

# 11<sup>th</sup> Maths Study Material

“If  $A$  and  $B$  are two sets, is it meaningful to write  $A \in B$ ?”.

For example, if  $A = \{1, 2\}$  and  $B = \{1, \{1, 2\}, 3, 4\}$ ,

then  $A \in B$ .

it is meaningful to write  $A \in B$ ?”.

$A \subseteq B$  and  $B \subseteq A$  What can you say about  $A$  and  $B$  ?

We can say two sets are equal i.e.  $A = B$

The number of elements in  $\mathcal{P}(A)$  is  $2^n$ , where  $n$  is the number of elements in  $A$ .

The number of Subsets of  $A = \mathcal{P}(A) = 2^n$

$$A \Delta B = (A - B) \cup (B - A).$$

$$A \Delta B = (A \cup B) - (A \cap B)$$

$$n(\emptyset) = 0 \text{ and } n(\{\emptyset\}) = 1.$$

For any two finite sets  $A$  and  $B$ ,  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ .

If  $A$  and  $B$  are disjoint finite sets, then  $n(A \cup B) = n(A) + n(B)$ .

For any three finite sets  $A$ ,  $B$  and  $C$ ,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C).$$

Is it correct to say  $A \times A = \{(a, a) : a \in A\}$ ?

Yes it is correct to say that  $A \times A = \{(a, a) : a \in A\}$

If  $A = \{1, 2, 3\}$  and  $B = \{2, 4, 6\}$  then find  $A \times B$

$$A \times B = \{(1, 2), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 2), (3, 4), (3, 6)\}.$$


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$$n(A \times B) = n(A)n(B), \text{ if } A \text{ and } B \text{ are finite.}$$


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$$n(A \times B \times C) = n(A)n(B)n(C), \text{ if } A, B \text{ and } C \text{ are finite.}$$


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Draw a real line and fix where the numbers  $\sqrt{2}$ ,  $e$  and  $\pi$



Give the examples which are not intervals

the set of all natural numbers,

the set of all non-negative integers,

set of all odd integers,

set of all even integers,

set of all prime numbers are not intervals.

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Give the examples which are finite and infinite intervals

- (i) The set of all real numbers greater than 0.
- (ii) The set of all real numbers greater than 5 and less than 7.
- (iii) The set of all real numbers  $x$  such that  $1 \leq x \leq 3$ .
- (iv) The set of all real numbers  $x$  such that  $1 < x \leq 2$ .

(i) is an infinite interval (ii), (iii) and (iv) are finite intervals.

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Infinity is a Number (or) not

Note that  $\infty$  is not a number.

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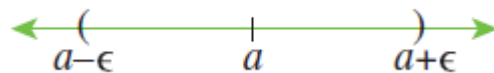
write the following intervals in symbolic form.

- (i)  $\{x : x \in \mathbb{R}, -2 \leq x \leq 0\}$  (ii)  $\{x : x \in \mathbb{R}, 0 < x < 8\}$   
 (iii)  $\{x : x \in \mathbb{R}, -8 < x \leq -2\}$  (iv)  $\{x : x \in \mathbb{R}, -5 \leq x \leq 9\}$ .

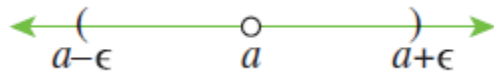
- (i)  $[-2,0]$  (ii)  $(0,8)$  (iii)  $(-8,-2]$  iv)  $[-5,9]$

### Define neighborhood and deleted neighborhood

**Neighbourhood** of a point ' $a$ ' is any open interval containing ' $a$ '. In particular, if  $\epsilon$  is a positive number, usually very small, then the  $\epsilon$ -neighbourhood of ' $a$ ' is the open interval  $(a - \epsilon, a + \epsilon)$ .



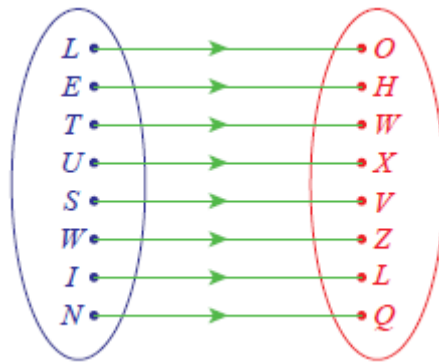
set  $(a - \epsilon, a + \epsilon) - \{a\}$  is called **deleted neighbourhood** of ' $a$ ' and it is denoted as  $0 < |x - a| < \epsilon$



### Give some examples of Relations in Mathematics

- (i) A number  $m$  is related to a number  $n$  if  $m$  divides  $n$  in  $\mathbb{N}$ .  
 (ii) A real number  $x$  is related to a real number  $y$  if  $x \leq y$ .  
 (iii) A point  $p$  is related to a line  $L$  if  $p$  lies on  $L$ .  
 (iv) A student  $X$  is related to a school  $S$  if  $X$  is a student of  $S$ .

### What is Cryptography ? Explain



Using this coding scheme, "LET US WIN" becomes "OHW XVZ LQ".

$\{(L, O), (E, H), (T, W), (U, X), (S, V), (W, Z), (I, L), (N, Q)\}$

If “KDUGZRUN” means “HARDWORK”,

“DFKLHYHPHQW” becomes “ACHIEVEMENT”

To decode, replace each letter by the letter three places before it.

“Is it  $f(x) = x - 3$ ?”.

**Yes it is**

Consider the equation:  $2x - y = 0$

The equation  $2x - y = 0$  represents a straight line.

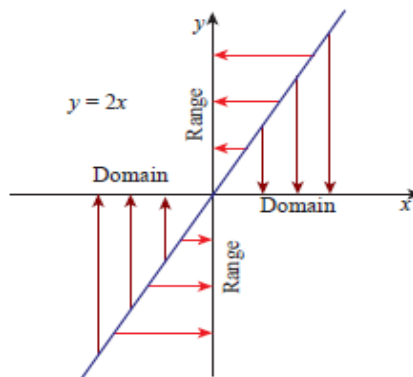
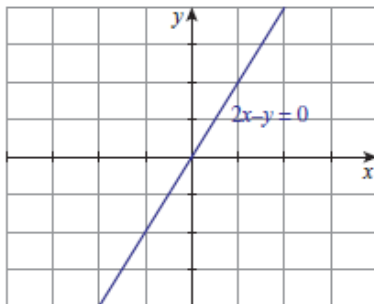
the points,  $(1, 2)$ ,  $(3, 6)$  lie on it

whereas  $(1, 1)$ ,  $(3, 5)$ ,  $(4, 5)$  are not lying on the straight line.

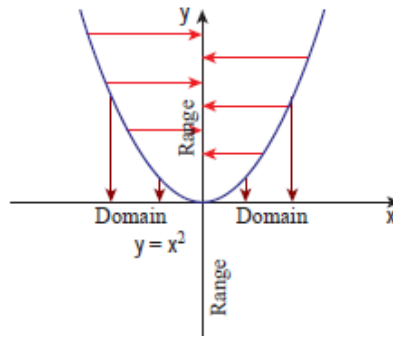
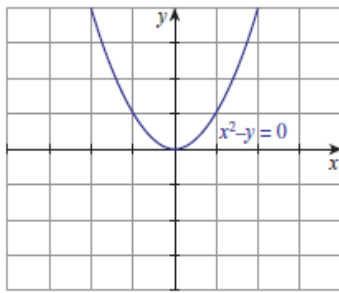
The analytical relation between  $x$  and  $y$  is given by  $y = 2x$ .

The set of all points that lie on the straight line is given as  $\{(x, 2x) : x \in \mathbb{R}\}$ .

this is a subset of  $\mathbb{R} \times \mathbb{R}$ .



Consider the equation:  $x^2 - y = 0$  the relation between  $x$  and  $y$  is  $y = x^2$ .



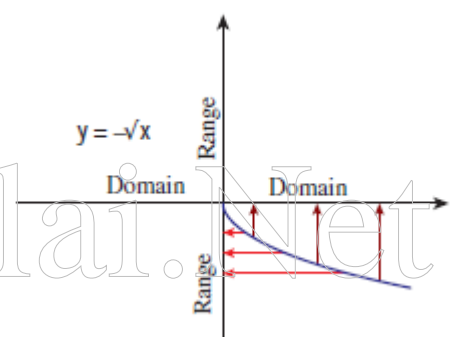
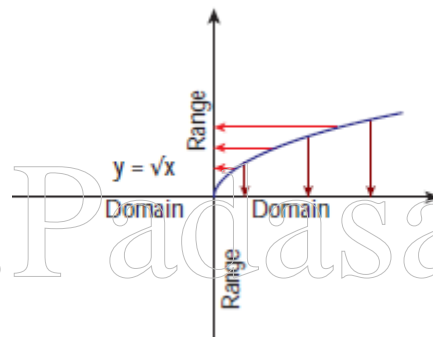
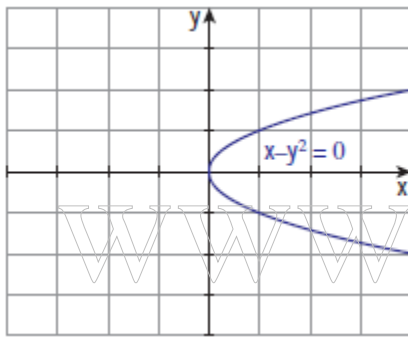
The set of all points on the curve is  $\{(x, x^2) : x \in \mathbb{R}\}$  this is a subset of  $\mathbb{R} \times \mathbb{R}$ .

Consider the equation:  $x - y^2 = 0$

the relation between  $x$  and  $y$  is  $y^2 = x$  or  $y = \pm\sqrt{x}$ ,  $x \geq 0$ .

The equation can also be re-written as  $y = +\sqrt{x}$  and  $y = -\sqrt{x}$ .

The set of all points on the curve is the union of the sets  $\{(x, \sqrt{x})\}$  and  $\{(x, -\sqrt{x})\}$ ,



## Define Relation

Let  $A$  and  $B$  be any two non-empty sets. A **relation**  $R$  from  $A$  to  $B$  is defined as a subset of the Cartesian product of  $A$  and  $B$ . Symbolically  $R \subseteq A \times B$ .

## Define reflexive, symmetric, and transitive

Let  $S$  be any non-empty set. Let  $R$  be a relation on  $S$ . Then

- $R$  is said to be **reflexive** if  $a$  is related to  $a$  for all  $a \in S$ .
- $R$  is said to be **symmetric** if  $a$  is related to  $b$  implies that  $b$  is related to  $a$ .
- $R$  is said to be **transitive** if " $a$  is related to  $b$  and  $b$  is related to  $c$ " implies that  $a$  is related to  $c$ .

Let us rewrite the definitions of these basic relations in a different form:

Let  $S$  be any non-empty set. Let  $R$  be a relation on  $S$ . Then  $R$  is

- reflexive if “ $(a, a) \in R$  for all  $a \in S$ ”.
- symmetric if “ $(a, b) \in R \Rightarrow (b, a) \in R$ ”.
- transitive if “ $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$ ”.

These three relations are called *basic relations*.

### Define equivalence relation

Let  $S$  be any set. A relation on  $S$  is said to be an *equivalence relation* if it is reflexive, symmetric and transitive.

### Example for reflexive

Let  $X = \{1, 2, 3, 4\}$

$R = \{(1, 1), (2, 1), (2, 2), (3, 3), (1, 3), (4, 4), (1, 2), (3, 1)\}$ .

As  $(1, 1), (2, 2), (3, 3)$  and  $(4, 4)$  are all in  $R$

it is reflexive. (Only if the above 4 pairs are in  $R$  it is reflexive)

**$R$  is not reflexive**

In the above relation for each pair  $(a, b) \in R$  the pair  $(b, a)$  is also in  $R$ .

i.e., In the above relation  $(2, 1), (1, 2) \in R$  and also  $(1, 3)$  and  $(3, 1) \in R$

So  $R$  is symmetric.

In the above relation As  $(2, 1), (1, 3) \in R$  and  $(2, 3) \notin R$ ,

we see that  $R$  is not transitive.

$R$  is not an equivalence relation.

$P$  denote the set of all straight lines in a plane.  $P = \{l, m, n, \dots\}$

Let  $R$  be the relation defined on  $P$  as  $\ell R m$  if  $\ell$  is parallel to  $m$ .

This relation is reflexive, symmetric and transitive.

Thus it is an equivalence relation.

Let  $A$  be the set consisting of children and elders of a family. (consist of both male and female)

Let  $R$  be the relation defined by  $aRb$  if  $a$  is a sister of  $b$ .

A woman is not a sister of herself.

So it is not reflexive.

(female is a sister of male and male cannot be sister of female)

So It is not symmetric also.

Clearly it is not transitive.

So it is not an equivalence relation.

Suppose If we consider the same relation on a set consisting only of females,

Let  $R$  be the relation defined by  $aRb$  if  $a$  is a sister of  $b$ .

It is reflexive

It becomes symmetric

It is not transitive

the set of natural numbers let  $R$  be the relation defined by  $xRy$

The relation is defined as  $x + 2y = 21$ .

The relation  $R$  is the set

$R = \{(1, 10), (3, 9), (5, 8), (7, 7), (9, 6), (11, 5), (13, 4), (15, 3), (17, 2), (19, 1)\}$ .

As  $(1, 1) \notin R$  it is not reflexive;

$(1, 10) \in R$  and  $(10, 1) \notin R$  it is not symmetric.

As  $(3, 9) \in R$ ,  $(9, 6) \in R$  but  $(3, 6) \notin R$ ,

the relation is not transitive.

The universal relation is always an equivalence relation.



An empty relation can be considered as symmetric and transitive.

If a relation contains a single element, then the relation is transitive.

A contains  $n$  elements and B contains  $m$  elements

A  $\times$  B has  $mn$  elements

The subsets of A  $\times$  B has  $2^{mn}$  elements

When A=B we see that the number of relations on a set containing  $n$  elements is  $2^{n^2}$ .

The number of reflexive relations on a set containing  $n$  elements is  $2^{n^2-n} = 2^{n(n-1)}$

The number of symmetric relations on a set containing  $n$  elements is  $2^{\frac{n^2+n}{2}} = 2^{n(n+1)/2}$

**Define inverse relation Give an example**

If  $R$  is a relation from  $A$  to  $B$ , then the relation  $R^{-1}$  defined from  $B$  to  $A$  by  $R^{-1} = \{(b, a) : (a, b) \in R\}$  is called the *inverse* of the relation  $R$ .

**Example**

For example, if  $R = \{(1, a), (2, b), (2, c), (3, a)\}$ ,

then  $R^{-1} = \{(a, 1), (b, 2), (c, 2), (a, 3)\}$ .

**Note:**

It is easy to see that the domain of  $R$  becomes the range of  $R^{-1}$  and the range of  $R$  becomes the domain of  $R^{-1}$ .

ie., Domain of  $R =$  Range of  $R^{-1}$

Range of  $R =$  Domain of  $R^{-1}$

**Define Identity relation**

If  $A = \{1, 2, 3, 4, 5, 6\}$



Then Identity relation  $\{(1,1) (2,2) (3,3) (4,4) (5,5) (6,6),\}$

Every element of A should be related to itself

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### Universal relation

Given  $A = \{1,2,3\}$

Then the relation  $R = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

R is universal relation

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For  $a, b \in \mathbb{Z}$ ,  $aRb$  if and only if  $a - b = 3k$ ,  $k \in \mathbb{Z}$  is an equivalence relation on  $\mathbb{Z}$ .

$$Z_0 = \{x \in \mathbb{Z} : xR0\} = \{\dots, -6, -3, 0, 3, 6, \dots\}$$

$$Z_1 = \{x \in \mathbb{Z} : xR1\} = \{\dots, -5, -2, 1, 4, 7, \dots\}$$

$$Z_2 = \{x \in \mathbb{Z} : xR2\} = \{\dots, -4, -1, 2, 5, 8, \dots\}$$

Thus  $\mathbb{Z} = Z_0 \cup Z_1 \cup Z_2$  and all are disjoint subsets.

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## FUNCTIONS

### Function is a relation

Every element of domain is mapped on to exactly one element in the range according to some rule, (or) law. The rule is called function

$F : A \rightarrow B$  A is called domain

B is called co domain

The set image of every element is called range.

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### Condition

1) Every element in the domain must have image.

2) An element in the domain cannot have two (or) more images.

Every function is a relation ( but ) a relation is not a function.

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Let  $f = \{(a,1) (b,2) (c,2) (d,4)\}$  If f is a function?

This is a function from the set  $\{a, b, c, d\}$  to  $\{1,2,4\}$

But This is not a function  $\{a, b, c, d, e\}$  to  $\{1,2,3,4\}$  because "e" has no image

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## Vertical line Test

If we draw a line parallel to y axis which meets the curve at only one point then it is a function .

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Note :

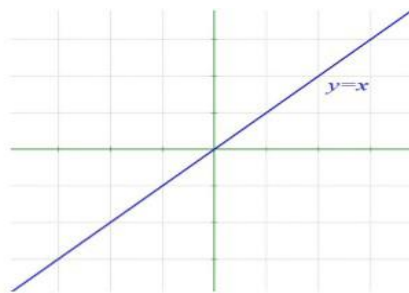
If the vertical line through a point x in the domain meets the curve at more than one point (or) does not meet the curve , then the curve will not represent a function.

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## Identity function

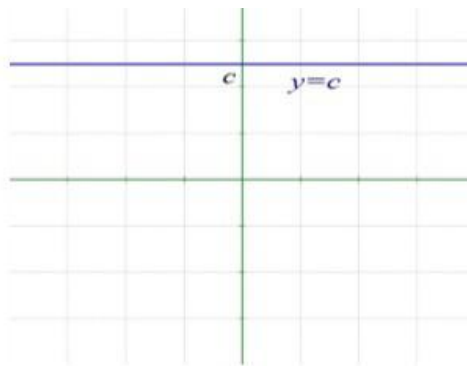
$f(x) = x$  is called identity function

The graph of the function is  $y=x$  Which is a straight line.



## Constant function

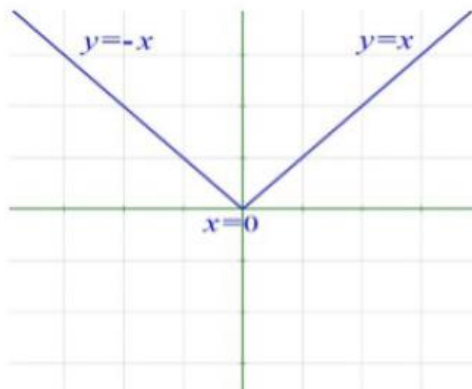
$f(x) = c$  is called constant function.



## Modulus (or) absolute value function

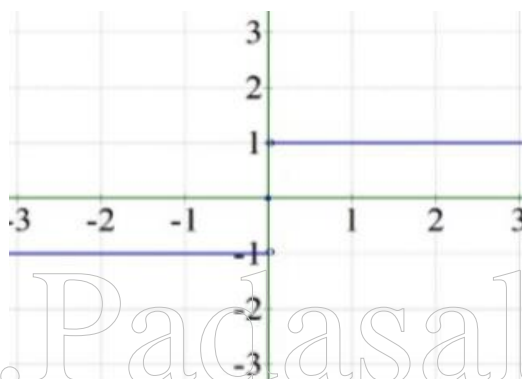
$f(x) = |x|$ , is called the *modulus function* or *absolute value function*.

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ x & \text{if } x > 0 \end{cases} \quad \text{or } |x| = \begin{cases} -x & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases} \quad \text{or } |x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$



### Signum function

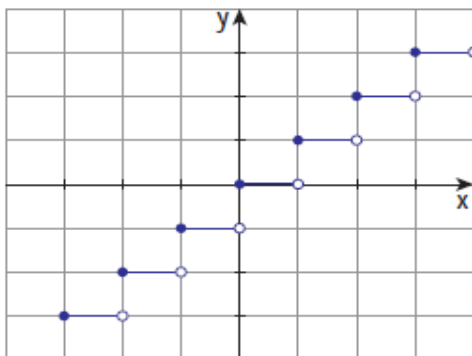
$f(x) = \begin{cases} \frac{x}{|x|} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$  is called the *signum function*.



### Step function

#### Greatest integer function (or) floor function

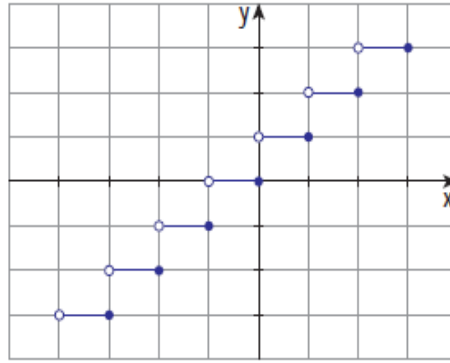
$f(x)$  is the greatest integer less than (or) equal to  $x$ . It is denoted by  $f(x) = \lfloor x \rfloor$ .



Let us note that  $\lfloor 1\frac{1}{5} \rfloor = 1$ ,  $\lfloor 7.23 \rfloor = 7$ ,  $\lfloor -2\frac{1}{2} \rfloor = -3$  (not  $-2$ ),  $\lfloor 6 \rfloor = 6$  and  $\lfloor -4 \rfloor = -4$ .

#### Smallest integer function (or) ceil function

$f(x)$  is the smallest integer greater (or) equal to  $x$ . It is denoted by  $f(x) = \lceil x \rceil$ .



Let us note that  $\lceil 1\frac{1}{5} \rceil = 2$ ,  $\lceil 7.23 \rceil = 8$ ,  $\lceil -2\frac{1}{2} \rceil = -2$  (not  $-3$ ),  $\lceil 6 \rceil = 6$  and  $\lceil -4 \rceil = -4$ .

## TYPES OF FUNCTIONS

### One to one function

A function  $f : A \rightarrow B$  is said to be *one-to-one* if  $x, y \in A, x \neq y \Rightarrow f(x) \neq f(y)$  [or equivalently  $f(x) = f(y) \Rightarrow x = y$ ].

### On to function

A function  $f : A \rightarrow B$  is said to be *onto*, if for each  $b \in B$  there exists at least one element  $a \in A$  such that  $f(a) = b$ . That is, the range of  $f$  is  $B$ .

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**Example 1.1**

Find the number of subsets of A if  $A = \{x: x=4n+1, 2 \leq n \leq 5, n \in \mathbb{N}\}$

**Solution**

n takes the value  $\{2,3,4,5\} = A$

When  $n = 2$   $x = 4(2)+1 = 8+1 = 9$

$n = 3$   $x = 4(3)+1 = 12+1 = 13$

$n = 4$   $x = 4(4)+1 = 16+1 = 17$

$n = 5$   $x = 4(5)+1 = 20+1 = 21$

The elements in the set  $A = \{9, 13, 17, 21\}$

The number of elements in  $A = n(A) = 4$

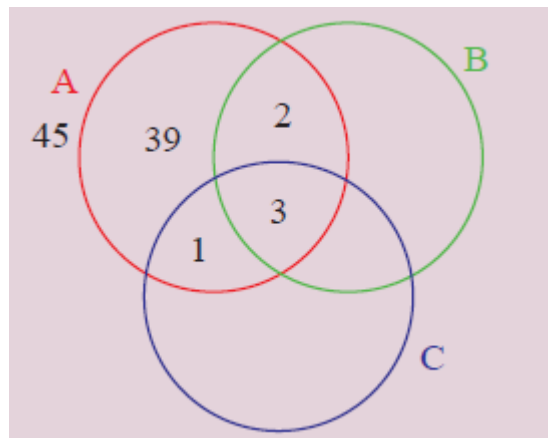
The number of subsets =  $n P(A) = 2^n$  Here  $n=4$  therefore  $n(PA) = 2^4 = 16$

**Example 1. 2**

In a Survey 5000 persons in a town , it was found that 45% of the person know Language A 25% know Language B, 10% know Language C, 5% know Language A and B 4% know Language B and C and 4% know Languages A and C. If 3% of the persons know all the three Languages , find the number of persons who knows only Language A

**Solution**

From the Venn diagram we see that

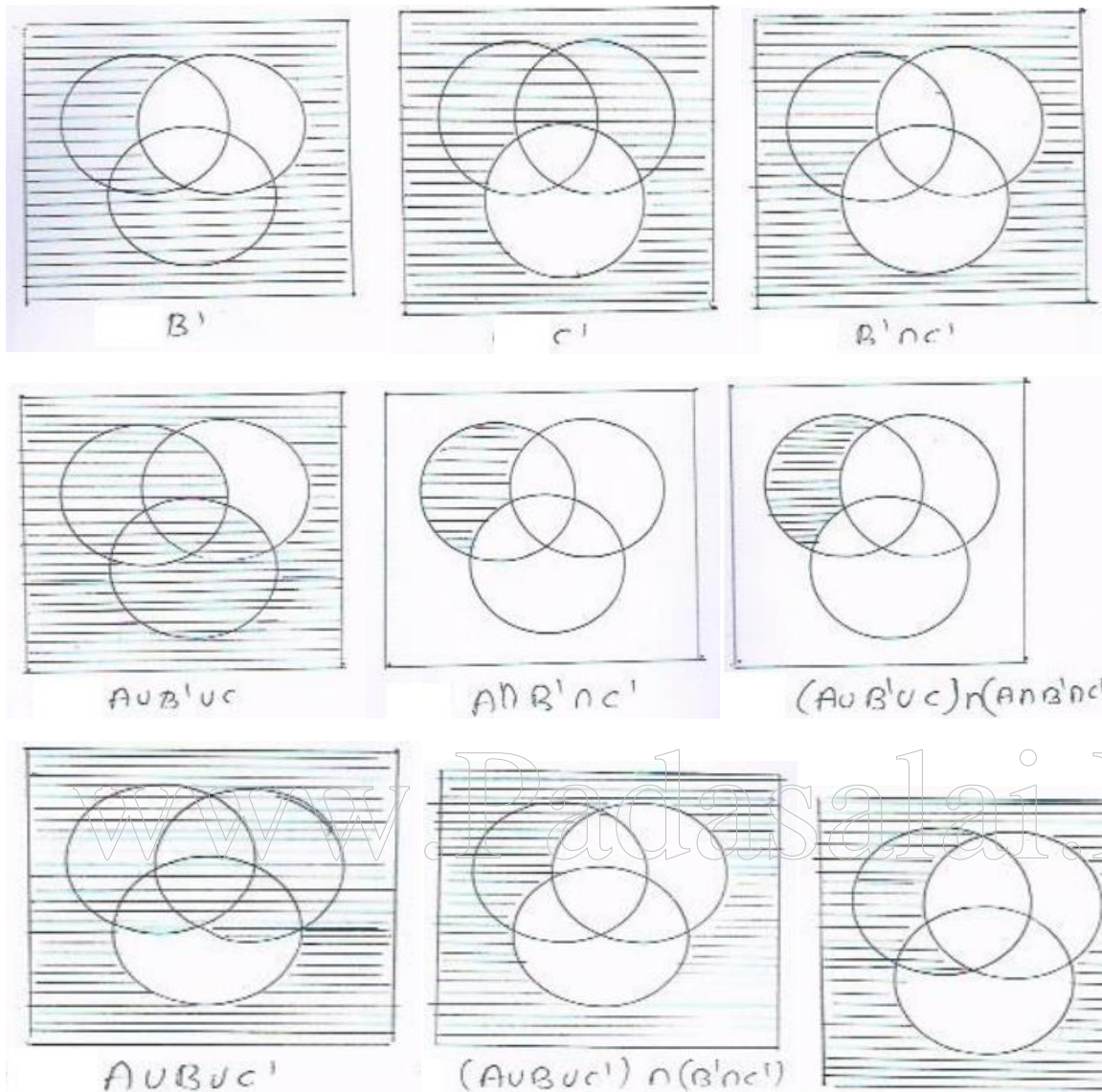
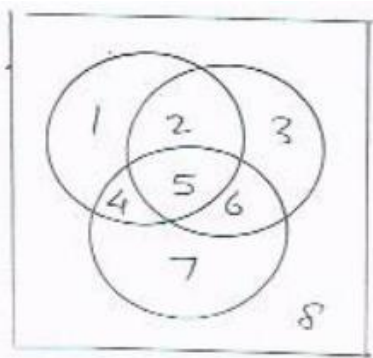


The percentage of persons who know Language A only = 39 %

The number of persons who knows Only Language A out of 5000 =  $5000 \times \frac{39}{100} = 50 \times 39 = 1950$

**Example 1.3**

Prove that  $((A \cup B' \cup C) \cap (A \cap B' \cap C')) \cup ((A \cup B \cup C') \cap (B' \cap C')) = (B' \cap C')$

**Another Method**

$$\text{Let } U = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$A = \{1, 2, 4, 5\}$$

$$B = \{2, 3, 5, 6\}$$

$$C = \{4, 5, 6, 7\}$$



**Example 1. 4**

If  $X = \{1, 2, 3, \dots, 10\}$  and  $A = \{1, 2, 3, 4, 5\}$  Find the number of sets  $B \subseteq X$  such that

$$A - B = \{4\}$$

**Solution**

$$B = C \cup \{1, 2, 3, 5\} \text{ Where } C = \{6, 7, 8, 9, 10\} \quad B \subseteq X \text{ such that } A - B = \{4\}$$

$$\text{The Number of subsets of } \{6, 7, 8, 9, 10\} = 2^5 = 32$$


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**Example 1.5**

If A and B are two sets so that  $n(B-A) = 2n(A-B) = 4n(A \cap B)$  and if  $n(A \cup B) = 14$  then find  $nP(A)$

**Solution**

$$\text{Let } n(A \cap B) = k, \quad n(A-B) = 2k \quad \text{and} \quad n(B-A) = 4k$$

$$\text{We know that } n(A \cup B) = n(A-B) + n(B-A) + n(A \cap B)$$

$$14 = 2k + 4k + k = 7k$$

$$7k = 14 \Rightarrow k = 2$$

$$\text{Therefore } n(B-A) = 4k = 4(2) = 8 \quad \text{and} \quad n(A-B) = 2k = 2(2) = 4$$

$$\text{We have a result } n(A) = n(A-B) + n(A \cap B)$$

$$= 4 + 2 = 6$$

$$\text{Therefore } n(A) = 6$$

$$\text{Hence } nP(A) = 2^6 = 64$$


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**Example 1. 6**

Two sets have m and k elements. If the total number of subsets of the first set is 112 more than that of the second set, find the values of m and k

**Solution**

$$\text{Let } n(A) = m \quad \text{and} \quad n(B) = k$$

A contain more number of elements than B

$$\text{Therefore } m > k$$

$$(\text{Number of subsets of A}) - (\text{Number of subsets of B}) = 112$$

$$2^m - 2^k = 112$$

Taking  $2^k$  as common term from the above equation

$$\text{We have } 2^k(2^{m-k} - 1) = 112 = 2^4 \times 7$$

$$\text{Since } 2^k = 2^4 \Rightarrow k = 4$$

$$2^{m-k} - 1 = 7$$

$$2^{m-k} = 7 + 1 = 8 = 2^3$$

$$m - k = 3 \quad (\text{Since } k = 4)$$

$$m - 4 = 3 \Rightarrow m = 7$$

### Example 1. 7

If  $n(A) = 10$  and  $n(A \cap B) = 3$ , find  $n((A \cap B)' \cap A)$

#### Solution

$$\begin{aligned} (A \cap B)' \cap A &= (A' \cup B)' \cap A \\ &= (A' \cap A) \cup (B' \cap A) \quad \text{By distributive law} \\ &= \phi \cup (B' \cap A) \quad (\text{Since } A' \cap A = \phi) \\ &= (B' \cap A) = A - B \end{aligned}$$

There fore  $n[(A \cap B)' \cap A] = n(A) - n(A \cap B)$  By formula

$$= 10 - 3 = 7$$

$$\text{Hence } n[(A \cap B)' \cap A] = 7$$

### Example 1. 8

If  $A = \{1, 2, 3, 4\}$  and  $B = \{3, 4, 5, 6\}$  Find  $n((A \cup B) \times (A \cap B) \times (A \Delta B))$

#### Solution

$$A \cup B = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(A \cup B) = 6$$

$$A \cap B = \{3, 4\} \Rightarrow n(A \cap B) = 2$$

$$\begin{aligned} A \Delta B &= (A - B) \cup (B - A) \\ &= \{1, 2\} \cup \{5, 6\} = \{1, 2, 5, 6\} \end{aligned}$$

$$n(A \Delta B) = 4$$

$$\begin{aligned} \text{Therefore } n((A \cup B) \times (A \cap B) \times (A \Delta B)) &= n(A \cup B) \times n(A \cap B) \times n(A \Delta B) \\ &= 6 \times 2 \times 4 = 48 \end{aligned}$$

### Example 9

#### Solution

If  $P(A)$  denotes the power set of  $A$  then Find  $n[P(P(P(\phi)))]$

We Know that  $\phi = \{ \}$

Therefore  $n(\phi) = 0$

$$n[P(\phi)] = 2^0 = 1$$

$$P(P(\phi)) = \{ \phi, \{\phi\} \}$$

$$n[P(P(\phi))] = 2^1 = 2$$

$$\text{then } n(P(P(P(\phi)))) = 2^2 = 4$$

## Exercise Problems

Ex 1.1

1.

(i) Write in the roster form  $\{x \in \mathbb{N} : x^2 < 121 \text{ and } x \text{ is a prime}\}$ .

Solution

Prime numbers are  $\{2, 3, 5, 7, 11, 13, \dots\}$

$$2^2 = 4 \quad 3^2 = 9 \quad 5^2 = 25 \quad 7^2 = 49 \quad 11^2 = 121 \text{ (this is not less than 121 we have } x=7 \text{ only)}$$

Hence the set  $A = \{2, 3, 5, 7\}$

(ii) Write in the roster form

the set of all positive roots of the equation  $(x-1)(x+1)(x^2-1) = 0$ .

Solution

$$x-1=0 \quad x=1$$

$$x+1=0 \quad x=-1$$

$$x^2-1=0 \quad x=\pm 1$$

The roots are  $\{1\}$  Since  $x$  is Positive ( $-1$  not to be included)

(iii) Write in the roster form

$$(iii) \{x \in \mathbb{N} : 4x + 9 < 52\}$$

Solution

$\mathbb{N}$  = set of natural number  $= \{1, 2, 3, 4, 5, \dots\}$

$$\text{When } n=1 \quad (\text{satisfying the condition } 4x+9 < 52) \Rightarrow 4(1)+9=13$$

$$\text{When } n=2 \quad 4(2)+9=17$$

When  $n=3$   $4(3)+9=21$  and so on..... finally  $n=10$   $4(10)+9=49$

Therefore the set  $A = \{1,2,3,4,5,6,\dots,10\}$

(OR)

$$4x+9 < 52 \Rightarrow 4x+9 < 52 \Rightarrow 4x < 43 \Rightarrow x < 43/4 \Rightarrow x < 10.75 \quad x \in \mathbb{N}$$

Hence the set  $A = \{1, 2,3,4,5,6,\dots,10\}$

---

(iv) Write in the roaster form.

$$(iv) \left\{ x : \frac{x-4}{x+2} = 3, x \in \mathbb{R} - \{-2\} \right\}.$$

Solution

$$\text{Since } \frac{x-4}{x+2} = 3 \quad (\text{By cross multiplication})$$

$$x-4 = 3(x+2)$$

$$x-4 = 3x+6$$

$$x-3x = 6+4 = 10$$

$$-2x = 10 \Rightarrow x = -5$$


---

2. Write the set  $\{-1, 1\}$  in set builder form.

Solution

$$x=1 \text{ and } x=-1$$

$$x-1=0 \text{ and } x+1=0$$

$$\text{Hence } x^2-1=0 \Rightarrow x^2=1$$

$$\text{Therefore the set } A = \{ x / x^2 = 1 \}$$


---

3. State whether the sets are finite or infinite.

(i)  $\{x \in \mathbb{N} : x \text{ is an even prime number}\}.$

Since  $\{2\}$  is the only even prime number Hence the set is finite

(ii)  $\{x \in \mathbb{N} : x \text{ is an odd prime number}\}.$

The set of odd prime  $\{3,5,7,11,13,\dots\}$  Hence the set is infinite

(iii)  $\{x \in \mathbb{Z} : x \text{ is even and less than } 10\}.$

The set of even numbers  $< 10$

$\{\dots, -6, -4, -2, 2, 4, 6, 8\}$  Hence the set is infinite

(iv)  $\{x \in \mathbb{R} : x \text{ is a rational number}\}$ .

Rational Number set is an infinite set  $\{x: p/q \in \mathbb{Q} \mid p, q \in \mathbb{Z} \text{ } q \neq 0\}$

Hence the set is infinite

(v)  $\{x \in \mathbb{N} : x \text{ is a rational number}\}$ .

Every Natural number is rational

$\mathbb{N} = \{1, 2, 3, 4, \dots\}$  Hence the set is infinite

4.

(i) By taking suitable sets  $A, B, C$ , verify  $A \times (B \cap C) = (A \times B) \cap (A \times C)$ .

Let  $A = \{1, 2, 3\}$   $B = \{3, 4\}$   $C = \{4, 5\}$

$B \cap C = \{4\}$

$A \times (B \cap C) = \{(1, 4), (2, 4), (3, 4)\}$  -----LHS

$A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$

$A \times C = \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\}$

$(A \times B) \cap (A \times C) = \{(1, 4), (2, 4), (3, 4)\}$  -----RHS

(ii) By taking suitable sets  $A, B, C$ , verify  $A \times (B \cup C) = (A \times B) \cup (A \times C)$ .

$B \cup C = \{3, 4, 5\}$

$A \times (B \cup C) = \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5)\}$  ...LHS

$A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$

$A \times C = \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\}$

$A \times (B \cup C) = \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5)\}$  ...RHS

(iii) By taking suitable sets  $A, B, C$ , verify  $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$

$A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$

$B \times A = \{(3, 1), (4, 1), (3, 2), (4, 2), (3, 3), (4, 3)\}$

$(A \times B) \cap (B \times A) = \{(3, 3)\}$  .....LHS

$$A \cap B = \{3\} \quad B \cap A = \{3\}$$

$$(A \cap B) \times (B \cap A) = \{(3,3)\} \text{-----RHS}$$


---

5. Justify the trueness of the statement

“An element of a set can never be a subset of it self”

Solution

If  $A = \{2\}$  The sub set of  $A = [\{\}, \{2\}]$   $A = [\{\}, A]$

In this case  $A$  is a subset of  $A$  itself

Hence element of a set can be a subset of itself

$\therefore$  the given statement is not true

---

6. If  $n(P(A)) = 1024$   $n(A \cup B) = 15$  and  $n(P(B)) = 32$  then find  $n(A \cap B)$

Solution

$$n(P(A)) = 1024 = 2^{10} \Rightarrow n(A) = 10$$

$$n(P(B)) = 32 = 2^5 \Rightarrow n(B) = 5$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$15 = 10 + 5 - n(A \cap B)$$

$$15 = 15 - n(A \cap B)$$

$$\therefore n(A \cap B) = 15 - 15 = 0$$


---

7.  $n(A \cap B) = 3$  and  $n(A \cup B) = 10$  then find  $n(P(A \Delta B))$

Solution

$$A \Delta B = (A - B) \cup (B - A)$$

$$= (A \cup B) - (A \cap B)$$

$$n(A \Delta B) = n(A \cup B) - n(A \cap B)$$

$$= 10 - 3 = 7$$

$$n(P(A \Delta B)) = 2^7 = 128$$


---

8. For a set  $A$   $A \times A$  contains 16 elements and two of its elements are (1,3) and (0,2) find the element of  $A$

Solution

We Know that  $= \{(a, b) / a, b \in A\}$

Since (1,3) and (0,2)  $\in A$

$$n(A \times A) = 16$$

$$n(A) \cdot n(A) = 16$$

$$n \times n = 16$$

Hence A contains only 4 elements (We know already 4 elements of A)

$$\text{Hence } A = \{0, 1, 2, 3\}$$

9. If A and B be two sets such that  $n(A)=3$  and  $n(B)=2$ , If  $(x,1)$   $(y,2)$   $(z,1)$  are in  $A \times B$  find A and B where x, y, z are distinct elements

Solution

$$A \times B = \{ (a,b) / a \in A \text{ and } b \in B \}$$

$$\therefore n(A) = 3 \quad \text{we have } A = \{x, y, z\}$$

$$\therefore n(B) = 2 \quad \text{we have } B = \{1, 2\}$$

10. If  $A \times A$  has 16 elements.  $S = \{(a, b) \in A \times A : a < b\}$   $(-1,2)$   $(0,1)$  are two elements of S

Then find the remaining elements of S.

Solution

$$A = \{-1, 0, 1, 2\}$$

$$n(A \times A) = 16$$

$$n(A) \cdot n(A) = 16 = 4^2$$

Hence A has 4 elements

The remaining elements of  $S = \{(-1,0) (-1,1) (0,1) (0,2) (1,2)\}$



# 11<sup>th</sup> Maths – Study Material

1.

Write the set  $B = \{3, 9, 27, 81\}$  in set-builder form.

2

Which of the following are empty sets? Justify.

$$A = \{x : x \in \mathbb{N} \text{ and } 3 < x < 4\} \quad B = \{x : x \in \mathbb{N} \text{ and } x^2 = x\}$$

3

Which of the following sets are finite or Infinite? Justify.

The set of all the points on the circumference of a circle.

$$B = \{x : x \in \mathbb{N} \text{ and } x \text{ is an even prime number}\}$$

4

Are sets  $A = \{-2, 2\}$ ,  $B = \{x : x \in \mathbb{Z}, x^2 - 4 = 0\}$  equal? Why?

5

Write  $\{x : -3 \leq x < 7\}$  as interval.

6

If  $A = \{1, 3, 5\}$ , how many elements has  $P(A)$ ?

7

If  $A = \{1, 2, 3, 6\}$ ,  $B = \{1, 2, 4, 8\}$  find  $B - A$

8

If  $A = \{p, q\}$ ,  $B = \{p, q, r\}$ , is  $B$  a superset of  $A$ ? Why?

9

Are sets  $A = \{1, 2, 3, 4\}$ ,  $B = \{x : x \in \mathbb{N} \text{ and } 5 \leq x \leq 7\}$  disjoint? Why?

10

If  $X$  and  $Y$  are two sets such that  $n(X) = 19$ ,  $n(Y) = 37$  and  $n(X \cap Y) = 12$ , find  $n(X \cup Y)$ .

11

Find the domain of the relation,  $R = \{(x, y) : x, y \in \mathbb{Z}, xy = 4\}$

12

Find the range of the following relations

$$R = \{(a, b) : a, b \in \mathbb{N} \text{ and } 2a + b = 10\} \quad R = \left\{ \left( x, \frac{1}{x} \right) : x \in \mathbb{Z}, 0 < x < 6 \right\}$$

13.

Let  $A = \{1,2\}$  and  $B = \{3,4\}$ . Write  $A \times B$ . How many subsets will  $A \times B$  have? List them.

14

Let  $A = \{1,2,3,\dots,14\}$ . Define a relation  $R$  from  $A$  to  $A$  by  $R = \{(x,y) : 3x-y = 0, \text{ where } x,y \in A\}$ . Write down its domain, co-domain and range.

15

If  $n(A) = 3$  and  $n(B) = 3$ , then find  $n(A \cap B)$ .

16

Examine the relation :  $R = \{(2,1), (3,1), (4,1)\}$  and state whether it is a function or not?

17

Under which condition a relation  $f$  from  $A$  to  $B$  is said to be a function?

18

Let  $A$  and  $B$  be two finite sets such that  $n(A - B) = 30$ ,  $n(A \cup B) = 180$ ,  $n(A \cap B) = 60$ , find  $n(B)$ .

19

If  $A \times B = \{(p,q), (p,r), (m,q), (m,r)\}$ , find  $A$  and  $B$ .

20

If  $X$  and  $Y$  are two sets such that  $n(X) = 17$ ,  $n(Y) = 23$  and  $n(X \cup Y) = 38$ , find  $n(X \cap Y)$ .

21

In a town of 840 persons, 450 persons read Hindi, 300 read English and 200 read both. Find the number of persons who read neither.

22

Show that  $A \cap B = A \cap C$  need not imply  $B = C$ .

23

If the ordered Pairs  $(x-1, y+3)$  and  $(2, x+4)$  are equal, find  $x$  and  $y$

(i)  $(3,3)$  (ii)  $(3,4)$  (iii)  $(1,4)$  (iv)  $(1,0)$

24

If,  $n(A) = 3$ ,  $n(B) = 2$ ,  $A$  And  $B$  are two sets Then no. of relations of  $A \times B$  have.

(i)  $(6)$  (ii)  $(12)$  (iii)  $(32)$  (iv)  $(64)$

25

Let  $R$  be a relation from  $Q$  to  $Q$  defined by  $R = \{(a, b) : a, b \in Q \text{ and } a - b \in \mathbb{Z}\}$   
 show that (i)  $(a, a) \in R$  for all  $a \in Q$  (ii)  $(a, b) \in R$  implies that  $(b, a) \in R$   
 (iii)  $(a, b) \in R$  and  $(b, c) \in R$  implies that  $(a, c) \in R$

26

If  $P = \{a, b, c\}$  and  $Q = \{d\}$ , form the sets  $P \times Q$  and  $Q \times P$  are these two Cartesian products equal?

27

If  $A$  and  $B$  are finite sets such that  $n(A) = m$  and  $n(B) = k$  find the number of relations from  $A$  to  $B$

28

The cartesian product  $A \times A$  has 4 elements among which are found  $(-1, 0)$  and  $(0, 1)$ . find the set and the remaining elements of  $A \times A$

29

If  $A = \{1, 2\}$ , find  $(A \times A \times A)$

30

If  $A$  and  $B$  are two sets containing  $m$  and  $n$  elements respectively how many different relations can be defined from  $A$  to  $B$ ?

31

If  $A = \{1, 2, 3\}$ ,  $B = \{3, 4\}$  and  $C = \{4, 5, 6\}$   
 find (i)  $A \times (B \cup C)$  (ii)  $A \times (B \cap C)$  (iii)  $(A \times B) \cap (B \times C)$

32

Let  $A = \{1, 2, 3, 4, 5\}$  and  $B = \{1, 3, 4\}$   
 let  $R$  be the relation, is greater than from  $A$  to  $B$ . Write  $R$  as a set of ordered pairs. find domain  $(R)$  and range  $(R)$

33

Let  $A = \{1, 2\}$ ,  $B = \{1, 2, 3, 4\}$ ,  $C = \{5, 6\}$  and  $D = \{5, 6, 7, 8\}$  verify that  
 (i)  $A \times (B \cap C) = (A \times B) \cap (A \times C)$  (ii)  $A \times C$  is subset of  $B \times D$

34.

Let  $R = \{(x, y) : y = x + 1\}$  and  $y \in \{0, 1, 2, 3, 4, 5\}$  list the element of  $R$

35

$A = \{1, 2, 3\}$ ,  $B = \{1, 2, 3, 4\}$ . and If  $R = \{(x, y) : (x, y) \in A \times B, y = x + 1\}$  then  
 (i) find  $A \times B$  (ii) write domain and Range

If  $A, B$  are two sets such that  $n(A \times B) = 6$  and some elements of  $A \times B$  are  $(-1, 2), (2, 3), (4, 3)$ , then find  $A \times B$  and  $B \times A$

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