1$$

$$= -1 + 2\cos C (\cos(A-B) - \cos C)$$

$$= -1 + 2\cos C [\cos(A-B) - \cos(\pi - A+B)]$$

$$= -1 + 2\cos C [\cos(A-B) - (-\cos(A+B))]$$

$$= -1 + 4\cos C \cos A \cos B.$$

Sub in (1)

$$= \frac{3}{2} + \frac{1}{2} (-1 + 4\cos A \cos B \cos C)$$

$$= 1 + 2\cos A \cos B \cos C.$$

8) P.T $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2\cos A \cos B \cos C$.

TBP sol: Procedure same as previous problem.

$$\text{LHS: } \sin^2 A + \sin^2 B + \sin^2 C = \frac{1-\cos 2A}{2} + \frac{1-\cos 2B}{2} + \frac{1-\cos 2C}{2}$$

$$= \frac{3}{2} - \frac{1}{2} (\cos 2A + \cos 2B + \cos 2C) \quad \text{--- (2)}$$

$$\text{Let } \cos 2A + \cos 2B + \cos 2C = 2\cos(A+B)\cos(A-B) + \cos 2C.$$

Sub in (2)

$$\frac{3}{2} - \frac{1}{2} (-1 + 4\cos A \cos B \cos C)$$

$$= 2 + 2\cos A \cos B \cos C.$$

$$= 2\cos(\pi-C)\cos(A-B) + \cos 2C$$

$$= -2\cos C \cos(A-B) + 2\cos^2 C - 1$$

$$= -1 - 2\cos C (\cos(A-B) - \cos C)$$

$$= -1 - 2\cos C (\cos(A-B) - \cos(\pi - A+B))$$

$$= -1 - 2\cos C [\cos(A-B) + \cos(A+B)]$$

$$= -1 - 2\cos C \cdot 2 \cos A \cos B$$

$$= -1 - 4\cos A \cos B \cos C.$$

9) P.T $\sin^2 A + \sin^2 B - \sin^2 C = 2 \sin A \sin B \cos C$.

TBP Sol: Procedure: Same as above problem.

$$\begin{aligned} \text{LHS: } \sin^2 A + \sin^2 B - \sin^2 C &= \frac{1 - \cos 2A}{2} + \frac{1 - \cos 2B}{2} - \frac{1 - \cos 2C}{2} \\ &= \frac{1}{2} - \frac{1}{2} (\cos 2A + \cos 2B - \cos 2C) \quad \text{--- } (*) \end{aligned}$$

$$\begin{aligned} \text{Let } \cos 2A + \cos 2B - \cos 2C &= 2 \cos (A+B) \cos (A-B) - \cos 2C \\ &= 2 \cos (\pi - C) \cos (A+B) - \cos 2C \\ &= -2 \cos C \cos (A+B) - (2 \cos^2 C - 1) \\ &= 1 - 2 \cos C (\cos (A+B) + \cos C) \\ &= 1 - 2 \cos C (\cos (A+B) + \cos (\pi - (A+B))) \\ &= 1 - 2 \cos C [\cos (A+B) - \cos (A+B)] \\ &= 1 - 2 \sin A \sin B \cos C. \end{aligned}$$

Sub in (*)

$$\begin{aligned} \sin^2 A + \sin^2 B - \sin^2 C &= \frac{1}{2} \cdot \frac{1}{2} (1 - 2 \sin A \sin B \cos C) \\ &= \frac{1}{2} - \frac{1}{2} \cdot 2 \sin A \sin B \cos C \\ &= \frac{1}{2} - \sin A \sin B \cos C. \end{aligned}$$

10 P.T $\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{A}{2} \tan \frac{C}{2} = 1$.

TBP Sol: Procedure: Apply $\tan \left(\frac{A+B}{2} \right) = \tan \left(\frac{\pi}{2} - \frac{C}{2} \right)$.

$$\because \frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}$$

$$\tan \left(\frac{A}{2} + \frac{B}{2} \right) = \tan \left(\frac{\pi}{2} - \frac{C}{2} \right)$$

$$\frac{\tan \frac{A}{2} + \tan \frac{B}{2}}{1 - \tan \frac{A}{2} \tan \frac{B}{2}} = + \cot \frac{C}{2}$$

$$\frac{\tan \frac{A}{2} + \tan \frac{B}{2}}{1 - \tan \frac{A}{2} \tan \frac{B}{2}} = \frac{1}{\tan \frac{C}{2}}$$

$$\tan \frac{A}{2} \tan \frac{C}{2} + \tan \frac{B}{2} \tan \frac{C}{2} = 1 - \tan \frac{A}{2} \tan \frac{B}{2}$$

$$\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{A}{2} \tan \frac{C}{2} = 1$$

11) P.T $\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$

TBP Sol: Already done.

11) If $A+B+C = \frac{\pi}{2}$ P.T $\sin 2A + \sin 2B + \sin 2C = 4 \cos A \cos B \cos C$.

TBP Sol: Procedure: Refer problem 3. $A+B = \frac{\pi}{2} - C$

$$\begin{aligned} \text{LHS: } \sin 2A + \sin 2B + \sin 2C &= 2 \sin(A+B) \cos(A-B) + \sin 2C \\ &= 2 \sin\left(\frac{\pi}{2} - C\right) \cos(A-B) + \sin 2C \\ &= 2 \cos C \cos(A-B) + 2 \sin C \cos C \\ &= 2 \cos C [\cos(A-B) + \sin C] \\ &= 2 \cos C [\cos(A-B) + \sin\left(\frac{\pi}{2} - A+B\right)] \\ &= 2 \cos C [\cos(A-B) + \cos(A+B)] \\ &= 2 \cdot \cos C \cdot 2 \cos A \cos B \\ &= 4 \cos A \cos B \cos C. \end{aligned}$$

12) If $A+B+C = \frac{\pi}{2}$ P.T $\cos 2A + \cos 2B + \cos 2C = 1 + 4 \sin A \sin B \sin C$.

TBP Sol: Procedure: Same as above problem.

$$\begin{aligned} \text{LHS: } \cos 2A + \cos 2B + \cos 2C &= 2 \cos(A+B) \cdot \cos(A-B) + \cos 2C \\ &= 2 \cos\left(\frac{\pi}{2} - C\right) \cos(A-B) + \cos 2C \\ &= 2 \sin C \cos(A-B) + (1 - 2 \sin^2 C) \\ &= 1 + 2 \sin C [\cos(A-B) - \sin C] \\ &= 1 + 2 \sin C [\cos(A-B) - \sin\left(\frac{\pi}{2} - A+B\right)] \\ &= 1 + 2 \sin C [\cos(A-B) - \cos(A+B)] \\ &= 1 + 2 \sin C [\cos(A-B) - \cos(A+B)] \\ &= 1 + 4 \sin C \cdot \sin A \cdot \sin B. \end{aligned}$$

13) If $A+B+C = \pi$ P.T $\sin(B+C-A) + \sin(C+A-B) + \sin(A+B-C) = 4 \sin A \sin B \sin C$.

TBP

Sol: Procedure: First reduce the sum to the form $\sin 2A + \sin 2B + \sin 2C$ by using $A+B+C = \pi$

$$\begin{aligned} \text{LHS } \sin(B+C-A) + \sin(C+A-B) + \sin(A+B-C) \\ &= \sin(\pi - A - A) + \sin(\pi - B - B) + \sin(\pi - C - C) \\ &= \sin 2A + \sin 2B + \sin 2C. \end{aligned}$$

Then Proceed as problem 3.

14) If $A+B+C=2S$ P.T $\sin(S-A) + \sin(S-B) + \sin(S-C) - \sin S$
TBP. $= 4 \sin A/2 \sin B/2 \sin C/2$

Sol: Procedure: Apply $\sin C + \sin D$ and $\sin C - \sin D$ formulas.

LHS: $\sin(S-A) + \sin(S-B) + \sin(S-C) - \sin S$.

$$= 2 \sin \frac{S-A+S-B}{2} \cos \frac{S-A-S+B}{2} + 2 \cos \frac{S-C+S}{2} \sin \frac{S-C-S}{2}$$

$$= 2 \sin \frac{2S-A-B}{2} \cos \frac{B-A}{2} + 2 \cos \frac{2S-C}{2} \sin(-C/2)$$

$$= 2 \sin \frac{C}{2} \cos \frac{B-A}{2} + 2 \cos \frac{A+B}{2} \sin(-C/2)$$

$$= 2 \sin C/2 \cos \frac{B-A}{2} - 2 \cos \frac{A+B}{2} \sin C/2$$

$$= 2 \sin C/2 \left[\cos \frac{B-A}{2} - \cos \frac{A+B}{2} \right] \quad \cos \left(\frac{B-A}{2} \right) = \cos \left(-\frac{(A-B)}{2} \right)$$

$$= 2 \sin C/2 \cdot 2 \sin A/2 \sin B/2$$

$$= 4 \sin A/2 \sin B/2 \sin C/2$$

$$= \cos \frac{A-B}{2}$$

15) If $\triangle ABC$ is a right triangle and if $\angle A = \pi/2$ P.T

1) $\cos^2 B + \cos^2 C = 1$

TBP 2) $\sin^2 B + \sin^2 C = 1$

3) $\cos B - \cos C = -1 + 2\sqrt{2} \cos B/2 \sin C/2$

Sol: $\because \angle A = \pi/2 \quad B+C = \frac{\pi}{2}$
 $B = \frac{\pi}{2} - C$

1) LHS: $\cos^2 B + \cos^2 C$
 $= (\cos(\frac{\pi}{2}-C))^2 + \cos^2 C$
 $= (\sin C)^2 + \cos^2 C$
 $= \sin^2 C + \cos^2 C = 1$

2) LHS $\sin^2 B + \sin^2 C$
 $= (\sin(\frac{\pi}{2}-C))^2 + \sin^2 C$
 $= \cos^2 C + \sin^2 C = 1$

3) LHS: $\cos B - \cos C = -2 \sin \frac{B+C}{2} \sin \frac{B-C}{2}$
 $= -2 \sin \frac{\pi}{4}$

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Trigonometric Equations.

- 1) Find the principal solution of i) $\sin \theta = \frac{1}{2}$ 2) $\sin \theta = -\frac{\sqrt{3}}{2}$
 3) $\csc \theta = -2$ 4) $\cos \theta = \frac{1}{2}$

T.B.P. Sol: 1) $\sin \theta = \frac{1}{2} > 0$ θ lies in the first quadrant

$$\sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$$

$\therefore \theta = \frac{\pi}{6}$ is the principal solution.

- 2) $\sin \theta = -\frac{\sqrt{3}}{2} < 0$ Principal value lies in IV quadrant

$$\sin \theta = -\frac{\sqrt{3}}{2} = -\sin\left(\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$$

Hence $\theta = -\frac{\pi}{3}$ is principal solution.

- 3) $\csc \theta = -2 \Rightarrow \sin \theta = -\frac{1}{2} < 0$
 θ lies in IV quadrant.

$$\sin \theta = -\frac{1}{2} = -\sin\left(\frac{1}{2}\right)$$

$$\theta = -\frac{\pi}{6}$$

$\therefore$ Principal solution $\theta = -\frac{\pi}{6}$.

- 4) $\cos \theta = \frac{1}{2} > 0$, θ lies in the I quadrant

$$\cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3} \text{ is the principal solution.}$$

- 2) Find the general solution of $\sin \theta = -\frac{\sqrt{3}}{2}$

T.B.P.

Sol: The general solution of $\sin \theta = \sin \alpha$, $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is
 $\theta = n\pi + (-1)^n \alpha$, $n \in \mathbb{Z}$.

$$\sin \theta = -\frac{\sqrt{3}}{2} = \sin\left(-\frac{\pi}{3}\right) \Rightarrow \alpha = -\frac{\pi}{3}$$

$\therefore$ The General solution is $\theta = n\pi + (-1)^n \left(-\frac{\pi}{3}\right)$, $n \in \mathbb{Z}$.

- 3) Find the general solution of i) $\sec \theta = -2$ ii) $\tan \theta = \sqrt{3}$

T.B.P. Sol: 1) $\sec \theta = -2 \Rightarrow \cos \theta = -\frac{1}{2}$

$$\Rightarrow \cos\left(\pi - \frac{\pi}{3}\right) = -\frac{1}{2} \Rightarrow \alpha = \frac{2\pi}{3}$$

General solution $\theta = 2n\pi \pm \frac{2\pi}{3}$, $n \in \mathbb{Z}$.

- 2) $\tan \theta = \sqrt{3}$, θ lies in I quadrant.

$$\tan \theta = \tan\left(\frac{\pi}{3}\right) \Rightarrow \alpha = \frac{\pi}{3}$$

General solution $\theta = n\pi + \frac{\pi}{3}$, $n \in \mathbb{Z}$.

- 4) Solve $3\cos^2 \theta = \sin^2 \theta$

T.B.P

Sol: $3\cos^2\theta = \sin^2\theta$

$$3\cos^2\theta = 1 - \cos^2\theta$$

$$4\cos^2\theta = 1 \Rightarrow \cos^2\theta = \frac{1}{4}$$

$$\cos\theta = \pm \frac{1}{2}$$

When $\cos\theta = \frac{1}{2} = \cos(\pi/3)$

$\therefore$ The general solution is $\theta = n\pi \pm \pi/3 \quad n \in \mathbb{Z}$.

When $\cos\theta = (-1/2) = \cos(\pi - \pi/3) = \cos(\frac{2\pi}{3})$

Then the general solution is $\theta = n\pi \pm \frac{2\pi}{3}$.

4) Solve $\sin x + \sin 5x = \sin 3x$.

T.B.P. Sol: LHS: $\sin 5x + \sin x = 2\sin 3x \cos 2x = \sin 3x$

$$\Rightarrow \sin 3x (2\cos 2x - 1) = 0$$

$$\sin 3x = 0 \quad (\text{or}) \quad 2\cos 2x = 1$$

$$\cos 2x = \frac{1}{2}$$

When $\sin 3x = 0 = \sin(0)$

$\therefore 3x = n\pi \Rightarrow x = \frac{n\pi}{3} \quad n \in \mathbb{Z}$.

When $\cos 2x = \frac{1}{2} = \cos(\pi/3)$

$$2x = \pi/3$$

$\therefore 2x = 2n\pi \pm \frac{\pi}{3}$

$$x = n\pi \pm \pi/6 \quad n \in \mathbb{Z}$$

5) Solve $\cos x + \sin x = \cos 2x + \sin 2x$.

T.B.P. Sol $\cos x + \sin x = \cos 2x + \sin 2x$.

$$\cos x - \cos 2x = \sin 2x - \sin x$$

$$2\sin\left(\frac{x+2x}{2}\right)\sin\left(\frac{2x-x}{2}\right) = 2\cos\left(\frac{2x+x}{2}\right)\sin\left(\frac{2x-x}{2}\right)$$

$$2\sin\frac{3x}{2}\sin\frac{x}{2} = 2\cos\frac{3x}{2}\sin\frac{x}{2}$$

$$\sin\frac{3x}{2}\sin\frac{x}{2} - \cos\frac{3x}{2}\sin\frac{x}{2} = 0$$

$$\sin\frac{x}{2}\left(\sin\frac{3x}{2} - \cos\frac{3x}{2}\right) = 0$$

$$\Rightarrow \sin\frac{x}{2} = 0 \quad (\text{or}) \quad \sin\frac{3x}{2} = \cos\frac{3x}{2}$$

If $\sin\frac{x}{2} = 0 \Rightarrow \frac{x}{2} = 0 \Rightarrow x = 0$ when $\sin\frac{3x}{2} = \cos\frac{3x}{2}$

$$\tan\frac{3x}{2} = 1 = \tan\frac{\pi}{4}$$

$$\therefore \frac{x}{2} = n\pi$$

$$x = 2n\pi \quad n \in \mathbb{Z}$$

$$\frac{3x}{2} = n\pi + \frac{\pi}{4} \Rightarrow 3x = 2n\pi + \frac{\pi}{2}$$

$$x = \frac{2n\pi}{3} + \pi/6$$

6) Solve the equation $\sin 9\theta = \sin \theta$.

TBP: Sol: $\sin 9\theta = \sin \theta$

$$\Rightarrow \sin 9\theta - \sin \theta = 0 = 2 \cos 5\theta \sin 4\theta = 0$$

$$\Rightarrow \cos 5\theta = 0 \text{ (or) } \sin 4\theta = 0$$

when $\sin 4\theta = 0 = \theta = 0$

$$\therefore 4\theta = n\pi$$

$$\theta = \frac{n\pi}{4} \quad n \in \mathbb{Z}$$

when $\cos 5\theta = 0$

$$5\theta = \frac{\pi}{2}$$

$$= (2n+1)\frac{\pi}{2}$$

$$\theta = (2n+1)\frac{\pi}{10} \quad n \in \mathbb{Z}$$

7) Solve: $\tan 2x = -\cot(x + \pi/3)$

TBP Sol: $\tan 2x = -\cot(x + \pi/3)$

$$= \tan\left(\frac{\pi}{2} + x + \pi/3\right)$$

$$\tan 2x = \tan\left(\frac{5\pi}{6} + x\right)$$

$$\therefore 2x = n\pi + \frac{5\pi}{6} + x$$

$$x = n\pi + \frac{5\pi}{6} \quad n \in \mathbb{Z}$$

8) Solve $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$.

TBP: Sol: $\sin x + \sin 3x - 3\sin 2x = \cos x + \cos 3x - 3\cos 2x$

$$2\sin 2x \cos x - 3\sin 2x = 2\cos 2x \cos x - 3\cos 2x$$

$$2\sin 2x \cos x - 2\cos 2x \cos x = 3(\sin 2x - \cos 2x)$$

$$2\cos x (\sin 2x - \cos 2x) = 3(\sin 2x - \cos 2x)$$

$$(\sin 2x - \cos 2x) [2\cos x - 3] = 0$$

$$\sin 2x - \cos 2x = 0 \quad \because 2\cos x - 3 \neq 0$$

$$\tan 2x = 1 = \tan \frac{\pi}{4}$$

$$2x = n\pi + \frac{\pi}{4}$$

$$x = \frac{n\pi}{2} + \frac{\pi}{8}$$

9) Solve $\sin x + \cos x = 1 + \sin x \cos x$

TBP Sol: Let $t = \sin x + \cos x$

$$1 + 2\sin x \cos x = (\sin^2 x + \cos^2 x) + 2\sin x \cos x$$

$$= (\sin x + \cos x)^2 = t^2$$

$$\Rightarrow \sin x \cos x = \frac{t^2 - 1}{2}$$

$$\therefore \text{L.H.S. } 1 + \frac{t^2 - 1}{2} - t = 0$$

$$2 + t^2 - 1 - 2t = 0$$

$$t^2 - 2t + 1 = 0 \Rightarrow (t-1)^2 = 0 \therefore t = 1$$

$$\text{Hence } \sin x + \cos x = 1$$

$$r = \sqrt{1+1} = \sqrt{2}$$

$$\sqrt{2} \left(\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x \right) = 1$$

$$\sqrt{2} \left(\cos \frac{\pi}{4} \sin x + \sin \frac{\pi}{4} \cos x \right) = 1$$

or)

$$\sqrt{2} \left(\sin \frac{\pi}{4} \sin x + \cos \frac{\pi}{4} \cos x \right) \quad \sin \left(\frac{\pi}{4} + x \right) = \frac{1}{\sqrt{2}} = \sin \frac{\pi}{4}$$

$$= \sqrt{2} \cos \left(\frac{\pi}{4} - x \right) = 1$$

$$\cos \left(\frac{\pi}{4} - x \right) = \frac{1}{\sqrt{2}} = \cos \frac{\pi}{4}$$

$$\frac{\pi}{4} - x = 2n\pi \pm \frac{\pi}{4}$$

$$(n) \quad x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$$

$$x = 2n\pi + \frac{\pi}{2} \quad (\text{or}) \quad x = 2n\pi$$

10) Solve: $2 \sin^2 x + \sin^2 2x = 2$.

TBP. Sol: L.H.S: $2 \sin^2 x + \sin^2 2x$

$$= 2 \sin^2 x + (2 \sin x \cos x)^2 = 2$$

$$= 2 \sin^2 x + 4 \sin^2 x \cos^2 x = 2$$

$$x - \cos^2 x + 2 \sin^2 x \cos^2 x = 1$$

$$\cos^2 x (2 \sin^2 x - 1) = 0$$

$$\cos^2 x = 0$$

$$\cos x = 0$$

$$x = (2n+1)\frac{\pi}{2} \quad n \in \mathbb{Z}$$

$$2 \sin^2 x = 1$$

$$\sin^2 x = \frac{1}{2}$$

$$\sin^2 x = \frac{1}{2} = \sin^2 \left(\frac{\pi}{4} \right)$$

$$x = n\pi \pm \frac{\pi}{4} \quad n \in \mathbb{Z}$$

11) Solve: P.T for any a, b $-\sqrt{a^2+b^2} \leq a \sin \theta + b \cos \theta \leq \sqrt{a^2+b^2}$

TBP: Sol: $a \sin \theta + b \cos \theta = \sqrt{a^2+b^2} \left[\frac{1}{\sqrt{a^2+b^2}} \sin \theta + \frac{1}{\sqrt{a^2+b^2}} \cos \theta \right]$

$$= \sqrt{a^2+b^2} \left[\cos x \sin \theta + \sin x \cos \theta \right]$$

$$= \sqrt{a^2+b^2} \left[\sin (\theta + x) \right]$$

$$|a \sin \theta + b \cos \theta| \leq \sqrt{a^2+b^2} \Rightarrow -\sqrt{a^2+b^2} \leq (a \sin \theta + b \cos \theta) \leq \sqrt{a^2+b^2}$$

where $\cos x = \frac{a}{\sqrt{a^2+b^2}}$
 $\sin x = \frac{b}{\sqrt{a^2+b^2}}$

12) Solve $\sqrt{3} \sin \theta - \cos \theta = \sqrt{2}$

TBP. Sol: $\sqrt{3} \sin \theta - \cos \theta = \sqrt{2}$ $r = \sqrt{3+1} = 2 > c = \sqrt{2}$

∴ The Eqn. may be written as

$$\frac{\sqrt{3}}{2} \sin \theta - \frac{1}{\sqrt{2}} \cos \theta = 1$$

$$\sin \theta \cos \frac{\pi}{6} - \cos \theta \sin \frac{\pi}{6} = 1$$

$$\sin(\theta - \pi/6) = \sin \frac{\pi}{4}$$

$$\therefore \theta - \frac{\pi}{6} = n\pi \pm (-1)^n \cdot \frac{\pi}{4}$$

$$\theta = n\pi + \frac{\pi}{6} \pm (-1)^n \frac{\pi}{4} \quad n \in \mathbb{Z}$$

13) Solve $\sqrt{3} \tan^2 \theta + (\sqrt{3}-1) \tan \theta - 1 = 0$

TBP Sol: $\sqrt{3} \tan^2 \theta + (\sqrt{3}-1) \tan \theta - 1 = 0$

$$\sqrt{3} \tan^2 \theta + \sqrt{3} \tan \theta - \tan \theta - 1 = 0$$

$$\sqrt{3} \tan \theta (\tan \theta + 1) - (\tan \theta + 1) = 0$$

$$(\tan \theta + 1) (\sqrt{3} \tan \theta - 1) = 0$$

$$\sqrt{3} \tan \theta - 1 = 0 \quad \tan \theta + 1 = 0$$

$$\sqrt{3} \tan \theta = 1$$

$$\tan \theta = \frac{1}{\sqrt{3}} = \tan \frac{\pi}{6}$$

$$\theta = n\pi + \frac{\pi}{6} \quad n \in \mathbb{Z}$$

$$\tan \theta = -1$$

$$= \tan(-\pi/4)$$

$$\theta = n\pi - \frac{\pi}{4} \quad n \in \mathbb{Z}$$

14) Solve: $\sin \theta + \sin 3\theta + \sin 5\theta = 0$

TBP: Sol: Procedure: Use $\sin c + \sin d$

$$(\sin 5\theta + \sin \theta) + \sin 3\theta = 0$$

$$2 \sin 3\theta \cos 2\theta + \sin 3\theta = 0$$

$$\sin 3\theta (2 \cos 2\theta + 1) = 0$$

$$\sin 3\theta = 0 \quad 2 \cos 2\theta + 1 = 0$$

$$3\theta = n\pi$$

$$\theta = \frac{n\pi}{3} \quad n \in \mathbb{Z}$$

$$2 \cos 2\theta = -1$$

$$\cos 2\theta = -1/2 = \cos(\pi - \pi/3)$$

$$2\theta = 2n\pi \pm \frac{2\pi}{3}, \quad n \in \mathbb{Z}$$

$$\theta = n\pi \pm \frac{\pi}{3}$$

$$\theta = (3n \pm 1) \frac{\pi}{3}$$

15) Find the general solution of $\tan \theta + \tan(\theta + \frac{\pi}{3}) + \tan(\theta + \frac{2\pi}{3}) = \sqrt{3}$

TBP: Sol: Procedure: ^{use} $\tan(A+B)$ and $\frac{\pi}{3}, \frac{2\pi}{3}$ values -

$$\tan \theta + \tan(\theta + \frac{\pi}{3}) + \tan(\theta + \frac{2\pi}{3}) = \sqrt{3} \quad \tan \frac{2\pi}{3} = \tan(\pi - \frac{\pi}{3}) = -\tan \frac{\pi}{3} = -\sqrt{3}$$

$$\frac{\tan \theta + \tan \theta + \tan \frac{\pi}{3}}{1 - \tan \theta \tan \frac{\pi}{3}} + \frac{\tan \theta + \tan \frac{2\pi}{3}}{1 - \tan \theta \tan \frac{2\pi}{3}} = \sqrt{3}$$

$$\tan \theta + \frac{\tan \theta + \sqrt{3}}{1 - \sqrt{3} \tan \theta} + \frac{\tan \theta - \sqrt{3}}{1 + \sqrt{3} \tan \theta} = \sqrt{3}$$

$$\frac{\tan \theta (1 - 3 \tan^2 \theta) + \tan \theta + \sqrt{3} \tan \theta + \sqrt{3} + 3 \tan \theta + \tan \theta - \sqrt{3} \tan \theta}{1 - 3 \tan^2 \theta} = \sqrt{3}$$

$$\frac{9 \tan \theta - 3 \tan^3 \theta}{1 - 3 \tan^2 \theta} = \sqrt{3} \Rightarrow \frac{3(3 \tan \theta - \tan^3 \theta)}{1 - 3 \tan^2 \theta} = \sqrt{3}$$

$$= \sqrt{3} \quad 3 \tan 3\theta = \sqrt{3}$$

$$\tan 3\theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

$$3\theta = n\pi + \frac{\pi}{6}$$

$$\theta = \frac{n\pi}{3} + \frac{\pi}{18} \quad n \in \mathbb{Z}$$

16) Solve: $2\cos^2 \theta + 3\sin \theta - 3 = 0$

TBP Sol: Procedure: change $\cos^2 \theta$ into $1 - \sin^2 \theta$ and then solve.

$$2\cos^2 \theta + 3\sin \theta - 3 = 0$$

$$2(1 - \sin^2 \theta) + 3\sin \theta - 3 = 0$$

$$2 - 2\sin^2 \theta + 3\sin \theta - 3 = 0$$

$$2\sin^2 \theta - 3\sin \theta + 1 = 0$$

$$a = 2, b = -3, c = 1$$

$$\sin \theta = \frac{3 \pm \sqrt{9 - 8}}{4} = \frac{3 \pm 1}{4} \quad \left(\frac{4}{4}, \frac{2}{4} \right)$$

$$\text{When } \sin \theta = 1 \Rightarrow \left(\frac{\pi}{2} \right)$$

$$\text{Solution } (-1)^n \cdot \frac{\pi}{2}$$

$$\text{When } \sin \theta = \frac{1}{2} = \sin \left(\frac{\pi}{6} \right)$$

$$\text{Solution } \theta = n\pi + (-1)^n \cdot \frac{\pi}{6} \quad n \in \mathbb{Z}$$

17) Solve $\cot \theta + \operatorname{cosec} \theta = \sqrt{3}$

TBP Sol: $\frac{\cos \theta}{\sin \theta} + \frac{1}{\sin \theta} = \sqrt{3}$

$$\cos \theta + 1 = \sqrt{3} \sin \theta$$

$$\sqrt{3} \sin \theta - \cos \theta = 1$$

$$r = \sqrt{3+1} = 2$$

$$\therefore \frac{\sqrt{3}}{2} \sin \theta - \frac{1}{2} \cos \theta = \frac{1}{2}$$

$$\sin \frac{\pi}{3} \sin \theta - \cos \frac{\pi}{3} \cos \theta = \frac{1}{2}$$

$$\cos \frac{\pi}{3} \cos \theta - \sin \frac{\pi}{3} \sin \theta = -\frac{1}{2}$$

$$\cos \left(\frac{\pi}{3} + \theta \right) = \cos \left(\pi - \frac{\pi}{3} \right)$$

$$\therefore \theta + \frac{\pi}{3} = 2n\pi \pm \frac{\pi}{3}$$

$$(i) \theta = 2n\pi + \frac{\pi}{3} \quad (ii) \theta = 2n\pi - \pi$$

18) solve: $\sin \theta + \sqrt{3} \cos \theta = 1$

TBP Sol: $r = \sqrt{1+3} = 2$

$$\frac{1}{2} \sin \theta + \frac{\sqrt{3}}{2} \cos \theta = \frac{1}{2}$$

$$\sin \frac{\pi}{6} \sin \theta + \cos \frac{\pi}{6} \cos \theta = \cos \left(\frac{\pi}{3} \right)$$

$$= \cos \frac{\pi}{3}$$

$$\cos \left(\theta - \frac{\pi}{6} \right)$$

$$\theta - \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{3}$$

$$\theta = 2n\pi + \frac{\pi}{2} \quad (ii) \theta = 2n\pi - \frac{\pi}{6}$$

19 solve: $\cos \theta + \sin \theta = \sqrt{2}$

TBP Sol: $r = \sqrt{1+1} = \sqrt{2}$

$$\cos \theta + \sin \theta = \sqrt{2}$$

$$\frac{1}{\sqrt{2}} \cos \theta + \frac{1}{\sqrt{2}} \sin \theta = 1$$

$$\cos \frac{\pi}{4} \cos \theta + \sin \frac{\pi}{4} \sin \theta = 1$$

$$\cos \left(\theta - \frac{\pi}{4} \right) = \cos 0$$

$$\theta - \frac{\pi}{4} = 2n\pi$$

$$\theta = 2n\pi + \frac{\pi}{4}$$

20) Solve: $2 \cos^2 x - 7 \cos x + 3 = 0$

TBP Sol: $2 \cos^2 x - 7 \cos x + 3 = 0$ $a = 2$
 $\cos x = \frac{7 \pm \sqrt{49 - 24}}{4}$ $b = -7$
 $c = 3$

$= \frac{7 \pm 5}{4} = 3, \frac{1}{2}$ $\because \cos \theta$ is between -1 and 1
 3 is not possible.

$\therefore \cos x = \frac{1}{2} = \cos \frac{\pi}{3}$

$x = 2n\pi \pm \frac{\pi}{3}$

21) Solve: $\cos 2\theta = \frac{\sqrt{5} + 1}{4}$

TBP Solve: $\cos 2\theta = \frac{\sqrt{5} + 1}{4}$

$\cos 2\theta = \cos \frac{\pi}{5}$

$\therefore 2\theta = 2n\pi \pm \frac{\pi}{5}$

$\theta = n\pi \pm \frac{\pi}{10}$

22) Solve $\sin 2\theta - \cos 2\theta - \sin \theta + \cos \theta = 0$

TBP Sol: $(\sin 2\theta - \sin \theta) - (\cos 2\theta - \cos \theta) = 0$

$2 \cos \frac{3\theta}{2} \sin \frac{\theta}{2} + 2 \sin \frac{3\theta}{2} \sin \frac{\theta}{2} = 0$

$2 \sin \frac{\theta}{2} \left(\cos \frac{3\theta}{2} + \sin \frac{3\theta}{2} \right) = 0$

$\sin \frac{\theta}{2} = 0$

$\frac{\theta}{2} = n\pi$

$\theta = 2n\pi$

$\cos \frac{3\theta}{2} + \sin \frac{3\theta}{2} = 0$

$\frac{-\cos \frac{3\theta}{2}}{\cos \frac{3\theta}{2}} + \frac{\sin \frac{3\theta}{2}}{\cos \frac{3\theta}{2}} = 0$

$\tan \frac{3\theta}{2} = -1$
 $= \tan \left(\pi - \frac{\pi}{4} \right)$

$\tan \frac{3\theta}{2} = \tan \frac{3\pi}{4}$

$\frac{3\theta}{2} = n\pi + \frac{3\pi}{4}$
 $= (4n + 3) \frac{\pi}{4}$

$\frac{3\theta}{2} = (4n + 3)\pi$
 $\theta = (4n + 3) \frac{\pi}{3}$

23) Solve $\cos x + \cos 3x - 2\cos 2x = 0$

Sol: $\cos x + \cos 3x - 2\cos 2x = 0$

TBP

$$2\cos 2x \cos x - 2\cos 2x = 0$$

$$2\cos 2x (\cos x - 1) = 0$$

$$2\cos 2x = 0$$

$$\cos 2x = 0$$

$$2x = (2n+1)\frac{\pi}{2}$$

$$x = (2n+1)\frac{\pi}{4}$$

$$\cos x - 1 = 0$$

$$\cos x = 1$$

$$\cos x = \cos 0$$

$$x = 2n\pi \pm 0$$

$$x = 2n\pi$$

24) Solve $\sin 5x - \sin x = \cos 3x$.

TBP Sol: $\sin 5x - \sin x = \cos 3x$

$$2\cos 3x \sin 2x - \cos 3x = 0$$

$$2\cos 3x (\sin 2x - 1) = 0$$

$$2\cos 3x = 0$$

$$\cos 3x = 0$$

$$3x = (2n\pi + 1)\frac{\pi}{2}$$

$$x = (2n+1)\frac{\pi}{6}$$

$$\sin 2x - 1 = 0$$

$$\sin 2x = 1$$

$$= \sin \frac{\pi}{2}$$

$$2x = 2n\pi + \frac{\pi}{2}$$

$$= (2n+1)\frac{\pi}{2}$$

$$x = (2n+1)\frac{\pi}{4}$$

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Properties of Triangle

Theorem: In any triangle, the lengths of the sides are proportional to the sines of the opposite angle:

$$(or) \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Proof:

Case i) $\angle A$ is acute.

Produce BO to meet the circle at D

$$\angle BDC = \angle BAC = A$$

$$\angle BCD = 90^\circ$$

$$\sin \angle BCD = \frac{BC}{BD} \quad (or) \quad \sin A = \frac{a}{2R} \quad (or) \quad \frac{a}{\sin A} = 2R.$$

Case ii) $\angle A$ is right angle

In this case O must be on the side BC of the $\triangle ABC$

$$\therefore \frac{a}{\sin A} = \frac{BC}{\sin 90^\circ} = \frac{2R}{1} \Rightarrow \frac{a}{\sin A} = 2R$$

Case iii) $\angle A$ obtuse

Produce BO to meet the circle at D

$$\angle BDC + \angle BAC = 180^\circ$$

$$\therefore \angle BDC = 180 - \angle BAC = 180 - A$$

$$\therefore \angle BCD = 90^\circ$$

$$\sin \angle BDC = \frac{BC}{BD} \quad (or) \quad \sin(180 - A) = \sin A = \frac{a}{2R}$$

$$\Rightarrow \frac{a}{\sin A} = 2R.$$

||| we can prove that $\frac{b}{\sin B} = 2R$ and $\frac{c}{\sin C} = 2R.$

$$\therefore \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R.$$

Note: $\frac{a}{b} = \frac{\sin A}{\sin B}, \frac{b}{c} = \frac{\sin B}{\sin C}, \frac{a}{c} = \frac{\sin A}{\sin C}.$

Napier's Formula

In a ΔABC where

$$1) \tan \frac{A-B}{2} = \frac{a-b}{a+b} \cot \frac{C}{2}$$

$$2) \tan \frac{B-C}{2} = \frac{b-c}{b+c} \cot \frac{A}{2}$$

$$3) \tan \frac{C-A}{2} = \frac{c-a}{c+a} \cot \frac{B}{2}$$

Proof:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

$$\text{Let } \frac{a-b}{b+c} \cot \frac{A-B}{2} = \frac{a-b}{a+b} \cot \frac{C}{2} = \frac{2R \sin A - 2R \sin B}{2R \sin A + 2R \sin B} \cdot \cot \frac{C}{2}$$

$$= \frac{2R (\sin A - \sin B)}{2R (\sin A + \sin B)} \cdot \cot \frac{C}{2}$$

$$= \frac{2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}}{2 \sin \frac{A+B}{2}} \cdot \cot \frac{C}{2}$$

$$= \cot \frac{A+B}{2} \tan \frac{A-B}{2} \cot \frac{C}{2}$$

$$= \cot \left(\frac{\pi}{2} - \frac{C}{2} \right) \cdot \tan \frac{A-B}{2} \cot \frac{C}{2}$$

$$= \tan \frac{C}{2} \cdot \tan \frac{A-B}{2} \cot \frac{C}{2}$$

$$\frac{a-b}{a+b} \cot \frac{C}{2} = \tan \frac{A-B}{2}$$

Cosine formula. $a^2 = b^2 + c^2 - 2bc \cos A$, $b^2 = c^2 + a^2 - 2ca \cos B$

$$(or) \cos A = \frac{b^2 + c^2 - a^2}{2bc}, \cos B = \frac{c^2 + a^2 - b^2}{2ca}, \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Proof In ΔABC Draw $AD \perp BC$.

$$AB^2 = AD^2 + BD^2$$

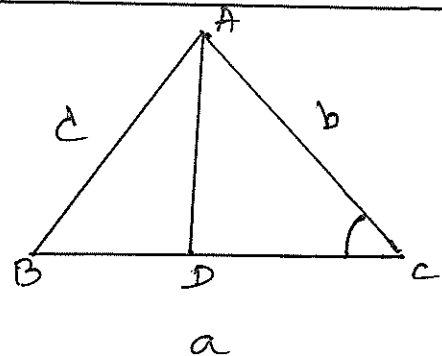
$$c^2 = AD^2 + BD^2 \quad \text{--- (1)}$$

$$\sin C = \frac{AD}{AC} \Rightarrow AD = AC \sin C = b \sin C \quad \text{--- (2)}$$

$$BD = BC - DC = a - b \cos C \quad \text{--- (3)}$$

$$\therefore c^2 = b^2 \sin^2 C + (a - b \cos C)^2$$

$$c^2 = b^2 \sin^2 C + (a^2 + b^2 \cos^2 C - 2ab \cos C) = a^2 + b^2 - 2ab \cos C$$



Note: 1) $a^2 = b^2 + c^2 - 2bc \cos A$ says that the square of the sides is sum of the squares of the other two sides diminished by twice the product of those two sides and the cosine of the included angle.

2) The law of Cosines can be viewed as generalisation of Pythagorean theorem.

3) $c^2 = a^2 + b^2 - 2ab \cos C$. Since $-\cos C < 1$

$$c^2 < a^2 + b^2 + 2ab$$

$$\Rightarrow c < a + b \quad \text{Why } a < b + c, b < c + a.$$

(ie) In any Δ the sum of the two sides is greater than the third side.

Projection Formula: In a ΔABC 1) $a = b \cos C + c \cos B$

2) $b = c \cos A + a \cos C$

3) $c = a \cos B + b \cos A$.

Proof: In ΔABC

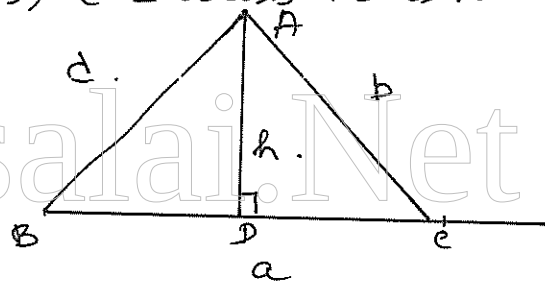
$a = BC$, Draw $AD \perp BC$

$$BC = BD + DC$$

$$= \frac{BD}{AB} \cdot AB + \frac{DC}{AC} \cdot AC$$

$$a = (\cos B) c + (\cos C) b.$$

$$a = c \cos B + b \cos C$$



Area of the ΔABC

In ΔABC , area of the Δ is

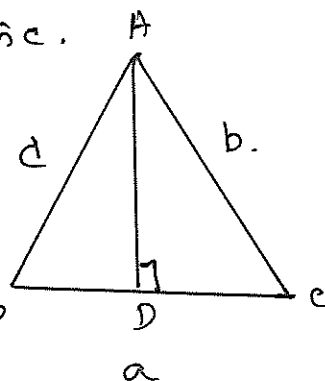
$$\Delta = \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B = \frac{1}{2} ab \sin C.$$

Proof: In ΔABC Draw $AD \perp BC$.

$$\frac{AD}{AC} = \sin C, \Rightarrow AD = b \sin C$$

$$\Delta = \frac{1}{2} b h \quad [b = \text{base}, h = \text{height}]$$

$$\Delta = \frac{1}{2} a \cdot b \sin C$$

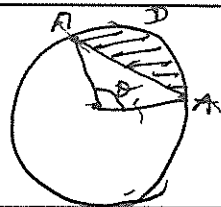


Note: 1) $\Delta = \frac{1}{2} ab \sin C$ says that the area is equal to one half of the product of two sides and the sine of their included angle.

2) Area of the segment $ABD = \text{Area of the sector} - \text{Area of the } \Delta OAB$

$$= \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin \theta$$

$$= \frac{1}{2} r^2 (\theta - \sin \theta)$$



Half angle formula: In ΔABC

$$1) \sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}} \quad 2) \cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}} \quad 3) \tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

Where s is the semi perimeter

Proof: $\sin \frac{A}{2} = \sqrt{\sin^2 \frac{A}{2}} = \sqrt{\frac{1 - \cos A}{2}} = \sqrt{\frac{1}{2} \left(1 - \frac{b^2 + c^2 - a^2}{2bc}\right)}$

$$= \sqrt{\frac{2abc - b^2 - c^2 + a^2}{4bc}} = \sqrt{\frac{a^2 - (b-c)^2}{4bc}}$$

$$= \sqrt{\frac{(a+b-c)(a-b+c)}{4bc}}$$

$$= \sqrt{\frac{(a+b+c-2c)(a+b+c-2b)}{4bc}}$$

$$= \sqrt{\frac{2(s-c)2(s-b)}{4bc}}$$

$$\sin \frac{A}{2} = \sqrt{\frac{(s-c)(s-b)}{bc}}$$

Note $\sin A = \frac{2}{bc} \sqrt{s(s-a)(s-b)(s-c)}$

Area of the Δ (Heron's Formula)

In ΔABC $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ where s is the semi perimeter of ΔABC

Proof: $\Delta = \frac{1}{2} ab \sin C = \frac{1}{2} ab \left(2 \sin \frac{C}{2} \cos \frac{C}{2}\right)$

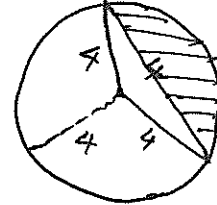
$$= ab \sqrt{\frac{(s-a)(s-b)}{ab}} \sqrt{\frac{s(s-c)}{ab}}$$

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

1) The government plans to have a circular zoological park of diameter 8 km. A separate area in the form of a segment formed by chord of length 4 km is to be allotted exclusively for a veterinary hospital in the park. Find the area of the segment to be allotted for the veterinary hospital.

$$\text{Area of the sector} = \frac{1}{2} r^2 (\theta - \sin \theta)$$

$\because \Delta$ is equilateral $\therefore \theta = \frac{\pi}{3}$



$$\begin{aligned} \text{Area of the sector} &= \frac{1}{2} 16 \left(\frac{\pi}{3} - \sin \frac{\pi}{3} \right) \\ &= 8 \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right) \\ &= \frac{8}{6} (2\pi - 3\sqrt{3}) \text{ m}^2 \end{aligned}$$

2) In a ΔABC P.T $b^2 \sin^2 C + c^2 \sin^2 B = 2bc \sin^2 A$.

TBP: Sol: We know $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$.

$$\therefore a = 2R \sin A, \quad b = 2R \sin B, \quad c = 2R \sin C.$$

$$\begin{aligned} \text{LHS} \quad b^2 \sin^2 C + c^2 \sin^2 B &= 4R^2 \sin^2 B \sin^2 C + 4R^2 \sin^2 C \sin^2 B \\ &= 4R^2 \sin^2 B \cdot 2 \sin C \cos C + 4R^2 \sin^2 C \cdot 2 \sin B \cos B \\ &= 8R^2 \sin B \sin C (\sin B \cos C + \cos B \sin C) \\ &= 8R^2 \sin B \sin C \cdot \sin(B+C) \\ &= 8R^2 \sin B \sin C \sin(\pi - A) \\ &= 8R^2 \sin C \sin B \sin A \\ &= 8R^2 \left(\frac{c}{2R} \cdot \frac{b}{2R} \right) \sin A \\ &= 2abc \sin A. \end{aligned}$$

3) Using sine formula prove projection formula (or)

TBP P.T $a = b \cos C + c \cos B$ by using sine formula.

Sol: Procedure
Sine formula $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$.

$$\text{RHS: } b \cos C + c \cos B$$

$$= 2R \sin B \cos C + 2R \sin C \cos B$$

$$= 2R (\sin B \cos C + \sin C \cos B)$$

$$= 2R \sin(B+C)$$

$$= 2R \sin(180 - A) = 2R \sin A = a.$$

$$(or) \quad b \cos C + c \cos B = a$$

4) using cosine formula derive projection formula.
TBP (e) P.T $a = b \cos C + c \cos B$ by using cosine formula.

Sol: Procedure: $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$, $\cos B = \frac{c^2 + a^2 - b^2}{2ca}$

LHS: $b \cos C + c \cos B$

$$= b \left(\frac{a^2 + b^2 - c^2}{2ab} \right) + c \left(\frac{c^2 + a^2 - b^2}{2ca} \right)$$

$$= \frac{a^2 + b^2 - c^2}{2a} + \frac{c^2 + a^2 - b^2}{2a}$$

$$= \frac{2a^2}{2a} = a.$$

5) using projection formula prove cosine formula.

(e) P.T $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ by using projection formula.

Sol: Projection formula

$$a = b \cos C + c \cos B \quad \text{--- (1)}$$

$$b = c \cos A + a \cos C \quad \text{--- (2)}$$

$$c = a \cos B + b \cos A \quad \text{--- (3)}$$

$$\text{(1)} \times a + \text{(2)} \times b + \text{(3)} \times (-c)$$

$$a^2 = a b \cos C + a c \cos B.$$

$$b^2 = b c \cos A + b a \cos C$$

$$-c^2 = -c a \cos B - c b \cos A$$

$$a^2 + b^2 - c^2 = 2ab \cos C$$

$$\frac{a^2 + b^2 - c^2}{2ab} = \cos C.$$

6) TBPPT $a \sin(A_2 + B) = (b + c) \sin A_2$

Sol: Procedure: $\frac{b}{a} = \frac{\sin B}{\sin A}$, $\frac{c}{a} = \frac{\sin C}{\sin A}$ and use S, D formula.

$$\sin A = 2 \sin A_2 \cos A_2$$

$$\text{RHS} = (b + c) \sin A_2 = a \left(\frac{b}{a} + \frac{c}{a} \right) \sin A_2$$

$$= a \left(\frac{\sin B}{\sin A} + \frac{\sin C}{\sin A} \right) \sin A_2$$

$$= a \left(\frac{2 \sin \left(\frac{B+C}{2} \right) \cos \left(\frac{B-C}{2} \right)}{2 \sin A_2 \cos A_2} \right) \sin A_2$$

$$\sin B + \sin C = 2 \sin \frac{B+C}{2} \cos \frac{B-C}{2}$$

$$\sin A = 2 \sin A_2 \cos A_2$$

$$\sin \frac{B+C}{2} = \sin \left(\frac{180-A}{2} \right) = \cos A_2$$

$$= \frac{a \cdot \cancel{2 \cos \frac{C}{2}} \cos \frac{B-C}{2}}{2 \cancel{\cos \frac{C}{2}}}$$

$$C = 180 - (A+B)$$

$$\begin{aligned} &= a \cos \frac{B-C}{2} = a \cos \left(\frac{B - (180 - A + B)}{2} \right) \\ &= a \cos (B - 180 + A + B) \\ &= a \cos \left(\frac{A + 2B - 180}{2} \right) \\ &= a \cos \left(\frac{A}{2} + B - 90 \right) \\ &= a \cos (90 - (\frac{A}{2} + B)) \\ &= a \sin (\frac{A}{2} + B) \end{aligned}$$

8) P.T. $a(\cos B + \cos C) = 2(b+c) \sin^2 \frac{A}{2}$

TBP

Sol: LHS: $a(\cos B + \cos C)$

$$= 2R \sin A \left(2 \cos \frac{B+C}{2} \cos \frac{B-C}{2} \right)$$

$$= 4R \cdot 2 \sin \frac{A}{2} \cos \frac{A}{2} \cdot \cos \frac{B+C}{2} \cos \frac{B-C}{2}$$

$$= 8R \sin \frac{A}{2} \cos \frac{A}{2} \cos \left(\frac{\pi}{2} - \frac{A}{2} \right) \cos \frac{B-C}{2}$$

$$= 8R \sin \frac{A}{2} \cos \frac{A}{2} \sin \frac{A}{2} \cos \frac{B-C}{2}$$

$$= 4R \sin^2 \frac{A}{2} \left(2 \cos \frac{A}{2} \cos \frac{B-C}{2} \right)$$

$$= 4R \sin^2 \frac{A}{2} \left(2 \cos \left(\frac{\pi}{2} - \frac{B+C}{2} \right) \cos \frac{B-C}{2} \right)$$

$$= 4R \sin^2 \frac{A}{2} \left(2 \sin \frac{B+C}{2} \cos \frac{B-C}{2} \right)$$

$$= 4R \sin^2 \frac{A}{2} [\sin B + \sin C]$$

$$= 2 \sin^2 \frac{A}{2} (2R \sin B + 2R \sin C)$$

$$= 2 \sin^2 \frac{A}{2} (b+c)$$

9) P.T. $a \cos A + b \cos B + c \cos C = 2a \sin B \sin C$

TBP

Sol: LHS: $a \cos A + b \cos B + c \cos C$

Use $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

$$= 2R \sin A \cos A + 2R \sin B \cos B + 2R \sin C \cos C$$

$$= R [\sin 2A + \sin 2B + \sin 2C]$$

$$= R [4 \sin A \sin B \sin C] \quad \text{①}$$

$$= 2 [2R \sin A \sin B \sin C]$$

$$= 2a \sin B \sin C$$

10) In a ΔABC if $\cos C = \frac{\sin A}{2 \sin B}$ S.T the triangle is isosceles.
TBD

Sol: Use $\frac{a}{b} = \frac{\sin A}{\sin B}$ and cosine formula.

$$\text{Given } \cos C = \frac{\sin A}{2 \sin B} = \frac{a}{2b}.$$

$$\frac{a^2 + b^2 - c^2}{2ab} = \frac{a}{2b}$$

$$a^2 + b^2 - c^2 = a^2$$

$$b^2 = c^2 \Rightarrow b = c \therefore \text{The triangle is isosceles.}$$

11) In a ΔABC P.T $\frac{\sin B}{\sin C} = \frac{c - a \cos B}{b - a \cos C}$.
TBP

Sol: apply projection formula $c = a \cos B + b \cos A$ $b = 2R \sin B$
 $b = c \cos A + a \cos C$ $c = 2R \sin C$

$$\text{RHS: } \frac{c - a \cos B}{b - a \cos C} = \frac{a \cos B + b \cos A - a \cos B}{c \cos A + a \cos C - a \cos C}$$

$$= \frac{b \cos A}{c \cos A} = \frac{b}{c} = \frac{2R \sin B}{2R \sin C} = \frac{\sin B}{\sin C}.$$

12) In a ΔABC $\frac{a+b}{a-b} = \tan \frac{A+B}{2} \cot \frac{A-B}{2}$
TBP

Sol: Procedure: Use sine formula and c, d formula.

$$\text{LHS: } \frac{a+b}{a-b} = \frac{2R \sin A + 2R \sin B}{2R \sin A - 2R \sin B}$$

$$= \frac{2R (\sin A + \sin B)}{2R (\sin A - \sin B)} = \frac{2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}}{2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}}$$

$$= \tan \frac{A+B}{2} \cot \frac{A-B}{2}$$

13) P.T $\frac{a \sin(B-C)}{b^2 - c^2} = \frac{b \sin(C-A)}{c^2 - a^2} = \frac{c \sin(A-B)}{a^2 - b^2}$
In any Δ .

Sol: Use sine formula

$$\text{Consider } \frac{a \sin(B-C)}{b^2 - c^2} = \frac{2R \sin A \sin(B-C)}{4R^2 \sin^2 B - 4R^2 \sin^2 C}$$

$$= \frac{2R \sin(\pi - B + C) \sin(B - C)}{2R^2 (\sin^2 B - \sin^2 C)}$$

$$= \frac{1}{2R} \frac{\sin(B + C) \sin(B - C)}{\sin^2 B - \sin^2 C}$$

$$= \frac{1}{2R} \frac{(\sin^2 B - \sin^2 C)}{(\sin^2 B - \sin^2 C)} = \frac{1}{2R}$$

11)ly

we can prove that $\frac{b \sin(C - A)}{c^2 - a^2} = \frac{c \sin(A - B)}{a^2 - b^2} = \frac{1}{2R}$

$$\therefore \frac{a \sin(B - C)}{b^2 - c^2} = \frac{b \sin(C - A)}{c^2 - a^2} = \frac{c \sin(A - B)}{a^2 - b^2} = \frac{1}{2R}$$

14) In any $\triangle ABC$ P.T $\frac{a^2 - c^2}{b^2} = \frac{\sin(A - C)}{\sin(A + C)}$
TBP.

Sol: Procedure: Sine Formula.

$$\text{LHS: } \frac{a^2 - c^2}{b^2} = \frac{4R^2 \sin^2 A - 4R^2 \sin^2 C}{4R^2 \sin^2 B}$$

$$= \frac{4R^2 (\sin^2 A - \sin^2 C)}{4R^2 \sin^2 B}$$

$$= \frac{\sin(A + C) \sin(A - C)}{\sin^2 B}$$

$$= \frac{\sin(\pi - B) \sin(A - C)}{\sin^2 B}$$

$$= \frac{\sin B \cdot \sin(A - C)}{\sin^2 B}$$

$$= \frac{\sin(A - C)}{\sin(\pi - A + B)} = \frac{\sin(A - C)}{\sin(A + C)}$$

15) If in a $\triangle ABC$ $\frac{\sin A}{a} = \frac{\sin(A - B)}{\sin(B - C)}$ P.T a^2, b^2, c^2 are in A.P.
TBP

Sol: Procedure Use $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$

$$\therefore \frac{\sin A}{\sin C} = \frac{\sin(A - B)}{\sin(B - C)}$$

$$\frac{\sin(\pi - B + C)}{\sin(\pi - A + B)} = \frac{\sin(A - B)}{\sin(B - C)}$$

$$\frac{\sin(B + C)}{\sin(A + B)} = \frac{\sin(A - B)}{\sin(B - C)}$$

$$\sin(B + C) \sin(B - C) = \sin(A + B) \sin(A - B)$$

$$\sin^2 B - \sin^2 C = \sin^2 A - \sin^2 B$$

$$kb^2 - ka^2 = ka^2 - kb^2$$

$$b^2 - c^2 = a^2 - b^2$$

$$2b^2 = a^2 + c^2 \Rightarrow a^2, b^2, c^2 \text{ are in AP.}$$

16) The angles of a triangle ABC, are in A.P and if $b:c = \sqrt{3}:\sqrt{2} \sin A$.

TBP) Sol: $\because A, B, C$ are in AP $2B = A + C$ and Sine formula.

In any Δ $A + B + C = 180^\circ$

$$2B + B = 180$$

$$3B = 180^\circ$$

$$B = 60^\circ$$

Given $b:c = \sqrt{3}:\sqrt{2}$

But $\frac{b}{\sin B} = \frac{c}{\sin C}$.

$$\frac{\sqrt{3}}{\sin 60} = \frac{\sqrt{2}}{\sin C} \Rightarrow \frac{\sqrt{3}}{\sqrt{3}/2} = \frac{\sqrt{2}}{\sin C}$$

$$2 = \frac{\sqrt{2}}{\sin C}$$

$$\sin C = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow C = 45^\circ$$

$$A + B + C = 180$$

$$A + 60 + 45 = 180 \therefore A = 75^\circ$$

17) In any ΔABC , P.T $b^2 \sin 2C + c^2 \sin 2B = 2bc \sin A$.

TBP

Sol: Procedure: Use Sine formula

$$\begin{aligned} \text{LHS: } b^2 \sin 2C + c^2 \sin 2B &= 4R^2 \sin^2 B \sin 2C + 4R^2 \sin^2 C \sin 2B \\ &= 4R^2 (2 \sin^2 B \sin C \cos C + 2 \sin^2 C \sin B \cos B) \\ &= 8R^2 \sin B \sin C (\sin B \cos C + \cos B \sin C) \\ &= 8R^2 \sin B \sin C \cdot \sin (B+C) \\ &= 8R^2 \sin B \sin C \sin (\pi - A) \\ &= 8R^2 \sin B \sin C \sin A \\ &= 2 \cdot 2R \sin B \cdot 2R \sin C \cdot \sin A \\ &= 2bc \sin A \end{aligned}$$

18) In a ΔABC P.T $\sin \left(\frac{B-C}{2} \right) = \frac{b-c}{a} \cos A/2$

TBP

Sol: Procedure: Use Sine formula and c, D formula, 2A formula

$$\begin{aligned} \text{RHS: } \frac{b-c}{a} \cos A/2 &= \frac{2R \sin B - 2R \sin C}{2R \sin A} \cdot \cos A/2 \\ &= \frac{2R (\sin B - \sin C)}{2R \sin A} \cdot \cos A/2 \end{aligned}$$

$$= \frac{2 \cos \frac{B+C}{2} \sin \frac{B-C}{2} \cdot \cos \frac{A}{2}}{2 \sin \frac{A}{2} \cos \frac{A}{2}}$$

$$= \frac{\cos \left(\frac{\pi}{2} - \frac{A}{2} \right) \cdot \sin \frac{B-C}{2}}{\sin \frac{A}{2}}$$

$$= \frac{\cancel{\sin \frac{A}{2}} \sin \frac{B-C}{2}}{\cancel{\sin \frac{A}{2}}} = \sin \frac{B-C}{2}$$

19) If the three angles in a triangle are in the ratio 1:2:3 then p.t the TBP corresponding sides are in the ratio 1:√3:2.

Sol: Procedure use Sine formula.

If angles are $\theta, 2\theta, 3\theta$

$$\theta + 2\theta + 3\theta = 180^\circ$$

$$\theta = 30^\circ$$

We know $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$\frac{a}{\sin 30^\circ} = \frac{b}{\sin 60^\circ} = \frac{c}{\sin 90^\circ}$$

$$\Rightarrow a:b:c = \sin 30^\circ : \sin 60^\circ : \sin 90^\circ$$

$$= \frac{1}{2} : \frac{\sqrt{3}}{2} : 1$$

$$= 1 : \sqrt{3} : 2$$

20) In a $\triangle ABC$ p.t $(b+c)\cos A + (c+a)\cos B + (a+b)\cos C = a+b+c$
TBP

Sol: Procedure: use cosine formula, arrange last steps as $a+b+c$.

$$\text{LHS} = (b+c)\cos A + (c+a)\cos B + (a+b)\cos C$$

$$= b\cos A + c\cos A + c\cos B + a\cos B + a\cos C + b\cos C$$

$$= (b\cos C + c\cos B) + (c\cos A + a\cos C) + (a\cos B + b\cos A)$$

$$= a+b+c.$$

21) In a $\triangle ABC$ p.t $\frac{a^2+b^2}{a^2+c^2} = \frac{1+\cos(A-B)\cos C}{1+\cos(A-C)\cos B}$
TBP

Sol: Procedure use Sine formula.

$$\text{LHS} = \frac{a^2+b^2}{a^2+c^2} = \frac{4R^2 \sin^2 A + 4R^2 \sin^2 B}{4R^2 \sin^2 A + 4R^2 \sin^2 C} = \frac{4R^2 (\sin^2 A + \sin^2 B)}{4R^2 (\sin^2 A + \sin^2 C)}$$

$$= \frac{1 - \cos^2 A + \sin^2 B}{1 - \cos^2 A + \sin^2 C}$$

$$\begin{aligned}
 &= \frac{1 - (\cos^2 A - \sin^2 B)}{1 - (\cos^2 A - \sin^2 C)} \\
 &= \frac{1 - \cos(A+B) \cdot \cos(A-B)}{1 - \cos(A+C) \cos(A-C)} \\
 &= \frac{1 - \cos(\pi - C) \cos(A-B)}{1 - \cos(\pi - B) \cos(A-C)} \\
 &= \frac{1 + \cos(A-B) \cos C}{1 + \cos(A-C) \cos B}
 \end{aligned}$$

22) _{TBP} Derive cosine formula using sine formula.

Sol: we know $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$.

$$\begin{aligned}
 \frac{b^2 + c^2 - a^2}{2bc} &= \frac{4R^2 \sin^2 B + 4R^2 \sin^2 C - 4R^2 \sin^2 A}{2 \cdot 2R \sin B \cdot 2R \sin C} \\
 &= \frac{4R^2 (\sin^2 B + \sin^2 C - \sin^2 A)}{2 \cdot 2R^2 \cdot \sin B \sin C} \\
 &= \frac{\sin^2 B + \sin^2 C - \sin^2 A}{2 \sin B \sin C} \\
 &= \frac{\sin^2 B + \sin^2(\pi - B) \sin^2(\pi - A)}{2 \sin B \sin C} \\
 &= \frac{\sin^2 B + \sin^2 B \cdot \sin^2(\pi - A)}{2 \sin B \sin C} \\
 &= \frac{\sin B (\sin B + \sin^2(\pi - A))}{2 \sin B \sin C} \\
 &= \frac{\sin(\pi - (\pi - A)) + \sin(\pi - A)}{2 \sin C} \\
 &= \frac{\sin(\pi - (\pi - A)) + \sin(\pi - A)}{2 \sin C} \\
 &= \frac{2 \sin C \cos A}{2 \sin C} = \cos A
 \end{aligned}$$

23) using Heron's formula s.t the equilateral Δ has the maximum area for any fixed perimeter
TBP.

Sol: Procedure: ^{use} If $xyz \leq k$ maximum occurs when $x = y = z$.
 Let $\triangle ABC$ be a triangle with constant perimeter $2s$. $\therefore s$ is const.

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

Δ is maximum when $(s-a)(s-b)(s-c)$ is maximum.

$$(ii) (s-a)(s-b)(s-c) \leq \left(\frac{(s-a) + (s-b) + (s-c)}{3} \right)^3 = \frac{s^3}{27}$$

$$(s-a)(s-b)(s-c) \leq \frac{s^3}{27} \quad \because GM \leq AM.$$

(iii) Equality occurs when $s-a = s-b = s-c$

(iv) when $a = b = c$ max of $(s-a)(s-b)(s-c) = \frac{s^3}{27}$

$\therefore$ For fixed perimeter $2s$, the area of the $\triangle$ is max when $a = b = c$.

$$\therefore \text{Max area} = \sqrt{\frac{s \cdot s^3}{27}} = \frac{s^2}{3\sqrt{3}} \text{ sq. units.}$$

24) An Engineer has to develop a triangular shaped park with a perimeter 120 m in a village. The park to be developed must be maximum TBP area. Find out the dimension of the park.

Sol: Perimeter of the $\triangle$ = 120 m.

(X) For a fixed perimeter $2s$, the area of the triangle is maximum when $a = b = c$.

$$\therefore \text{Side of the } \triangle = \frac{120}{3} = 40 \text{ m.}$$

$$a = b = c = 40 \text{ m.}$$

25) A rope of length 12 m is given. Find the largest area of the triangle TBP formed by this rope and find the dimensions of the triangle so formed.

Sol: Perimeter of the $\triangle$ = 12 m.

(X) For a fixed perimeter $2s$ the area of the triangle is maximum when $a = b = c$.

$$\therefore \text{Side of the } \triangle = \frac{12}{3} = 4$$

$$\therefore a = b = c = 4.$$

$$\text{Area of the } \triangle = \frac{s^2}{3\sqrt{3}}$$

$$\boxed{2s = 12}$$

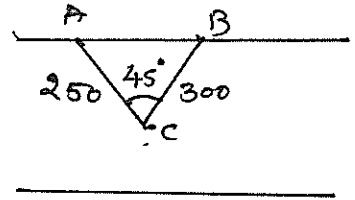
$$\boxed{s = 6}$$

$$= \frac{36^2}{3\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3} \text{ sq. mts}$$

- 26) Two trees A and B are on the same side of a river. From a point C in the river the distance of trees A and B are 250m and 300m respectively. The angle C is 45° . Find the distance between the trees. $\sqrt{2}: 1.414$.

Sol: Procedure: Use cosine formula.

$$\begin{aligned}
 AB^2 &= AC^2 + BC^2 - 2AB \cdot AC \cdot \cos 45^\circ \\
 &= 250^2 + 300^2 - 2 \times 250 \times 300 \times \frac{1}{\sqrt{2}} \\
 &= 62500 + 90000 - 75000\sqrt{2} \\
 &= 62500 + 90000 - 75000 \times 1.414 \\
 &= 152500 - 106050 \\
 &= 46450 \\
 AB &= \sqrt{46450} = 215.52 \text{ m.}
 \end{aligned}$$



- 27) A tree stands vertically on a hill side which makes an angle 15° with the horizontal. From a point on the ground 35 m down the hill from the base of the tree. The angle of elevation of the top of the tree is 60° . Find the height of the tree.

Sol: Let PA be the tree AR be the Ground

In $\triangle ARQ$

$$\angle RAQ = 60^\circ \quad \angle ARQ = 90^\circ \quad \angle AQP$$

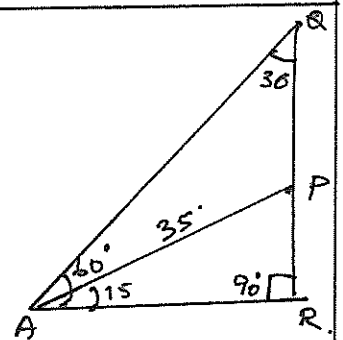
$$\text{In } \triangle APQ \quad \angle PAQ = 60 - 15 = 45^\circ$$

$$\text{By Sine Rule: } \frac{35}{\sin 30} = \frac{PQ}{\sin 45}$$

$$= \frac{35}{\frac{1}{2}} = \frac{PQ}{\frac{1}{\sqrt{2}}}$$

$$\therefore \frac{70}{\frac{1}{\sqrt{2}}} = PQ$$

$$\therefore PQ = \frac{70 \times \sqrt{2}}{\frac{1}{\sqrt{2}}} = 35\sqrt{2} \text{ m.}$$



28) Two ships leave a port at the same time. One goes 24 km/hr in the direction $N 45^\circ E$ and the other travels 32 km/hr in the direction $S 75^\circ E$. Find the distance between the ships at the end of 3 hrs.

Sol: Procedure: Use cosine formula.

$$OP = 24 \times 3 = 72 \text{ km}$$

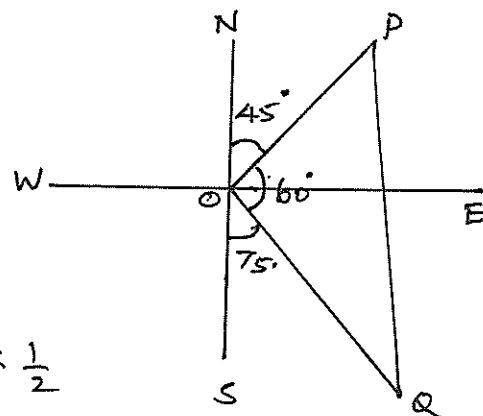
$$OQ = 32 \times 3 = 96 \text{ km.}$$

By cosine formula

$$\begin{aligned} PQ^2 &= OP^2 + OQ^2 - 2 \cdot OP \cdot OQ \cdot \cos 60^\circ \\ &= 72^2 + 96^2 - 2 \times 72 \times 96 \times \frac{1}{2} \\ &= 5184 + 9216 - 6912 \end{aligned}$$

$$= 7488$$

$$PQ = \sqrt{7488} = 86.53 \text{ km.}$$



Solutions of 51e

1. In ΔABC $a=3$, $b=5$, $c=7$. Find the values of $\cos A$, $\cos B$, $\cos C$
TBP

Sol: $a=3$, $b=5$, $c=7$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{25 + 49 - 9}{70} = \frac{65}{70} = \frac{13}{14}$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ca} = \frac{49 + 9 - 25}{2 \times 7 \times 3} = \frac{33}{42} = \frac{11}{14}$$

$$\text{likh} \cos C = -\frac{1}{2}$$

2. In ΔABC $A=30^\circ$, $B=60^\circ$ and $c=10$ Find a and b .
TBP

Sol: $A=30^\circ$, $B=60^\circ$ $c=10$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\begin{aligned} C &= 180 - (30 + 60) \\ &= 90^\circ \end{aligned}$$

$$\frac{a}{\sin 30^\circ} = \frac{b}{\sin 60^\circ} = \frac{10}{\sin 90^\circ}$$

$$\frac{a}{\frac{1}{2}} = \frac{b}{\frac{\sqrt{3}}{2}} = 10 \Rightarrow a=5$$

$$b = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}$$

3) In ΔABC , $a=2\sqrt{2}$, $b=2\sqrt{3}$, $C=75^\circ$ find the other sides and angles.
TBP

Sol: $a = 2\sqrt{2}$, $b = 2\sqrt{3}$, $c = 75^\circ$

$$\cos c = \frac{a^2 + b^2 - c^2}{2ab} \Rightarrow \cos 75^\circ = \frac{8 + 12 - c^2}{8\sqrt{6}}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{20-c^2}{4\sqrt{6}\sqrt{3}}$$

$$\frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{20-c^2}{4\sqrt{6}\sqrt{3}}$$

$$4\sqrt{3}(\sqrt{3}-1) = 20-c^2$$

$$c^2 = 20 - (12 - 4\sqrt{3})$$

$$= 8 + 4\sqrt{3} = 4(\sqrt{3}+2)$$

$$c^2 = 20 - 12 + 4\sqrt{3}$$

$$= 8 + 4\sqrt{3}$$

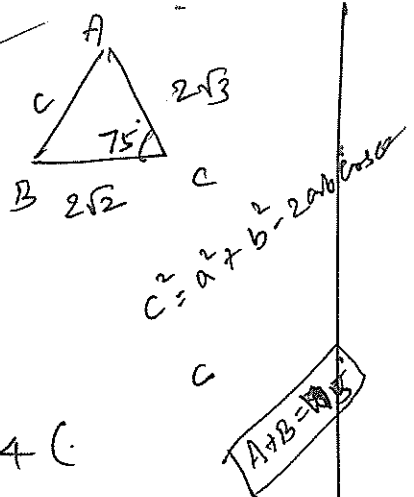
$$= 4(2 + \sqrt{3}) = 2c$$

$$(\sqrt{3}+2)^2 = 2 \times 2\sqrt{3}$$

$$\frac{(\sqrt{3}+2)^2}{2+6+2\sqrt{12}} = \frac{8+4\sqrt{3}}{8+4\sqrt{3}}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{12 + 8 + 4\sqrt{3} - 8}{2 \cdot 2\sqrt{3}}$$



4) Find the area of the Δ whose sides are 13 cm, 14 cm, 15 cm

TBP: Sol: $2s = a + b + c \Rightarrow s = \frac{13 + 14 + 15}{2} = 21$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{21 \times 8 \times 7 \times 6}$$

$$= \sqrt{7 \times 3 \times 4 \times 2 \times 7 \times 3 \times 2}$$

$$= 7 \times 3 \times 2 \times 2 = 84 \text{ sq. cm.}$$

5) In any Δ ABC P.T $a \cos A + b \cos B + c \cos C = \frac{8\Delta^2}{abc}$.

TBP Sol: We know $a \cos A + b \cos B + c \cos C = 2a \sin B \sin C$.

$$\text{LHS: } a \cos A + b \cos B + c \cos C = 2a \sin B \sin C$$

$$= 2a \cdot \left(\frac{2\Delta}{ca}\right) \left(\frac{2\Delta}{ab}\right)$$

$$= \frac{8\Delta^2}{abc}$$

b) Suppose that there are two cell phone towers within range of cell phone. Two towers are located at 6 km apart along a straight highway running east to west and the cell phone is north of the highway. The signal is 5 km from the first tower and $\sqrt{31}$ km from the second tower. Determine the cell phone north and east of the first tower and how far it is from the highway.

Sol: Let θ be the position of the cell phone from north to east of the first tower.

By cosine formula.

$$(\sqrt{31})^2 = 25 + 36 - 2 \times 5 \times 6 \cos \theta$$

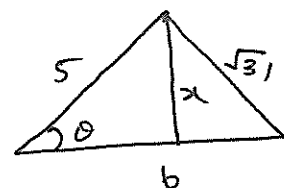
$$60 \cos \theta = 61 - 31$$

$$= 30$$

$$\cos \theta = \frac{30}{60} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

Let x be the distance of the cell phone's position from the highway

$$\begin{aligned} \sin \theta &= \frac{x}{5} \Rightarrow x = 5 \sin 60 \\ &= 5 \cdot \frac{\sqrt{3}}{2} \text{ KM.} \end{aligned}$$



7) Suppose that a boat travels 10 km from the port towards north and TBP then turns 60° to its left. If the boat travels further 8 km, how far from the port is the boat.

Sol: By using cosine formula

$$BP^2 = 10^2 + 8^2 - 2 \cdot 10 \cdot 8 \cos 120$$

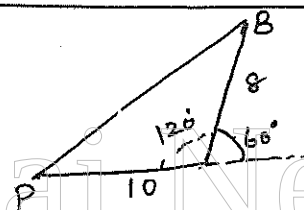
$$= 100 + 64 - 160 (-\cos \frac{\pi}{3})$$

$$= 164 + 160 \times \frac{1}{2}$$

$$= 244$$

$$= 4 \times 61$$

$$BP = 2\sqrt{61} \text{ KM.}$$



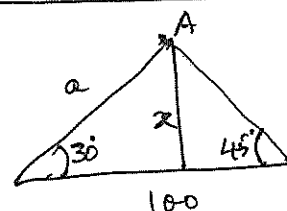
8) Suppose two radar station located 100 km apart, each detect a fighter aircraft between them. The angle of elevation measured by the first TBP station is 30° whereas the angle of elevation measured by the second station is 45° . Find the altitude of the aircraft at the instant.

$$\text{Sol: } \angle A = 180 - (30 + 45) = 105^\circ$$

$$\frac{a}{\sin A} = \frac{100}{\sin 105} \Rightarrow \frac{100}{\frac{\sqrt{3}+1}{2\sqrt{2}}} \times \frac{1}{\sqrt{2}}$$

$$= \frac{200(\sqrt{3}-1)}{2} = 100(\sqrt{3}-1)$$

$$\sin 30^\circ = \frac{x}{a} \Rightarrow x = \frac{100(\sqrt{3}-1)}{2} \cdot \frac{1}{2} = 50(\sqrt{3}-1) \text{ KM.}$$



9) If the sides of $\triangle ABC$ are $a=4$, $b=6$, $c=8$ then S.T $4\cos B + 3\cos C = 2$

TBP

$$\text{Sol: } \cos B = \frac{c^2 + a^2 - b^2}{2ca} = \frac{64 + 16 - 36}{2 \cdot 8 \cdot 4}$$

$$= \frac{44}{64} = \frac{11}{16}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{16 + 36 - 64}{2 \times 4 \times 6} = \frac{-12}{2 \times 4 \times 6} = -\frac{1}{4}$$

$$\text{H.S} = 4 \cos B + 3 \cos C = 4 \times \frac{11}{16} + 3 \times \left(-\frac{1}{4}\right) = \frac{11}{4} - \frac{3}{4} = \frac{8}{4} = 2$$

10) In a ΔABC $a = \sqrt{3}-1$, $b = \sqrt{3}+1$, $C = 60^\circ$ Find the other side and other TBP two angles.

$$\text{Sol: } a = \sqrt{3}-1 \quad b = \sqrt{3}+1 \quad c = 60^\circ$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$= (\sqrt{3}-1)^2 + (\sqrt{3}+1)^2 - 2(\sqrt{3}+1)(\sqrt{3}-1) \cos 60^\circ$$

$$= 2(3+1) - 2(3-1) \cdot \frac{1}{2}$$

$$= 8 - 2$$

$$= 6$$

$$c = \sqrt{6}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin 60^\circ}$$

$$\frac{\sqrt{3}+1}{\sin B} = \frac{\sqrt{6} \cdot \sqrt{3}}{\sqrt{3}/2} = \sqrt{2} \cdot 2$$

$$\sin B = \frac{\sqrt{3}+1}{2\sqrt{2}} \Rightarrow B = 105^\circ$$

$$\frac{\sqrt{3}-1}{\sin A} = \frac{\sqrt{6}}{\sqrt{3}/2} = \frac{\sqrt{3} \cdot \sqrt{2} \times 2}{\sqrt{3}}$$

$$\sin A = \frac{\sqrt{3}-1}{2\sqrt{2}} \Rightarrow A = 15^\circ$$

11) In a ΔABC P.T if $a = 12$, $b = 8$, $C = 30^\circ$ area is 24 sq. cm.
TBP

$$\text{Sol: } \Delta = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} \cdot 12 \cdot 8 \sin 30^\circ$$

$$= \frac{1}{2} \cdot 12 \cdot 8 \cdot \frac{1}{2} = 24 \text{ sq. cm.}$$

12) In a ΔABC , $a = 18$, $b = 24$ cm, $c = 30$ area of the Δ is 216 sq. cm.
TBP

$$\text{Sol: } s = \frac{18 + 24 + 30}{2} = 36$$

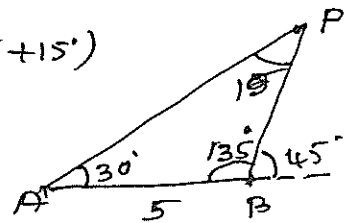
$$\begin{aligned}
 \Delta &= \sqrt{s(s-a)(s-b)(s-c)} \\
 &= \sqrt{36 \times 18 \times 12 \times 6} \\
 &= \sqrt{36 \times 9 \times 2 \times 2 \times 6 \times 6} \\
 &= 6 \times 3 \times 2 \times 6 \\
 &= 216 \text{ sq. cm.}
 \end{aligned}$$

- 13) Two soldiers A and B in two different underground bunkers on a straight road, spot an intruder at the top of a hill. The angle of elevation of the intruder from A and B to the ground level in the same direction are 30° and 45° resp. If A and B stand 5 km apart find the distance of the intruder from B.

Sol: From the diag. $\angle P = \theta = 180 - (135 + 15^\circ)$
 $= 15^\circ$

$$\frac{5}{\sin \theta} = \frac{x}{\sin 30^\circ}$$

$$\frac{5}{\sin 75^\circ} = \frac{x}{\frac{1}{2}} \Rightarrow 2x = \frac{5}{\sin 15^\circ}$$

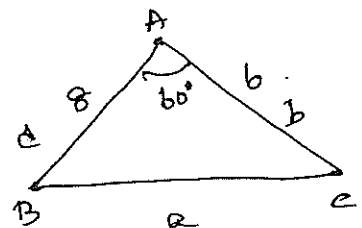


$$\begin{aligned}
 2x &= \frac{5 \times 2\sqrt{2}}{\sqrt{3}-1} \\
 x &= \frac{5\sqrt{2}}{\sqrt{3}-1}
 \end{aligned}$$

- 14) A researcher wants to determine the width of a pond from east to west which cannot be done by actual measurement from a point P. He finds the distance to the eastern most point of the pond to be 8 km while the distance to western most point from P to be 6 km if the angle between the two sight is 60° find the width of the pond.

Sol: $\angle A = 60^\circ$ $b = 6$, $c = 8$

$$\begin{aligned}
 a^2 &= b^2 + c^2 - 2bc \cos A \\
 &= 36 + 64 - 2 \times 6 \times 8 \times \cos 60^\circ \\
 &= 100 - 48 \times \frac{1}{2} \\
 &= 52 \\
 &= 4 \times 13 \\
 a &= 2\sqrt{13} \text{ km.}
 \end{aligned}$$



- 15) Two Navy helicopters A and B are flying over the Bay of Bengal at same altitude from the sea level to search a missing boat TBP. Pilots of Both the helicopters sight the boat at the same time while they are apart 10 km from each other. If the distance of the boat from A is 6 km if the line segment AB subtends 60° at the boat Find the distance of the boat from B.

Sol: $\angle C = 60^\circ$ $b = 6$, $c = 10$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{6}{\sin B} = \frac{10}{\sin 60^\circ} \Rightarrow \sin B = \frac{6 \times \sqrt{3}}{10 \times 2} = \frac{3\sqrt{3}}{10}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = a = b \cos C + c \cos B$$

$$a = 6 \cos 60^\circ + 10 \sqrt{1 - \sin^2 B}$$

$$= 6 \times \frac{1}{2} + 10 \sqrt{\frac{100 - 27}{100}}$$

$$= 3 + 10 \frac{(\sqrt{73})}{10} = 3 + \sqrt{73} \text{ km.}$$

- 16) A straight tunnel is to be made through a mountain. A surveyor observes the two extremities A and B of the tunnel to be built from a point P in front of the mountain TBP. If $AP = 3 \text{ km}$, $BP = 5 \text{ km}$, $\angle APB = 120^\circ$ then find the length of the tunnel to be built.

Sol: $AP = 3$, $BP = 5$ $\angle APB = 120^\circ$

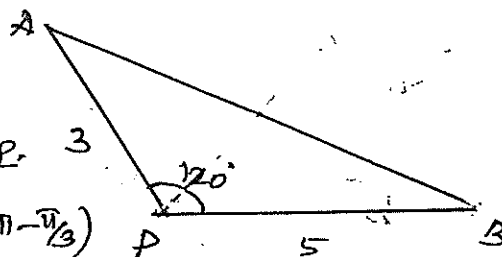
$$AB^2 = AP^2 + BP^2 - 2 \cdot AP \cdot BP \cos 120^\circ$$

$$= 9 + 25 - 2 \cdot 3 \cdot 5 \cos (180^\circ - 60^\circ)$$

$$= 9 + 25 + 2 \cdot 3 \cdot 5 \cdot \frac{1}{2}$$

$$= 49$$

$$AB = 7$$



- 17) A farmer wants to purchase triangular shaped land with sides 120 feet and 80 feet and angle included between these two sides is 60° . If the land costs Rs 500 per sq. feet find the amount need to purchase the land. Also find the perimeter of the land. TBP

Sol: $AB = 120$ $AC = 60$ $\angle A = 60^\circ$

$$A = \frac{1}{2} AB \cdot AC \cdot \sin 60$$

$$= \frac{1}{2} \times 120 \times 60 \times \frac{\sqrt{3}}{2}$$

$$= 1800 \times 1.732 \text{ sq feet}$$

Total cost = $500 \times 1800 \times 1.732$

$$= 900000 \times 1.732$$

$$= 1558800000 \text{ Rs.}$$

$$= 15,58,800$$

$$a^2 = b^2 + c^2 - 2bc \cos 60^\circ$$

$$= 14400 + 3600 - 2 \times 120 \times 60 \cdot \frac{1}{2}$$

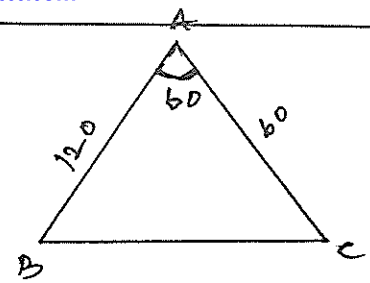
$$= 18000 - 7200$$

$$a^2 = 10800 = 400 \times 27$$

$$a = 20\sqrt{27}$$

∴ Perimeter = $120 + 60 + 20\sqrt{27}$

$$= 180 + 20\sqrt{27} \text{ feet.}$$



- 18) A man starts his morning walk at a point A reaches to point B and C and finally back to A s.t $\angle A = 60^\circ$ $\angle B = 45^\circ$ $AC = 4 \text{ km}$
Find the total distance he covered during his morning walk.

Sol: $b = 4 \text{ km}$ $\angle A = 60$ $\angle B = 45^\circ$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 60} = \frac{4}{\sin 45}$$

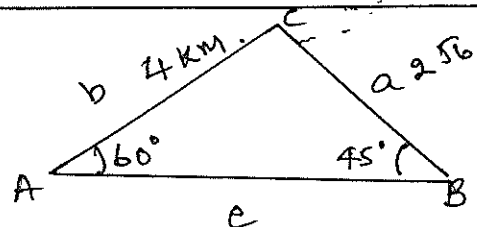
$$\frac{2a}{\sqrt{3}} = \frac{4}{\sqrt{2}}$$

$$a = 2\sqrt{6}$$

Total distance

$$= 4 + 2\sqrt{6} + 2\sqrt{3} + 2$$

$$= 2\sqrt{6} + 2\sqrt{3} + 6$$



$$c = a \cos B + b \cos A$$

$$= 2\sqrt{6} \cdot \cos 45 + 4 \cdot \cos 60$$

$$= 2\sqrt{6} \times \frac{1}{\sqrt{2}} + 4 \times \frac{1}{2}$$

$$= 2\sqrt{3} \times \sqrt{2} + 2$$

$$= 2\sqrt{3} + 2$$

$$= 2(\sqrt{3} + 1)$$

19) A plane is 1 km from one landmark and 2 km from another. From the plane's point of view the land between them subtends an angle 60° . How far apart are landmarks?

Sol: $\angle A = 60^\circ$ $b = 2$, $c = 1$

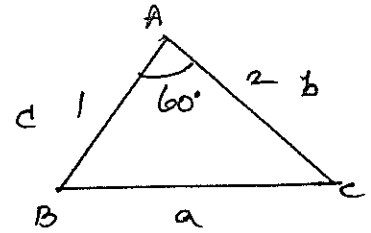
$$a^2 = b^2 + c^2 - 2bc \cos 60^\circ$$

$$= 4 + 1 - 2 \cdot 2 \cdot 1 \cdot \frac{1}{2}$$

$$= 5 - 2$$

$$= 3$$

$$a = \sqrt{3}$$



(If $\theta = 45^\circ$ the book answer is correct).

20) Two vehicles leave the same place P at the same time moving along two different roads. One vehicle moves at an average speed of 60/km per hr. and the other vehicle moves at an average speed of 80/km per hr. After half an hour the vehicles reach the destinations A and B. If AB subtends 60° at the initial point P then find AB.

Sol: $AP = 30$, $PB = 40$ $\angle P = 60^\circ$

By cosine formula

$$AB^2 = AP^2 + PB^2 - 2AP \cdot PB \cos 60^\circ$$

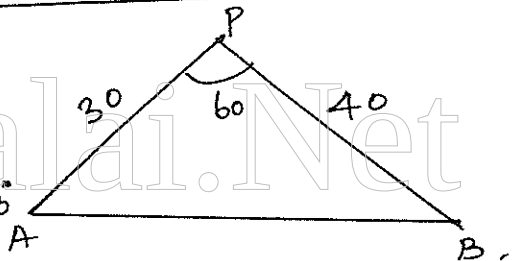
$$= 30^2 + 40^2 - 2 \cdot 30 \cdot 40 \cdot \frac{1}{2}$$

$$= 900 + 1600 - 1200$$

$$= 1300$$

$$= 100 \times 13$$

$$AB = 10\sqrt{13} \text{ km.}$$



(Note: The book answer is correct only when after one hour)

21) In any $\triangle ABC$ that the area $\Delta = \frac{b^2 + c^2 - a^2}{4 \cot A}$.

Sol: RHS :

$$\frac{b^2 + c^2 - a^2}{4 \cot A} = \frac{2bc \cos A}{2 \cdot \frac{\cos A}{\sin A}}$$

$$= \frac{1}{2} bc \sin A$$

$$= \Delta$$

Exercise - 3.2

4) In a circle of diameter 40 cm a chord of length 20 cm. Find the length of the minor arc of the chord.

Sol: diameter of the circle $r = 20$ cm.

Let $AB = 20$ cm

$\therefore$ The Δ is equilateral Δ .

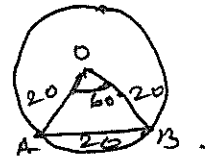
$$\therefore \angle AOB = 60^\circ$$

TBP

$$l = r\theta = 20 \times \frac{\pi}{3}$$

$$= 20 \times \frac{1}{3} \times \frac{22}{7}$$

$$= 20.95 \text{ cm}$$



5) Find the degree measure of the angle subtended at the centre of circle of radius 100 cm by an arc of length 22 cm.

Sol: $r = 100$ cm

Length of the arc $= 11$ cm.

$$\theta = \frac{l}{r} = \frac{22}{100}$$

$$\pi = 180^\circ$$

TBP

$$\theta = 180 \times \frac{22}{100} \times \frac{1}{\pi}$$

$$= \frac{18 \times 7}{10} = 12^\circ \frac{3}{5}$$

$$\approx 12^\circ 36'$$

9) An airplane propeller rotates 1000 times per minute. Find the number of degrees that a point on the edge of the propeller will rotate in one second.

Sol: One complete rotation: 360°

$$(1^{\text{st}}) \text{ number of degrees taken in } 1' = 1000 \times 360^\circ$$

$$60'' = 1000 \times 360^\circ$$

"

$$1'' = \frac{1000 \times 360^\circ}{60}$$

$$= 6000^\circ$$

TBP

"

$$\therefore x = 0, \pi, \frac{3\pi}{2}, \pi.$$

$$2) 2\cos^2 x + 1 = -3\cos x.$$

$$2\cos^2 x + 3\cos x + 1 = 0$$

$$(\cos x + 1)(2\cos x + 1) = 0$$

$$\cos x + 1 = 0 \quad 2\cos x + 1 = 0$$

$$\cos x = -1 \quad 2\cos x = -1$$

$$\cos x = -\frac{1}{2}$$

TBP

$$\text{When } \cos x = -1$$

$$\cos x = \cos(\pi)$$

$$x = \pi$$

$$\text{When } 2\cos x + 1 = 0$$

$$\cos x = -\frac{1}{2}$$

$$x = \pi + \frac{\pi}{3} \text{ or } \pi - \frac{\pi}{3}$$

$$= \frac{4\pi}{3} \text{ (or) } \frac{2\pi}{3}.$$

$$\therefore \text{The values are } x = \pi, \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$3) 2\sin^2 x + 1 = 3\sin x.$$

$$2\sin^2 x + 1 = 3\sin x$$

$$2\sin^2 x - 3\sin x + 1 = 0$$

$$(\sin x - 1)(2\sin x - 1) = 0$$

$$\sin x - 1 = 0$$

$$\sin x = 1$$

$$x = \frac{\pi}{2}$$

$$2\sin x - 1 = 0$$

$$2\sin x = 1$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}$$

$$\therefore \text{The values are } x = \frac{\pi}{6}, \frac{\pi}{2}$$

$$4) \cos 2x = 1 - 3\sin x$$

$$\cos 2x = 1 - 3\sin x$$

$$1 - 2\sin^2 x = 1 - 3\sin x.$$

$$-2\sin^2 x + 3\sin x = 0$$

$$2\sin^2 x = 3\sin x$$

$$\sin x(2\sin x - 3) = 0$$

$$\sin x = 0$$

$$2\sin x = 3$$

$$\sin x = \frac{3}{2} \text{ not possible.}$$

$$x = 0, \pi$$

$$\therefore \text{The values are } x = 0, \pi.$$

Exercise - 3.7

3) If $x + y + z = xyz$, p.t $\frac{2x}{1-x^2} + \frac{2y}{1-y^2} + \frac{2z}{1-z^2} = \frac{2x}{1-x^2} \cdot \frac{2y}{1-y^2} \cdot \frac{2z}{1-z^2}$

Sol: Given $x + y + z = xyz$

Let $x = \tan A$, $y = \tan B$, $z = \tan C$

TBP

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$.

Let $\tan(A+B+C) = 0 \Rightarrow \tan[(A+B)+C]$

$$\frac{\tan(A+B) + \tan C}{1 - \tan(A+B)\tan C} = 0$$

$$\Rightarrow \tan(A+B) + \tan C = 0$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} + \tan C = 0$$

$$\Rightarrow \tan A + \tan B + \tan C = \tan A \tan B \tan C.$$

$\therefore \tan(A+B+C) = 0$

$$\Rightarrow \tan(2A+2B+2C) = 0$$

$$\Rightarrow \tan 2A + \tan 2B + \tan 2C = \tan 2A \cdot \tan 2B \cdot \tan 2C \quad \text{--- (1)}$$

$\therefore x = \tan A$, $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A} = \frac{2x}{1-x^2}$

Similarly $\tan 2B = \frac{2y}{1-y^2}$

$\tan 2C = \frac{2z}{1-z^2}$

Sub. in (1)

$$\frac{2x}{1-x^2} + \frac{2y}{1-y^2} + \frac{2z}{1-z^2} = \frac{2x}{1-x^2} \cdot \frac{2y}{1-y^2} \cdot \frac{2z}{1-z^2}.$$

Exercise 3.9

6) In a $\triangle ABC$ $\angle A = 60^\circ$ p.t $b + c = 2a \cos\left(\frac{B+C}{2}\right)$

Sol: $A + B + C = 180^\circ$

$B + C = 180 - 60 = 120$

TBP

$$\frac{B+C}{2} = 60^\circ$$

We know $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$.

$$\Rightarrow a = 2R \sin A, b = 2R \sin B, c = 2R \sin C.$$

$$\begin{aligned} \text{RHS: } 2a \cos\left(\frac{B+C}{2}\right) &= 2 \cdot 2R \sin A \cos\left(\frac{B+C}{2}\right) \\ &= 2 \cdot 2R \sin 60^\circ \cos\left(\frac{B+C}{2}\right) \end{aligned}$$

$$\begin{aligned}
 &= 4R \cdot \sin\left(\frac{B+C}{2}\right) \cdot \cos\left(\frac{B-C}{2}\right) \\
 &= 2R \cdot 2\sin\left(\frac{B+C}{2}\right) \cos\left(\frac{B-C}{2}\right) \\
 &= 2R \cdot [\sin B \cos C] \\
 &= 2R \left[\frac{b}{2R} + \frac{c}{2R} \right] = 2R \left[\frac{b+c}{2R} \right] \\
 &= b+c \\
 &= \text{LHS.}
 \end{aligned}$$

Exercise - 3.8

1) Find the principle solution and general solution of the following.

1) $\sin \theta = -\frac{1}{2}$ 2) $\cot \theta = \sqrt{3}$ 3) $\tan \theta = \frac{1}{\sqrt{3}}$ 4)

1) $\sin \theta = -\frac{1}{2}$ θ lies in IV quadrant $\therefore$ range is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

TBP

$$\sin \theta = \frac{\pi}{4}$$

$$\therefore \theta = -\frac{\pi}{4}$$

2. $\cot \theta = \sqrt{3}$ θ lies in the I quadrant.

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\theta = \frac{\pi}{6}$$

3. $\tan \theta = -\frac{1}{\sqrt{3}}$ θ lies in the IV quadrant.

$$= \frac{\pi}{6}$$

$$\theta = -\frac{\pi}{6}$$

2) Solve the following equations for which solutions lies in the

1) $\sin^4 x = \sin^2 x$ 2) $2\cos^2 x + 1 = -3\cos x$ 3) $2\sin^2 x + 1 = 3\sin x$ $0 < \theta \leq 2\pi$

4) $\cos 2x = 1 - 3\sin x$.

Sol: 1) $\sin^4 x = \sin^2 x \Rightarrow \sin^2 x (\sin^2 x - 1) = 0$

$$\sin^2 x = 0 \quad \sin^2 x - 1 = 0$$

$$\sin x = 0 \quad \sin x = \pm 1$$

If $\sin x = 0$ $x = 0, \pi$

$\sin x = 1$ $x = \frac{\pi}{2}$

$\sin x = -1$ $x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

TBP

Exercise: 3.9

8) In a ΔABC PT $(a^2 - b^2 + c^2) \tan B = (a^2 + b^2 - c^2) \tan C$.

Sol: $(a^2 - b^2 + c^2) \tan B = (a^2 + b^2 - c^2) \tan C$.

$$\tan B \cdot \cot C = \frac{a^2 + b^2 - c^2}{a^2 - b^2 + c^2}$$

$$= \frac{k^2 \sin^2 A + k^2 \sin^2 B - k^2 \sin^2 C}{k^2 \sin^2 A - k^2 \sin^2 B + k^2 \sin^2 C}$$

$$= \frac{\sin^2 A + \sin^2 B - \sin^2 C}{\sin^2 A - \sin^2 B + \sin^2 C}$$

$$= \frac{\sin(A+B) \cdot \sin(A-C) + \sin^2 B}{\sin(A+B) \sin(A-B) + \sin^2 C}$$

$$= \frac{\sin B \cdot \sin(A-C) + \sin^2 B}{\sin C \sin(A-B) + \sin^2 C}$$

$$= \frac{\sin B [\sin(A-C) + \sin B]}{\sin C [\sin(A-B) + \sin C]}$$

$$= \frac{\sin B [\sin(A-C) + \sin(A+B)]}{\sin C [\sin(A-B) + \sin(A+B)]}$$

$$= \frac{\sin B}{\sin C} \cdot \frac{2 \sin A \cos C}{2 \sin A \cos B}$$

$$= \tan B \cdot \cot C$$

Exercise - 3.10

1) Determine whether the following measurements produce one Δ , two Δ s, no triangle $\angle B = 88^\circ$, $a = 23$, $b = 2$ Solve if solution exists.

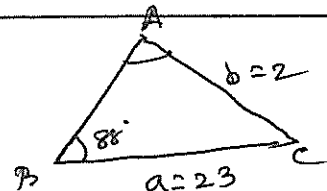
Sine formula $\frac{a}{\sin A} = \frac{b}{\sin B}$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\sin A = \frac{a (\sin B)}{b} = \frac{23 \sin 88^\circ}{2}$$

$$= 23 \times 0.99$$

$\sin A = 22.99$, which is not possible
solution does not exist



16) Suppose that a satellite in space, an earth station and the centre of earth all lie in the same plane. Let r be the radius of earth and R be the radius of distance from the centre of earth to the satellite. Let d be the distance from the earth station to the satellite. Let α be the angle of elevation from the earth station to the satellite. If the line segment connecting earth station and satellite subtends angle at the centre of earth then p-t $d = \sqrt{1 + \left(\frac{r}{R}\right)^2 - 2\frac{r}{R} \cos \alpha}$

Sol: Let S be the satellite, $CE = r$, $SE = d$
 $SC = R$.

By cosine formula $d^2 = r^2 + R^2 - 2rR \cos \alpha$

$$\frac{d^2}{R^2} = \frac{r^2}{R^2} + 1 - 2\frac{r}{R} \cos \alpha$$

$$= 1 + \frac{r^2}{R^2} - 2\frac{r}{R} \cos \alpha$$

$$d^2 = R^2 \left[1 + \left(\frac{r}{R}\right)^2 - 2\frac{r}{R} \cos \alpha \right]$$

$$d = R \sqrt{1 + \left(\frac{r}{R}\right)^2 - 2\frac{r}{R} \cos \alpha}$$



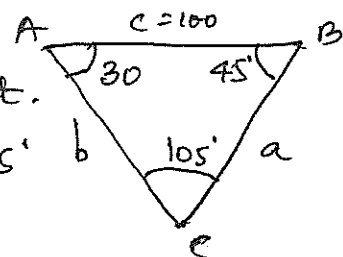
12) A fighter jet has to hit a small target by flying a horizontal distance when the target is sighted, the pilot measures the angle of depression to be 30° . If after 100 km, the target has an angle of depression of 45° . How far is the target from the fighter jet at that instant?

Sol: Let C be the position of the target.

A, B are the positions of the fighter jet.

$$\angle A = 30^\circ \quad \angle B = 45^\circ \quad \therefore \angle C = 105^\circ$$

$$AB = 100 \text{ km}$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{a}{\sin 30^\circ} = \frac{b}{\sin 45^\circ} = \frac{c}{\sin 105^\circ}$$

$$\text{we know } \sin 105^\circ = \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

$$\therefore \frac{a}{\cancel{1/2}} = \frac{100 \times \cancel{2\sqrt{2}}}{\sqrt{3} + 1} = \frac{100\sqrt{2}}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

$$= \frac{100(\sqrt{6} - \sqrt{2})}{3 - 1}$$

$$\therefore a = 50(\sqrt{6} - \sqrt{2}) \text{ km}$$

Inverse Trigonometric function:

1. Find the principle value of 1) $\sin^{-1}(\frac{1}{\sqrt{2}})$ 2) $\cos^{-1}(\frac{\sqrt{3}}{2})$ 3) $\operatorname{cosec}^{-1}(-1)$
TBP 4) $\sec^{-1}(-\sqrt{2})$ 5) $\tan^{-1}(\sqrt{3})$

Sol: 1) $\sin^{-1}(\frac{1}{\sqrt{2}})$

Let $y = \sin^{-1}(\frac{1}{\sqrt{2}})$ where $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$$\sin y = \frac{1}{\sqrt{2}} \Rightarrow y = \frac{\pi}{4}$$

$\therefore$ The principle value of $\sin^{-1}(\frac{1}{\sqrt{2}}) = \frac{\pi}{4}$.

2) $\cos^{-1}(\frac{\sqrt{3}}{2})$

Let $y = \cos^{-1}(\frac{\sqrt{3}}{2})$ where $0 \leq y \leq \pi$

$$\cos y = \frac{\sqrt{3}}{2}$$

$$y = \frac{\pi}{6}$$

$\therefore$ The principle value of $\cos^{-1}(\frac{\sqrt{3}}{2}) = \frac{\pi}{6}$.

3) $\operatorname{cosec}^{-1}(-1)$

Let $y = \operatorname{cosec}^{-1}(-1)$ where $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$$\operatorname{cosec} y = -1$$

$$= -\operatorname{cosec}(\frac{\pi}{2})$$

$$= \operatorname{cosec}(-\frac{\pi}{2})$$

$$\left[\operatorname{cosec}(-\theta) = -\operatorname{cosec} \theta \right]$$

$\therefore$ The principle value of $\operatorname{cosec}^{-1}(-1) = -\frac{\pi}{2}$

4) $\sec^{-1}(-\sqrt{2})$

Let $y = \sec^{-1}(-\sqrt{2})$ where $0 \leq y \leq \pi$

$$\sec y = -\sqrt{2}$$

$$= \sec(-\frac{\pi}{4}) \quad (\theta \text{ lies in II quadrant})$$

$$= \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$\therefore$ The principle value of $\sec^{-1}(-\sqrt{2})$ is $\frac{3\pi}{4}$

5) $\tan^{-1}(\sqrt{3})$

Let $y = \tan^{-1}(\sqrt{3})$ where $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

$$\tan y = \sqrt{3}$$

$$y = \frac{\pi}{3}$$

$\therefore$ The principle value of $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$.

2) A man standing directly opposite to one side of a road of width x meter views a circular shaped traffic green signal of diameter a meter on the other side of the road. The bottom of the green signal is b meter height from the horizontal level of viewer's eye. If α denotes the angle subtend by the diameter of the green signal at the viewer's eye, then P.T $\alpha = \tan^{-1}\left(\frac{a+b}{x}\right) - \tan^{-1}\left(\frac{b}{x}\right)$

Sol: Diameter of the signal = a

$$DA = b$$

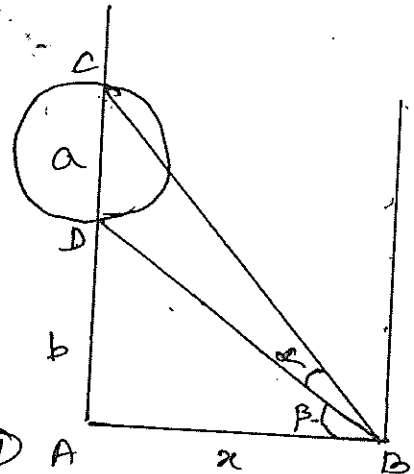
$$\angle CBD = \alpha$$

$$AB = x \text{ mts.}$$

$$\text{Let } \angle DBA = \beta.$$

$$\text{From } \triangle ABD \quad \tan \beta = \frac{b}{x}.$$

$$\beta = \tan^{-1}\left(\frac{b}{x}\right) \quad \text{--- (1)}$$



$$\text{From } \triangle ABC \quad \tan(\alpha + \beta) = \frac{a+b}{x}$$

$$\alpha + \beta = \tan^{-1}\left(\frac{a+b}{x}\right) \quad \text{--- (2)}$$

$$\text{(2) - (1)}$$

$$\alpha + \beta - \beta = \tan^{-1}\left(\frac{a+b}{x}\right) - \tan^{-1}\left(\frac{b}{x}\right)$$

$$\alpha = \tan^{-1}\left(\frac{a+b}{x}\right) - \tan^{-1}\left(\frac{b}{x}\right)$$

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