XII-MATHEMATICS

MARKS : 90

TIME : 2 hrs 30 Mins.

UNIT TEST (1-5)

I. Choose the correct answer 20×1=20

1. If $A=\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$ and $A^T=A^{-1}$, then the value of x is

a) $-\frac{4}{5}$ b) $-\frac{3}{5}$ c) $\frac{3}{5}$ d) $\frac{4}{5}$

2. If A^TA^{-1} is Symmetric, then $A^2=$ ___. a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$

3. If $\rho(A)=\rho(A|B)$, then the system $AX=B$ is a) in Consistent b) Consistent c) No solution d) trivial solution

4. Transforming a non-singular matrix A to the form I_n by applying elementary row operation is called a) Gauss elimination method b) Gauss Jordan method c) Matrix Inversion Method d) Rank method

5. $i^{-1924}+i^{2018}=$ a) 0 b) 1 c) i d) $-i$

6. z is purely imaginary iff $z=$ ___. a) $\bar{z}$ b) $-\bar{z}$ c) iz d) $-iz$

7. If $\omega=\text{cis}\frac{2\pi}{3}$, then the number of distinct roots of

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=0.$$ a) 1 b) 2 c) 3 d) 4

8. If $z=\frac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$, then $|z|=$ a) 0 b) 1 c) 2 d) 3

9. Every polynomial equation of degree $n\geq 1$ has atleast one root in
a) $\mathbb{R}$ b) $\mathbb{Q}$ c) $\mathbb{Z}$ d) $\mathbb{C}$

10. If the equations $x^2+px+q=0$ and $x^2+p'x+q'=0$ have a common root then it must be = ___. a) $\frac{p'-p}{q-q'}$ b) $\frac{q-q'}{p'-p}$ c) $\frac{q-q'}{pq'-p'q}$ d) $\frac{pq'-p'q}{q+q'}$

11. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

a) $[1,2]$ b) $[-1,1]$ c) $[0,1]$ d) $[-1,0]$.

12. If $\sin^{-1}\frac{x}{5} + \text{cosec}^{-1}\frac{5}{4} = \frac{\pi}{2}$, then the value of x is

a) 2 b) 3 c) 4 d) 5

13. Radius of the circle $ax^2 + ay^2 + 2g'x + 2f'y + c = 0$ is

a) $\sqrt{g'^2 + f'^2 - c'}$ b) $\frac{1}{a}\sqrt{g'^2 + f'^2 - c'}$ c) $\frac{1}{a}\sqrt{g'^2 + f'^2 - c'a}$ d) $\sqrt{ag'^2 + af'^2 - ac'}$

14. Centre line of sinusoidal function $y = d + a\cos(bt - c)$ is

a) $x = d$ b) $x = a$ c) $y = d$ d) $y = a$

15. x-intercept of $\cos^{-1}x$ is a) $\frac{\pi}{2}$ b) π c) 1 d) 0

16. If $x^3 + 12x^2 + 10\alpha x + 1999$ definitely has a positive root, iff

a) $\alpha \leq 0$ b) $\alpha > 0$ c) $\alpha < 0$ d) $\alpha \geq 0$

17. Length of Latus rectum of the ellipse is

a) $\frac{2a}{b^2}$ b) $\frac{2b}{a}$ c) $4a$ d) $\frac{2b^2}{a}$

18. eccentricity of the circle is ____.

a) 1 b) >1 c) <1 d) 0

19. If the co-ordinates at one end of a diameter of the circle $x^2 + y^2 - 8x - 4y + c = 0$ are $(11,2)$, the co-ordinates of the other end are

(a) $(-5,2)$ b) $(2,-5)$ c) $(5,-2)$ d) $(-2,5)$

20. If $x + y = k$ is normal to the parabola $y^2 = 12x$ then the value of k is

a) 3 b) -1 c) 1 d) 9.

II. i) Answer any 7 questions. ii) Question number 30 is compulsory $7 \times 2 = 14$

21. State and prove left cancellation law for matrices.

22. Find the rank of $\begin{bmatrix} 1 & 2 \\ -1 & 5 \\ 3 & 1 \end{bmatrix}$

23. Simplify: $i\, i^2 i^3 \ldots\ldots i^{2020}$

24. If $z = x+iy$, find $Re(i\bar{z})$

25. If p and q are the roots of the equation $lx^2+nx+n=0$, Show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$.

26. Prove that a line cannot intersect a circle at more than two points.

27. Define amplitude and period of a graph

28. Draw the graph of secant function.

29. Solve: $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x$ for $x > 0$.

30. The orbit of Halley's Comet is an ellipse 36.18 astronomical units long and by 9.12 astronomical units wide. Find its eccentricity

III i) Answer any 7 questions ii) Question number 40 is compulsory $7 \times 3 = 21$

31. If $A = \begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal. find a, b and c, and hence A^{-1}.

32. Solve: $x+y-2z=0, 2x-3y+z=0, 3x-7y+10z=0, 6x-9y+10z=0$

33. Find z^{-1}, if $z = (2+3i)(1-i)$

34. Find the sum of squares of roots of the equation $2x^4-8x^3+6x^2-3=0$

35. Solve: $x^4-9x^2+20=0$

36. Find the domain of $\sin^{-1}(2-3x^2)$.

37. The line $3x+4y-12=0$ meets the coordinate axes at A and B. Find the equation of the circle drawn on AB as diameter.

38. Find the length of Latus rectum of the Parabola $y^2=4ax$.

39. Find the equation of the ellipse whose $e = \frac{1}{2}$, one of the foci is (2,3) and a directrix is $x=7$. Also find the length of minor axis.

40. Prove that Modulus of the product is equal to product of the moduli.

IV Answer all the questions : $7 \times 5 = 35$

41.a) Investigate for what values of λ and μ the system of linear equations $x+2y+z=7$, $x+y+\lambda z=\mu$, $x+3y-5z=5$ has i) no solution (ii) a unique solution (iii) an infinite number of solutions.

(OR)

b) By using Gaussian elimination method, balance the chemical reaction equation: $C_5H_8 + O_2 \rightarrow CO_2 + H_2O$.

42. a) Let z_1, z_2 and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1 + z_2 + z_3 \neq 0$. Prove that $\left|\frac{z_1z_2 + z_2z_3 + z_3z_1}{z_1 + z_2 + z_3}\right| = r$

(OR)

b) Prove that $\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$ where either $x^2 + y^2 \leq 1$ (or) $xy > 0$

43. a). Prove that the values of $\sqrt[4]{-1}$ are $\pm\frac{1}{\sqrt{2}}(1 \pm i)$

(OR)

b). Evaluate: $\sin\left[\sin^{-1}\left(\frac{3}{5}\right) + \sec^{-1}\left(\frac{5}{4}\right)\right]$

44. a) Solve: $(x-4)(x-7)(x-2)(x+1) = 16$ (OR)

b) solve: $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

45). a) Solve: $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$ (OR)

b) Find the locus of P if $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$

46) a) Find the equation of the circle through the points $(1,0), (-1,0), (0,1)$

(OR)

b) Find the centre, vertices, and the foci, length of latus rectum for $4x^2 + y^2 + 24x - 2y + 21 = 0$.

47) a) A semielliptical archway over a one-way road has a height of 3m and a width of 12m. The truck has a width of 3m and a height of 2.7m. Will the truck clear the opening of the archway?

(OR)

b) Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$. The tower is 150m tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the radius of the top and base of the tower.