

SETS

Definition: Any well defined Collection of objects is called set.

Some Standard sets:

1. The set of all natural numbers (or Positive integers) = N .
 $N = \{1, 2, 3, \dots\}$

2. The set of all whole numbers - W
 $W = \{0, 1, 2, 3, 4, \dots\}$

3. Set of Integers - Z (or) I

$$Z = \{\dots -3, -2, -1, 0, 1, 2, 3, \dots\}$$

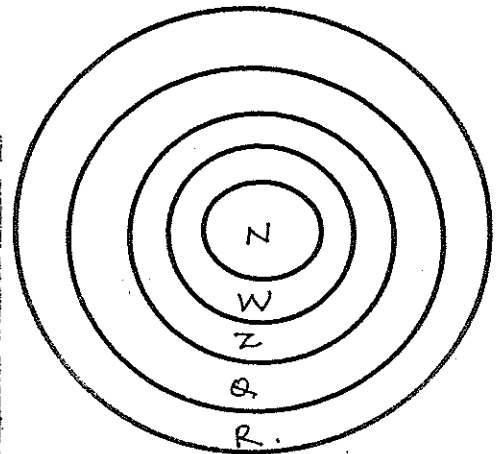
4. Set of rational numbers - Q

$$Q = \left\{ x : x = \frac{m}{n} \text{ where } m \text{ and } n \text{ are integers, } n \neq 0 \right\}$$

5. Set of Real numbers - R .

$$R = \{x : x \text{ is real number}\}$$

$$R = \{x : x \text{ is either rational (or) irrational number}\}$$



Null set (or) void set: A set containing no elements is called empty set ($\emptyset$)

Eg: The collection of all integers whose square is < 0 is empty set.

Finite and infinite sets. A set is called finite if the process of counting of its different elements comes to an end. Otherwise it is called infinite. The empty set is always taken as finite.

Order of a finite set: The number of different elements in a finite set S is called order of S . (or) $n(S)$

Cardinal number: The number of distinct elements in a finite set is called cardinal number.

1. The order of the empty set is zero
2. A set whose order is 1 is singleton set. (or) Singleton set is the set which contains only one distinct element.

Equivalent set: Two finite sets A and B are said to be equivalent ($A \sim B$) iff they contain the number of distinct elements are same.
 (or) $n(A) = n(B)$ (or) cardinal numbers are same

Equal set Two sets A and B are said to be equal ($A = B$) iff every member of A is a member of B and every member of B is a member of A .

Singleton Set: A set which contains only one element is known as singleton set.

Universal Set:

Properties of power set: 1) If $A \subseteq B$ Then $P(A) \subseteq P(B)$

$$2) P(A) \cap P(B) = P(A \cap B)$$

$$3) P(A \cup B) \neq P(A) \cup P(B)$$

1) Natural number: $N = \{1, 2, 3, \dots\}$.

2) Whole Number : $W = \{0, 1, 2, 3, \dots\}$. $N \subset W$.

3) Integer $= Z = \{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$.

4) Rational Number : $Q = \left\{ \frac{p}{q} \right\}$, $p, q \in Z$ and $q \neq 0$. $W \subset Z$.

Note: All integers are also rational number $\because$ it can be represented by $\frac{n}{1}$.

$Z \subset Q$. $\therefore N \subset W \subset Z \subset Q$.

5) Irrational number: A number which cannot be written in the form $\frac{p}{q}$ where p and q ^{both} are integers and $q \neq 0$ called irrational number.

$$Q' = \{x : x \in R, x \notin Q\}.$$

Note: A non terminating, non repeating decimal number are irrational number.

Intersection of two sets: Let A and B be any two sets then the set consisting of the elements which belong to both A and B is called the intersection of A and B ($A \cap B$)

Disjoint sets: Two sets A and B are called disjoint iff $A \cap B = \emptyset$ (i.e.) iff they have no elements in common.

Difference of two sets: Let A, B be any two sets then $A - B$ is the set consisting of all the elements which belong to A but do not belong to B .

Symmetric difference of two sets: Let A, B be two sets. Symmetric difference of A and B denoted by $A \Delta B$ is defined as the set $(A - B) \cup (B - A)$.

Complement of a set: Let A be any given set and U be the universal set, then the set consisting of all the elements of U which do not belong to A is called the complement of A . and is denoted by $\bar{A}$ (or) A' .

Basic laws of Set Theory: If A and B are any two sets and U is the universal set then

- 1) Idempotent laws: $A \cup A = A$ and $A \cap A = A$.
- 2) Identity laws: $A \cup U = U$ and $A \cap U = A$ and $A \cup \emptyset = A$
 $A \cap \emptyset = \emptyset$.
- 3) Commutative law $A \cup B = B \cup A$, $A \cap B = B \cap A$.
- 4) Complement laws: $A \cup \bar{A} = U$, $A \cap \bar{A} = \emptyset$
- 5) Associative law: If A, B, C are any three sets then
 - 1) $(A \cup B) \cup C = A \cup (B \cup C)$
 - 2) $(A \cap B) \cap C = A \cap (B \cap C)$
- 6) Distributive law: If A, B, C are any three sets then
 - 1) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
 - 2) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- 7) De Morgan's law: If A and B are any two sets then $(A \cup B)' = A' \cap B'$
 - 1) $(A \cap B)' = A' \cup B'$
 - 2) $A - (B \cup C) = (A - B) \cap (A - C)$
 - 3) $A - (B \cap C) = (A - B) \cup (A - C)$



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Sub set: Let A and B are two sets s.t every member of A is a member of B , then A is called the subset of B . $A \subset B$.
 If A is the subset of B we say that A is contained in B (or) B contains A then B is the super set.

$$N \subset W \subset Z \subset Q \subset R.$$

Note: 1. Two sets A and B are equal iff $A \subset B$, $B \subset A$.

2. Empty set is also a subset of every set.

Proper Sub set: Let A be the subset of B , we say that A is a proper subset of B if $A \neq B$ (or) if there exists at least one element in B which does not belong to A . A subset which is not proper, is called improper subset.

1. Every set is an improper subset of itself (X)

2. When A is empty, then the null set is the proper subset.

Power set: The set formed by all the subsets of a given set A is called power set of A . denoted by $P(A)$

Note: If $n(A) = n$, then $n[P(A)] = 2^n$ (X)

Working Rule to write the power set

1. First write ϕ

2. write down the singleton ^{sub}sets of each containing only one element -

3. write all the subsets two elements from the set A .

Continue this way and in the end write A itself

Enclose all these in braces to get the power of A .

Comparable set: Two sets A and B are said to be comparable iff either $A \subset B$ (or) $B \subset A$.

Clearly equal sets are always comparable set

Universal Sets: All sets under investigation are regarded as subsets of a fixed set. we call this set the universal set U .

Venn Diagram: we can illustrate the relationship between the sets

with the help of the diagrams known as Venn diagrams.

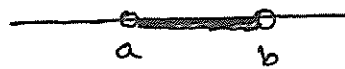
Generally we represent the universal set U by the rectangle enclosing a certain region and its subsets by closed curve within the rectangle.



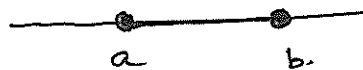
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Intervals as subsets of $\mathbb{R}$.

1) The set $\{x \in \mathbb{R} : a < x < b\}$ is called open interval and is denoted by (a, b)



2) The set $\{x \in \mathbb{R} : a \leq x \leq b\}$ is called closed interval and denoted by $[a, b]$



3) The set $\{x \in \mathbb{R}, a < x \leq b\}$ is an interval left open and right closed.



4) The set $\{x \in \mathbb{R}, a \leq x < b\}$ is an interval left closed and right open



5) The set $\{x \in \mathbb{R} : x < a\}$ is an interval which is denoted by $(-\infty, a)$ It is open on both sides.

6) The set $\{x \in \mathbb{R} : x \leq a\}$ is an interval which is denoted by $(-\infty, a]$ It is closed on the right and open on the left

7) The set $\{x \in \mathbb{R} : x > a\}$ is an interval which is denoted by (a, ∞) both open

8) The set $\{x \in \mathbb{R} : x \geq a\}$ is an interval which is denoted by $[a, \infty)$ left closed Right open.

Note) The first four is finite interval.

2) The last four is infinite interval.

3) Each interval (finite, or infinite) is an infinite set containing infinitely many rational and infinitely many irrational numbers.

Operation on two sets: 1) union of two sets:

Let A, B be any two sets then the set consisting of elements which belong to A or B or both is called union of A and B . $(A \cup B)$



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Applications of sets: Relation involving order of sets

1) If A and B are finite sets and $A \cap B = \emptyset$ then

$$n(A \cup B) = n(A) + n(B)$$

$$n(\bar{A}) = n(U) - n(A) \Rightarrow n(A) + n(\bar{A}) = n(U)$$

In particular $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$\text{When } A \cap B = \emptyset \quad n(A \cup B) = n(A) + n(B)$$

2) If A and B are any two sets for which $A \cap B = \emptyset$ then $A \cup B$ is the union of mutually disjoint sets $A - B$, $A \cap B$, $B - A$. Hence if both A and B are finite sets

$$n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$$

3) If A and B are any two ^{finite} ~~events~~ sets for which $A \cap B = \emptyset$ then A is union of disjoint sets $A - B$ and $A \cap B$ and B is the union of disjoint sets $B - A$ and $A \cap B$ then

$$\left. \begin{aligned} n(A - B) &= n(A) - n(A \cap B) \\ n(B - A) &= n(B) - n(A \cap B) \end{aligned} \right\} \textcircled{x}$$

4) If A and B and C are any three finite sets then

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

Laws of difference of sets: For any two sets A and B we have

- 1) $A - B = A \cap B'$
- 2) $B - A = B \cap A'$
- 3) $A - B \subseteq A$
- 4) $B - A \subseteq B$
- 5) $A - B = A \Leftrightarrow A - B = \emptyset$
- 6) $(A - B) \cup B = A - B \cup B$
- 7) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

For any three sets A, B, C

- 1) $A - (B \cap C) = (A - B) \cup (A - C)$
- 2) $A - (B \cup C) = (A - B) \cap (A - C)$
- 3) $A - (B - C) = (A \cap B) - (A \cap C)$
- 4) $A \cap (B - C) = (A \cap B) - (A \cap C)$
- 5) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

Properties of Complement sets.

- 1) $(A')' = A = U - A'$ (law of double complementation)
- 2) $A \cup A' = U$
- 3) $A \cap A' = \emptyset$
- 4) $\emptyset' = U$
- 5) $U' = \emptyset$
- 6) $(A \cup B)' = U - (A \cup B)$

De Morgan's law

- 1) If A and B are any two sets then 1) $(A \cup B)' = A' \cap B'$
2) $(A \cap B)' = A' \cup B'$
- 2) $A \Delta B = B \Delta A$
- 3) $(A \Delta B) \Delta C = A \Delta (B \Delta C)$
- 4) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

Important Results:

$$1) n(A \cup B) = n(A-B) + n(B-A) + n(A \cap B)$$

$$2) n(A) = n(A-B) + n(A \cap B)$$

$$3) n(B) = n(B-A) + n(A \cap B)$$

$$4) n(A \Delta B) = n[(A-B) \cup (B-A)] \quad \left. \begin{aligned} &= n(A-B) + n(B-A) \\ &= n(A) + n(B) - 2n(A \cap B) \end{aligned} \right\} \because A-B, B-A \text{ are disjoint}$$

$$5) n(A' \cup B') = n[(A \cap B)'] = n(U) - n(A \cap B), \quad n(A' \cap B') = n(A \cup B)'$$

$$6) n(A-B) = n(A \cap B)' = n(A) - n(A \cap B) \quad = n(U) - n(A \cap B)$$

If A, B, C are three finite sets. then

$$1) n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$2) n(A \text{ only}) = n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$$

$$3) n(\bar{A} \cap \bar{B} \cap \bar{C}) = n(U) - n(A \cup B \cup C)$$

Identity: If 1) $A \cup \phi = A$, $A \cap U = A$

Idempotent $A \cup A = A$, $A \cap A = A$.

Absorption: $A \cup (A \cap B) = A$

$$A \cap (A \cup B) = A$$

$$4) n(A \cap B' \cap C') = n\{A \cap (B \cup C)'\} = n(A) - n\{A \cap (B \cup C)\}$$

$$5) \{(A \cup B' \cup C) \cap (A \cap B' \cap C')\} \cup \{(A \cup B \cup C') \cap (B' \cap C')\} = B' \cap C'$$

Relations

Ordered pair: An ordered pair consists of two objects or elements in a given fixed order.

Equal of ordered Pairs: Two ordered pairs (a, b_1) and (a_2, b_2) are equal iff $a_1 = a_2$ and $b_1 = b_2$

Cartesian product of sets: For two nonempty sets A and B the set of all ordered pairs (a, b) s.t. $a \in A$ and $b \in B$ is called Cartesian product of the sets A and B denoted by $A \times B$.

$$(ie) A \times B = \{ (a, b) : a \in A \text{ and } b \in B \}$$

If there are three sets A, B and C and $a \in A$

$b \in B$ and $c \in C$ then $A \times B \times C = \{ (a, b, c) : a \in A, b \in B, c \in C \}$

Properties of Cartesian product:

1. For three sets A, B and C

$$1) n(A \times B) = n(A) \cdot n(B)$$

2. $A \times B = \emptyset$ if either A or B is any empty set

$$3. A \times (B \cup C) = (A \times B) \cup (A \times C)$$

$$4. A \times (B \cap C) = (A \times B) \cap (A \times C)$$

$$5. A \times (B - C) = (A \times B) - (A \times C)$$

$$6. (A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$$

$$7. A \subseteq B \text{ and } C \subseteq D \text{ then } (A \times C) \subseteq (B \times D)$$

$$8. A \subseteq B \text{ then } A \times A \subseteq (A \times B) \cap (B \times A)$$

$$9. A \times B = B \times A \iff A = B$$

10. If either A or B is an infinite set then $A \times B$ is infinite

$$11. A \times (B' \cup C') = (A \times B) \cap (A \times C) \quad A \times (B' \cap C') = (A \times B) \cup (A \times C)$$

$$12. A \times (B' \cap C') = (A \times B) \cup (A \times C)$$

✓ 13. If A and B are any two non empty sets having n elements in common then $A \times B$ and $B \times A$ have n^2 elements in common.

$$14. A \neq B \text{ then } A \times B \neq B \times A$$

$$15. A = B \text{ then } A \times B = B \times A, \text{ 16) } A \subseteq B \text{ then } A \times C \subseteq B \times C \text{ for any set C.}$$

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Relation: If A and B are two nonempty sets then the relation R from A to B is a subset of $A \times B$.

If $R \subseteq A \times B$ and $(a, b) \in R$, then we say that a is related to b by the relation R .

Domain and Range: Let R be the relation from the set A to B . Then the set of all first components or Co-ordinates of the ordered pairs belonging to R is called the domain of R while the set of all second elements or Co-ordinates of the ordered Pairs belonging to R is the Range of R .

(u) The set of elements of A is domain and the set of image elements is range.

The set of elements of B is Co-domain
Range $\subseteq$ of codomain.

Types of Relation:

1. Void Relation: As $\phi \subseteq A \times A$ for any set A , ϕ is the relation (or) empty relation on A called empty relation or void relation.
2. Universal Relation: $A \times A \subseteq A \times A$, so $A \times A$ is a relation on A called the universal relation.
3. Identity relation: The relation $I = \{(a, a) : a \in A\}$ is called identity relation.
4. Reflexive relation: A relation R is said to be reflexive relation if every element of A related to itself;
 $(a, a) \in R \quad \forall a \in A \Rightarrow R$ is reflexive.
5. Symmetric Relation: A relation R is said to be symmetric relation iff $(a, b) \in R \Rightarrow (b, a) \in R \quad \forall a, b \in R$.
6. Anti symmetric relation: A relation R is said to be anti symmetric relation iff $(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b, \forall a, b \in A$.
7. Transitive Relation: A relation R is said to be transitive relation iff $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$
 $\forall a, b, c \in A$.

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8) Equivalence Relation: A relation R is said to be an equivalence relation if it is reflexive, symmetric and transitive on A .

9. Partial order relation: A relation R is said to be partial order relation, if it is reflexive, symmetric and antisymmetric.

10). Total order Relation: A relation R on a set A is said to be a total order relation on A if R is partial order relation on A .

Inverse Relation: If A and B are two non empty sets and R be a relation from A to B s.t. $R = \{(a, b) : a \in A, b \in B\}$ then the inverse of R denoted by R^{-1} is the relation from B to A and is defined by $R^{-1} = \{(b, a) : (a, b) \in R\}$.

Equivalence classes of an Equivalence Relation.

Let R be equivalence relation in A ($A \neq \emptyset$) Let $a \in A$.

Then the equivalence class of a denoted by $[a]$ is defined as the set of all those points of A which are related to a under the relation R .

Composition of Relation: Let R and S be two relations from the set A to B and B to C respectively, then we can define the relation $S \circ R$ from A to C s.t. $(a, c) \in S \circ R \Leftrightarrow \exists b \in B$ s.t. $(a, b) \in R$ and $(b, c) \in S$.

Congruence Modulo n : Let n be an arbitrary but fixed integer. Two integers a and b are said to be congruence modulo n if $a - b$ is divisible by n .

$$a \equiv b \pmod{n} \Leftrightarrow a - b \text{ is divisible by } n.$$

1. If R and S are two equivalence relation on a set A then $R \cap S$ is also equivalence relation on A .
2. The union of two equivalence relation on a set is not necessarily an equivalence relation on a set.
3. If R is an equivalence relation on a set A then R^{-1} is also an equivalence relation on A .
4. If a set A has n elements, then the number of reflexive relations from A to A is $2^{n^2 - n}$.

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Examples: Empty relation: 1) Consider a relation R in the set $A = \{1, 2, 3, 4\}$ given by $R = \{(a, b) : a \in A, b \in A, a \neq b \text{ and } a = b\}$. This is the empty set as no pair (a, b) satisfies the condition $a \neq b, a = b$.

2) Consider the relation R in the set $A = \{1, 3, 5, 7\}$ given by $R = \{(a, b) : a - b = 9\}$. This is empty set as no pair satisfies the condition $a - b = 9$ in A .

Universal Relation: 1) Let $A = \{1, 2, 3\}$ then

$R = \{(1, 1) (1, 2) (1, 3), (2, 1) (2, 2) (2, 3) (3, 1) (3, 2) (3, 3)\}$ is universal relation in A .

2) Let us consider the relation R in the set $A = \{1, 3, 5, 7\}$ given by

$$R = \{(a, b) : |a - b| \geq 0\}.$$

As all pairs (a, b) in $A \times A$ satisfies $|a - b| \geq 0 \therefore R = \{(a, b) : |a - b| \geq 0\}$ is the whole set $A \times A$. $\therefore R$ is universal set.

Identity relation: If $A = \{1, 2, 3, 4, 5, 6\}$ then $I = \{(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6)\}$

In any identity relation on A , every element of A should be related to itself only.

Inverse relation: If $A = \{1, 2, 3, 4\}$ $B = \{a, b, c\}$

$$R = (1, a) (2, b) (3, a) (4, b) \text{ then}$$

$$R^{-1} = (a, 1) (b, 2) (a, 3) (b, 4)$$

We conclude that the domain of R is identical with Range of R^{-1} and the Range of R is identical with the domain of R^{-1} .

(i.e) Domain of $R = \text{Range of } R^{-1}$, Range of $R = \text{Domain of } R^{-1}$.

Reflexive relation: 1. Let A be the set of triangles in a plane. The relation R in A 'is similar to' is reflexive because every triangle is similar to itself.

2. Let A be the set of all lines in a plane and R means 'is parallel to'. Now every line $a \in A$ is parallel to itself.

3. The relation ' $>$ ' on the set R is not reflexive because no real number can be not greater than to itself.

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Symmetric Relation: i) Let A be the set of lines in a plane and let R means 'is perpendicular to'. Then R is symmetric. (ie) if the line a is $\perp$ to the line b then it implies that the line b is $\perp$ to a .

$\therefore R$ is symmetric.

2. Let N be the set of natural numbers and R means a divides b . Then the relation R is anti symmetric. $\because$ a divides b and b divides a only when $a = b$.

Transitive Relation: 1. Let A be the set of all lines in a plane and R be the relation in A defined by 'is parallel to'. Then if the line a is parallel to b and b is parallel to c then a is parallel to c . Hence R is transitive.

2) Let A be the set of members of a family and R means 'is the brother of'. Now if a is a brother of b and b is the brother of c then a is the brother of c . $\therefore R$ is transitive.

3) If the R is taken as a subset of $A \times A$ then $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$. R is called transitive relation.

Equivalence relation: A relation R on a set A is called an equivalence relation on A when R is reflexive, symmetric, and transitive.

(ie) if 1) $(a, a) \in R, a \in A$

2) $(a, b) \in R, \Rightarrow (b, a) \in R$, where ever $a, b \in A$.

3) $(a, b) \in R$, and $(b, c) \in R \Rightarrow (a, c) \in R \forall a, b, c \in A$

Example: Let A be the set of all triangles in a plane and let R be defined by 'is Congruent to'.

1. Since every triangle is congruent to itself R is reflexive.

2. Since if triangle a is congruent to b then the triangle b is congruent to triangle a . Then R is symmetric.

3. If the triangle a is congruent to triangle b and triangle b is congruent to the triangle c then the triangle a is congruent to c . Then R is transitive.

$\therefore$ The relation defined above is reflexive, symmetric, transitive. Hence R is an equivalence relation.

Horizontal test to find the function through its graphs -

Let the function be given as a curve in the plane. If the horizontal through a point y in a co-domain meets the curve at some point then the x -coordinate of all the points gives pre image of y

1. If the horizontal line through a point y in the co-domain does not meet the curve, then there will be no pre image for y and hence the function is not onto
2. If the horizontal line through at least one point meets the curve at more than one point, then the function is not one-one
3. If for all y in the range the horizontal line through y meets the curve at only one point then the function is one-one.

Method to solve problems Based on types of relation.

For Reflexive: we have to show that $\forall a \in A, (a, a) \in R$. For this we take an arbitrary element of set A in form of variable (say x) and then check (x, x) satisfies the given condition of R or not (ie $(x, x) \in R$ or not). If it satisfies this then R is reflexive.

For Symmetric: For $a, b \in A$ if $(a, b) \in R$ then $(b, a) \in R$ for this take two arbitrary elements of A in the form of two variables (x and y) s.t $(x, y) \in R$ and then check (y, x) satisfies the given condition of R or not (ie) if $(x, y) \in R \Rightarrow (y, x) \in R$ then R is symmetric otherwise not.

For Transitive: $\forall a, b, c \in A, (a, b) \in R, (b, c) \in R$ then $(a, c) \in R$. For this we take three arbitrary elements $\in A$ in the form of x, y, z s.t $(x, y) \in R$ and $(y, z) \in R$ then check whether (x, z) satisfies the condition of R or not. (ie) $(x, z) \in R$. If the condition satisfies then R is transitive otherwise not.

Functions

Definition: A relation R from a set A to the set B is said to be the function if every element of set A has one and only one image in set B .

(oo) A function f from $A \rightarrow B$ s.t the domain of f is A and no two distinct ordered pairs in f have the same first element.

Eggs: The relation 'son of' is a function But the relation 'father of' is not a function.

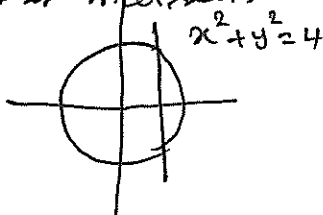
Note: Function is also a mapping.

Characteristics of a function $f: A \rightarrow B$:

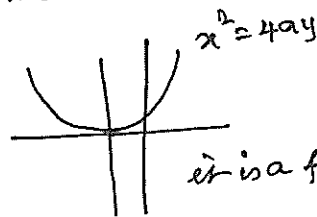
- 1) For each element $x \in A$, there is a unique element image y in B .
- 2) $y \in B$ is the value of the function.
3. $f: A \rightarrow B$ is not a function, if there is an element in A which has more than one image in B but more than one element of A may be associated with same elements of B .
4. $f: A \rightarrow B$ is not a function if an element of A does not have an image in B .

Identification of a function with its Graphs:

If we draw a vertical line (or) $\parallel$ to y axis, then if this line intersects the graph of the expression in more than one point then it is a relation. If it intersects at only one point then it is a function.



relation, not a function



it is a function.

Every function is a relation.

Relation need not be a function.

classification: Real functions are generally classified under two categories algebraic function and transcendental function.

Algebraic function: 1) polynomial function: If the function $y = f(x)$ is

$$\begin{aligned} \text{given by } f(x) &= a_0 x^n + a_1 x^{n-1} + \dots + a_{n-1} x + a_n \\ &= \sum_{i=0}^n a_i x^{n-i} \end{aligned}$$

where $a_0, a_1, a_2, \dots, a_n$ are real numbers and n is any non-negative integer.

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then $f(x)$ is called a polynomial function in x . then the degree of the polynomial is n .

Eg: $y = f(x) = 3x^5 - 4x^2 - 2x + 1$ is a polynomial fn. of degree 5.

2) Rational function: If the function $y = f(x)$ is given by $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomial function then $f(x)$ is called rational fn. in x .

3) Irrational function: The algebraic function containing two or more terms having non integral rational power of x is called irrational function. Eg: $y = f(x) = 2\sqrt{x} - 3\sqrt[3]{x} + b$

In General Algebraic fn. is in the form $y = f(x)$

II Transcendental function: A function which is not algebraic fn. is known as Transcendental function

Eg: Trigonometric, Inverse Trigonometric, Exponential, logarithmic function.

Explicit Function: A function is said to be explicit function if it is expressed as $y = f(x)$

Implicit function: A function is said to be implicit function if it is expressed $f(x, y) = c$. where c is constant.

Eg $\cos(x+y) - (\cos x - y) = 2$

Intervals of the function:

1) closed interval: The set of real numbers x s.t $a \leq x \leq b$ is called closed interval and is denoted by $[a, b]$

2) open interval: The set of all real numbers s.t $a < x < b$ is called open interval and is denoted by (a, b)

3) Semi open = $[a, b) = a \leq x < b$.

semi closed $(a, b] = a < x \leq b$.

Periodic Function: A function $f(x)$ is said to be a periodic fn. of x provided there exist a real number $T > 0$ s.t

$$f(T+x) = f(x) \quad \forall x \in \mathbb{R}.$$

The smallest positive real number T , satisfying the above condition is known as period or the fundamental period of $f(x)$.



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- Important points
- 1) Constant function is a periodic function with no fundamental period.
 2. If $f(x)$ is a periodic function with period T then $\frac{1}{f(x)}$, $\sqrt{f(x)}$ is also periodic with same period T .
 3. If $f(x)$ is periodic with T_1 and $g(x)$ is periodic with T_2 then $f(x) + g(x)$ is periodic with period equal to LCM of T_1 and T_2 provided there is positive k s.t $f(k+x) = g(x)$, $g(k+x) = f(x)$.
 4. $\sin x$, $\cos x$, $\csc x$, $\sec x$ are periodic function with period 2π
 5. $\tan x$, $\cot x$ is periodic fns with period π .
 6. $|\sin x|$, $|\cos x|$, $|\tan x|$, $|\cot x|$, $|\sec x|$ and $|\csc x|$ is also a periodic fns with period π .
 7. $\sin^n x$, $\cos^n x$, $\sec^n x$, $\csc^n x$ are periodic function with period 2π when n is odd (or) even.
 8. $\tan^n x$, $\cot^n x$ are periodic with period π
 9. $|\sin x| + |\cos x|$, $|\tan x + \cot x|$, $|\sec x + \csc x|$ are periodic with period $\pi/2$

Even Function: A real function $f(x)$ is an even function if
 $f(-x) = f(x)$ Eg 1) $f(x) = x^2$ $f(x) = \cos x$
 $f(-x) = x^2$ $f(-x) = \cos(-x)$
 $= \cos x$.

Odd Function: A real function $f(x)$ is an odd function if $f(-x) = -f(x)$ Eg $f(x) = x^3$ $f(x) = \sin x$
 $f(-x) = -x^3$ $f(-x) = \sin(-x)$
 $= -\sin x$.

Properties: 1 Even $\pm$ Even = Even

2 odd $\pm$ odd = odd.

3 Even $\times$ odd = odd fns.

4 Even $\times$ even = Even

5 odd $\times$ odd = even.

6 If f or g , $g \pm f$ is even if any one of f and g or both are even.

7. f or g , $g \pm f$ is odd if f and g are both are odd.

8. The graph of the even fns is symmetrical to Y axis

The graph of the odd fns is symmetrical about origin (or) opposite quadrants.

9. An even fns can never be one to one, odd function may or may not be one-one.



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Types of function: (Mappings)

one-to-one or injective function: A function $f: A \rightarrow B$ is defined to be one to one if different elements in A have different images in B . (ie) if no two different elements in A have same image in B . in other words Every element in A has unique image in B .

$$\text{For every } x_1, x_2 \in A, f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

$$\text{if } x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$$

Eg: 1) $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 2x + 3$

$$f(x_1) = 2x_1 + 3, f(x_2) = 2x_2 + 3.$$

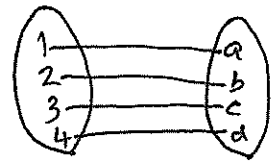
$$\text{If } f(x_1) = f(x_2)$$

$$2x_1 + 3 = 2x_2 + 3$$

$$2x_1 = 2x_2$$

$$x_1 = x_2$$

then f is one to one.



Eg: 2 $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = |x| + x$.

$$\text{For } x_1 \neq x_2 \text{ (ie) } -2 \neq +3$$

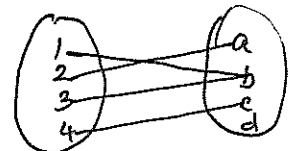
$$f(-2) = -x + x = 2 - 2 = 0$$

$$f(+3) = -x + 3 = 0 \Rightarrow f(-2) = f(+3)$$

But $-2 \neq +3$
 $\therefore f$ is not one-one.

Many to one: A function $f: A \rightarrow B$ is said to be many to one function if at least two elements in A have same image in B .

$$\text{For } x_1 \neq x_2 \Rightarrow f(x_1) = f(x_2)$$



$$\text{Eg: } f(x) = x^2$$

$$\text{Let } -1, +1 \in \mathbb{R} \text{ and } -1 \neq +1 \text{ (ie) } x_1 \neq x_2$$

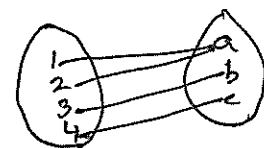
$$f(-1) = 1$$

$$f(+1) = 1$$

$$\text{(ie) } f(x_1) = f(x_2) \therefore f \text{ is many to one}$$

On to function: A function $f: A \rightarrow B$ is said to be on to (surjective) if every element of B is the image of some elements in A under f .

(ie) Range = Codomain.



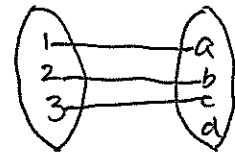
(Note) It may be one to one (or) many to one



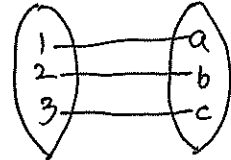
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Into function: A function $f: A \rightarrow B$ is called into function if there is at least one element of B which has no pre image in A .

Eg: $A = \{1, 2, 3\}$ $B = \{a, b, c, d\}$
 $f: (1, a) (2, b) (3, c)$



Bijective function: A function $f: A \rightarrow B$ is called bijective if f is both one to one and on.



Composition of functions:

Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be two functions. Then the composition of function f and g denoted by $f \circ g$ is defined by $f \circ g: A \rightarrow C$. (15)

Note: 1) In general $f \circ g \neq g \circ f$. Composition of function is not commutative.

2) $f \circ g = g \circ f$ if f and g are inverses to each other.

3) If f and g are two bijections then composition of function is also bijection.

4. Composition of function with the identity function is the function itself.

Invertible function: A function $f: A \rightarrow B$ is said to be invertible

if there exists a function $g: B \rightarrow A$ s.t. $g \circ f = f \circ g = I$.

$\therefore$ Function g is called the inverse of f and is denoted by f^{-1} .

$\therefore g = f^{-1}$.

Note: $f: A \rightarrow B$, $g: B \rightarrow A$ be two functions s.t. $g \circ f = f \circ g = I$

then f must be both one to one and onto.

Otherwise f^{-1} does not exist.

Inverse function (if exist) is unique.

Procedure to find out the inverse of a function.

Let $f: A \rightarrow B$ be bijection. In order to find f^{-1}

Put $y = f(x)$, solve $y = f(x)$ and find the x value in terms of y . Now replace x by $f^{-1}(y)$.

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1) Find the inverse fun. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a fn. defined by

(*) $f(x) = 2x + 1$. Find f^{-1} .

Sol: Let $g = f^{-1}$

$$g \circ f = I$$

$$(g \circ f)(x) = x$$

$$g \circ f(x) = x$$

$$g(2x+1) = x.$$

$$\text{Let } 2x+1 = y$$

$$x = \frac{y-1}{2}$$

$$\therefore g(x) = \frac{y-1}{2}$$

$$f^{-1}(y) = \frac{y-1}{2} \Rightarrow f^{-1}(x) = \frac{x-1}{2}.$$

2) Let $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x+1$ find f^{-1} .

Let $g = f^{-1}$.

(*)

$$(g \circ f)(x) = I = x.$$

$$g \circ f(x) = x.$$

$$g(x+1) = x.$$

$$\text{Let } x+1 = y \Rightarrow x = y-1.$$

$$g(y) = y-1.$$

$$f^{-1}(y) = y-1.$$

$$f^{-1}(x) = x-1.$$

3) Find the domain of the rational fn. $f(x) = \frac{x^2+x+2}{x^2-x}$

$$q(x) = x^2 - x = x(x-1) = 0$$

$$x = 0, 1$$

$\therefore$ Domain of fn. $S = \mathbb{R} - \{0, 1\}$.

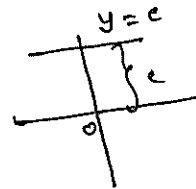


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Types of function:

1. Constant function:

Let c be a fixed real number. The function that associates to each real number x , this fixed number c is called a constant function. (ex) $y = f(x) = c \quad \forall x \in \mathbb{R}$.



⊗ Here range set is singleton set. ⊗

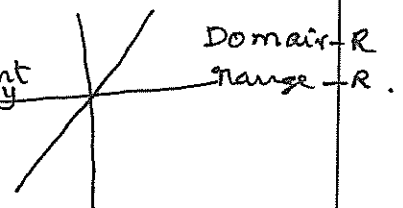
$$\text{Domain} = \mathbb{R}$$

$$\text{range} = \{c\}$$

2) Identity fn: The function that associates to each real number x for the same is called the identity fn.

$$(ex) \quad y = f(x) = x \quad \forall x \in \mathbb{R}.$$

For each element of x there is a unique element y at the same time all Real value of x has image and Range = Codomene



Hence Constant fn. is both one-one and onto. $\therefore f^{-1}$ exist.

3) Linear function: A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined in the form $f(x) = ax + b$ where a and b constant is called linear fn.

$$\left. \begin{array}{l} \text{Domain} = \mathbb{R} \\ \text{range} = \mathbb{R} \end{array} \right\}$$

Note 1. Graph of the linear fn. is st. line

2. Inverse of linear fn. always exist and f^{-1} is also linear.

$\therefore$ inverse fn. of Linear fn. is one to one and onto

4) Polynomial fn: Let $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ where $a_0, a_1, \dots, a_n$ are real numbers, $a_n \neq 0$. Then f is the polynomial function.

1) $\therefore f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^3 + 5x^2 + 3$ is a cubic polynomial fn. or polynomial fn. of degree two.

2) $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = ax^2 + bx + c$ $a, b, c \neq 0$ is quadratic polynomial (or) polynomial fn. of degree two.



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Rational fn: Let $p(x)$ and $q(x)$ be any two polynomial fn.
Let S be the sub set of R obtaining after removing all values of x for which $q(x) = 0$ from R .

The ~~set~~ function $f: S \rightarrow R$ defined by $f(x) = \frac{p(x)}{q(x)}$ $q(x) \neq 0$ is called the rational fn.

Exponential fn: For any number $a > 0, a \neq 1$. the function $f: R \rightarrow R$ defined by $f(x) = a^x$ is called an exponential function.

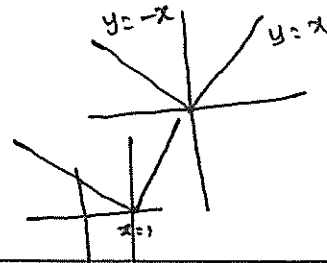
For exponential function the range is always R^+ .
(Set of all +ve real numbers) Domain R , Range $(0, \infty)$

Logarithmic functions: The general form is $f(x) = \log_a x$
Domain $(0, \infty)$ of a log fn becomes Co-domain of exponential fn. $a \neq 1$ and any +ve num.
Codomain $(-\infty, \infty)$ of log fn. becomes the domain of exponential fn.
The inverse of exponential fn is logarithmic fn.

Modulus fn $y = f(x) = |x|$ where $|x|$ denotes the absolute value of x that is $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

Domain R ,
Range $= [0, \infty)$

For $f(x) = |x-1|$

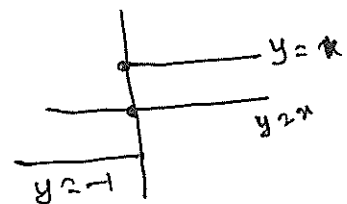


Signum fn: $f(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ (or) $f(x) = \begin{cases} \frac{x}{|x|} & x \neq 0 \\ 0 & x = 0 \end{cases}$

$y = f(x) = \text{sgn}(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$

Domain $= R$

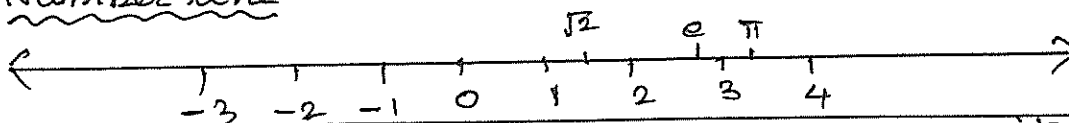
Range $\{-1, 0, 1\}$





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Maths - one Mark Tips.

Functions and Graphs.1. Number line

2. Interval : Subset of real line is called an interval if it contains at least two numbers and contains all the real numbers lying between any two of its elements.

Eg: the set of all real numbers x s.t. $x > 6$

$$x \text{ s.t. } -2 \leq x \leq 5$$

$$x \text{ s.t. } x < 5.$$

3) Set of all natural numbers is not an interval.

Set of all non zero real numbers is also not an interval.

Since: 1) Between any two natural number there are infinitely many real numbers which are not included in the given set.

2. Here 0 is absent. It fails to contain every real number between any two real numbers say -1 and 1.

4) A finite interval is said to be closed if it contains both of its end points and open if it contains neither of its end points and it is denoted by $[]$, $()$ respectively.

$$[2, 5] = 2 \leq x \leq 5, \quad (2, 5) = 2 < x < 5.$$

5. We can't write closed interval by $-\infty$ or ∞ . They are not representatives of Real numbers.

6) Neighbourhood : 1) In a number line the neighbourhood of a point (real number) is defined by an open interval of small length.

2) In a plane neighbourhood of a point is defined as an open disc with small radius.

3) In a space neighbourhood of a point is defined as an open sphere with very small radius.

7) A variable is an independent variable when it has any arbitrary value. (radius)

A variable is said to be dependent when its value depends on the other variable.
(V, A, S)



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8) Cartesian product:

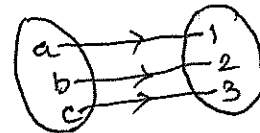
Let $A = \{a_1, a_2, a_3\}$, $B = \{b_1, b_2\}$. The Cartesian product of these two sets A and B is denoted by $A \times B$ is

$$A \times B = \{(a_1, b_1), (a_1, b_2), (a_2, b_1), (a_2, b_2), (a_3, b_1), (a_3, b_2)\}$$

$$A \times B \neq B \times A$$

9) Relation: Let A and B be any two sets. A relation from $A \rightarrow B$ is the subset of the Cartesian product of $A \times B$.

10) Function: Relation may be function (or) may not be function. In the ordered pair set if the first elements are different and the second element are also different then the relation said to be function.



In the ordered pair set if the first elements are same and the second element are different it is not a function.



X. Function must be relation

11) Domain: Co-domain, Range

Domain is the set A

Codomain is the set B.

The image set of the elements A is Range.

X. The range may or may not be equal to Co-domain.

12) If the domain is not stated explicitly for the function $y = f(x)$. The domain is assumed to be the largest set of x values which the formula gives real y values.

$$f(x) = \sqrt{4 - x^2} \quad [-2, 2]$$

In this function $-2 \leq x \leq 2$ $f(x)$ gives real value otherwise imaginary.

$\therefore [-2, 2]$ is the domain.

13) Similarly for the suitable value of x the value of $f(x)$ (i.e) image set is the range.

$$f(x) = \sqrt{9 - x^2} \quad [-3, 3] \quad \text{(i.e) images}$$

for any suitable value of x the result will be in $[0, 3]$

$\therefore [0, 3]$ is the range.

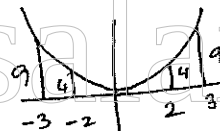
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Function	Domain (x)	Range (y or f(x))
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \frac{1}{x}$	$\mathbb{R} - \{0\}$ Non zero Real numbers	$\mathbb{R} - \{0\}$
$y = \sqrt{1-x^2}$	$[-1, 1]$	$[0, 1]$
$y = \sin x$	$(-\infty, \infty)$ $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ principal domain	$[-1, 1]$
$y = \cos x$	$(-\infty, \infty)$ $[0, \pi]$ principal domain	$[-1, 1]$
$y = \tan x$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ principal domain	$(-\infty, \infty)$
$y = e^x$	$(-\infty, \infty)$	$(0, \infty)$
$y = \log_e x$	$(0, \infty)$	$(-\infty, \infty)$

14) Graph of the function: The graph of a function f is the graph of the equation $y = f(x)$

1) $f(x) = x^2$

for every x there is a unique y

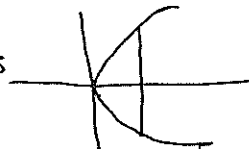


If we draw a vertical line it cuts the curve at only one place

$\therefore$ The function $y = x^2$ is a graph.

2) $y^2 = x$ or $y = \pm \sqrt{x}$

If we draw a vertical line it cuts the curve at two points



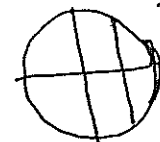
(Note: For the function, in the ordered pair first value are different, 2 second elements may be same)

(e) For unique value y x two different value of y . Hence it is not a graph. at $x=4$, $y = \pm \sqrt{4}$ $(2, \sqrt{4})$ $(2, -\sqrt{4})$

3) $x^2 + y^2 = 4$

$x^2 + y^2 = 4$ is a circle with radius 2.

If we draw a vertical line at $x=1$ it cuts the curve at two places (e) For $x=1$ $y = \pm \sqrt{3}$ $(1, \sqrt{3})$ $(1, -\sqrt{3})$.



Hence it is not a function.

4) 1) $y = x^3$ 2) $y = \cos x$ 3) $y = \sin x$ 4) $y = e^x$

$y = x^3$

$y = \cos x$

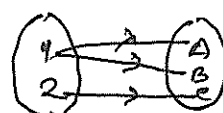
$y = \sin x$

$y = e^x$

are graph of the function.

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15) Types of functions

1. onto functions: If the range and co-domain are equal then the function is onto. otherwise it is called into fn.
(or) the image set is equal to co-domain then it is called onto.
(or) every element in the co-domain has pre image.
(or) ~~any~~ ^{if any} single element in the co-domain ~~is not connected with~~ ^{is connected with} the element of A. (not onto)
2) In the ordered pair, first elements are different second element are same.
3) any ~~two~~ element in the set A has ~~same~~ ^{Two} images in B. But range and co-domain should be same 
4) onto function is also known as surjective function.

2) one to one function:

- A function is said to be one to one if each element of the range is associated with exactly one element of the domain.
- (or) two different elements in the domain A have different images in the co-domain B. But in co-domain there may be some element which has no pre image.
- 2) in ordered pair set the first elements are different and the second elements are also different.

(or) $a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2)$ 

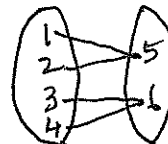
Example 1) let $A = \{1, 2, 3, 4\}$ $B = \{5, 6\}$

$f(1) = 5, f(2) = 5, f(3) = 6, f(4) = 6$. then S.T f is

onto function.

Ordered Pair

$(1, 5) (2, 5) (3, 6) (4, 6)$



Range = $\{5, 6\}$

Co-domain = $\{5, 6\}$. $\therefore$ The given function is onto.

Example 2) let $A = \{1, 2, 3, 4\}$ $B = \{5, 6, 7\}$, $f(1) = 5, f(2) = 5$

$f(3) = 6, f(4) = 6$ then S.T it is not onto (or) into

Range = $\{5, 6\}$, Co domain = $\{5, 6, 7\}$ Range $\neq$ Co domain $\therefore$ it is not onto

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Example for not onto: one to one

S.T the function $y = x^2$ is not onto.

For $x = 1$ $y = 1$ $(1, 1)$ $(-1, 1)$

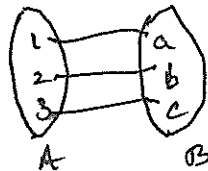
$x = -1$ $y = 1$ In the ordered pair the first element

are different but second elements are same.

(ie) different element has same image

Example for one to one

$A = \{1, 2, 3\}$ $B = \{a, b, c\}$ S.T the function is defined by $\{(1, a), (2, b), (3, c)\}$ is one to one fn.



different elements in A has different images in B $\therefore$ it is one to one.

- 16) A function is said to be injective if it is one-to-one
 17) A function is said to be bijective if it is both one to one and onto. and surjective if it is onto.

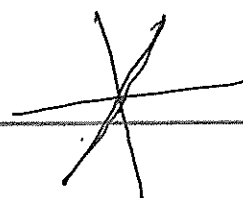
Example: S.T the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x + 1$ is bijective

for the set of all $\mathbb{R}$, for the different real numbers have different images under $f(x) = x + 1$ and no same element has the different images $\therefore$ it is onto

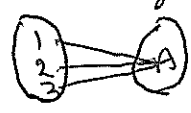
Further $f(a_1) = f(a_2) \Rightarrow a_1 + 1 = a_2 + 1 \Rightarrow a_1 = a_2$ Hence it is one to one. $\therefore$ it is bijective

18) $f: A \rightarrow A$ is defined by $f(x) = x \quad \forall x \in A$ then the function is said to be identity function and is denoted by I .

If every element has the same image then it is identity fn.



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- 19) For the function f , if f^{-1} exist it must be one to one and onto
(a) f^{-1} exists iff f is one to one and onto.
- 20) For the inverse function f^{-1} the co domain of f becomes domain of f^{-1} .
- 21) Since all the functions are relation, the inverse function is also a relation.
- 22) If the function is not one to one and onto f^{-1} does not exist.
- 23) In general $f \circ g \neq g \circ f$ (a) the composition of function need not be commutative. There may be some special cases have commutative.
- 24) By the definition of identity fun $f \circ g = g \circ f = I$.
- 25) Let $f: A \rightarrow B$ and be a function. If then exists a fun $g: B \rightarrow A$ s.t $f \circ g = I_B$ and $g \circ f = I_A$ then g is called called the inverse of f and is denoted by f^{-1}
- 26) The domain and the co-domain of both f and g are same
 $f \circ g = g \circ f = I$.
- 27) If f^{-1} exists then f is said to be invertible
28. $f \circ f^{-1} = f^{-1} \circ f = I$. (commutative admissible for I)
- 29) $(f \pm g)(x) = f(x) \pm g(x)$
- 30) $f \cdot g(x) = f(x) \cdot g(x)$
- 31) $\frac{f}{g}(x) = f(x) / g(x)$
32. $(cf)(x) = c \cdot f(x)$
33. Product of two functions is different from composition of function
34. Constant fun: If the range of the function is singleton set then the function is called constant fun.
- Eg: 'is a son of' is a constant fun between sons and father here father is a range
- 



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35) Linear function: A fn. $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined in the form $f(x) = ax + b$ then the function is called linear function.

The graph of the linear fn. is a st. line

Inverse of linear fn. always exists and also linear.

36) polynomial function:

If $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ where $a_0, a_1, \dots, a_n$ are all real numbers $a_n \neq 0$ then f is a polynomial fn. of degree n

37) Rational fn.: Let $p(x)$ and $q(x)$ are any two polynomial fn. let S be the subset of $\mathbb{R}$ obtained after removing all values of x for which $q(x) = 0$ from $\mathbb{R}$.

The function $f: S \rightarrow \mathbb{R}$ defined by $f(x) = \frac{p(x)}{q(x)}$ $q(x) \neq 0$ is called a rational fn.

Note: To find the domain of a rational fn. put $q(x) = 0$ and find x value. Except these x value the remaining value in $\mathbb{R}$ is the domain of the rational fn.

38) Exponential Function: For any number $a \neq 1$ and $a > 0$ the fn. $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = a^x$ is called the exponential fn.

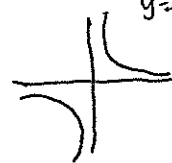
For Exponential fn. the range is always $\mathbb{R}^+$.

Note: The curve of $f(x) = e^x$ lies between the curves 2^x and 3^x .

39) Logarithmic fn.: The inverse of exponential fn. is a logarithmic fn. The general form of logarithmic fn is $f(x) = \log_a x$ $a \neq 1$ and $a > 0$

40) Reciprocal of a fn.: The fn. $f: S \rightarrow \mathbb{R}$ defined by $g(x) = \frac{1}{f(x)}$ is called reciprocal fn. of $f(x)$

a) The graph of reciprocal fn. $g(x) = \frac{1}{x}$ does not meet neither axes for finite real number.



b) Reciprocal fn. is associated with product of two fns

(ie) if f and g are reciprocals of each other then $f(x) \cdot g(x) = 1$

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c) Inverse fn associated with composition of fn.

(or) if f and g are two fns $f \circ g = g \circ f = I$.

41) Absolute value fn (or) modulus fn

If $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = |x|$ then the function is called absolute fn of x

$$\text{where } |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

42) Step function: a) Greatest integer fn

The fn whose value at any real number x is the greatest integer less than or equal to x is called greatest integer fn and is denoted by $[x]$.

$f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = [x]$

$[2.5] = 2$. The domain of the fn is $\mathbb{R}$ and the range of the fn is $\mathbb{Z}$

b) Least integer fn: The fn whose value at any real number x is the smallest integer greater than or equal to x is called the least

integer fn and is denoted by $f(x) = \lceil x \rceil$

$$\lceil 2.5 \rceil = 3$$

43) Signum fn: If $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by

$$f(x) = \begin{cases} |x| & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Then f is called the signum fn

Domain is $\mathbb{R}$ and the range $\{-1, 0, 1\}$

44) If $f(-x) = f(x)$ for all x in the domain then the fn is called even fn

If $f(-x) = -f(x)$ for all x in the domain then the fn is called odd fn.

The graph of even fn is symmetric about y axis

The graph of odd fn is symmetric about origin.

contact - 9444209677

T.G.Venkatesan



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Properties of Equivalence relation:

- 1) The inverse of an equivalence relation is also an equivalence relation.
2. The intersection of two equivalence relation on a set A is also an equivalence relation
3. The union of two equivalence relation on a set A not necessarily equivalence relation on A .

Note: The relation 'congruence modulo n ' is an equivalence relation in the set $\mathbb{Z}$ of all integers (Proof important)

Function as a relation.

- 1) A relation f from non empty set A to non empty set B is said to be a function if every element of set A has only one image in set B .
- 2) and no element in A does not have image in B .

Note: Every function is relation. But the converse need not be true

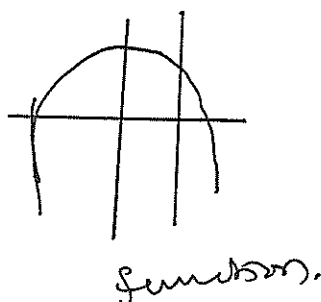
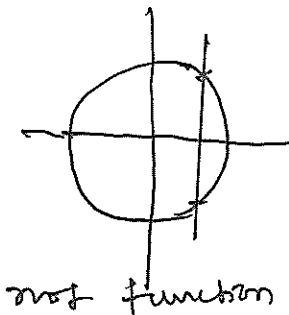
(e) every relation need not be function.

$f: \{(1, 2) (2, 3) (3, 4)\}$ is a function $A = \{1, 2, 3\}$
 $B = \{2, 3, 4\}$.

$g: \{(1, 2) (1, 3) (2, 4)\}$ is not a function. Since $3 \in A$ does not have image and $1 \in A$ has two images $2, 3 \in B$.

Identification of a function with its Graph.

Draw a vertical line (ie) any line parallel to y axis. This line intersect the graph of the curve at more than one point (ie) for one value of x it has two value of y (ie) for unique element in A has two images in B . Hence it is a relation but not function. If the vertical line cuts the curve at only one point (ie) for unique value of x has unique image in B . Hence this relation is a function. This is known as vertical line test.



Working Rule for finding the domain of Real function -

For Rational function.

1. Put the denominator equal to zero and find the values of x
2. The value of x are the values at which rational function is not defined. Then omit the set of these values from $\mathbb{R}$ to get required domain.

For Square root function.

We know that expression under the square root should not be negative. So for finding the domain of square root function put the expression without root greater than or equal to 0 and find then the value of x for which it is positive.

Working rule for finding Range of Real functions -

1. Find the domain of the function $y = f(x)$
2. Transform the equation $y = f(x)$ as $x = g(y)$
(or) convert x in terms of y .
3. Find the values of y from $x = g(y)$ s.t the values of x are real and lying in the domain of f .
4. The value of y obtained be the range of f

Properties of logarithmic function

1. $\log_a 1 = 0$ $a > 0$ and $a \neq 1$
2. $\log_a a = 1$
3. $\log_a (xy) = \log_a x + \log_a y$
4. $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
5. $\log_a x^m = m \log_a x$
6. $\log_a x^m = \frac{m}{n} \log_a (x^n)$
7. $\log_a m = \frac{1}{\log_m a}$
8. $\log_a b \cdot \log_b x = \log_a x$

Note! $\log_a x$ and a^x are inverses to each other. So their graphs are mirror image each other in the $y = x$.

Algebra of real function -

1. Addition of two real function:

The sum of the two functions $(f+g)$ defined by

$$(f+g)(x) = f(x) + g(x) \quad \forall x \in D_1 \cap D_2 \quad D - \text{Domain}$$

∴ The domain of $f+g = D_1 \cap D_2$

2) Subtraction of two real function.

The difference of two real functions $(f-g)$ is defined by

$$(f-g)(x) = f(x) - g(x) \quad \forall x \in D_1 \cap D_2$$

∴ The domain of $f-g$ is $D_1 \cap D_2$

3) Multiplication of two real functions.

The product of the function (fg) is defined by $(fg)(x) = f(x) \cdot g(x)$
 $\forall x \in D_1 \cap D_2$

∴ The domain of $fg = D_1 \cap D_2$

4. Quotient of two real function.

The quotient of the function is defined by $\frac{f}{g}(x) = \frac{f(x)}{g(x)}$

$$\forall x \in D_1 \cap D_2 - \{x | g(x) \neq 0\}$$

Multiplication of real function by a scalar.

Multiplication of real function by a constant is defined as $(cf)(x) = c \cdot f(x) \quad \forall x \in D_1$

The domain is D_1

Methods to check whether a function is one-one or many to one

1. Consider any two arbitrary elements say $a_1, a_2 \in (\text{domain})$

2) Put $f(a_1) = f(a_2)$ and simplify the equation

3) If we get $a_1 = a_2$ f is one-one and if we get

$a_1 \neq a_2$ f many to one.

Note! In order to prove f is not one-one it is sufficient to show $f(1) = 1$ and $f(2) = 1$

Methods to check whether the function is onto or into

1) Let $f: A \rightarrow B$ be the given function. Consider y be any arbitrary element in B . 2) Put $y = f(x)$ and simplify it to obtain x in terms of y then let $x = g(y)$ under the given condition simplify then we get y then f is onto or f is not onto

(or) If $\text{range} = \text{codomain}$ (ie every element in the codomain has pre-image in domain) then f is onto.

Note: If a function is one-one and onto (bijective) then only f^{-1} exists.

Composition of function:

Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be any two functions then the composition of f and g denoted by $g \circ f$ is defined as $g \circ f: A \rightarrow C$ given by

$$g \circ f(x) = g[f(x)] \quad x \in A.$$

Clearly domain of $g \circ f$ = domain of f .

Note: 1) $g \circ f(x) = g[f(x)]$ First f rule applied and then g rule is applied

2) Generally $g \circ f \neq f \circ g$

3) If $f: R \rightarrow R$ and $g: R \rightarrow R$ are real function then $g \circ f, f \circ g$ exists.

4) Two functions f and g having the same domain D are said to be equal if $f(x) = g(x) \quad \forall x \in D$.

Composition of function is associative.

If for any three function f, g and h are three functions s.t. $(f \circ g) \circ h = f \circ (g \circ h)$ exist and $(f \circ g) \circ h = f \circ (g \circ h)$

Properties 1. If $f: A \rightarrow B$ and $g: B \rightarrow C$ are one-one then $g \circ f: A \rightarrow C$ is also one-one

2) If $f: A \rightarrow B, g: B \rightarrow C$ are onto then $g \circ f: A \rightarrow C$ is also onto.

3) If $f: A \rightarrow B$ and $g: B \rightarrow C$ are two functions

then $g \circ f: A \rightarrow C$ is onto $\Rightarrow g: B \rightarrow C$ is onto

$g \circ f: A \rightarrow C$ is one-one $\Rightarrow f: A \rightarrow B$ is one-one.

Note: If $g \circ f$ is one-one, then it is not necessary that both f and g are one-one

If $g \circ f$ is onto then it is not necessary that both g and f are onto.

Method to show that the given function is invertible -

1. Show that f is one-one

2. Show that f is onto Then f is invertible.

$\implies$

To find f^{-1} let us take $y = f(x)$ and find x in terms of y - then

$$f^{-1}(y) = \text{Put the } x \text{ value in terms of } y.$$

Replace y by x

$$\therefore f^{-1}(x) = \dots$$

Properties of f^{-1} .

- 1) The inverse is unique
- 2) The inverse of a bijective function is also bijective
- 3) $(f^{-1})^{-1} = f$.

Graphing functions - The following type of transformation used in graphs

- 1) Reflection: a) The graph $y = -f(x)$ is the reflection of a graph of f about the x axis.
- b) The graph $y = f(-x)$ is the reflection of a graph of f about the y axis.
- c) The graph $f^{-1}(x)$ is the reflection of the graph f about the line $y = x$.

Translation: $y = f(x+c)$ $c > 0$ causes the shift to the left

$$y = f(x-c) \quad c > 0 \text{ causes the shift to the right}$$

$$y = f(x) + d \quad d > 0 \text{ causes to shift to the upward}$$

$$y = f(x) - d \quad d > 0 \text{ causes to shift to the downward}$$

Dilation 1) If we multiply a function by a constant > 1 the graph moves away from the x axis

2) If we multiply a function by a constant < 1 then the graph moves towards the x axis.

1) The number of elements in set A is m and in set B is n then the number of relations are 2^{mn} .

2. The number of elements in set A is n the number of relation on the set A itself $= 2^{n^2}$

3) If the number of elements in a set A is n the number of reflexive relation is $2^{n(n-1)}$

4) If the number of elements in a set A is n then the number of Symmetric relation is $2^{\frac{n(n+1)}{2}}$

1) If A and B are two non-empty finite sets containing m and n elements respectively.

The number of functions from $A \rightarrow B = n^m$

The number of one-one function (injective) = $\begin{cases} nC_m \cdot m! & n \geq m \\ 0 & n < m \end{cases}$

The number of onto function (surjective) = $\begin{cases} \sum_{r=1}^n (-1)^{n-r} \cdot nCr \cdot r^m & m \geq n \\ 0 & m < n \end{cases}$

The number of one-one and onto bijective $\left\{ \begin{array}{ll} n! & m = n \\ 0 & m \neq n \end{array} \right.$

1) If f is injective $n(A) \leq n(B)$

2) If f is surjective $n(A) \geq n(B)$

3) If f is bijective $n(A) = n(B)$

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DOMAIN, RANGE, CONTINUITY, DIFFERENTIABILITY AND GRAPH OF SOME DIFFERENT FUNCTIONS. (TABLE)

Continuity and Differentiability of Different Functions

Type of Functions	Curve	Domain and Range	Continuity and Differentiability
Identity function	$f(x) = x$	Domain $= R$, Range $=] - \infty, \infty [$	
Exponential function	$f(x) = a^x, a > 0, a \neq 1$	Domain $= R$, Range $=] 0, \infty [$	Continuous and differentiable in their domain
Logarithmic function	$f(x) = \log_a x, x, a > 0$ and $a \neq 1$	Domain $= (0, \infty)$, Range $= R$	
Root function	$f(x) = \sqrt{x}$	Domain $= [0, \infty)$, Range $= [0, \infty)$	Continuous and differentiable in $(0, \infty)$
Greatest integer function	$f(x) = [x]$	Domain $= R$, Range $= I$	Other than integral value it is continuous and differentiable
Least integer function	$f(x) = (x)$	Domain $= R$, Range $= I$	
Fractional part function	$f(x) = \{x\} = x - [x]$	Domain $= R$, Range $= [0, 1)$	
Signum function	$f(x) = \frac{ x }{x}$ $= \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$	Domain $= R$, Range $= \{-1, 0, 1\}$	Continuous and differentiable everywhere except at $x = 0$
Constant function	$f(x) = c$	Domain $= R$, Range $= \{c\}$, where $c \rightarrow \text{constant}$	
Polynomial function		Domain $= R$	Continuous and differentiable in their domain

Type of Functions	Curve	Domain and Range	Continuity and Differentiability
Sine function	$y = \sin x$	Domain $= R$, Range $= [-1, 1]$	
Cosine function	$y = \cos x$	Domain $= R$, Range $= [-1, 1]$	
Tangent function	$y = \tan x$	Domain $= R - \left\{ (2n+1)\frac{\pi}{2} \right\}$ Range $= R$	Continuous and differentiable in their domain
Cosecant function	$y = \operatorname{cosec} x$	Domain $= R - n\pi$, Range $= (-\infty, -1] \cup [1, \infty)$	
Secant function	$y = \sec x$	Domain $= R - \left\{ (2n+1)\frac{\pi}{2} \right\}$, Range $= (-\infty, -1] \cup [1, \infty)$	
Cotangent function	$y = \cot x$	Domain $= R - \{n\pi\}$, Range $= R$	
Arc sine function	$y = \sin^{-1} x$	Domain $= [-1, 1]$, Range $= \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$	
Arc cosine function	$y = \cos^{-1} x$	Domain $= [-1, 1]$, Range $= [0, \pi]$	
Arc tangent function	$y = \tan^{-1} x$	Domain $= R$, Range $= \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) - \{0\}$	
Arc cosecant function	$y = \operatorname{cosec}^{-1} x$	Domain $= (-\infty, -1] \cup [1, \infty)$, Range $= \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) - \{0\}$	Continuous and differentiable in their domain
Arc secant function	$y = \sec^{-1} x$	Domain $= (-\infty, -1] \cup [1, \infty)$, Range $= [0, \pi] - \left\{ \frac{\pi}{2} \right\}$	
Arc cotangent function	$y = \cot^{-1} x$	Domain $= R$, Range $= (0, \pi)$	

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11) $y = \sin x$. Domain $(-\infty, \infty)$ Range $[-1, 1]$ Principle Domain $[-\pi/2, \pi/2]$	12) $y = \cos x$ Domain $(-\infty, \infty)$ Range $[-1, 1]$ Principle Domain $[0, \pi]$	13) $y = \tan x$. Domain $R - \{(2k+1)\pi/2\}$ Range $(-\infty, \infty)$ $k \in \mathbb{Z}$
14) $y = \csc x$ Domain $R - \{(2k+1)\pi/2\}$ Range $(-\infty, -1] \cup [1, \infty)$ $k \in \mathbb{Z}$	15) $y = \sec x$. Domain $R - \{k\pi\}$ Range $(-\infty, -1] \cup [1, \infty)$	16) $y = \cot x$. Domain $R - \{k\pi\}$ Range $(-\infty, \infty)$ $k \in \mathbb{Z}$
1. <u>Relation</u>: Let A and B be any two sets. A relation from $A \rightarrow B$ is a subset of a Cartesian product $A \times B$	3) <u>Graph of the function</u>. 1) For every x there is unique image. But any element in A does not possess two images.	4) <u>One to one</u>: A function is said to be one-one if each element of the range is associated with exactly one element of the domain. (ie) all the element in B has preimage. Codomain = Range [Injective].
2) <u>Function</u>: One to one, many to one But not one to many and every element of A should have image. b) The relation 'son of' is a function (many to one) c) The relation 'father of' is not a function (one to many)	2) <u>Vertical line test</u>: If the vertical line cuts the curve in two points it is not the graph of the function. $x^2 + y^2 = 4$	5) <u>on to</u>: If the range of the fn is equal to codomain then the fn is called an onto fn. It may be one to one (or) many to one [Surjective]. It both one-one and onto it is bijective.
a) Relation need not be function or if many function. 3) <u>Function</u> should be relation.	Graph of the function. $y = x^2$	Note: If the function is one-one and onto then it exists s.t. $f \circ f^{-1} = I$ (or) $f^{-1} \circ f = I$ (or) f is bijective. Composition of fn. does not be commutative always. $f \circ g \neq g \circ f$

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XI Std: Sets, Relations and functions.Roster form (or) Tabulated form.1, 2, 3 Marks.

1. Represent the following sets in the roster form.

1. $\{x: x \text{ is a vowel in English alphabets}\}$

2. $\{x: x \text{ is an integer where } -1 \leq x \leq 7\}$

3. $\{x: x \text{ is two digit number s.t the sum of the digits is 10}\}$

4. $\{x: x \text{ is a natural number, } x \text{ is a perfect square, } x \leq 100\}$

5. $\{x: x \text{ is a word letter in the word MAHARASHTRA}\}$

6. $\{x: x \text{ is a positive integer and a multiple of 5}\}$

Sol: 1) $\{a, e, i, o, u\}$

2) $\{-1, 0, 1, 2, 3, 4, 5, 6, 7\}$

3) $\{19, 91, 28, 82, 37, 73, 46, 64, 55\}$

4. $\{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$

5. $\{M, A, H, R, S, T\}$

6. $\{5, 10, 15, 20, 25, \dots\}$

2) Exhibit in tabulation form.

1) $A = \{x: x \text{ is a natural number } 3 < x < 7\}$

2. $B = \{x: x \text{ is a prime number, } x < 20\}$

3. $C = \{x: x \text{ is a prime number which is a divisor of } 60\}$

1) $A = \{4, 5, 6\}$

2. $B = \{2, 3, 5, 7, 11, 13, 17, 19\}$

3. $C: \text{Divisor of } 60 \text{ are } 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}$

$= \{ \text{prime numbers} \}$

$= \{2, 3, 5\}$

3) Represent the following sets in the set Builder form.

1. $\{2, 3, 5\}$

2) $\{M, A, T, H, E, I, C, S\}$

3) $\{2, 4, 6, 8, 10\}$

4) $\{3, 9, 27, 81\}$

5) $\{5, 10, 15, 20\}$

6) $\{0, 3, 6, 9, 12, 15, 18, \dots\}$

1. $\{x: x \text{ is a prime divisor of } 30\}$

4) $\{x: x = 5n, n \in \mathbb{N}, n \leq 4\}$

2. $\{x: x \text{ is the letter of the word MATHEMATICS}\}$

5) $\{x \in \mathbb{N} : x = 3n, n \in \mathbb{N}, n \text{ is a whole number}\}$

3. $\{x: x = 3^n \text{ where } n \text{ is a natural number } < 4\}$

4) Represents the following sets in the set builder form.

$$A = \{7, 8, 9, 10, 11\} \quad B = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\} \quad C = \{1, 4, 9, \dots, 100\}$$

$$A = \{x : x \text{ is the natural number between 6 and 12}\}$$

$$B = \{x : x = \frac{1}{n} \mid n \text{ is a natural number}\}$$

$$C = \{x : x = n^2 \mid n \text{ is a natural number}\}$$

5) write the roster form as well as set builder form of set containing the elements 0, 2, 4, 6, 8

$$\text{Roster form: } A = \{0, 2, 4, 6, 8\}$$

$$\text{Set Builder form } B = \{x : x = 2(n-1) \mid n \leq 5\}$$

6) write the set of all vowels in English alphabet which precedes s

$$A = \{a, e, i, o\}$$

7) write the set in the Roster form $\{x : x \text{ is a positive integer and divisor of } 9\}$

$$A = \{1, 3, 9\}$$

8) write the set of all natural numbers x s.t. $4x+9 < 50$ in the roster form.

$$\begin{array}{ll} \text{when } x=1 & 4+9=13 \\ x=2 & 8+9=17 \\ x=3 & 12+9=21 \\ \vdots & \end{array}$$

$$\therefore A = \{1, 2, 3, \dots, 10\}$$

$$x=10 \quad 40+9=49 < 50$$

9) write the following in Roster form.

$$1. \{x : x \in \mathbb{Z} \text{ and } |x| \leq 2\} \quad 2) x : x = \frac{n}{1+n^2} \text{ and } 1 \leq n \leq 3 \quad n, N.$$

$$1) x : x \in \mathbb{Z} \text{ and } |x| \leq 2$$

$$|x| \leq 2 \Rightarrow -2 \leq x \leq 2$$

$$\therefore A = \{-2, -1, 0, 1, 2\}$$

$$2) x : x = \frac{n}{1+n^2} \quad 1 \leq n \leq 3$$

$$x=1 = \frac{1}{2} \quad \therefore A = \left\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}\right\}$$

$$2 = \frac{2}{5}$$

$$3 = \frac{3}{10}$$

(*) write in the Roster form.

1) $A = \{a_n : n \in \mathbb{N}, a_{n+1} = 3a_n, a_1 = 1\}$

2) $B = \{a_n : n \in \mathbb{N}, a_{n+2} = a_{n+1} + a_n \text{ and } a_1 = a_2 = 1\}$.

1) $a_1 = 1$ and $a_{n+1} = 3a_n$.

$$a_2 = a_{1+1} = 3a_1 = 3$$

$$a_3 = a_{2+1} = 3a_2 = 3 \cdot 3 = 3^2$$

$$a_4 = a_{3+1} = 3a_3 = 3 \cdot 3^2 = 3^3 \text{ and soon.}$$

$$\therefore A = \{1, 3, 3^2, 3^3, \dots\}$$

2) $a_1 = a_2 = 1$

$$a_3 = a_{1+2} = a_2 + a_1 = 2$$

$$a_4 = a_{2+2} = a_3 + a_2 = 2 + 1 = 3$$

$$a_5 = a_{3+2} = a_4 + a_3 = 3 + 2 = 5 \text{ and soon.}$$

$$\therefore B = \{1, 1, 2, 3, 5, \dots\}$$

11) write the set $A = \{14, 21, 28, 35, \dots, 98\}$ in the set builder form.

The given numbers are natural number > 13 and < 99 and multiply of 7.

$$\{x : x \text{ is a natural number multiply of 7 and } 13 < x < 99\}$$

$$\{x = 7n \mid n \in \mathbb{N} \text{ and } 2 \leq x \leq 14\}$$

12) Describe the set in the set Builder form.

$$B = \{53, 59, 61, 67, 71, 73, 79, 83, 89, 97\}$$

$$\{x : x = \text{prime numbers between 50 and } 100\}$$

13) write the set $D = \{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}, \dots\}$ in the set builder form.

$$D = \{x : x = \frac{n}{n+1} \mid n \in \mathbb{N} \text{ and } n \leq 5\}$$

14) Using Roster (or) listing method to express the set

$$A = \{x : x = n^3, n \in \mathbb{N} \text{ and } x < 80\}$$

$$A = \{1, 8, 27, 64\}$$

15) List elements of the set $A = \{x : x \text{ is an integer } -\frac{1}{2} \leq x < \frac{1}{2}\}$

$$A = \{0, 1, 2, 3, 4\}$$

16) Express the set $D = \{x : x = \frac{n^2-1}{n^2+1}, n \in \mathbb{N} \text{ and } x < \frac{1}{2}\}$ in roster form.

$$x=1, \quad x=0$$

$$x=2, \quad x=\frac{3}{5}$$

$$x=3, \quad x=\frac{8}{10}$$

$$\therefore A = \{0, \frac{3}{5}, \frac{4}{5}, \frac{8}{10}\}$$

17) Describe the following set in Roster form $\{x : x \text{ is a letter of the word PROPORTION}\}$.

$$A = \{P, R, O, T, I, N\}$$

Symmetrical difference of two sets. $A \Delta B = (A-B) \cup (B-A)$
(or) $= (A-B) - (A \cap B)$

18) P.T. $A - (B \cap C) = (A-B) \cup (A-C)$

Proof: Let $x \in A - (B \cap C)$

$$\Rightarrow x \in A, \text{ and } x \notin B \cap C.$$

$$\Rightarrow x \in A \text{ and } x \notin B \text{ (or) } x \notin C.$$

$$\Rightarrow x \in A \text{ and } x \notin B \text{ (or) } x \in A \text{ and } x \notin C$$

$$\Rightarrow x \in A-B \text{ (or) } x \in A-C$$

$$\Rightarrow x \in (A-B) \cup x \in (A-C)$$

$$\Rightarrow A - (B \cap C) \subseteq (A-B) \cup (A-C) \quad \text{--- (1)}$$

$\therefore$ Suppose the symbol $\cap$ (or) $\cup$ are in brackets ^{LHS} Change the symbol in RHS.
Suppose $-$ is in bracket in LHS. No change the symbol in RHS.

$$\text{Let } x \in (A-B) \cup (A-C)$$

$$\Rightarrow x \in A-B \text{ (or) } x \in A-C$$

$$\Rightarrow x \in A, x \notin B \text{ (or) } x \in A, x \notin C$$

$$\Rightarrow x \in A, (x \notin B \text{ or } x \notin C)$$

$$\Rightarrow x \in A, x \notin (B \cap C)$$

$$\Rightarrow x \in A - (B \cap C)$$

$$\therefore (A-B) \cup (A-C) \subseteq A - (B \cap C) \quad \text{--- (2)}$$

$$\therefore \text{From (1) and (2) } A - (B \cap C) = (A-B) \cup (A-C)$$

19) P.T. $A \cap (B-C) = (A \cap B) - (A \cap C)$

Proof: Let $x \in A \cap (B-C)$

$$\Rightarrow x \in A, x \in B-C$$

$$\Rightarrow x \in A, (x \in B, x \notin C)$$

$$\Rightarrow (x \in A, x \in B) \text{ and } x \notin C.$$

$$\Rightarrow x \in A \cap B \text{ and } x \notin A \cap C$$

$$\Rightarrow x \in (A \cap B) - (A \cap C)$$

$$\therefore A \cap (B-C) \subseteq (A \cap B) - (A \cap C) \quad \text{--- (1)}$$

Let $x \in (A \cap B) - (A \cap C)$

$$\Rightarrow x \in A \cap B \text{ and } x \notin A \cap C$$

$$\Rightarrow x \in (A \text{ and } x \in B) \text{ and } (x \in A \text{ and } x \notin C)$$

$$\Rightarrow x \in A \text{ and } (x \in B \text{ and } x \notin C)$$

$$\Rightarrow x \in A \text{ and } x \in B-C.$$

$$\Rightarrow x \in A \cap (B-C)$$

$$\therefore (A \cap B) - (A \cap C) \subseteq A \cap (B-C) \quad \text{--- (2)}$$

$$\text{From (1) and (2) } A \cap (B-C) = (A \cap B) - (A \cap C)$$

20) P.T. $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

Proof: LHS: $A \cap (B \Delta C) = A \cap \{(B-C) \cup (C-B)\}$

$$= A \cap (B-C) \cup A \cap (C-B)$$

$$= \{(A \cap B) - (A \cap C)\} \cup \{(A \cap C) - (A \cap B)\}$$

$$= (A \cap B) \Delta (A \cap C) = \text{RHS.}$$

21) If $A = \{x: x \text{ is a factor of } 20\}$ and $B = \{5, 10, 15, 20\}$ find $A-B$ and $B-A$.

Sol: $A = \{1, 2, 4, 5, 10, 20\}$, $B = \{5, 10, 15, 20\}$

$$A-B = \{1, 2, 4\}$$

$$B-A = \{15\}$$

22) If S is the set of all prime numbers and $M = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ then evaluate 1) $M \cup (S \cap M)$ 2) $(S-M) \cap (M-S)$

Sol: 1) $S = \{2, 3, 5, 7, 11, 13, 17, 19, \dots\}$

$$M = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$S \cap M = \{2, 3, 5, 7\}$$

$$M \cup (S \cap M) = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} = M$$

$$2) S-M = \{11, 13, 17, 19, \dots\}$$

$$M-S = \{0, 1, 4, 6, 8, 9\}$$

$$(S-M) \cap (M-S) = \emptyset$$

23) Verify $A - (B \cup C) = (A-B) \cap (A-C)$ with sets $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$ and $C = \{1, 3, 5\}$.

Sol: $A = \{1, 2, 3\}$
 $B = \{3, 4, 5\}$
 $C = \{1, 3, 5\}$

LHS $B \cup C = \{1, 3, 4, 5\}$ $A - (B \cup C) = \{2\}$

RHS $A - B = \{1, 2\}$

$A - C = \{2\}$

$\therefore (A-B) \cap (A-C) = \{2\}$
 $\therefore \text{LHS} = \text{RHS}$

24) If $U = \{1, 2, 3, 4, 6, 7, 8, 9, 10\}$, $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 4, 6, 8, 10\}$ and $C = \{1, 2, 3, 4\}$ then

Find) 1) U' , 2) $A \cup A'$, 3) $A \cap (B-C)$, 4) $A - (B \cap C)$, 5) $A' \cap (B \cup C)$
 6) A' , 7) $A \cap A'$, 8) $A - (B \cup C)$, 9) $A' \cup (B \cup C)$
 10) $A' \cup (B' \cap C')$

25) For any two subsets A, B of the universal set U then S.T
 $A \cup (A \cap B) = A$

Sol: Let x be any arbitrary element of $A \cup (A \cap B)$

$$\begin{aligned} x \in A \cup (A \cap B) &= \{x : x \in A, \text{ (or) } x \in A \cap B\} \\ &= \{x : x \in A \text{ or } x \in A, x \in B\} \\ &= \{x : x \in A\} \\ &= A. \end{aligned}$$

26) For any two sets A and B P.T $A \cap (A \cup B) = A$.

$$\begin{aligned} \text{Sol: Let } x \in A \cap (A \cup B) &= \{x : x \in A, x \in A \cup B\} \\ &= \{x : x \in A, x \in A \text{ or } x \in B\} \\ &= \{x : x \in A\} \\ &= A. \end{aligned}$$

27) For any two sets A and B P.T $A - B \cap B - A = \emptyset$.

$$\begin{aligned} \text{Sol: } x \in A - B &\Rightarrow x \in A \text{ and } x \notin B. \\ &\Rightarrow x \notin B - A. \\ \text{any } x \in B - A &\Rightarrow x \in B \text{ and } x \notin A \\ &\Rightarrow x \notin A - B. \\ \therefore (A - B) \cap (B - A) &= \emptyset. \end{aligned}$$

$$\textcircled{\otimes} A \Delta B = (A - B) \cup (B - A)$$

$$\text{(or) } (A \cup B) - (A \cap B)$$

28) State which of the sets given below are finite or infinite.

- 1) $\{x : x \text{ is a prime number, } x \text{ is even}\} \rightarrow$ infinite set
- 2) Set of all rivers in India $\rightarrow$ finite set
- 3) Set of all concentric circles $\rightarrow$ infinite set
- 4) $\{x : x \text{ is a multiple of 2, } x \text{ is an integer}\} \rightarrow$ infinite set
- 5) The set of months in a year $\rightarrow$ finite set
- 6) $\{1, 2, 3, \dots\} \rightarrow$ infinite set
- 7) $\{1, 2, 3, \dots, 100\} \rightarrow$ finite set
- 8) The set of prime numbers less than 99 $\rightarrow$ finite set
- 9) The set of lines which are parallel to x axis $\rightarrow$ infinite set
- 10) The set of circles in a plane passing through the origin $\rightarrow$ infinite set

- (1) The set of lines which are parallel to y-axis $\rightarrow$ infinite set
 (2) The set of letters in English alphabets $\rightarrow$ finite set
 (3) The set of numbers which multiplied by 5 $\rightarrow$ infinite set
 (4) The set of animals living on the earth $\rightarrow$ infinite set
 (5) The set of all historical monuments in India $\rightarrow$ finite set
 (6) $\{x: x \text{ is an integer } x < 5\} \rightarrow$ finite set
 (7) $\{x: x \text{ is a real number } 1 < x < 2\} \rightarrow$ infinite set
 (8) $\{x: x \text{ is a natural number } > 500\} \rightarrow$ infinite set
 (9) $\{x: x \text{ is an integer, } x \text{ is a factor of } 1000\} \rightarrow$ finite set.

30) In a survey, it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 liked products A and B, 14 liked B and C, 12 liked C and A and 8 liked all the three products. Find how many liked C only.

$$n(A) = 21, n(B) = 26, n(C) = 29, n(A \cap B) = 14, n(B \cap C) = 14, n(C \cap A) = 12$$

$$n(A \cap B \cap C) = 8$$

$$\therefore n(A \cap B) = n(A) - n(A \cap B)$$

$$\text{Now } n(A \cap B \cap C) = n[(A \cup B) \cap C] = n(C) - n\{(A \cup B) \cap C\}$$

$$= n(C) - n\{(A \cap C) \cup (B \cap C)\}$$

$$= n(C) - \{n(A \cap C) + n(B \cap C) - n(A \cap B \cap C)\}$$

$$= 29 - (12 + 14 - 8) = 29 - 18 = 11.$$

31) Let A and B are two sets s.t. $A \times B$ consists of 6 elements. If the three elements of $A \times B$ are $(1, 4), (2, 6), (3, 6)$. Find $A \times B$ and $B \times A$

$$\text{Let } A = \{1, 2, 3\}$$

$$A \times B = \{(1, 4), (2, 4), (3, 4), (1, 6), (2, 6), (3, 6)\}$$

Sol.

$$B = \{4, 6\}$$

$$B \times A = \{(4, 1), (4, 2), (4, 3), (6, 1), (6, 2), (6, 3)\}$$

32) Let A and B be two sets s.t. $n(A) = 3, n(B) = 2$, if $(x, 1), (y, 2), (z, 1)$ are in $A \times B$. Find A and B. where x, y, z are distinct. (Already done)

33) Let $A = \{1, 2\}, B = \{3, 4\}$ write $A \times B$. How many subsets will $A \times B$ have.

$$n(A) = 2, n(B) = 2, n(A \times B) = 4$$

$$n[P(A \times B)] = 2^4 = 16.$$

34) If $A = \{x: x^2 - 5x + 6 = 0\}, B = \{2, 4\}, C = \{4, 5\}$ then find $A \times (B \cap C)$

$$x^2 - 5x + 6 = 0 \Rightarrow (x-2)(x-3) = 0 \Rightarrow x = \{2, 3\}$$

$$\therefore A = \{2, 3\}, B = \{2, 4\}, C = \{4, 5\}.$$

$$B \cap C = \{4\}, A \times (B \cap C) = \{(2, 4), (3, 4)\}$$

35) If $A = \{a, b, c, d, e\}$ $B = \{a, c, e, g\}$ $C = \{b, e, f, g\}$. Then verify that
 i) $A \cap (B - C) = (A \cap B) - (A \cap C)$ 2) $A - (B \cap C) = (A - B) \cup (A - C)$

1) $A \cap (B - C) = (A \cap B) - (A \cap C)$

LHS $B - C = \{a, c, f\}$

$A \cap (B - C) = \{a, c\}$ — ①

RHS $A \cap B = \{a, c, e\}$

$A \cap C = \{b, e\}$

$\therefore (A \cap B) - (A \cap C) = \{a, c\}$ — ②

$\therefore LHS = RHS$

LHS $B \cap C = \{e, g\}$

$A - (B \cap C) = \{a, b, c, d\}$ — ①

RHS $A - B = \{b, d\}$

$A - C = \{a, c, d\}$

$(A - B) \cup (A - C) = \{a, b, c, d\}$ — ②

$A - (B \cap C) = (A - B) \cup (A - C)$

$LHS = RHS$

Let $A = \{1, 2, 4, 5\}$ $B = \{2, 3, 5, 6\}$ $C = \{4, 5, 6, 7\}$

36) verify the following identities. 1) $A \cap (B - C) = (A \cap B) - (A \cap C)$

2) $A - (B \cup C) = (A - B) \cap (A - C)$

3) $A - (B \cap C) = (A - B) \cup (A - C)$

Solution: 1) $A \cap (B - C) = (A \cap B) - (A \cap C)$

LHS $B - C = \{2, 3\}$

$A \cap (B - C) = \{2\}$

RHS $A \cap B = \{2, 5\}$

$(A \cap C) = \{4, 5\}$

$(A \cap B) - (A \cap C) = \{2\}$ — ②

From ① and ② $A \cap (B - C) = (A \cap B) - (A \cap C)$

2) $A - (B \cup C) = (A - B) \cap (A - C)$

LHS $B \cup C = \{2, 3, 4, 5, 6, 7\}$

$A - (B \cup C) = \{1\}$ — ①

RHS $A - B = \{1, 4\}$

$A - C = \{1, 2, 5\}$

$(A - B) \cap (A - C) = \{1\}$ — ②

From ① and ② $A - (B \cup C) = (A - B) \cap (A - C)$

If $A = \{1, 2, 3\}$ $B = \{2, 3, 4, 5\}$ $C = \{2, 4, 6, 8\}$ verify that

37) 1) $A - (A - B) = A \cap B$

2) $A \cap (B - C) = (A \cap B) - (A \cap C)$

1) LHS $A - B = \{1\}$

$A - (A - B) = \{2, 3\}$

RHS $A \cap B = \{2, 3\}$

$\therefore LHS = RHS$

2) LHS $B - C = \{3, 5\}$

$A \cap (B - C) = \{3\}$ — ①

$A \cap B = \{2, 3\}$

$(A \cap B) - (A \cap C) = \{3\}$

$A \cap C = \{2, 4\}$ — ②

$\therefore A \cap (B - C) = (A \cap B) - (A \cap C)$

For any two sets A and B $A \Delta B = (A \cup B) - (A \cap B)$

38) (or) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

38) Let $S = \{x \mid x \text{ is a positive multiple of 3 less than } 100\}$.

$P = \{x \mid x \text{ is a prime number less than } 20\}$ then $n(S) + n(P) = ?$

$$S = \{3, 6, 9, 12, \dots, 99\} \Rightarrow n(S) = 33$$

$$P = \{2, 3, 5, 7, 11, 13, 17, 19\} \Rightarrow n(P) = 8$$

$$\therefore n(S) + n(P) = 33 + 8 = 41.$$

39) A set contains n elements. Its power set contains 2^n elements.

40) If $n(A) = 10$, $n(B) = 6$, $n(C) = 5$ for three disjoint sets A, B, C then $n(A \cup B \cup C) = ?$

$$\begin{aligned} n(A \cup B \cup C) &= n(A) + n(B) + n(C) \\ &= 10 + 6 + 5 = 21. \end{aligned}$$

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Book Problems

1. Find the number of subsets of $A = \{x: x = 4n+1, 2 \leq n \leq 5, n \in \mathbb{N}\}$.

Sol: $A = \{x: x = 4n+1, n = 2, 3, 4, 5\}$

TBP $= \{9, 13, 17, 21\} \Rightarrow n(A) = 4.$

Number of subsets $= 2^4 = 16.$

2. In a survey of 5000 persons in a town, it was found that 45% of the persons know language A, 25% know language B, 10% know language C, 5% know language A and B, 4% know language B and C and 3% know all the three languages, find the number of persons who know only language A.

Sol. $45\% \text{ of } 5000 = 2250, 25\% \text{ of } 5000 = 1250, 10\% \text{ of } 5000 = 500$

$4\% \text{ of } 5000 = 200, 3\% \text{ of } 5000 = 150.$

(ie) $n(A) = 2250, n(B) = 1250, n(C) = 500, n(A \cap B) = 250$

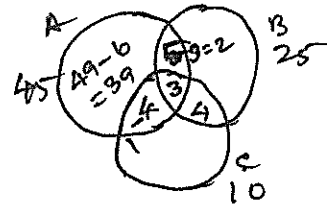
$n(B \cap C) = 200, n(A \cap C) = 200, n(A \cap B \cap C) = 150.$

Number of persons who know A only

$$\begin{aligned} n(A \cap B' \cap C') &= n\{A \cap (B \cup C)'\} \\ &= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C) \\ &= 2250 - 250 - 200 + 150 \\ &= 1950. \end{aligned}$$

(or) 39% of 5000 people speaks A only

$$= 5000 \times \frac{39}{100} = 1950.$$



3) If $X = \{1, 2, 3, 4, \dots, 10\}$ and $A = \{1, 2, 3, 4, 5\}$ Find the number of subsets of $B \subseteq X$ s.t. $A - B = \{4\}$

Sol: $X = \{1, 2, 3, 4, \dots, 10\}, A = \{1, 2, 3, 4, 5\}$

Let $B = C \cup A = C \cup \{1, 2, 3, 4, 5\} = \{1, 2, 3, 5, 6, 7, 8, 9, 10\}$

$\therefore B = \{6, 7, 8, 9, 10\}$

$\therefore A - B = \{4\} \therefore$ The number of subsets of $\{6, 7, 8, 9, 10\}$
 $n = 5 \therefore$ subsets $= 2^5 = 32.$

4. If A and B are two sets so that $n(B-A) = 2n(A-B) = 4n(A \cap B)$ and if $n(A \cup B) = 14$. Then find $n[P(A)]$.

Sol: Let $n(A \cap B) = k$.

$$n(A \cup B) = n(A-B) + n(B-A) + n(A \cap B)$$

$$\therefore n(B-A) = 2k$$

$$n(A-B) = k$$

$$n(A) = n(A-B) + n(A \cap B)$$

$$\therefore n(A \cup B) = n(A-B) + n(B-A) + n(A \cap B) = k + 2k + k = 4k$$

$$14 = 4k$$

$$\therefore k = 2$$

$$\therefore n(B-A) = 4$$

$$n(A-B) = 2$$

$$n(A) = n(A-B) + n(A \cap B)$$

$$= 2 + 2 = 4$$

$$\therefore n[P(A)] = 2^4 = 16$$

5) Two sets have m and k elements. If the total number of subsets of the first set is 112 more than that of the second set. Find the value of k and m .

Sol: Let A, B be two sets s.t. $n(A) = m$ and $n(B) = k$. $m > k$.

$$n[P(A)] = 2^m, n[P(B)] = 2^k$$

$$2^m - 2^k = 112$$

$$2^k [2^{m-k} - 1] = 112$$

$$= 2^4 \times 7$$

$$\Rightarrow 2^k = 2^4$$

$$\text{and } 2^{m-k} - 1 = 7$$

$$k = 4$$

$$2^{m-4} = 8 = 2^3$$

$$m - 4 = 3$$

$$m = 7$$

$$\begin{array}{r} 2 \overline{) 112} \\ \underline{26} \\ 28 \\ \underline{24} \\ 4 \end{array}$$

$$2$$

6) If $n(A) = 10$ and $n(A \cap B) = 3$, find $n[(A \cap B)' \cap A]$

$$\text{Sol: } (A \cap B)' = A' \cup B'$$

$$(A \cap B)' \cap A = (A' \cup B') \cap A$$

$$= (A' \cap A) \cup (B' \cap A)$$

$$= \emptyset \cup (B' \cap A)$$

$$= B' \cap A = A - B$$

$$\therefore n[(A \cap B)' \cap A] = n(A - B)$$

$$= n(A) - n(A \cap B)$$

$$= 10 - 3$$

$$= 7$$

12) If A and B be two sets s.t. $n(A) = 3$, and $n(B) = 2$, If $(x,1)$ $(y,2)$ $(z,1)$ are in $A \times B$. find A and B where x, y, z are distinct elements.

Sol: $A = \{x, y, z\}$ $n(A) = 3$.

$B = \{1, 2\}$ $n(B) = 2$ and $A \times B = \{(x,1) (y,1) (z,1) (x,2) (y,2) (z,2)\}$

13) If $A \times A$ has 16 elements $S = \{(a,b) \in A \times A : a < b\}$; TBP $(-1, 2)$ $(0, 1)$ are two elements of S, then find the remaining elements of S. Solution Let $A = \{-1, 0, 1, 2\}$ ✓
 $S = \{(-1, 0) (-1, 1) (-1, 2) (0, 1) (0, 2) (1, 2)\}$

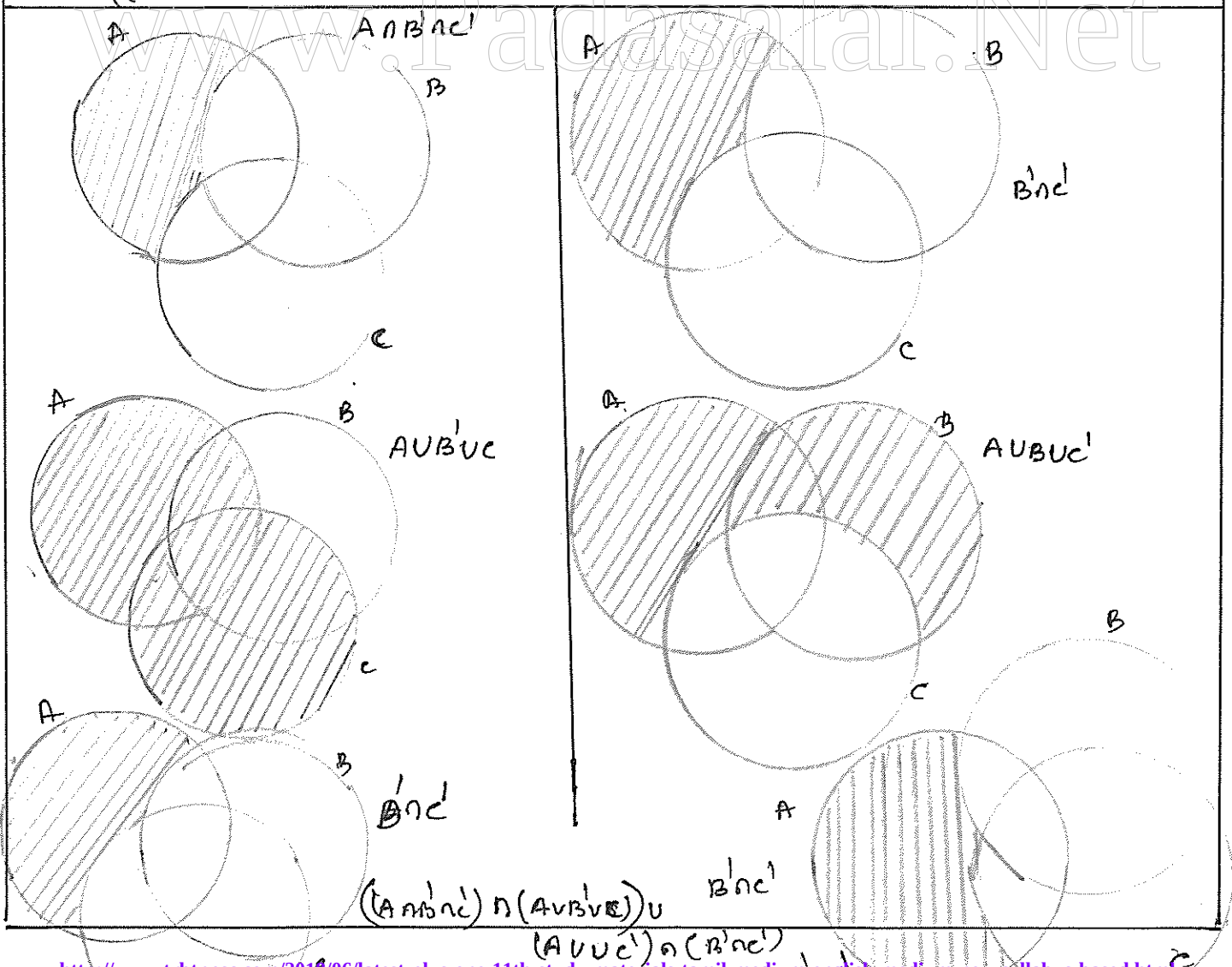
14) P.T. $((A \cup B' \cup C') \cap (A \cap B' \cap C')) \cup ((A \cup B \cup C') \cap (B' \cap C')) = B' \cap C'$.

Sol: $A \cap B' \cap C' \subseteq A \subseteq A \cup B' \cup C' \Rightarrow (A \cap B' \cap C') \cap (A \cup B' \cup C') = A \cap B' \cap C'$

Also $B' \cap C' \subseteq C' \subseteq A \cup B \cup C' \Rightarrow (A \cup B \cup C') \cap (B' \cap C') = B' \cap C'$

But $A \cap B' \cap C' \subseteq B' \cap C'$

$\therefore (A \cap B' \cap C') \cap (A \cup B' \cup C') \cup ((A \cup B \cup C') \cap (B' \cap C')) = B' \cap C'$.



7) If $A = \{1, 2, 3, 4\}$ $B = \{3, 4, 5, 6\}$ find $n[(A \cup B) \times (A \cap B) \times (A \Delta B)]$

Sol: $A \cup B = \{1, 2, 3, 4, 5, 6\}$ $A \cap B = \{3, 4\}$ $n(A \Delta B) = 4$

TBP $n(A \cup B) = 6$

$n(A \cap B) = 2$

$A \Delta B$ is A symmetric diff. B.

$\therefore n[(A \cup B) \times (A \cap B) \times (A \Delta B)] = 6 \times 2 \times 4 = 48$

$A \Delta B = (A - B) \cup (B - A)$

8) If $P(A)$ denotes the power set of A then find $n[P(P(P(\emptyset)))]$

Sol: $P(\emptyset)$ contains 1 element

$P(P(\emptyset)) = 2$

TBP $P(P(P(\emptyset))) = 2^2 = 4$ $\therefore n[P(P(P(\emptyset)))] = 4$

9) If $n(P(A)) = 1024$ $n(A \cup B) = 15$ $n(P(B)) = 32$ find $n(A \cap B)$

Sol: If $n[P(A)] = 1024 = 2^{10} \therefore n(A) = 10$

$n[P(B)] = 32 = 2^5 = n(B) = 5$

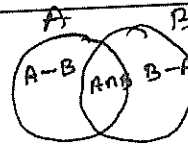
$$\begin{array}{r} 2 \overline{) 1024} \\ 2 \overline{) 512} \\ 16 \overline{) 256} \\ 16 \end{array}$$

TBP $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cap B) = 10 + 5 - n(A \cup B)$
 $= 15 - 15 = 0$

10) If $n(A \cap B) = 3$ and $n(A \cup B) = 10$ then find $n(P(A \Delta B))$

$A \Delta B = (A - B) \cup (B - A)$
 $= (A \cup B) - (A \cap B)$



TBP $n(A \Delta B) = n(A \cup B) - n(A \cap B)$

$= 10 - 3 = 7$

$n[P(A \Delta B)] = 2^7 = 128$

11) For a set A, $A \times A$ contains 16 elements and two of its elements are (1, 3) and (0, 2) Find the element A.

Let $A = \{1, 3, 0, 2\}$

$A = \{1, 3, 0, 2\}$

$A \times A$ contains 16 elements also contains (1, 3), (0, 2) ...

$\therefore A = \{1, 3, 0, 2\}$

4) If $A = \{1, 4\}$, $B = \{2, 3, 6\}$ $C = \{2, 3, 7\}$ verify

1) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

LHS: $B \cup C = \{2, 3, 6, 7\}$

$A \times (B \cup C) = \{(1, 2), (1, 3), (1, 6), (1, 7), (4, 2), (4, 3), (4, 6), (4, 7)\}$ — ①

RHS: $A \times B = \{(1, 2), (1, 3), (1, 6), (4, 2), (4, 3), (4, 6)\}$

$A \times C = \{(4, 2), (1, 3), (1, 7), (4, 6), (4, 3), (4, 7)\}$

$(A \times B) \cup (A \times C) = \{(1, 2), (1, 3), (1, 6), (1, 7), (4, 2), (4, 3), (4, 6), (4, 7)\}$ — ②

From ① and ② $A \times (B \cup C) = (A \times B) \cup (A \times C)$

2) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

LHS $B \cap C = \{2, 3\}$

$A \times (B \cap C) = \{(1, 2), (1, 3), (4, 2), (4, 3)\}$ — ①

$A \times B = \{(1, 2), (1, 3), (1, 6), (4, 2), (4, 3), (4, 6)\}$

$A \times C = \{(1, 2), (1, 3), (1, 7), (4, 2), (4, 3), (4, 7)\}$

$(A \times B) \cap (A \times C) = \{(1, 2), (1, 3), (4, 2), (4, 3)\}$ — ②

From ① and ② $A \times (B \cap C) = (A \times B) \cap (A \times C)$

3) $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$

LHS $A \times B = \{(1, 2), (1, 3), (1, 6), (4, 2), (4, 3), (4, 6)\}$

$B \times A = \{(2, 1), (2, 4), (3, 1), (3, 4), (6, 1), (6, 4)\}$

$(A \times B) \cap (B \times A) = \{\emptyset\}$ — ①

RHS $A \cap B = \{\emptyset\}$

$B \cap A = \{\emptyset\}$

$(A \cap B) \times (B \cap A) = \{\emptyset\}$ — ②

From ① and ② $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$

4) $C - (B - A) = (C \cap A) \cup (C \cap B')$

LHS $B - A = \{2, 3, 6\}$

$C - (B - A) = \{7\}$ — ①

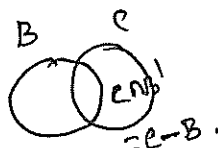
RHS: $(C \cap A) = \{\emptyset\}$

$C \cap B' = C - B = \{7\}$

$\therefore (C \cap A) \cup (C \cap B') = \{7\}$ — ②

$\therefore$ From ① and ②

$C - (B - A) = (C \cap A) \cup (C \cap B')$



5) $(B - A) \cap C = (B \cap C) - A = B \cap (C - A)$

$B - A = \{2, 3, 6\}$

$(B - A) \cap C = \{2, 3\}$ — ①

$B \cap C = \{2, 3\}$

$(B \cap C) - A = \{2, 3\}$ — ②

$C - A = \{2, 3, 7\}$

$B \cap (C - A) = \{2, 3\}$ — ③

From ①, ②, ③

$(B - A) \cap C = (B \cap C) - A = B \cap (C - A)$

6) $(B - A) \cup C = (B \cup C) - (A - C)$

LHS: $B - A = \{2, 3, 6\}$

$(B - A) \cup C = \{2, 3, 6, 7\}$ — ①

RHS $B \cup C = \{2, 3, 6, 7\}$

$A - C = \{1, 4\}$

$(B \cup C) - (A - C) = \{2, 3, 6, 7\}$ — ②

$\therefore$ From ① and ②

$(B - A) \cup C = (B \cup C) - (A - C)$

1) write the following in roster form.

1) $x \in \mathbb{N} : x^2 < 121$ and x is prime

$A = \{2, 3, 5, 7\}$

2) the set of all positive roots of the equation $(x-1)(x+1)(x^2-1) = 0$

The roots are ± 1

$A = \{1\}$

3) $\{x \in \mathbb{N} : 4x + 9 < 52\}$

$x = 1, 4x + 9 = 13$

$x = 2, \quad = 17$

$x = 3, \quad = 21$

$x = 4, \quad = 25$

$x = 5, \quad = 29$

$x = 6, \quad = 33$

$x = 7, \quad = 37$

$x = 8, \quad = 41$

$x = 9, \quad = 45$

$x = 10, \quad = 49$

$\therefore x : A = \{1, 2, 3, \dots, 10\}$

4.

Example 1.3 Prove that

$$((A \cup B' \cup C) \cap (A \cap B' \cap C')) \cup ((A \cup B \cup C') \cap (B' \cap C')) = B' \cap C'.$$

Solution:

We have $A \cap B' \cap C' \subseteq A \subseteq A \cup B' \cup C$ and hence $(A \cup B' \cup C) \cap (A \cap B' \cap C') = A \cap B' \cap C'$. Also, $B' \cap C' \subseteq C' \subseteq A \cup B \cup C'$ and hence $(A \cup B \cup C') \cap (B' \cap C') = B' \cap C'$. Now as $A \cap B' \cap C' \subseteq B' \cap C'$, we have

$$((A \cup B' \cup C) \cap (A \cap B' \cap C')) \cup ((A \cup B \cup C') \cap (B' \cap C')) = B' \cap C'.$$

Note: Try to simplify the above expression using Venn diagram.

Example 1.4 If $X = \{1, 2, 3, \dots, 10\}$ and $A = \{1, 2, 3, 4, 5\}$, find the number of sets $B \subseteq X$ such that $A - B = \{4\}$

Solution:

For every subset C of $\{6, 7, 8, 9, 10\}$, let $B = C \cup \{1, 2, 3, 5\}$. Then $A - B = \{4\}$. In other words, for every subset C of $\{6, 7, 8, 9, 10\}$, we have a unique set B so that $A - B = \{4\}$. So number of sets $B \subseteq X$ such that $A - B = \{4\}$ and the number of subsets of $\{6, 7, 8, 9, 10\}$ are the same. So the number of sets $B \subseteq X$ such that $A - B = \{4\}$ is $2^5 = 32$.

Example 1.5 If A and B are two sets so that $n(B - A) = 2n(A - B) = 4n(A \cap B)$ and if $n(A \cup B) = 14$, then find $n(\mathcal{P}(A))$.

Solution:

To find $n(\mathcal{P}(A))$, we need $n(A)$.

Let $n(A \cap B) = k$. Then $n(A - B) = 2k$ and $n(B - A) = 4k$.

Now $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B) = 7k$.

It is given that $n(A \cup B) = 14$. Thus $7k = 14$ and hence $k = 2$.

So $n(A - B) = 4$ and $n(B - A) = 8$. As $n(A) = n(A - B) + n(A \cap B)$, we get $n(A) = 6$ and hence $n(\mathcal{P}(A)) = 2^6 = 64$.

Example 1.6 Two sets have m and k elements. If the total number of subsets of the first set is 112 more than that of the second set, find the values of m and k .

Solution:

Let A and B be the two sets with $n(A) = m$ and $n(B) = k$. Since A contains more elements than B , we have $m > k$. From the given conditions we see that $2^m - 2^k = 112$. Thus we get, $2^k(2^{m-k} - 1) = 2^4 \times 7$.

Then the only possibility is $k = 4$ and $2^{m-k} - 1 = 7$. So $m - k = 3$ and hence $m = 7$.

Relations

1.10) check the relation $R = \{(1,1) (2,2) (3,3) - \dots (n,n)\}$ defined on the TBP set $S = \{1, 2, 3, - \dots n\}$ for the three basic relation.

Solution: 1) As $(a,a) \in R$ for all $a \in S$, R is reflexive.

2) $\therefore$ For $(a,b) \in R, (b,a) \in R$.

Also for every pair $(a,b) \in R, (b,a) \in R$ Hence R is symmetric.

3) No pairs $(a,b) (b,c)$ are in $R \therefore (a,c) \notin R$.

$\therefore R$ is not transitive is not true $\therefore R$ is transitive

$\therefore R$ is reflexive, symmetric and transitive this relation is an equivalence relation.

1.11) Let $S = \{1, 2, 3\}$ $P = \{(1,1) (1,2) (1,3) (3,1)\}$

TBP 1. Is P is reflexive? If not state the reason and write the minimum set of ordered pairs to be included to P so as to make it reflexive.

Ans: P is not reflexive $\therefore (3,3) \notin P$.

If we include $(3,3)$ in P then it is reflexive.

2. Is P is symmetric? If not, state the reason, write minimum number of ordered pairs to be included to P so as to make it symmetric and write minimum number of ordered pairs to be deleted from P so as to make it symmetric.

Ans: P is not symmetric $\therefore$ For $(1,2) \in P, (2,1) \notin P$

Suppose if we include $(2,1)$ in P is symmetric (or)

if we delete $(1,2)$ from P then it is symmetric.

3) Is P is transitive? state the reason, write the minimum number of ordered pair to be included to P so as to make it transitive and write minimum number of ordered pairs to be deleted from P so as to make it transitive.

Ans. For $(1,3), (3,1) \in P, (3,3) \notin P \therefore$ it is not transitive.
if we include $(3,3)$ ^{to $(3,2)$} is transitive (or) if we delete $(1,3)$ ^{it is} transitive.

4) Is P is an equivalence relation? if not, write the minimum ordered pair to be included to P so that to make P as an equivalence relation.

Ans: P is not equivalence relation.

If we include $(3,3)$ ^{$(3,2)$ $(2,3)$} and $(2,1)$ in P then only it is equivalence relation.

1.12) Let $A = \{0, 1, 2, 3\}$ Construct relations on A of the following types,

1) not reflexive, not symmetric, not transitive
 TBP Ans: Construct the relation $\{(1, 2) (2, 3)\}$. is not reflexive not symmetric and not transitive.

2) not reflexive, not symmetric and transitive.
 Ans: $\{(1, 2)\}$ is transitive but not reflexive and not symmetric.

3) not reflexive, symmetric, not transitive
 Ans: $\{(1, 2) (2, 1)\}$ is symmetric, not reflexive, not transitive.

4) not reflexive, symmetric, transitive.
 Ans: $\{(1, 2) (2, 1) (1, 1) (2, 2)\}$ not reflexive, but symmetric and transitive.

5) reflexive, not symmetric, not transitive.
 Ans: $\{(0, 0) (1, 1) (2, 2) (3, 3) (1, 2) (1, 3)\}$ reflexive, not symmetric, not transitive.

6) reflexive, symmetric, not transitive.
 Ans: $\{(0, 0) (1, 1) (2, 2) (3, 3) (1, 2) (2, 3) (2, 1) (3, 2)\}$
 reflexive, symmetric, not transitive.

7) reflexive, symmetric, transitive.
 Ans: $\{(0, 0) (1, 1) (2, 2) (3, 3)\}$.
 Reflexive, symmetric and transitive.

1.13) In the set $\mathbb{Z}$ of integers define $m R n$ if $m - n$ is multiple of 12
 TBP P.T R is an equivalence relation.

1) Reflexive: $m R m = m - m = 0$ and $0 \times 12 = 0$ is multiple of 12 hence $m R m$ is reflexive

2) Symmetric: $m R n = m - n = 12k$
 $n R m \quad n - m = 12(-k)$ Both are multiple of 12
 Hence $m R n = n R m \therefore$ Symmetric.

3) Transitive: Let $m R n = m - n = 12k$.
 $n R p = n - p = 12l \Rightarrow$ multiple of 12
 $= m - p = 12(k+l) = m R p$

$\therefore$ transitive.

Hence R is an equivalence relation.

1. Let R be a relation defined on the set of natural number N as follows $R = \{(x, y), x \in N, y \in N, 2x + y = 4\}$ find the domain and range of the relation R . Also verify whether R is reflexive, symmetric and transitive.

Sol: $R = \{(x, y) \mid x \in N, y \in N, 2x + y = 4\}$.

Domain $\{1, 2, 3, 4, \dots, 20\}$

Range $\{39, 37, 35, 33, 31, \dots, 1\}$

$\therefore R = \{(1, 39) (2, 37) (3, 35) (4, 33) \dots (19, 3) (20, 1)\}$.

1) R is not reflexive $\because (2, 2) \notin R$.

2) R is not symmetric $\because$ For $(1, 39) \in R$ $(39, 1) \notin R$.

3) R is not transitive $\because (11, 19)$ and $(19, 3) \in R$.
 $(11, 3) \notin R$.

Hence R is neither reflexive, nor symmetric, nor transitive.

- 2) On the set of natural numbers let R be the relation defined by aRb if $2a + 3b = 30$. Write down the relation by listing all the pairs. Check whether it is

1) reflexive 2) symmetric, 3) transitive 4) equivalence.

$R = \{(a, b) \mid a \in N, b \in N, 2a + 3b = 30\}$.

Domain $\{6, 9, 12\}$

Range $= \{6, 4, 2\}$

$R = \{(6, 6) (6, 4) (6, 2) (9, 6) (9, 4) (9, 2) (12, 6) (12, 4) (12, 2)\}$.

1) Not reflexive $\because (9, 9) \notin R$.

2) Not Symmetric $\because (6, 4) \in R$ But $(4, 6) \notin R$.

3) It is transitive : $(6, 6) \in R$ $(6, 4) \in R \Rightarrow (6, 4) \in R$.

$(9, 6) \in R$ $(6, 4) \in R \Rightarrow (9, 4) \in R$.

$(12, 6) \in R$ $(6, 4) \in R \Rightarrow (12, 4) \in R$.

$\therefore R$ is ^{not} reflexive, ^{not} symmetric, and transitive it is an ^{not} equivalence relation.

3) on the set of natural numbers let R be the relation by aRb if $a+b \leq 6$ write down the relation by listing all the pairs. check whether it is 1) reflexive 2) symmetric 3) transitive 4) Equivalence.

Sol: $R: \{(a,b) \mid a \in \mathbb{N}, b \in \mathbb{N}, a+b \leq 6\}$.

Domain: $\{1, 2, 3, 4, 5\}$

Range: $\{5, 4, 3, 2, 1\}$

$R = \{(1,5) (1,4) (1,3) (1,2) (1,1) (2,5) (2,4) (2,3) (2,2) (2,1) (3,5) (3,4) (3,3) (3,2) (3,1) (4,5) (4,4) (4,3) (4,2) (4,1) (5,5) (5,4) (5,3) (5,2) (5,1)\}$.

1. Reflexive: $\because \{(1,1) (2,2) (3,3) (4,4) (5,5)\} \in R$. It is reflexive

2) Symmetric: $(5,3) \in R$ and $(3,5) \in R$.
(or) $\forall (a,b) \in R, (b,a) \in R$. $\therefore$ It is symmetric.

3) transitive: $\forall (a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$.

(or) $(3,4), (4,2) \in R \Rightarrow (3,2) \in R$.

Hence transitive.

$\therefore$ It is an equivalence relation.

4) In the set $\mathbb{Z}$ of integers define on $\mathbb{R}$ if $x-y$ is divisible by 7. P.T
 R is an equivalence relation. $R = \{(x,y), x \in \mathbb{Z}, y \in \mathbb{Z}, x-y \text{ is divisible by } 7\}$.

Sol: Let $\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \dots\}$.

1) Reflexive: $x \in \mathbb{Z}, x-x=0$ which is divisible by 7.

$\therefore R$ is reflexive

2) Symmetric: If $x \in \mathbb{Z}, y \in \mathbb{Z}$ and $x-y$ is divisible by 7
then $-(x-y) = y-x$ is also divisible by 7.

$\therefore$ It is symmetric.

3) Transitive: If $x \in \mathbb{Z}, y \in \mathbb{Z}$ and $z \in \mathbb{Z}$.

and $x-y$ is divisible by 7

$y-z$ is divisible by 7

$\Rightarrow x-y+y-z$ is also divisible by 7

$\Rightarrow x-z$ is divisible by 7. Hence it is transitive

$\therefore R$ is an equivalence relation.

5) In a set of all natural number N , let a relation R be defined by
 $R = \{(a, b) \mid a - b \text{ is divisible by } 3, a \in N, b \in N\}$ P.T R is an equivalence relation.

Solution: $R = \{(a, b) \mid a \in N, b \in N, a - b \text{ is divisible by } 3\}$.

1) Reflexive: $\forall a \in N, a - a = 0$ which is divisible by 3.
 Hence R is Reflexive.

2) Symmetric: $\forall a, b \in N, a - b$ is divisible by 3
 Hence $-(a - b) = b - a$ is also divisible by 3
 Hence R is Symmetric.

3) Transitive: $\forall a, b, c \in N, a - b$ is divisible by 3
 $b - c$ is divisible by 3
 Hence $(a - b) + (b - c)$ is divisible by 3.
 $a - c$ is divisible by 3
 $\therefore R$ is Transitive.

Hence R is an equivalence relation.

b) S.T relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R: (a, b) : |a - b|$ is even is an equivalence relation.

Solution: Let 1. Reflexive
 $a \in A, (a, a) \in R \forall a \in A$.

$|a - a| = 0$ which is even $\therefore R$ is reflexive

2) Symmetric: $\forall a, b \in A, |a - b|$ is even then
 $\therefore (a, b) \in R \Rightarrow (b, a) \in R. |b - a|$ is also even.

3) Transitive: Let $a, b, c \in A, (a, b) \in R$ and $(b, c) \in R$
 $|a - b|$ is even and $|b - c|$ is even.

then $|a - b| + |b - c|$ is also even

$|a - c|$ is even $\therefore R$ is Transitive.

Hence R is an equivalence relation.

7) Show that the relation R 'is perpendicular to' on the set of all straight line in a plane is symmetric, but it is neither reflexive nor transitive

Sol: R is the relation 'is perpendicular to' on the set of lines L in a plane is symmetric because $l_1 \perp l_2$ and $l_2 \perp l_1$
 $\forall (l_1, l_2) \in R$.

For $(l_1, l_1) \in R$ l_1 is not $\perp$ to l_1 . $\therefore$ it is not reflexive

For $l_1, l_2, l_3 \in R$ $l_1 \perp l_2$, and $l_2 \perp l_3$ But l_1 not $\perp$ to l_3
(ex) $(l_1, l_2) \in R$ and $(l_2, l_3) \in R$ but $(l_1, l_3) \notin R$.

$\therefore R$ is not transitive.

Hence R is symmetric but neither reflexive nor transitive.

8) S.T the relation 'is greater than' in a set R of all real numbers is transitive but it is neither reflexive nor transitive.

Sol: 1) Reflexive: As no real number can be greater than itself $\therefore$ it is not reflexive.

2) Symmetric: $(a, b) \in R$ $a > b$. but $b \nmid a$
 $\therefore (a, b) \in R$ $(b, a) \notin R$.

$\therefore$ it is not symmetric.

3) Transitive: As $a, b, c \in R$.

$$a > b, b > c \Rightarrow a > c$$

$\therefore$ the relation greater than is transitive.

Hence the relation 'greater than' is transitive but neither reflexive nor transitive.

9) Discuss the relation reflexive, symmetric, transitive for the relation R defined on the set of all positive integers m, n if m divides n .

Sol: Reflexive: Since every positive integer divides itself
(ex) $\forall a \in \mathbb{Z}^+ (a, a) \in R$. So R is reflexive.

Symmetric: $\because 2$ divides 6 but 6 does not divide 2
 $\therefore (a, b) \in R$ a divides b but b does not divide a Hence R is not symmetric.

Transitive: 2 divides 4 and 4 divides $8 \Rightarrow 2$ divides 8

$$(ex) \forall a, b, c \in \mathbb{Z}^+ \text{ and } a \text{ divides } b, b \text{ divides } c \Rightarrow a \text{ divides } c$$

$\therefore R$ is transitive.

$\therefore R$ is reflexive and transitive.

10) The non empty set consisting of children in a family and a relation R defined as aRb if a is a brother of b then R is

Sol: $R: \{(a, b) \mid a \text{ is a brother of } b\}$.

1) Reflexive: $a \in R$, $(a, a) \notin R$ "a being not at all brother to it self -
 $\therefore$ not reflexive.

2) Symmetric: $(a, b) \in R$, a is a brother b but b may or may not be brother of a (ie) b may be sister of a .
 $\therefore$ it is not symmetric.

3) Transitive: a is a brother of b and b is a brother of $c \Rightarrow a$ is a brother of c

$(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R$.
 Hence transitive.

$\therefore$ It is ^{is not} symmetric but transitive.

11) Let A be the set consisting of all the female members of a family. The relation R is defined by aRb if a is not a sister of b .

Sol: $R: \{(a, b) \mid a \text{ is not a sister of } b\}$.

1) Reflexive $a \in R$, $(a, a) \notin R$. a being a not at all not sister of a .
 $\therefore$ Not reflexive

2) Symmetric: $a, b \in R$, a is not a sister b hence b is also not a sister of a
 $\therefore (a, b) \in R$ and $(b, a) \in R$.

Hence Symmetric.

3) Transitive: a is not a sister of b and b is not a sister of c
 a may be sister of c

(ie) $(a, b) \in R, (b, c) \in R$ and $(a, c) \notin R$.

$\therefore R$ is not reflexive.

12) Let A be the set consisting of all the members of a family. The relation R defined by aRb if a is not a sister of b .

Sol: $R = \{(a, b) \mid a \in A, b \in A, a \text{ is not a sister of } b\}$.

1) Reflexive $(a, a) \in R$ $\because$ a being a not sister of a itself
 $\therefore R$ is reflexive.

2) Symmetric: If a being a not sister of b , b may be sister of a .
 $\therefore (a, b) \in R, (b, a) \notin R$.
Hence R is not symmetric.

3) Transitive: $\because$ If a being a not sister of b , and b being a not sister of $c \Rightarrow a$ is also not sister of c .
 $(a, b) \in R, (b, c) \in R$ but $(a, c) \notin R$.
 $\therefore R$ is not transitive.

13) Let $X = \{a, b, c, d\}$ and $R = \{(a, a), (b, b), (a, c)\}$ write down the minimum number of ordered pairs to be included to R to make it 1) reflexive 2) symmetric 3) transitive 4) equivalence.

Sol: $X = \{a, b, c, d\}$

$R = \{(a, a), (b, b), (a, c)\}$

1) If we include $(c, c), (d, d)$ to R it is reflexive.

2) If we include (c, a) is symmetric. $\therefore \{(a, a), (b, b), (c, c), (a, c), (c, a)\}$ symmetric.

3) If we include $(c, d), (d, a), (a, d)$ and hence it is transitive.
 $(a, c) \in R$ and $(c, d) \in R \Rightarrow (a, d) \in R$.
 $(c, d), (d, a), (a, d), (d, c)$
 $\therefore$ if we include $(c, c), (d, d), (c, a)$ it is equivalence.

14) Let $A = \{a, b, c\}$ and $R = \{(a, a), (b, b), (a, c)\}$ write down the minimum number of ordered pairs to be included to R to make it 1) reflexive 2) symmetric 3) transitive 3) equivalence.

Sol: Reflexive: $A = \{a, b, c\}$

$R = \{(a, a), (b, b), (a, c)\}$.

$\therefore$ If we include (c, c) then $R = \{(a, a), (b, b), (c, c), (a, c)\}$ then R is reflexive.

Symmetric: If we include (c, a) then in R $\{(a, a), (b, b), (c, c), (a, c), (c, a)\}$ then R is symmetric.

Transitive = $\{(a, a) (b, b) (c, c), (a, c) (c, a)\}$

If we include $(c, b), (b, c) (a, b), (b, a)\}$.

then R is symmetric and transitive.

$\therefore R$ is equivalence.

15) Let P be the set of all triangles in a plane and R be the relation defined on P as $a R b$ if a is similar to b . $P \times P$ is an equivalence relation.

Sol: Let P be the set of all triangles in a plane. and

$R = \{(T_1, T_2) : T_1 \text{ is similar to } T_2 \text{ and } T_1, T_2 \in P\}$.

Since the triangles are said to be similar if the corresponding angles are equal and ratio of the corresponding sides are equal.

Reflexive: Let $T_1 \in P$. We know that every triangle is similar to itself.

$\therefore (T_1, T_1) \in R$. Hence R is reflexive.

Symmetric: Let $T_1, T_2 \in P$ s.t. $(T_1, T_2) \in R \Rightarrow T_1$ is similar to T_2 .

$\Rightarrow T_2$ is also similar to T_1 Hence $(T_2, T_1) \in R$.

$\therefore R$ is symmetric.

Transitive: Let $T_1, T_2, T_3 \in P$ s.t.

$(T_1, T_2) \in R, (T_2, T_3) \in R \Rightarrow (T_1, T_3) \in R$.

$\because T_1$ is similar to T_2 and T_2 is similar to T_3
 $\Rightarrow T_1$ is similar to T_3 . Hence R is transitive.

$\therefore R$ is Reflexive, Symmetric, Transitive. Hence R is equivalence relation.

16) Let T be the set of all triangles in a plane with R is a relation on T by $R = \{(T_1, T_2) : T_1 \text{ is congruent to } T_2 \text{ and } T_1, T_2 \in T\}$ s.t. R is an equivalence relation. Consider three right angled triangles T_1, T_2, T_3 with sides T_1 with 3, 4, 5, T_2 with 5, 12, 13 and T_3 with 6, 8, 10. which are related.

Sol: Let T be the set of all Δ s in a plane.

$R = \{(T_1, T_2) : T_1 \text{ is congruent to } T_2 \text{ and } T_1, T_2 \in T\}$.

$\because$ If the two triangles are said to be congruent if they have same size and same shape.

Reflexive: Let $T_1 \in T$. We know that every triangle is congruent to itself.

$(T_1, T_1) \in R$. Hence R is reflexive.

Symmetric: Let $T_1, T_2 \in T$. $(T_1, T_2) \in R \Rightarrow T_1$ is congruent to T_2 then T_2 is also congruent to T_1 . $\therefore (T_2, T_1) \in R$ Hence R is symmetric.

Transitive: $T_1, T_2, T_3 \in T$. If T_1 is congruent to T_2 and T_2 is congruent to T_3
 $\Rightarrow T_1$ is congruent to T_3 Hence $(T_1, T_3) \in R$.

∴ R is transitive. ∴ R is reflexive, symmetric and transitive.
 R is an equivalence relation. ∵ $\frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{1}{2}$ ∴ The ratio of the corresponding sides of T_1 and T_2 are equal. ∴ T_1 is related to T_2 .

17) P.T The relation 'friendship' is not an equivalence relation on the set of all people in Chennai.

Sol: Let P be the set of all people in Chennai
 and R be the friendship.

Reflexive Let $P_1 \in P$. P_1 is friend to itself. Hence R is reflexive.

Symmetric: Let $P_1 \in P$ be the friend of $P_2 \in P$ $(P_1, P_2) \in R$

Then P_2 is also a friend of $P_1 \Rightarrow (P_2, P_1) \in R$.

Hence R is symmetric.

Transitive: If $P_1 \in P$ is a friend of $P_2 \in P$ and P_2 is a friend of $P_3 \in P$ but not necessarily P_1 is friend of P_3 .

(ex) $(P_1, P_2) \in R$, $(P_2, P_3) \in R$ but $(P_1, P_3) \notin R$.

Hence R is not transitive.

∴ R is not an equivalence relation.

18) Let $A = \{0, 1, 2, 3\}$ and define a relation R on A as follows
 $R = \{(0, 0) (0, 1) (0, 3) (1, 0) (1, 1) (2, 2) (3, 0) (3, 3)\}$.

Sol: $R = \{(0, 0) (0, 1) (0, 3) (1, 0) (1, 1) (2, 2) (3, 0) (3, 3)\}$,

1. Reflexive: $\{(0, 0) (1, 1) (2, 2) (3, 3)\} \forall a \in A (a, a) \in R$ Hence reflexive.

2) Symmetric: ∵ $(1, 0) (0, 1) \in R$ and $(0, 3) (3, 0) \in R$. R is symmetric.

3) Transitive: ∵ $(1, 0) \in R$ $(0, 3) \in R$, $(1, 3) \notin R$. Hence it is not transitive.

19) State the reason for the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2) (2, 1)\}$ is not transitive.

∵ $(1, 2) \in R$, and $(2, 1) \in R$ $(1, 1) \notin R$. Hence it is not transitive.

20) Let $R = \{(x, x^3) : x \text{ is a prime number less than } 5\}$ be a relation
 find the Range of R .

Sol: $A = \{2, 3\}$,

$R = \{(2, 8), (3, 27)\}$ ∴ Range = $\{8, 27\}$.

21) Let R is a relation is a divisor of 1 from the set $A = \{1, 2, 3\}$ to $B = \{4, 10, 15\}$
 then write the set of ordered pairs corresponding to R .

$$R = \{(1,4), (1,10), (1,15), (2,4), (2,10), (3,15)\}.$$

22) S.T the relation R in the set A of real numbers defined by $R = \{(a,b) | a \leq b\}$ is reflexive and transitive but not symmetric.

Sol: $A = \{\text{set of real numbers}\}.$

$$R = \{(a,b) | a \leq b\}.$$

1) Reflexive: $a \in R \Rightarrow (a,a) \in R$ (e) $a \leq a$ Hence R is reflexive.

2) Symmetric: $\{(a,b) | a \leq b\}$ but $\{(b,a) \notin R\}$ $b \not\leq a$
Hence R is not symmetric.

3) Transitive: $\forall a, b \in R$ and $a \leq b, b \leq c \Rightarrow a \leq c.$

$$\therefore (a,b) \in R, (b,c) \in R \Rightarrow (a,c) \in R.$$

Hence R is transitive.

23) S.T the relation R in the set A of real numbers defined as $R = \{(a,b) : a, b \in \mathbb{R} \text{ and } a \leq b^3\}$. S.T R is ^{neither} reflexive and ^{nor} symmetric nor transitive.

Sol: Let $A = \{\text{set of all real numbers}\}, R = \{(a,b) : a, b \in \mathbb{R} \text{ and } a \leq b^3\}$

Reflexive: Consider $\frac{1}{3} \in A.$

$$\left(\frac{1}{3}, \frac{1}{27}\right) \notin R \because \frac{1}{3} \not\leq \frac{1}{27} \text{ Hence } (a,a) \notin R. \text{ Hence not reflexive.}$$

Symmetric: Consider $-2, 3 \in R.$

$$(-2, 3) \in R \because -2 \leq 27$$

$$\text{But } (3, -2) \notin R \Rightarrow 27 \not\leq -2 \text{ Hence } R \text{ is not symmetric.}$$

Transitive: Consider $3, \frac{3}{2}, \frac{27}{8} \in \mathbb{R}$

$$(3, \frac{3}{2}) \in R \because 3 \leq \frac{27}{8}$$

$$\text{and } \left(\frac{3}{2}, \frac{27}{8}\right) \in R \because \frac{3}{2} \leq \frac{64}{27}$$

$$\text{But } (3, \frac{4}{3}) \notin R \because 3 \not\leq \frac{64}{27} \text{ Hence } R \text{ is not transitive.}$$

24) Let the relation R on the set A of real numbers defined as $(a, b) \in R \Rightarrow 1 + ab > 0 \forall a, b \in A$. S.T R is reflexive, symmetric but not transitive.

Sol: $A = \{ \text{Set of real numbers} \}$,

$$R = \{ (a, b) \mid 1 + ab > 0 \forall a, b \in A \}$$

1) Reflexive $\forall a \in A, (a, a) \in R \because 1 + a^2 > 0$.

2) Symmetric $\forall a, b \in A (a, b) \in R \Rightarrow (b, a) \in R \because 1 + ab > 0$
 $1 + ba > 0$
 $\forall a, b \in A$.
 $\therefore R$ is symmetric.

3) Transitive: $(1, \frac{1}{2}) \in R \because 1 + 1 \cdot \frac{1}{2} > 0$

$$(\frac{1}{2}, -1) \in R \because 1 + (-1) \cdot \frac{1}{2} > 0$$

$$\text{But } (1, -1) \notin R \because 1 + (-1)(1) = 0 \not> 0$$

$\therefore R$ is not transitive.

25) S.T the relation less than in the set of natural numbers is not reflexive, not symmetric, transitive.

Sol: Let $A = \{ \text{Set of natural numbers} \}$.

1) Reflexive $a \in A, (a, a) \notin R \because a$ is not less than itself.
 $\therefore R$ is not reflexive.

2) Symmetric, $a, b \in A, (a, b) \in R \because a < b$.
 and $a < b$. But $(b, a) \notin R \because b \not< a$.
 $\therefore$ it is not symmetric.

3) Transitive: $\forall a, b, c \in A$ and $(a < b < c) \Rightarrow a < c$
 $\therefore (a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$.

Hence R is not reflexive, not symmetric but transitive.

26) Consider the set $N \times N$ the set of all ordered pairs of natural numbers. Let R be the relation in $N \times N$ which is defined as $(a, b) R (c, d)$ if and only if $ad = bc$. P.T R is an equivalence relation.

Sol: $\forall (a, b) \in N \times N$.

$(a, b) R (a, b) \Rightarrow ab = ba \Rightarrow (b, a) \in R$. Hence R is symmetric.

2) Symmetric. $\forall (a, b) \in N \times N$

$$(a, b) R (c, d) \Rightarrow ad = bc$$

$$(c, d) R (a, b) \Rightarrow cb = ad \therefore R \text{ is symmetric.}$$

Transitive: $(a, b) R (c, d)$ and $(c, d) R (e, f)$ then $(a, b) R (e, f)$

$$(a, b) R (c, d) = ad = bc$$

$$(c, d) R (e, f) = cf = de$$

$$= adcf = bcde$$

$$\div cd = af = be \Rightarrow (a, b) R (e, f)$$

$\therefore R$ is transitive.

$\therefore R$ is equivalence relation.

27) Consider the set $N \times N$, the set of ordered pairs of natural numbers. Let R be the relation in $N \times N$ which is defined by $(a, b) R (c, d)$ iff $a + d = b + c$. $\therefore R$ is an equivalence relation.

Sol: Let R be the relation on $N \times N$ defined by

$$(a, b) R (c, d) \Leftrightarrow a + d = b + c$$

1. Reflexive: $\forall (a, b) \in N \times N$
 $(a+b) R (a+b) = a+b = b+a \therefore R$ is reflexive

2. Symmetric: $\forall (a, b), (c, d) \in N \times N$
 $(a+b) R (c+d) = (a+d) = b+c$
 $(c+d) R (a+b) = c+b = d+a \therefore R$ is symmetric.

3) Transitive: $\forall (a, b), (c, d), (e, f) \in N \times N$

$$(a, b) R (c, d) \Rightarrow a + d = b + c$$

$$(c, d) R (e, f) \Rightarrow c + f = d + e$$

$$(+)$$

$$a + d + c + f = b + c + d + e$$

$$a + f = b + e \Rightarrow (a, b) R (e, f)$$

$\therefore R$ is Transitive.

$\therefore R$ is equivalence Relation.

28) S.T the relation R defined in the set A of all polygons are $R = \{(P_1, P_2) : P_1 \text{ and } P_2 \text{ have same number of sides}\}$ is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4, 5?

Sol: Let $R = \{(P_1, P_2) : P_1, P_2 \text{ have the same number of sides}\}$

1. Reflexive: $\therefore (P, P) \in R$ as the same polygon have same number of sides with itself.

2. Symmetric: Let $(P_1, P_2) \in R \Rightarrow P_1$ and P_2 have the same number of sides.
 P_2 and P_1 have the same number of sides.
 $(P_2, P_1) \in R$ hence R is symmetric.

3. Transitive: Let $(P_1, P_2), (P_2, P_3) \in R$. $\therefore P_1$ and P_2 have the same number of sides.
 P_2 and P_3 have the same number of sides.
 $\Rightarrow P_1$ and P_3 have the same number of sides.

$(P_1, P_3) \in R$ Hence R is reflexive. ($\therefore$ Triangle is also polygon with 3 sides)
The elements in A related to the right angled triangle T with sides 3, 4, 5 are those polygons which have 3 sides. Hence, the set of all elements in A related to T is the set of triangle.

Theorem: 1) The number of relations from a set containing m elements to a set containing n elements is 2^{mn} . In particular the number of relations on a set containing n elements is 2^{n^2}

2) The number of reflexive relations on a set containing n elements is $2^{n^2 - n}$

3) The number of symmetric relations on a set containing n elements is $2^{\frac{n^2 + n}{2}}$

Theorem: If R is the relation from A to B then the relation R^{-1} defined from B to A by $R^{-1} = \{(b, a) : (a, b) \in R\}$ is called the inverse relation of R .

The domain of R becomes range of R^{-1} and the range of R becomes domain of R^{-1}

29) Let A be the set of first 10 natural numbers and let R be relation on A defined by $\{(x, y) \in R \Leftrightarrow x + 2y = 10\}$. Express R and R^{-1} as sets of ordered pairs. Also determine 1) domain of R and R^{-1} 2) Range of R and R^{-1} .

Sol: $R = \{(x, y), x \in A, y \in A, x + 2y = 10\}$

$$\therefore y = \frac{10 - x}{2} \quad \text{when } x = 2 \quad y = 4$$

$$x = 4 \quad y = 3$$

$$x = 6 \quad y = 2$$

$$x = 8 \quad y = 1$$

$$\therefore R = \{(2, 4), (4, 3), (6, 2), (8, 1)\}$$

$$R^{-1} = \{(4, 2), (3, 4), (2, 6), (1, 8)\}$$

$$\text{Domain of } R = \{2, 4, 6, 8\} = \text{range of } R^{-1}$$

$$\text{Range of } R = \{4, 3, 2, 1\} = \text{Domain of } R^{-1}$$

30) If $A = \{a, b\}$ $B = \{2, 3\}$ find the number of relations from A to B .

$$\text{Sol: Number of relations} = 2^{2 \times 2} = 2^4 = 16.$$

31) If $n(A) = 3$ and $B = \{2, 3, 4, 6, 7, 8\}$ then find the number of relations from A to B .

$$\text{Sol: } n(A) = 3 \quad n(B) = 6$$

$$\therefore \text{Number of relations} = 2^{3 \times 6} = 2^{18}$$

Functions.

32) Find the domain of each of the following:

1) $\frac{1}{\sqrt{x-2}}$ 2) $\sqrt{4-x^2}$

1) Sol: For $x-2 > 0$ $f(x)$ is real value $f(x) = \frac{1}{\sqrt{x-2}}$

(or) $x > 2 \Rightarrow x \in (2, \infty)$ $f(x) = \frac{1}{\sqrt{x-2}}$ is real

 $\therefore$ the domain of $\frac{1}{\sqrt{x-2}}$ is $(2, \infty)$

2) $f(x) = \sqrt{4-x^2}$ For $f(x) \geq 0$ $4-x^2 \geq 0$

(or) $x^2 - 4 \leq 0 \Rightarrow (x+2)(x-2) \leq 0$

$x \in [-2, 2]$

 $\therefore$ The domain is $[-2, 2]$ 33) Find the domain for which the functions $f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ are equal.

Sol: $f(x) = 2x^2 - 1$ $g(x) = 1 - 3x$

Given $f(x) = g(x)$

$2x^2 - 1 = 1 - 3x \Rightarrow 2x^2 + 3x - 2 = 0$

$2x^2 + 4x - x - 2 = 0$

$2x(x+2) - 1(x+2) = 0$

$(x+2)(2x-1) = 0$

$x = -2, \frac{1}{2}$

 $\therefore$ domain for $f(x) = g(x)$

is $\left\{\frac{1}{2}, -2\right\}$

34) Find the domain of $f(x) = \frac{1}{\sqrt{x+|x|}}$

Sol: Find $f(x) = \frac{1}{\sqrt{x+|x|}}$ $|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

$x+|x| = \begin{cases} x+x & x \geq 0 \\ 0 & x < 0 \end{cases} \Rightarrow x+|x| = \begin{cases} 2x \geq 0 \\ 0 < 0 \end{cases}$

 $\therefore$ when $\sqrt{x+|x|} > 0$ $f(x)$ is real.

$x+|x| > 0$

"

$2x > 0 \Rightarrow x > 0 \therefore x \in (0, \infty)$

Hence the domain $= (0, \infty)$ Note Domain of modulus function is always $\mathbb{R}$ (ee) $(-\infty, \infty)$

35) Find the domain of $f(x) = \frac{1}{x+2}$

Sol: For $f(x)$ is real, $x+2 \neq 0 \Rightarrow x \neq -2$

$\therefore$ Domain: $\mathbb{R} - \{-2\}$

36) Find the domain of $f(x) = \frac{x-1}{x-3}$.

Sol: For $f(x)$ is real $f(x) \neq \frac{0}{0}$ $x-3 \neq 0 \Rightarrow x \neq 3$.

$\therefore$ Domain $(f) = \mathbb{R} - \{3\}$

37) Find the domain of $\frac{2x-3}{x^2-3x+2}$.

Sol: $f(x) = \frac{2x-3}{x^2-3x+2}$

Put $x^2-3x+2=0$ $\begin{matrix} 2 \\ \wedge \\ -2 \end{matrix}$

$$(x-2)(x-1)=0$$

$$x=1, 2$$

$\therefore$ Except $x=1, 2$ other all values of x gives $f(x)$ is real
 $\therefore$ domain $\mathbb{R} - \{1, 2\}$.

38) Find the domain of $\frac{x^2+3x+5}{x^2-5x+4}$

Sol: Let $f(x) = \frac{x^2+3x+5}{x^2-5x+4}$

Put $x^2-5x+4=0$

$$(x-4)(x-1)=0 \Rightarrow x=1, 4$$

$\therefore \mathbb{R} - \{1, 4\}$ is the domain

39) Find the domain of $\sqrt{4-x} + \frac{1}{\sqrt{x^2-1}}$

Sol: clearly $f(x)$ is satisfying $\forall x$ when

$$4-x \geq 0 \text{ and } x^2-1 \geq 0$$

$$x \leq 4 \text{ and } (x+1)(x-1) > 0$$

when $x < -1$ or $x > 1$

$$\therefore x \in (-\infty, -1) \text{ and } x \in (1, 4)$$

$\therefore$ The domain of f is $(-\infty, -1) \cup (1, 4)$

40) Find the domain of $f(x) = \frac{x^2+3x+5}{x^2-5x+4}$.

Sol: For $f(x)$ is real the value of x without $x^2-5x+4=0$

$$(x-1)(x-4)=0 \quad x=1, 4$$

$\therefore f(x)$ is real $\forall x \in \mathbb{R} - \{1, 4\}$.

Hence the domain = $\mathbb{R} - \{1, 4\}$.

41) Find the domain of the function $f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$

Sol: Put $x^2 - 8x + 12 = 0$

$$(x-6)(x-2) = 0 \Rightarrow x = 2, 6$$

$\therefore$ The domain of $f(x) = \mathbb{R} - \{2, 6\}$.

42) Find the domain and Range of the function $f(x) = -|x|$

Sol: $f(x) = -|x|$

x can have all real values

$\therefore f(x)$ is defined $\forall \mathbb{R}$.

$\therefore$ Domain of $f(x) = \mathbb{R}$.

Range: $\because |x|$ cannot be negative

$$-|x| \leq 0 \therefore \text{Range} = (-\infty, 0]$$

43) Find the domain and range of the real function $f(x) = \sqrt{x-1}$

Sol: f is defined for $x-1 \geq 0 \Rightarrow x \geq 1$

Hence the domain is $[1, \infty)$

Range: $\forall x \in [1, \infty)$ the range is $[0, \infty)$

44) Find the domain and Range of the real function $f(x) = |x-1|$

Sol: $f(x) = |x-1| \geq 0 \forall$ real value of x

Hence the domain is $\mathbb{R}$.

Range: $\forall x \in \mathbb{R}$ $|x-1|$ cannot be negative

Hence the range is $[0, \infty)$ (i.e) non negative real number.

45) Find the domain and range of 1) $f(x) = \sqrt{9-x^2}$ $x \in \mathbb{R}$,
2) $f(x) = \sqrt{\frac{x^2-1}{x-1}}$ $x \in \mathbb{R}, x \neq 1$

Sol: 1) $f(x) = \sqrt{9-x^2}$

$$9-x^2 \geq 0 \quad x^2 \leq 9 \quad (\text{or}) \quad -3 \leq x \leq 3.$$

$\therefore$ Domain of $f = [-3, 3]$

Range Take $f(x) = y$

$$y = \sqrt{9-x^2} \Rightarrow y^2 = 9-x^2 \Rightarrow x^2 = 9-y^2$$

$$\text{Now } x^2 \geq 0 \quad 9-y^2 \geq 0 \quad y^2 \leq 9 \Rightarrow -3 \leq y \leq 3$$

$\therefore$ Range for $y \geq 0 \therefore$ Range $[0, 3]$.

$$2) f(x) = \frac{x^2-1}{x-1} \quad x \in \mathbb{R} \quad x \neq 1$$

When $x-1 \neq 0$ $f(x)$ is defined
 $x \neq 1$ $f(x)$ is defined.

$\therefore$ Domain of $f = \mathbb{R} - \{1\}$.

Range: $y = \frac{x^2-1}{x-1} = \frac{(x+1)(x-1)}{(x-1)}$

$$y = x+1$$

to get $y=2$ we have to put $x=1$ which is absurd.

$\therefore$ Range = $\mathbb{R} - \{2\}$.

46) Let $f = \left\{ \left[x, \frac{x^2}{1+x^2} \right] \right\} \quad x \in \mathbb{R}$ be a function $\mathbb{R} \rightarrow \mathbb{R}$. Determine the range of $\mathbb{R}$.

Sol: $f(x) = \frac{x^2}{1+x^2} \quad \forall x \in \mathbb{R} \quad f(x)$ is defined.

Let $y = \frac{x^2}{1+x^2} \Rightarrow y(1+x^2) = x^2$
 $y + yx^2 = x^2$

$$y = x^2(1-y)$$

$$x^2 = \frac{y}{1-y} \quad \therefore x = \pm \sqrt{\frac{y}{1-y}}$$

For Real value of x $y \geq 0$
 $1-y > 0 \Rightarrow 1 > y \Rightarrow y < 1$

$$\therefore 0 \leq y < 1$$

$\therefore$ Range is $\{y = f(x) : 0 \leq y < 1\}$.

47) Find the domain and Range of the function $f(x) = \frac{x-2}{3-x}$.

Sol: Let $f(x) = \frac{x-2}{3-x}$

$f(x)$ is defined for all $x \in \mathbb{R}$ except $3-x \neq 0$
 $x \neq 3$.

$\therefore$ domain of $f = \mathbb{R} - \{3\}$.

Range: Let $y = f(x) = \frac{x-2}{3-x}$

$$yx - 3y = x - 2$$

$$yx - x = 3y - 2$$

$$x(y-1) = 3y-2$$

$$x = \frac{3y-2}{y-1}$$

$$\Rightarrow 3y - yx = x - 2$$

$$x = \frac{3y+2}{y+1}$$

clearly x has real value when

$$y+1 \neq 0 \Rightarrow y \neq -1$$

$\therefore$ The range is $\mathbb{R} - \{-1\}$.

48) Find the domain and range of the function $f(x) = \frac{4-x}{x-4}$

Sol: clearly $f(x)$ is defined $\forall x \in \mathbb{R}$ except $x-4 \neq 0 \Rightarrow x \neq 4$.

$\therefore$ Domain of $f = \mathbb{R} - \{4\}$.

$$\text{Range: } f(x) = -\frac{(x-4)}{x-4} = -1$$

$\therefore$ The range of $f = \{-1\}$

49) Find the Domain and range of $f(x) = \frac{1}{2-\sin 3x}$.

Sol: $f(x) = \frac{1}{2-\sin 3x}$.

$$\therefore -1 \leq \sin 3x \leq 1 \quad \forall x \in \mathbb{R}$$

$$-1 \leq -\sin 3x \leq 1 \quad \forall x \in \mathbb{R}$$

$$1 \leq 2 - \sin 3x \leq 3 \quad \text{add 2 on both sides.}$$

$$\Rightarrow 2 - \sin 3x \neq 0 \quad \therefore \frac{1}{2 - \sin 3x} \text{ is defined } \forall x \in \mathbb{R}.$$

$\therefore$ domain of $f = \mathbb{R}$.

Range: $\therefore 1 \leq 2 - \sin 3x \leq 3$
 $\frac{1}{3} \leq \frac{1}{2 - \sin 3x} \leq 1 \quad (\text{on reciprocal}) \quad \forall x \in \mathbb{R}.$

$\therefore$ The range is $[\frac{1}{3}, 1]$.

Find the Range of $\frac{1}{1-2\cos x}$

Sol: $-1 \leq \cos x \leq 1$

$$-2 \leq 2\cos x \leq 2$$

$$-1 \leq 1 - 2\cos x \leq 3$$

$$-\frac{1}{3} \geq \frac{1}{1-2\cos x} \geq \frac{1}{3}$$

Range
 $(-\infty, -\frac{1}{3}] \cup [\frac{1}{3}, \infty)$

50) Find the domain of the function $f(x) = \frac{1}{1-2\cos x}$

Sol: $f(x) = \frac{1}{1-2\cos x}$

$f(x)$ is defined only $\forall x \in \mathbb{R}$ except $1-2\cos x \neq 0$

$$(i) 2\cos x = 1$$

$$\cos x = \frac{1}{2}$$

$$(ii) \text{ except } x = (2n\pi \pm \pi/3) \quad n \in \mathbb{Z}.$$

Hence the domain is $\mathbb{R} - \{(2n\pi \pm \pi/3) \mid n \in \mathbb{Z}\}$.

51) Find the range of the function $f(x) = \frac{1}{1-3\cos x}$.

Sol: $f(x) = \frac{1}{1-3\cos x}$

we know $-1 \leq \cos x \leq 1$

$$-3 \leq -3\cos x \leq 3$$

$$-2 \leq 1-3\cos x \leq 4$$

Taking the reciprocals

$$-\frac{1}{2} \geq \frac{1}{1-3\cos x} \geq \frac{1}{4}$$

$\therefore$ Range is $(-\infty, -\frac{1}{2}] \cup [\frac{1}{4}, \infty)$

52) Find the domain of $f(x) = \frac{1}{1-2\sin x}$.

Sol: Let $f(x) = \frac{1}{1-2\sin x}$

We know that $-1 \leq \sin x \leq 1$

$$-2 \leq -2\sin x \leq 2 \quad \forall x \in \mathbb{R}$$

$$-1 \leq 1-2\sin x \leq 3 \quad \forall x \in \mathbb{R}$$

$$\therefore \sin x \neq 1 \quad -1 \geq \frac{1}{1-2\sin x} > \frac{1}{3}$$

Suppose $1-2\sin x = 0$

$$2\sin x = 1$$

$$\sin x = \frac{1}{2}$$

$$x = n\pi + (-1)^n \cdot \frac{\pi}{3}, \quad n \in \mathbb{Z}$$

$$\therefore \text{Domain of } f = \mathbb{R} - \left\{ n\pi + (-1)^n \cdot \frac{\pi}{3} \right\} \quad n \in \mathbb{Z}.$$

53) Find the largest possible domain for the real valued $f(x)$

$$f(x) = \frac{\sqrt{9-x^2}}{\sqrt{x^2-1}}$$

Sol: $f(x) = \frac{\sqrt{9-x^2}}{\sqrt{x^2-1}}$

$9-x^2$ will not be negative for $9-x^2 \geq 0$
 $x^2 \leq \pm 3 \quad -3 \leq x \leq 3$

$$\therefore x \in [-3, 3] \quad \text{--- (1)}$$

x^2-1 will not be negative and $\neq 0$

Suppose $x^2-1 = 0 \quad x = \pm 1$

$\therefore x$ lies outside $[-1, 1]$

$$\therefore x \in (-\infty, -1) \cup (1, \infty) \quad \text{--- (2)}$$

Combining (1) and (2)

The domain of f is $[-3, 3] \cap [(-\infty, -1) \cup (1, \infty)]$

$$\Rightarrow \text{--- (1) } [-3, -1) \cup (1, 3]$$

54) Find the largest possible domain of the real valued $f(x)$

$$f(x) = \frac{\sqrt{4-x^2}}{\sqrt{x^2-9}}$$

Sol: $f(x) = \frac{\sqrt{4-x^2}}{\sqrt{x^2-9}}$

$4-x^2$ will not be negative Suppose $4-x^2=0$ $x=\pm 2$
 $\therefore x \in [-2, 2]$ — ①

and x^2-9 will not be -ve and $\neq 0$.

Suppose $x^2-9=0$
 $x^2=9 \Rightarrow x=-3, 3$

$\therefore x$ lies outside $[-3, 3]$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$ — ②

Combining ① and ② $x \in [-2, 2] \cup (-\infty, -3) \cup (3, \infty)$

$\therefore x \in (-3, -2] \cup [2, 3)$

$\therefore$ The domain is $(-3, -2] \cup [2, 3)$

55) Find the Range of the function $\frac{1}{2\cos x - 1}$

Sol: $f(x) = \frac{1}{2\cos x - 1}$

We know $-1 \leq \cos x \leq 1$

$-2 \leq 2\cos x \leq 2$. Add (-1) on both sides

$-3 \leq 2\cos x - 1 \leq 1$

$-\frac{1}{3} \geq \frac{1}{2\cos x - 1} \geq 1$

(or) $\frac{1}{2\cos x - 1} \leq -\frac{1}{3} \Rightarrow (-\infty, -\frac{1}{3}]$

$\frac{1}{2\cos x - 1} \geq 1 \Rightarrow [1, \infty)$

$\therefore$ The range is $(-\infty, -\frac{1}{3}] \cup [1, \infty)$

56) Find the largest possible domain for the real valued function f defined by $f(x) = \sqrt{x^2 - 5x + 6}$

Sol: $x^2 - 5x + 6$ should not be negative (or) $x^2 - 5x + 6 \geq 0$.

$x^2 - 5x + 6 = 0 \Rightarrow (x-2)(x-3) = 0$
 $\Rightarrow x = 2, 3$

$\therefore$ The domain is outside $(2, 3)$ (or) $(-\infty, 2] \cup [3, \infty)$

57) Determine whether the function $f: A \rightarrow B$ defined by $f(x) = 4x + 7$ $x \in A$ is one-one.

Sol: Let $f: A \rightarrow B$ defined by $f(x) = 4x + 7$

Model-I Let $x_1, x_2 \in A$ s.t. $f(x_1) = f(x_2)$

$$4x_1 + 7 = 4x_2 + 7$$

$$4x_1 = 4x_2$$

$$x_1 = x_2 \quad \therefore f \text{ is one-one function.}$$

58) S.T the function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$

Sol: $\therefore f(x) = 1$ for $x > 0$ (ie) $\forall$ positive real values $f(x) = 1$
 (ie) more than one element have the same image

$f(x) = -1$ for $x < 0$ (ie) $\forall$ negative real values $f(x) = -1$.

(ie) more than one element have same image

$\therefore f$ is not one-one.

59) check which of the following function is onto and into

i) $f: A \rightarrow B$ given by $f(x) = 3x$ where $A = \{0, 1, 2\}$ and $B = \{0, 3, 6\}$.

ii) $f: \mathbb{Z} \rightarrow \mathbb{Z}$ given by $f(x) = 3x + 2$ $\{\mathbb{Z}$ is the set of all integers

Sol: i) $f: A \rightarrow B$, $f(x) = 3x$, $A = \{0, 1, 2\}$
 $f(0) = 0$, $f(1) = 3$, $f(2) = 6$, $B = \{0, 3, 6\}$.

$\therefore$ Range and Codomain are equal (or) Every element B has pre image in A . $\therefore$ it is onto.

ii) $f: \mathbb{Z} \rightarrow \mathbb{Z}$, $f(x) = 3x + 2$ $\mathbb{Z}$ = set of all integers

$$\text{let } y = f(x) \Rightarrow y = 3x + 2$$

$$3x = y - 2$$

$$x = \frac{y-2}{3}$$

When $y = 0$ $x = -\frac{2}{3} \notin \mathbb{Z}$. $\therefore 0 \in$ codomain does not have any pre image

Model-I

$\therefore f$ is not onto function. $\therefore$ it is into fn.

60) Let $\mathbb{R}$ be the set of all non-zero real number. Then S.T $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = \frac{1}{x}$ is one-one and onto

Sol: $f: \mathbb{R} \rightarrow \mathbb{R}$ Let $x_1, x_2 \in \mathbb{R}$ $x_1 \neq 0$, $x_2 \neq 0$

Model-I

$$f(x_1) = f(x_2)$$

$$\frac{1}{x_1} = \frac{1}{x_2} \Rightarrow x_1 = x_2 \quad \therefore f \text{ is one-to-one.}$$

Model-II

$$\text{Let } y = f(x) = \frac{1}{x} \Rightarrow x = \frac{1}{y}$$

$$f(x) = f\left(\frac{1}{y}\right) = \frac{1}{\frac{1}{y}} = y \Rightarrow \text{every element in the co-domain has pre image } \therefore \text{it is onto}$$

61) S.T the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x^2$ is neither one-one nor onto

Sol: $f: \mathbb{R} \rightarrow \mathbb{R}$ $f(x) = x^2$

For $f(1) = 1$ and $f(2) = 4$

$f(-1) = 1$ $f(-2) = 4$.

$\therefore$ more than one element in the domain have same image

$\therefore$ it is not one-one.

$\therefore$ in the co-domain the negative real numbers have no pre images $\therefore$ it is not onto.

62) S.T the function $f: \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = 2x$ is one to one but not onto.

Sol: $f: \mathbb{N} \rightarrow \mathbb{N}$ $f(x) = 2x$.

$\forall x \in \mathbb{N}$ there is a unique image in co-domain

(or) for $f(x_1) = f(x_2)$

$2x_1 = 2x_2$

$x_1 = x_2$

$\therefore f$ is one-one.

But $\forall$ odd natural numbers in co-domain there will be no pre image in domain.

(ee) Let $y = f(x) = 2x \Rightarrow x = \frac{y}{2}$

For $y = 1$ $x = \frac{1}{2} \notin \mathbb{N}$.

$\therefore f$ is not onto.

63) S.T the function $f: \mathbb{N} \rightarrow \mathbb{N}$ given by $f(1) = f(2) = 1$ and $f(x) = x-1$ for every $x > 2$ is onto but not one-one.

Sol: $f: \mathbb{N} \rightarrow \mathbb{N}$ $f(1) = 1$

$f(2) = 1$ $\therefore$ for More than one element in the domain has same image $\therefore$ it is not one-one.

For $x > 2$ let $y = f(x) = x-1$

$x = y+1$

$f(y+1) = y+1-1$

$= y$

$\therefore f$ is onto.

64) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 + 1$. then find the pre images of 17 and -3

Sol: Consider $x^2 + 1 = 17$

$\Rightarrow x^2 = 16$

$x = \pm 4$

and $x^2 + 1 = -3$

$x^2 = -4$

Pre image of (-3) does not exist.

65) $A = \{1, 2, 3\}$ $B = \{4, 5, 6, 7\}$ and $f = \{(1, 4) (2, 5) (3, 6)\}$ is a function from A to B . State whether f is one-one or not

Sol: $f = \{(1, 4) (2, 5) (3, 6)\}$

$\therefore$ Since every element of A has unique image in B f is one-one

66) Check the injectivity of the following function $f: \mathbb{R} \rightarrow \mathbb{R}$ is given by $f(x) = x^3$

Sol: Let $x_1, x_2 \in \mathbb{R}$ $f(x_1) = f(x_2)$

$$x_1^3 = x_2^3$$

$$x_1 = x_2 \quad \text{Taking cube roots on both sides.}$$

$\therefore f$ is injective

67) S.T The function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = |x|$ is neither one-one nor onto

Sol: $f(x) = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases} \therefore \text{Range is } [0, \infty)$

For more than one element in $\mathbb{R}$ there is unique image.

(e.g) $x = 1$ $f(1) = 1$ $\therefore$ it is not one-one.

$x = -1$ $f(-1) = 1$.

Also Range $[0, \infty) \neq \mathbb{R} \therefore$ Range $\neq$ Codomain Hence it is not onto.

68) S.T The relation $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = \cos x \quad \forall x \in \mathbb{R}$ is neither one to one nor onto

Sol: Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $f(x) = \cos x$.

$$f(0) = \cos 0 = 1$$

$$f(2\pi) = \cos 2\pi = 1$$

$\therefore$ For 2 values of x there is unique image and $\therefore$ it is not one-one

Also $\{-1, 1\} \neq \mathbb{R}$ (e.g) Range is not equal to co-domain
Hence it is not onto.

69) State whether the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 3 - 4x$ is onto or not

Sol: Let $y \in \mathbb{R}$ be the Codomain.

$$y = f(x) = 3 - 4x$$

$$x = \frac{3-y}{4}$$

$$= f\left(\frac{3-y}{4}\right) = 3 - 4\left(\frac{3-y}{4}\right) = y.$$

$\therefore f$ is onto.

7b) let us consider some illustration.

1. $X = \{1, 2, 3, 4\}$, $Y = \{a, b, c, d, e\}$

BP

$$f = \{(1, a) (2, c) (3, e) (4, b)\}$$

✓ clearly it is one-one $\because$ every element of X has unique image in Y .

But not onto $\because$ in Y , d ~~does not have any pre image~~

2. $X = \{1, 2, 3, 4\}$ $Y = \{a, b\}$

$$f = \{(1, a) (2, a) (3, a) (4, a)\}$$

It is not one-one $\because$ more than one element in X have same image

Also it is not onto $\because$ in Y , b does not have any pre image

3. $X = \{1, 2, 3, 4\}$ $Y = \{a\}$.

$$f = \{(1, a) (2, a) (3, a) (4, a)\}$$

f is not one-one since more than one element in X have same image

But onto since co-domain = range.

4. $X = \{1, 2, 3, 4\}$ $Y = \{a, b, c, d, e\}$.

$$f = (1, a) (2, b) (3, b) (4, b)$$

f is neither one-one nor onto.

$\because$ in X , 3 and 4 have same image and d and e in Y does not have any pre image

5) $X = \{1, 2, 3, 4\}$ $Y = \{a, b, c, d\}$

$$f = \{(1, a) (2, b) (3, c) (4, d)\}$$

f is both one-one and onto

Because Every element in X has unique image in Y
and Range = codomain.

6) $X = \{1, 2, 3, 4\}$ $Y = \{a, b, c, d, e\}$

$$f = (1, a) (2, c) (3, e)$$

This is not at all a function $\because$ $4 \in X$ has no image

But it is a relation.

- (2) Find linear maps $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ so that the following hold, if possible. If it is not possible, explain why.
- (a) T is both 1-1 and onto.
 - (b) T is 1-1 but not onto.
 - (c) T is not 1-1, but is onto.
 - (d) T is neither 1-1 nor onto.
- (3) Do the same as Exercise 2, but this time for $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$.
- (4) Do the same as Exercise 2, but this time for $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$.

Problems :

1. (2) Show that if $\{\vec{a}_1, \dots, \vec{a}_n\}$ spans $\mathbb{R}^m$ and is linearly independent, $n = m$.
2. (2) Show that if $\{\vec{a}_1, \dots, \vec{a}_n\}$ spans $\mathbb{R}^m$ and is linearly dependent, $n > m$.
3. (2) Show that if $\{\vec{a}_1, \dots, \vec{a}_n\}$ does not span $\mathbb{R}^m$ and is linearly independent, $n < m$.
4. (2) Given any $n, m \geq 1$, find a linearly dependent set of vectors $\{\vec{a}_1, \dots, \vec{a}_n\} \subseteq \mathbb{R}^m$ which does not span $\mathbb{R}^m$.

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71) Let $A = \{1, 2, 3\}$ and $B = \{a, b, c, d\}$ Give a function for each of the TBP following

- 1) neither one-one nor onto
- 2) not one-one but onto
- 3) one-one but not onto
- 4) one-one and onto

1) neither one-one nor onto.

$$f = \{(1, a), (2, b), (2, c), (3, d)\}$$

more than one element in A have the same image. $\therefore$ it is not one-one

$\therefore d$ has no pre image in A it is not onto.

2) not one-one but onto

$$f = \{(1, a), (2, a), (3, b), (3, c), (3, d)\}$$

not one-one because more than one element in the set A has same image the set B. But every element in the set B has pre-image in the set A. $\therefore$ it is onto.

3) one-one but not onto

$$f = \{(1, a), (2, b), (3, d), (4, c)\}$$

it is one-one and onto

$\therefore$ Every element in the set A has unique image in the set B. also every element in the set B has pre image in the set A (i.e) range = Co domain Hence it is onto.

72) Check whether the following functions are one-one and onto TBP:

1) $f: \mathbb{N} \rightarrow \mathbb{N}$ define by $f(n) = n+2$

2) $f = \mathbb{N} \cup \{-1, 0\} \rightarrow \mathbb{N}$ defined by $f(n) = n+2$.

Sol: 1. $f: \mathbb{N} \rightarrow \mathbb{N}$ $f(n) = n+2$

Let $f(n) = f(m)$

$n+2 = m+2 \Rightarrow n = m$ $\therefore f$ is one-one

But $f(n) = n+2 \Rightarrow f(1) = 3$

$f(2) = 4$ and soon.

$\therefore \{1, 2\}$ in the Codomain have no pre image in the domain

Hence it is not onto.

2) If m is the codomain $m-2$ is in the domain

$f(m-2) = m-2+2 = m$ thus m has pre image in domain

Hence this function is onto.

$\{a_1, \dots, a_n\}$ spans $\mathbb{R}^m$	$\{a_1, \dots, a_n\}$ is LI	Example	Necessary conditions
True	True	$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$	$n = m$
True	False	$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$	$n > m$
False	True	$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$	$n < m$
False	False	$\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$	$n, m \geq 1$

We leave it to the reader to verify that these necessary conditions are necessary for the given properties to hold (See Problems). Note that these necessary conditions are not sufficient. For example, for the (True, True) case, $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ has the same number of vectors as number of components, but $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ is not linearly independent.

Exercises :

(1) Determine if the following sets of vectors span $\mathbb{R}^m$, where m is the number of components they each have. Also, determine if the set is linearly independent.

- (a) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$
- (b) $\left\{ \begin{bmatrix} 1 \\ 5 \end{bmatrix}, \begin{bmatrix} -4 \\ 7 \end{bmatrix} \right\}$
- (c) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix} \right\}$
- (d) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} -4 \\ -8 \\ -20 \end{bmatrix} \right\}$
- (e) $\left\{ \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -7 \\ 5 \end{bmatrix} \right\}$
- (f) $\left\{ \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 7 \\ 8 \end{bmatrix} \right\}$
- (g) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$
- (h) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -6 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ 5 \end{bmatrix} \right\}$
- (i) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 8 \\ 9 \end{bmatrix} \right\}$

73) check whether following f for one-to-oneness and onto ness.

BP 1) $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{1}{x}$.

2) $f: \mathbb{R} - \{0\} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{1}{x}$.

Sol: 1) First $f(x)$ is not at all a function $\because$ at $x=0$ $f(x)$ not defined.

2) $f: \mathbb{R} - \{0\} \rightarrow \mathbb{R}$

$f(x) = \frac{1}{x}$ for all element $x \in \mathbb{R} - \{0\}$ has unique image in $\mathbb{R}$ $\therefore$ it is one-one.

But $0 \in \mathbb{R}$ (codomain) has no preimage in $\mathbb{R} - \{0\}$ $\therefore$ it is not onto.

74) If $f: \mathbb{R} - \{-1, 1\} \rightarrow \mathbb{R}$ is defined by $f(x) = \frac{x}{x^2-1}$. Verify whether BP. f is one to one or not.

Sol: Let $f(x) = f(y)$

$$\frac{x}{x-1} = \frac{y}{y-1} \Rightarrow x(y^2-1) = y(x^2-1)$$

$$xy^2 - x = x^2y - y$$

$$xy^2 - x^2y = x - y = 0$$

$$xy(y-x) + (y-x) = 0$$

$$(y-x)(xy+1) = 0$$

$$\Rightarrow y-x=0 \text{ (or) } xy=-1$$

$$\text{If } y-x=0$$

$$f(x) = f(y) = x = y$$

$$\text{But if } x=y \Rightarrow x^2 = -1$$

$$x = \text{imaginary} \therefore x \neq y$$

$$(e) \text{ If } f(x) = f(y) \Rightarrow x \neq y \therefore f \text{ is not one-one.}$$

75) If $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined as $f(x) = 2x^2 - 1$ find the BP image of 17, 4 and -2.

$$1) f(x) = 2x^2 - 1 = 17$$

$$2x^2 = 18$$

$$x^2 = 9$$

$$x = \pm 3$$

Hence the pre image of 17 is 3 and -3

$$2) f(x) = 2x^2 - 1 = 4$$

$$2x^2 = 5$$

$$x^2 = \frac{5}{2}$$

$$x = \pm \sqrt{\frac{5}{2}}$$

Hence the pre image of 4 is $\sqrt{\frac{5}{2}}$ and $-\sqrt{\frac{5}{2}}$

$$3) f(x) = 2x^2 - 1 = -2 \quad x^2 = -\frac{1}{2} \quad x \text{ is imaginary}$$

$$2x^2 = -1 \therefore \text{Pre image of } -2 \text{ does not exist.}$$

76) If $f: [-2, 2] \rightarrow B$ is given by $f(x) = 2x^3$ then find B so that f is onto.

TBP.

Sol: The minimum value of $f(-2) = -16$

and maximum value of $f(2) = 16$.

Hence $B = [-16, 16]$. and also f is onto.

77) Check whether the function $f(x) = x|x|$ defined on $[-2, 2]$ is one-one or not. If it is one-one find the suitable co-domain so that the function becomes bijection.

Sol: Let $x, y \in [-2, 2]$. s.t. $f(x) = f(y)$ If $y = 0$ then $x = 0$
and $f(x) = x|x|$. $x|x| = y|y|$ $y \neq 0$ then $x \neq 0$

$$\frac{x}{y} = \frac{|y|}{|x|} = \left| \frac{y}{x} \right| > 0 \text{ (always)}$$

$\Rightarrow \frac{x}{y} > 0$ (ie) both x, y must be negative or both are positive.

(ie) $f(x) = f(y) \Rightarrow x^2 = y^2$ Hence f is one-one.
 $x = y$

Then $f(x) = \begin{cases} +x^2 & x \geq 0 \\ -x^2 & x < 0 \end{cases}$

$\therefore$ The range is $[-4, 4]$

When $x = -2$ $f(x) = -4$ $\therefore f$ becomes bijection

$x = 2$ $f(x) = 4$ from $[-2, 2] \rightarrow [-4, 4]$.

78) S.T the relation $xy = -2$ is a function for a suitable domain TBP and find the domain and range of function.

Sol: Let us consider $f: \mathbb{R} - \{0\} \rightarrow \mathbb{R} - \{0\}$

$$\text{and } xy = -2 \\ x = -\frac{2}{y}$$

For $\forall x \in \mathbb{R} - \{0\}$ it has unique image in $\mathbb{R} - \{0\}$ $\therefore$ This relation is a function. also the range of the function is $\mathbb{R} - \{0\}$

79) State whether the following relations are functions or not. If it is a function check for one-oneness and onto-ness. TBP
If it is not function state why? 1) $A = \{a, b, c\}$ and $f = (a, c)(b, c)(c, b)$
 $f: A \rightarrow A$
2) $X = \{x, y, z\}$ $f = (x, y)(x, z)(z, x)$ $f: X \rightarrow X$

Sol: 1. $A = \{a, b, c\}$ $f: A \rightarrow A$ $f = (a, c)(b, c)(c, b)$ Since every element of A has image the given relation is a function.
but it is not one-one because more than one element have same image.
2. $X = \{x, y, z\}$ $f: X \rightarrow X$ $f = (x, y)(x, z)(z, x)$.
 f is not at all a function because $\forall x \in X$ has no image.

80) write the values of f at $x = -4, 1, -2, 7, 0$ if

$$f(x) = \begin{cases} -x+4 & -\infty < x \leq -3 \\ x+4 & -3 < x < -2 \\ x^2-x & -2 \leq x < 1 \\ x-x^2 & 1 \leq x < 7 \\ 0 & \text{otherwise} \end{cases}$$

Sol: $f(-4) = 4+4 = 8$

$f(1) = 1-1 = 0$

$f(-2) = 4+2 = 6$

$f(7) = 0$

$f(0) = 0$

81) write the values of f at $-3, 5, 2, -1, 0$ if

$$f(x) = \begin{cases} x^2+x-5 & x \in (-\infty, 0) \\ x^2+3x-2 & x \in (3, \infty) \\ x^2 & x \in (0, 2) \\ x^2-3 & \text{otherwise} \end{cases}$$

Sol: $f(-3) = 9-3-5 = 1$

$f(5) = 25+15-2 = 38$

$f(2) = 4-3 = -1$

$f(-1) = 1-1-5 = -5$

$f(0) = -3$

82) If $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = \cos x$ $g(x) = 3x^2$ s.t $f \circ g \neq g \circ f$.

Sol: $g \circ f(x) = g(\cos x)$
 $= 3\cos^2 x$

$f \circ g(x) = f(3x^2) = \cos(3x^2)$ $\therefore f \circ g \neq g \circ f$

83) If $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ are defined respectively as $f(x) = x^2+3x+1$ and $g(x) = 2x-3$ find 1) $f \circ g$ and 2) $g \circ f$.

Sol: $f \circ g(x) = f(2x-3)$

$= (2x-3)^2 + 3(2x-3) + 1$

$= 4x^2 - 12x + 9 + 6x - 9 + 1$

$= 4x^2 - 6x + 1$

$g \circ f(x) = g(x^2+3x+1) = 2(x^2+3x+1)-3$

$= 2x^2 + 6x + 2 - 3$

$= 2x^2 + 6x - 1$

84) If $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = (3-x^3)^{1/3}$ find $f \circ f(x)$

Sol: Let $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = (3-x^3)^{1/3}$.

$$\begin{aligned} f \circ f(x) &= f(3-x^3)^{1/3} \\ &= \left[3 - (3-x^3)^{1/3} \right]^{1/3} \\ &= \left[3 - (3-x^3) \right]^{1/3} = (x^3)^{1/3} = x. \end{aligned}$$

85) Find $g \circ f$ and $f \circ g$ if 1) $f(x) = |x|$ and $g(x) = |5x-2|$

$$\begin{aligned} \text{Sol: } g \circ f(x) &= g(|x|) \\ &= |5|x|-2| = |5x-2| \\ f \circ g(x) &= f(|5x-2|) \\ &= ||5x-2|| = |5x-2| \end{aligned}$$

86) Consider $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = 4x+3$. S.T f is invertible and find f^{-1} .

Sol: To show f is invertible to show f is one-one and onto.

$$\text{Let } f(x) = f(y) \quad \forall x, y \in \mathbb{R}.$$

$$4x+3 = 4y+3$$

$$4x = 4y$$

$$x = y \Rightarrow \text{is one-one.}$$

$$\text{Let } y = f(x) = y = 4x+3$$

$$4x = y-3$$

$$x = \frac{y-3}{4}$$

$$\therefore f(x) = f\left(\frac{y-3}{4}\right)$$

$$= 4\left(\frac{y-3}{4}\right) + 3$$

$$= y.$$

$\therefore f$ is onto.

$$f^{-1}(y) = \frac{y-3}{4} \text{ replace } y \text{ by } x$$

$$f^{-1}(x) = \frac{x-3}{4}.$$

87) S.T $f: [-1, 1] \rightarrow \mathbb{R}$ given by $f(x) = \frac{x}{x+2}$ is one-one. Find the inverse of the function $f: [-1, 1] \rightarrow \text{Range of } f$.

Sol: $f: [-1, 1] \rightarrow \mathbb{R}$ is defined by $f(x) = \frac{x}{x+2}$

$$\text{Let } x_1, x_2 \in [-1, 1] \text{ s.t. } f(x_1) = f(x_2)$$

$$\frac{x_1}{x_1+2} = \frac{x_2}{x_2+2}$$

$$x_1 x_2 + 2x_1 = x_1 x_2 + 2x_2$$

$$2x_1 = 2x_2$$

$$x_1 = x_2 \quad \therefore f \text{ is one-one.}$$

Let y be the range

$$y = \frac{x}{x+2}$$

$$xy + 2y = x$$

$$2y = x - xy$$

$$x = \frac{2y}{1-y} = \frac{2y}{1-y} \quad \therefore f(x) = f\left(\frac{2y}{1-y}\right)$$

$$= \frac{\frac{2y}{1-y}}{\frac{2y}{1-y} + 2} = \frac{2y}{2y + 2 - 2y} = \frac{2y}{2} = y.$$

$\therefore f$ is onto also

(not necessary to prove)

$$\therefore f^{-1}(y) = \frac{2y}{1-y} \Rightarrow f^{-1}(x) = \frac{2x}{1-x} \quad x \neq 1.$$

88) Let $f: \{(1,2) (3,4) (2,2)\}$ and $g: \{(2,1) (3,1) (4,2)\}$ find
TBP $f \circ g$ and $g \circ f$.

Sol: Domain of $f: \{1, 2, 3\}$ Range of $f: \{2, 4\}$.

Domain of $g: \{2, 3, 4\}$ Range of $g: \{1, 2\}$.

$\therefore$ The range of f contained in domain of g then $g \circ f$ defined.
The range of g contained in domain of f then $f \circ g$ defined.

$$g \circ f(1) = g(2) = 1 \quad \therefore g \circ f = \{(1,1) (2,1) (3,2)\}$$

$$g \circ f(2) = g(2) = 1$$

$$g \circ f(3) = g(4) = 2$$

$$\text{Further } f \circ g(2) = f(1) = 2$$

$$f \circ g(3) = f(1) = 2$$

$$f \circ g(4) = f(2) = 2$$

$$f \circ g = \{(2,2), (3,2) (4,2)\}$$

89) Let $f: \{(1,4) (2,5) (3,5)\}$ and $g: \{(4,1) (5,2) (6,4)\}$ find $g \circ f$.
TBP Can you find $f \circ g$?

Sol: Domain of $f: \{1, 2, 3\}$ Range of $f: \{4, 5\}$.

Domain of $g: \{4, 5, 6\}$ Range of $g: \{1, 2, 4\}$

Range of f contained in domain of g

$$\therefore g \circ f \text{ exist. } \begin{aligned} g \circ f(1) &= g(4) = 1 = (1, 1) \\ g \circ f(2) &= g(5) = 2 = (2, 2) \\ g \circ f(3) &= g(5) = 2 = (3, 2) \end{aligned}$$

$$\therefore g \circ f = \{(1, 1), (2, 2), (3, 2)\}.$$

But range of $g = \{1, 2, 3\}$ does not contained in domain of f

$\therefore f \circ g$ does not exist.

90) Let f and g be two functions from $\mathbb{R} \rightarrow \mathbb{R}$ defined by
TBP $f(x) = 3x - 4$ and $g(x) = x^2 + 3$ find $f \circ g$ and $g \circ f$

$$\begin{aligned} \text{Sol: } g \circ f(x) &= g(3x - 4) \\ &= (3x - 4)^2 + 3 = 9x^2 - 24x + 16 + 3 \\ &= 9x^2 - 24x + 19. \end{aligned}$$

$$\begin{aligned} f \circ g(x) &= f(x^2 + 3) \\ &= 3(x^2 + 3) - 4 = 3x^2 + 9 - 4 \\ &= 3x^2 + 5 \end{aligned}$$

91) Let $f, g: \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = 2x - |x|$ and $g(x) = 2x + |x|$
TBP find $f \circ g$.

$$\text{Sol: We know that } |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$f(x) = 2x - x = x \quad x \geq 0$$

$$2x - (-x) = 3x < 0$$

$$g(x) = 2x + x = 3x \quad x \geq 0$$

$$2x - x = x \quad x < 0$$

$$f \circ g(x) = f(3x) \quad x \geq 0$$

$$f(x) \quad x < 0$$

$$f \circ g = 3x \quad x \geq 0$$

$$3x \quad x < 0$$

$$\therefore f \circ g = 3x \quad \forall x.$$

92) If $f, g: \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x + |x|$ and $g(x) = \frac{x + |x|}{|x| - x}$ find
TBP $g \circ f, f \circ g$

$$\text{Sol: We know that } |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$\therefore f(x) = |x| + x = \begin{cases} x+x = 2x & x \geq 0 \\ -x+x = 0 & x < 0 \end{cases}$$

$$g(x) = |x| - x = \begin{cases} x-x = 0 & x \geq 0 \\ -x-x = -2x & x < 0 \end{cases}$$

$$f \circ g(x) = f(g(x)) = \begin{cases} f(0) = 0 & x \geq 0 \\ f(-2x) = 2(-2x) = -4x & \text{when } x < 0, -2x > 0 \end{cases}$$

$$g \circ f(x) = g(f(x)) = \begin{cases} g(0) = 0 & x \geq 0 \\ g(0) = 0 & \text{when } x < 0, -2x > 0 \end{cases}$$

$$\therefore g \circ f(x) = 0 \quad \forall x \in \mathbb{R}.$$

93) If $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 3x - 5$ P.T f is bijection and find its inverse.

Sol: Let $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 3x - 5$

$$\text{Let } f(x) = f(y)$$

$$3x - 5 = 3y - 5$$

$$3x = 3y$$

$$x = y$$

$\therefore f$ is one-one.

$$\text{Let } y = f(x) = 3x - 5 \Rightarrow y + 5 = 3x$$

$$\frac{y+5}{3} = x$$

$$\therefore f(x) = f\left(\frac{y+5}{3}\right) = 3 \cdot \frac{y+5}{3} - 5 = y$$

$\therefore f$ is onto

$\therefore f$ is bijection. and

$$f^{-1}(y) = \frac{y+5}{3}$$

$$f^{-1}(x) = \frac{x+5}{3}$$

94) Let f, g and h are the functions from $\mathbb{R} \rightarrow \mathbb{R}$ s.t
TBP.

$$1) (f+g) \circ h = (f \circ h) + (g \circ h)$$

$$2) (f \cdot g) \circ h = (f \circ h) \cdot (g \circ h)$$

$$1) \text{ LHS: } (f+g) \circ h(x) = (f+g)h(x)$$

$$\begin{aligned} \text{RHS: } (f \cdot g)h(x) &= f(h(x)) \cdot g(h(x)) = f(h(x)) + g(h(x)) \\ &= (f \circ h)(x) \cdot (g \circ h)(x) = (f \circ h)(x) + (g \circ h)(x) \\ &= (f \circ h) + (g \circ h) \end{aligned}$$

95) Let f and g be two functions given by

$$f = \{(2,4)(5,6)(8,-1)(10,-3)\} \text{ and } g = \{(2,5)(7,1)(8,4)(10,13)(11,-5)\}$$

Find the domain of $f+g$

Sol: Domain of $f+g = D_1 \cap D_2$

$$D_1 = \{2, 5, 8, 10\} \quad D_2 = \{2, 7, 8, 10, 11\}$$

$$D_1 \cap D_2 = \{2, 8, 10\}$$

96) Let f and g be two real functions defined by

$$f = \{(0,1)(2,0)(3,-4)(4,2)(5,1)\} \quad g = \{(1,0)(2,2)(3,-1)(4,4)(5,3)\}$$

Find the domain of fg .

Domain of $fg = D_1 \cap D_2$

$$D_1 = \{0, 2, 3, 4, 5\} \quad D_2 = \{1, 2, 3, 4, 5\}$$

$$D_1 \cap D_2 = \{2, 3, 4, 5\}$$

97) Find the set of values of x for which the function $f(x) = 3x^2 - 1$
 $g(x) = 3 + x$ are equal.

$$3x^2 - 1 = 3 + x$$

$$3x^2 - x - 4 = 0$$

$$3x^2 + 3x - 4x - 4 = 0$$

$$3x(x+1) - 4(x+1) = 0$$

$$(x+1)(3x-4) = 0$$

$$x = -1, \frac{4}{3}$$

$$\therefore \{-1, \frac{4}{3}\}$$

In a survey, it is found that 21 like product A, 26 like product B and 29 like product C. If 14 people like products A and B, 12 people like products C and A, 14 people like products B and C and 8 people like all three. Find how many like product C only.

Sol: $n(C) = 29$, $n(C \cap A) = 12$ $n(B \cap C) = 14$ $n(A \cap B \cap C) = 8$

$$\begin{aligned} n(C \text{ only}) &= n(C) - n(C \cap A) - n(C \cap B) + n(A \cap B \cap C) \\ &= 29 - 12 - 14 + 8 \\ &= 37 - 26 = 11. \end{aligned}$$

From 50 students taking examinations in Maths, physics, chemistry each of the student pass at least one of the subject. 37 passed in Maths 24 in physics, 43 in chemistry. Almost 19 passes in M and P. and 29 in M and C and 20 P and C. Find the largest possible number that could have passed all three subjects.

Sol: $n(M) = 37$, $n(P) = 24$ $n(C) = 43$ $n(M \cap P) \leq 19$, $n(M \cap C) \leq 29$
and $n(P \cap C) \leq 20$ and $n(M \cap P \cap C) = 50$

$$\begin{aligned} n(M \cup P \cup C) &= n(M) + n(P) + n(C) - n(M \cap P) - n(M \cap C) - n(P \cap C) \\ &\quad + n(M \cap P \cap C) \leq 50 \end{aligned}$$

$$37 + 24 + 43 - 19 - 29 - 20 + n(M \cap P \cap C) \leq 50$$

$$32 + n(M \cap P \cap C) \leq 50$$

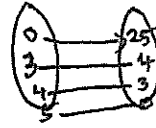
$$\Rightarrow n(M \cap P \cap C) = 14.$$

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1/2 Marks.

1. Write the domain of the relation R defined on the set Z of integers as follows $(a, b) \in R \Leftrightarrow a^2 + b^2 = 25$

$$\text{Domain} = \{0, \pm 3, \pm 4, \pm 5\}$$



1, 2, and >5 are not matched with any element in the co-domain.

- 2) If $R = \{(x, y) \mid x^2 + y^2 \leq 4, x, y \in Z\}$ is a relation on a Z . Write the domain of R .

$$\text{Domain} = \{0, \pm 1, \pm 2\}$$

- 3) Write the identity relation on a set $A = \{a, b, c\}$

$$= \{(a, a), (b, b), (c, c)\}$$

- 4) Write smallest reflexive relation on the set $A = \{1, 2, 3, 4\}$

$$\{(1, 1), (2, 2), (3, 3), (4, 4)\}$$

- 5) If $R = \{(x, y) \mid x + 2y = 8\}$ is a relation then write the range of R .

$$y = \frac{8-x}{2}$$

$$x=2 \quad y=3$$

$$x=4 \quad y=2$$

$$x=6 \quad y=1$$

$\therefore$ The range is $\{1, 2, 3\}$.

- 6) If R is a symmetric relation on a set A then write the relation between R and R^{-1}

$$R = R^{-1}$$

- 7) $R = \{(x, y) \mid x^2 - y^2 < 1\}$ be the relation on set $A = \{1, 2, 3, 4, 5\}$ write R as the set of pairs.

$$\{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)\}$$

- 8) If $A = \{2, 3, 4\}$, $B = \{1, 3, 7\}$ and $R = \{(x, y) \mid x < y, x \in A, y \in B\}$ is a relation then write R^{-1}

$$R = \{(2, 3), (2, 7), (3, 7), (4, 7)\} \quad R^{-1} = \{(3, 2), (7, 2), (7, 3), (7, 4)\}$$

- 9) Let $A = \{3, 5, 7\}$, $B = \{2, 4, 9\}$ and R is a relation given by 'is less than' write R as set of ordered pairs.

$$= \{(3, 4), (3, 9), (5, 9), (7, 9)\}$$

- 10) Let $A = \{3, 5, 7\}$, $B = \{2, 6, 10\}$ and R be a relation from $A \rightarrow B$ defined by $R = \{(x, y) \mid x \text{ and } y \text{ are relatively prime}\}$ then write R and R^{-1}

$$R = \{(3,2) (3,10) (5,2) (5,6) (7,2) (7,6) (7,10)\}$$

$$R^{-1} = \{(2,3) (10,3) (2,5) (6,5) (2,7) (6,7) (10,7)\}$$

11) $A = \{1, 2, 3, 4, 5, 6, 7\}$ and if $R = \{x, y\} : y \text{ is one half of } x; x, y \in A$ is a relation on A then write the R as a set of ordered pairs.

$$R = \{(2,1) (4,2) (6,3) (8,4)\}. \text{ (ie) } y \text{ is one half (ie) half of } x\}$$

12) Let $A = \{2, 3, 4, 5\}$ and $B = \{1, 3, 4\}$ if R is a relation $A \rightarrow B$ given by aRb iff a is a divisor of b. Write the set of ordered pairs.

$$R = \{(2,4) (4,4) (3,3)\}.$$

13) State the reason for the relation R on the set $\{1, 2, 3\}$ given by $R = \{(2,1) (1,2)\}$ is not to be transitive.

$\because (2,1) \in R \text{ and } (1,2) \in R \Rightarrow (2,2) \notin R \therefore \text{it is not transitive}$

14) Let $R = \{(a, a^3) : a \text{ is a prime number less than } 5\}$ be a relation. Find the range of R.

2, 3 are the prime numbers < 5

$$\therefore 2^3 = 8, 3^3 = 27 \text{ Range} = \{8, 27\}.$$

15) For the set $A = \{1, 2, 3\}$ define the relation R on the set A as follows.

$$R = \{(1,1) (2,2) (3,3) (1,3)\} \text{ write the ordered pairs to be added to R to makes the smallest equivalence relation.}$$

Suppose if we add (3,1) $R = \{(1,1) (2,2) (3,3) (1,3) (3,1)\}$.

Now R is clearly reflexive and symmetric and $\because (1,3) \in R, (3,1) \in R,$

$\therefore$ transitive Hence R is equivalence. $\Rightarrow (1,1) \in R.$

16) Let $A = \{0, 1, 2, 3\}$ and R is a relation on A defined by

$$R = \{(0,0) (0,1) (0,3) (1,0) (1,1) (2,2) (3,0) (3,3)\} \text{ is R is reflexive, symmetric, and transitive.}$$

$\because \{(0,0) (1,1) (2,2) (3,3)\} \in R$ it is reflexive.

$(0,1) (1,0) (0,3) (3,0) \in R$ it is symmetric.

$(1,0) \in R, (0,3) \in R$ but $(1,3) \notin R$. Hence it is not transitive.

17) Let the relation R be defined on the set $A = \{1, 2, 3, 4, 5\}$ by

$$R = \{(a,b) : |a^2 - b^2| < 8\} \text{ write R as set of ordered pairs.}$$

$$A = \{1, 2, 3, 4, 5\} \quad R = \{(1,1) (1,2) (2,1) (2,2) (2,3) (3,1) (3,2) (3,3) (3,4) (4,3) (4,4) (5,5)\}.$$

18) Let the relation R be defined on N by aRb iff $2a+3b=30$ then write R as a set of ordered pairs.

Sol: $2a = 30 - 3b \Rightarrow 3b = 30 - 2a$ $a=3$ $(3,8)$
 $a = \frac{30-3b}{2}$ $= 24$ $a=b$ $(6,6)$
 $b=8$ $a=9$ $(9,4)$
 $b = \frac{18}{3}$ $a=12$ $(12,2)$

$\therefore \{(3,8) (6,6) (9,4) (12,2)\}$

19) Write the smallest equivalence relation on the set $A = \{1, 2, 3\}$.

$R = \{(1,1) (2,2) (3,3)\}$ Here we need not worry about transitive. transitive is possible only when $(1,2) \dots$ are there.

M.C.Q.

20. Let R be the relation on the set N given by $R = \{(a,b) \mid a = b-2, b > 6\}$ then

1) $(2,4) \in R$ 2) $(3,8)$ 3) $(6,8)$ 4) $(8,7)$

$b-a=2$ $b > 6$. $\therefore b > 6$ ①, ③, ④ are possible.

$b-a=2 \Rightarrow 8-6=2$ ③ is the answer.

21) Which of the following is not an equivalence relation on Z ?

1) $aRb \Leftrightarrow a+b$ is an even integer 2) $aRb = a-b$ is an even integer

3) $aRb \Leftrightarrow a < b$ 4) $aRb = a = b$.

③ when $a \neq a$ will not be reflexive $\therefore$ it is not equivalence. otherwise ①, ②, ④ are equivalence relation.

22) R is a relation on the set Z of integers and it is given by $(x,y) \in R$

$\Leftrightarrow |x-y| \leq 1$ Then R is

1) Reflexive and transitive

2) reflexive and symmetric

3) Symmetric and transitive

4) an equivalence relation.

2) Reflexive and symmetric $\therefore |1-1| \leq 1$ and $|1-2| \leq 1$ and $|2-1| \leq 1$
 $R = \{(1,1) (2,2) (3,3) (1,2) (2,1)\}$ $|1-2| \leq 1, |2-1| \leq 1$

23) The relation R defined on the set $A = \{1, 2, 3, 4, 5\}$ by $R = \{(a,b) \mid a^2 - b^2 < 16\}$ given by.

1) $(1,1) (2,1) (3,1) (4,1) (2,3)$

2) $(2,2) (3,2) (4,2) (2,4)$

3) $(3,3) (4,3) (5,4) (3,4)$

4) none of these

24) Let R be the relation over the set of all st. lines in a plane s.t

$l_1 R l_2 \Leftrightarrow l_1 \perp l_2$ then R is

1) symmetric

2) reflexive

3) transitive

4) equivalence relation.

l_1 is not $\perp$ to l_1 not reflexive

$\therefore$ reflexive

$l_1 \perp l_2$ $l_2 \perp l_3$

$\Rightarrow l_1 \parallel l_3$

not transitive

$l_1 \perp l_2 \Rightarrow l_2 \perp l_1$ Symmetric

25) If $A = \{a, b, c\}$ then the relation $R = \{(b, c)\}$ on A is

- 1) reflexive only 2) symmetric only 3) transitive only 4) reflexive and transitive only.

26) Let $A = \{2, 3, 4, 5, \dots, 17, 18\}$ Let $\simeq$ be the equivalence relation on $A \times A$ Cartesian product of A with itself defined by $(a, b) \simeq (c, d)$ iff $ad = bc$.
Then the number of ordered pairs of the equivalence classes of $(3, 2)$ is

- 1) 4 2) 5 3) 6 4) 7.

27) The relation R in $N \times N$ s.t. $(a, b) R (c, d) \iff a + d = b + c$ is

- 1) reflexive but not symmetric 2) reflexive and transitive but not symmetric
3) equivalence relation 3) none of these

$$(a, b) R (a, b) = a + b = b + a \text{ reflexive } \cdot \in N$$

$$(a, b) R (c, d) = a + d = b + c \text{ Symmetric } \cdot \in N$$

$$(a, b) R (c, d), (c, d) R (e, f) \Rightarrow (a, b) R (e, f) = a + f = b + e \text{ transitive } \cdot \in N$$

$\therefore$ equivalence relation.

28) If $A = \{1, 2, 3\}$ $B = \{1, 4, 6, 9\}$ and R is a relation from A to B defined by $x > y$. The range of R is

- 1) $\{1, 4, 6, 9\}$ 2) $\{4, 6, 9\}$ 3) $\{1\}$ 4) none of these

29) R is defined from $\{2, 3, 4, 5\}$ to $\{5, 6, 7\}$ by $x R y \iff x$ is relatively prime to y then the domain of R is

- 1) $\{2, 3, 5\}$ 2) $\{3, 5\}$ 3) $\{2, 3, 4\}$ 4) $\{2, 3, 4, 5\}$.

$$R = \{(2, 3) (2, 7) (3, 7) (3, 10) (4, 3) (4, 6) (4, 7) (4, 10) (5, 6)\}$$

$\therefore$ Domain is $\{2, 3, 4, 5\}$.

30) Let R be the relation on N defined by $x + 2y = 8$ the domain of R is

- 1) $\{2, 4, 8\}$ 2) $\{2, 4, 6, 8\}$ 3) $\{2, 4, 6\}$ 4) $\{1, 2, 3, 4\}$.

$$2y = 8 - x = x = \{2, 4, 6\}$$

31) R is a relation from $\{11, 12, 13\}$ to $\{8, 10, 12\}$ defined by $y = x - 3$. Then R^{-1} is

- 1) $\{(8, 11) (10, 13)\}$ 2) $\{(11, 8) (13, 10)\}$ 3) $\{(10, 13) (8, 11) (8, 10)\}$ 4) none of these

$$R = \{(11, 8) (13, 10)\} \Rightarrow R^{-1} = \{(8, 11) (10, 13)\}$$

32) Let $R = \{(a, a) (b, b) (c, c) (a, b)\}$ be a relation on set $A = \{a, b, c\}$

then R is 1) identity relation 2) reflexive 3) symmetric

4) equivalence.

2) reflexive $\because A = \{a, b, c\}$ and $\{(a, a) (b, b) (c, c)\} \in R$.

33) Let $A = \{1, 2, 3\}$ and $R = \{(1, 2) (2, 3) (1, 3)\}$ be a relation on A . Then R is
 1) neither reflexive nor transitive 2) neither symmetric nor transitive
 3) transitive 4) none of these

3) transitive $(1, 2) \in R, (2, 3) \in R \Rightarrow (1, 3) \in R$.

34) If R is a relation on the set $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ given by $xRy \Leftrightarrow y = 3x$
 then $R =$ 1) $\{(3, 1) (6, 2) (9, 3)\}$ 2) $\{(3, 1) (6, 2) (9, 3)\}$
 3) $\{(3, 1) (2, 6) (3, 9)\}$ 4) none of these

(x , one time, y , 3 times)

35) If R is a relation on the set $A = \{1, 2, 3\}$ given by $R = \{(1, 1) (2, 2) (3, 3)\}$
 then R is
 1) reflexive 2) symmetric 3) transitive 4) All of these

$R = \{1, 2, 3\}$ & $\{(1, 1) (2, 2) (3, 3)\} \therefore R$ is reflexive, symmetric and transitive (or) All.
 If there is $(2, 3) \notin R$ then only we have check symmetric or transitive.

36) If $A = \{a, b, c, d\}$ then the relation $\{(a, b) (b, a) (a, a)\}$ on A is
 1) symmetric and transitive only 2) reflexive and transitive only
 3) symmetric only 4) transitive only.

$(a, b) \in R \Rightarrow (b, a) \in R \therefore$ Symmetric.

37) If $A = \{1, 2, 3\}$ then the relation $R = \{(2, 3)\}$ on A is
 1) symmetric and transitive only 2) symmetric only
 3) transitive only 4) none of these

38) Let R be the relation on the set $A = \{1, 2, 3, 4\}$ given by
 $R = \{(1, 2) (2, 2) (1, 1) (4, 4) (1, 3) (3, 3) (3, 2)\}$ then

- 1) R is reflexive and symmetric but not transitive
- 2) R is reflexive and transitive but not symmetric
- 3) R is symmetric and transitive but not reflexive
- 4) R is equivalence relation.

$\{(1, 1) (2, 2) (3, 3) (4, 4)\} \in R \therefore$ reflexive

$(3, 2) \in R$ but $(2, 3) \notin R \therefore$ not symmetric.

$(1, 3) \in R (3, 2) \in R \Rightarrow (1, 2) \in R \therefore$ transitive

39) The relation $R = \{(1, 1) (2, 2) (3, 3)\}$ on the set $\{1, 2, 3\}$ is
 1) Symmetric only 2) reflexive only 3) equivalence relation
 4) transitive only

- 40) Let $A = \{1, 2, 3\}$ and Consider the relation $R = \{(1,1) (2,2) (3,3) (1,2) (2,3) \text{ and } (1,3)\}$
- 1) reflexive but not symmetric
 - 2) reflexive but not transitive
 - 3) symmetric and transitive
 - 4) neither symmetric nor transitive

$\because \{(1,1) (2,2) (3,3)\} \in R$ - reflexive

$(1,2) \in R$ but $(2,1) \notin R$ not symmetric

$(1,2) \in R$ $(2,3) \in R \Rightarrow (1,3) \in R \therefore$ transitive.

41. The relation S defined on the set R of all real numbers by the rule aRb iff $a \geq b$ is

1. an equivalence relation
- 2) reflexive, transitive but not symmetric
- 3) Symmetric transitive but not reflexive
- 4) neither transitive nor reflexive but symmetric.

$a \geq a \therefore$ reflexive (this possible only for equality)

$a \geq b$ but $b \not\geq a$ not symmetric

$a \geq b, b \geq c \Rightarrow a \geq c \in R$ Transitive

- 42) Let R be a relation on the set N of natural numbers defined by nRm iff n divides m then R is

- 1) Reflexive and symmetric
- 2) Transitive and symmetric
- 3) Equivalence
- 4) Reflexive, transitive but not symmetric.

n divides $n \therefore$ reflexive

n divides $m \not\Rightarrow m$ divides n Hence not symmetric.

$m | n$ and $n | l \Rightarrow m | l \therefore$ transitive.

- 43) Let L denotes the set of all straight lines in a plane. Let a relation R be defined by lRm iff l is $\perp$ to m then R is

- 1) Reflexive
- 2) Symmetric
- 3) Transitive
- 4) None of these

l is not $\perp$ to l . not reflexive

$l \perp m \Rightarrow m \perp l$ symmetric

$l \perp m$ and $m \perp n$ but l is not $\perp$ to n not transitive.

- 44) Let T be the set of all triangles in the Euclidean plane and let a relation R on T defined as aRb if a is congruent to b for all $a, b \in T$ then R is

1. Symmetric but not transitive
- 2) transitive but not symmetric
- 3) neither symmetric nor transitive
- 4) both symmetric and transitive

45) Consider a non empty set consisting of children in a family and a relation R defined as aRb if a is a brother of b then R is

- 1) Symmetric but not transitive 2) transitive but not symmetric
3) neither symmetric nor transitive 3) both symmetric and transitive

2) transitive but not symmetric.

$\because$ a is a brother of b , b is a brother of $c \Rightarrow a$ is a brother of c .

(b may be sister also)

$\therefore$ not symmetric.

46) For real number x and y define xRy iff $x-y+\sqrt{2}$ is an irrational number. Then the relation R is

- 1) reflexive 2) symmetric 3) transitive 4) none of these.

Clearly R is reflexive $\because xRx = x-x+\sqrt{2} = \sqrt{2}$ is irrational

Functions.

47) $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x + \sqrt{x^2}$ is

- 1) injective 2) surjective 3) bijective 4) None of these

$\because f(-1) = 0$ and $f(-2) = 0$ it is not injective

There is no pre images for all negative numbers which are in co-domain

$\therefore$ it is not onto. $\therefore$ it is not bijective. Hence none of these is answer.

48) The function $f(x) = 2^x + 2^{|x|}$ if $f: \mathbb{R} \rightarrow \mathbb{R}-\mathbb{R}$.

- 1) one to one 2) many-one and onto 3) one-one and into
4) many one and into.

$$f(x) = \begin{cases} 2^x + 2^{-x} & x < 0 \\ 2^x + 2^x & x \geq 0 \end{cases} \quad f(-1) = 2^{-1} + 2 = 5/2 \quad \text{one-one}$$

$$f(1) = 2 + 2 = 4$$

$\forall x \in \mathbb{R} f(x) = +ve.$ $\because$ -ve values in co-domain has no pre image
hence it is not onto $\because$ one-one and into

49) Let $f: \mathbb{R} - (-b) \rightarrow \mathbb{R} - (1)$ be defined by $f(x) = \frac{x+a}{x+b}$ $a \neq b$ then

1. f is one one but not onto 2) f is onto but not one-one
3. f is both one-one and onto 4) none of these.

$$\forall a, b \in \mathbb{R} \quad f(a) = \frac{2a}{a+b} \quad \text{and} \quad f(b) = \frac{a+b}{2b} \quad \text{Suppose} \quad \frac{a+b}{2b} = +1$$

$$a+b = 2b$$

$\because f$ is one-one and clearly f is onto

$\therefore f$ is both one one and onto.

$$\because a \neq b, \quad a-b \neq 0$$

$$\because \frac{a+b}{2b} \neq +1$$

50) The function $f: A \rightarrow B$ defined by $f(x) = x^2 + 6x - 8$ is bijection if

- 1) $A = (-\infty, 3)$ $B = (-\infty, 1]$ 2) $A = [-3, \infty)$ and $B = (-\infty, 1)$
 3) $A = [-\infty, 3]$ $B = [1, \infty)$ 4) $A = [3, \infty)$ and $B = [1, \infty)$

Put the values of A in $f(x)$ and find B .
 see which one is satisfied

$$x^2 - 6x + 8 = 0$$

$$(x-4)(x-2) = 0$$

$$x = (2, 4)$$

51) Let $A: \{x \in \mathbb{R} : -1 \leq x \leq 1\} = B$ then the mapping $f: A \rightarrow B$ given by $f(x) = x|x|$ 1) injective but not surjective 2) surjective but not injective
 3) bijective 4) none of these.

$$f(x) = \begin{cases} x^2 & x \geq 0 \\ -x^2 & x < 0 \end{cases}$$

$\therefore$ it is bijective

clearly it is one-one $f(1) = 1$
 $f(-1) = -1$
 and every element in $-1 \leq x \leq 1$ have image
 $\therefore$ onto.

52) The function $f: [0, \infty) \rightarrow \mathbb{R}$ given by $\frac{x}{x+1} = f(x)$ is

- 1) one-one and onto 2) one-one but not onto
 3) onto but not one-one 4) neither one-one nor onto.

For every element $[0, \infty)$ there is a unique image in $\mathbb{R}$. $\therefore$ one-one
 but the negative value in the co-domain ($\mathbb{R}$) does not have any pre image $\therefore$ it is not onto.

$\therefore$ it is one-one but not onto

53) The range of the function $f(x) = {}^{7-x}P_{x-3}$ is

- 1) $\{1, 2, 3, 4, 5\}$ 2) $\{1, 2, 3, 4, 5, 6\}$ 3) $\{1, 2, 3, 4\}$ 4) $\{1, 2, 3\}$.

We can find nPr for $n \geq r$ only. $nPr = \frac{n!}{(n-r)!}$

Put $x=3$ ${}_4P_0 = 1$ $x=4$, ${}_3P_1 = \frac{3!}{2!} = 3$ $x=5$, ${}_2P_2 = 2$

range = $\{1, 2, 3\}$.

54) A function f from the set of natural numbers to set of integers defined by $f(n) = \frac{n-1}{2}$ when n is odd

$= -\frac{n}{2}$ when n is even.

- is 1) neither one-one nor onto 2) one-one but not onto
 3) onto but not one-one 4) one and onto

Every different odd numbers in $\mathbb{N}$ has unique image and every element in integers have pre image in $\mathbb{N}$. $\therefore$ it is one-one and onto.

55) Which of the following functions from $\mathbb{Z}$ to itself are bijections

- 1) $f(x) = x^3$ 2) $f(x) = x+2$ 3) $f(x) = 2x+1$ 4) $f(x) = x^2+x$.

$f(x) = x^2+x$ is neither one-one and onto.

$f(x) = 2x+1$ there is no pre image for 0 in co-domain

$f(x) = x+2$ it is one-one and onto (bijective)

$f(x) = x^3$ = All elements in the Co domain does not have pre image

56) Which of the following function from $A: \{x: -1 \leq x \leq 1\}$ to itself are bijection.

- 1) $f(x) = \frac{x}{2}$ 2) $g(x) = \sin(\frac{\pi x}{2})$ 3) $h(x) = |x|$ 4) $k(x) = x^2$

For $k(x) = x^2$ there is no different image for $f(-1)$ and $f(1)$ $\therefore$ not one-one

$h(x) = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$ For -1 there is no pre image $\therefore$ not onto

$f(x) = \frac{x}{2}$: for -1 is no pre image $\therefore$ not onto

$g(x) = \sin(\frac{\pi x}{2})$ it is clearly one-one and onto.

57) Let $A: \{x: -1 \leq x \leq 1\}$ and $f: A \rightarrow A$ s.t $f(x) = x|x|$ Then f is

- 1) $\checkmark$ bijection 2) injective but not surjective
3) Surjective but not injective 4) neither injective nor surjective

$f(x) = \begin{cases} x^2 & x \geq 0 \\ -x^2 & x < 0 \end{cases}$ clearly bijection.

58) If a function $f: \mathbb{R} \rightarrow A$ given by $\frac{x^2}{x^2+1}$ is a surjection then $A =$

- 1) $\mathbb{R}$ 2) $[0, 1]$ 3) $(0, 1]$ 4) $[0, 1)$

59) If $f: \mathbb{R} \rightarrow \mathbb{R}$ is given by $f(x) = 3x-5$ then $f^{-1}(x) =$

- 1) $\frac{1}{3x-5}$ 2) $\frac{x+5}{3}$ 3) does not exist 4) does not exist because of not onto.

$$\text{Let } y = f(x) = 3x-5 \Rightarrow \frac{y+5}{3} = x \Rightarrow f^{-1}(y) = \frac{y+5}{3}$$

$$f^{-1}(x) = \frac{x+5}{3}$$

60) A function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = b^x + b^{|x|}$ is

- 1) one-one and onto 2) many one and onto
3) $\checkmark$ one-one and into & many one and into

61) $f: \mathbb{Z} \rightarrow \mathbb{Z}$ be given by $f(x) = \begin{cases} \frac{x}{2} & \text{if } x \text{ is even} \\ 0 & \text{if } x \text{ is odd} \end{cases}$

- 1) onto but not one-one 2) one-one but not onto
3) one-one and onto 4) $\checkmark$ neither one-one nor onto.

For different odd numbers no different image $\therefore$ not one-one
and different odd number with the Co domain has no pre image Hence not onto

62) which of the following function from $A = \{x \in \mathbb{R} : -1 \leq x \leq 1\}$ to itself are bijections

- 1) $f(x) = |x|$
- 2) $f(x) = \sin \frac{\pi x}{2}$
- 3) $f(x) = \sin \frac{\pi x}{4}$
- 4) none of these

63) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{x^2-8}{x^2+2}$ then f is

- 1) one-one but ^{not} onto
- 2) one-one and onto
- 3) onto but not one-one
- 4) neither one-one nor onto

$$f(x_1) = f(x_2)$$

$$\frac{x_1^2-8}{x_1^2+2} = \frac{x_2^2-8}{x_2^2+2} \Rightarrow \frac{x_1^2-8}{x_1^2+2} - \frac{x_2^2-8}{x_2^2+2} = 0$$

$$\frac{x_1^2(x_2^2+2) - (x_2^2-8)(x_1^2+2)}{(x_1^2+2)(x_2^2+2)} = 0$$

$$x_1^2 x_2^2 + 2x_1^2 - x_2^2 x_1^2 - 8x_1^2 - x_2^2 x_1^2 - 8x_2^2 + 8x_1^2 + 16 = x_1^2 x_2^2 - 8x_1^2 + 2x_2^2 - 16$$

$$10x_1^2 = 10x_2^2 \Rightarrow x_1 = x_2 \quad \therefore \text{one-one}$$

$$y = f(x) = \frac{x^2-8}{x^2+2} \Rightarrow x^2 y + 2y = x^2 - 8$$

$$x^2(y-1) = -2y-8$$

$$x^2 = \frac{-2y-8}{y-1}$$

$$\therefore f\left(\frac{-2y-8}{y-1}\right) = \left(\frac{-2y-8}{y-1} - 8\right) / \left(\frac{-2y-8}{y-1} + 2\right)$$

$$= \frac{-2y-8-8y+8}{-2y-8+2y+2} = \frac{-10y}{-2y-6} = \frac{10y}{2y+6} = y \quad \therefore \text{onto}$$

$\therefore$ one-one and onto.

64) let $f(x) = x^3$ be the function with domain $\{0, 1, 2, 3\}$ then the domain of f^{-1} is

- 1) $\{3, 2, 1, 0\}$
- 2) $\{0, -1, -2, -3\}$
- 3) $\{0, 1, 8, 27\}$
- 4) $\{0, -1, -8, -27\}$

$$f(x) = x^3$$

$$f(0) = 0, f(1) = 1, f(2) = 8, f(3) = 27$$

$$f: (0, 0) (1, 1) (2, 8) (3, 27) \text{ domain of } f^{-1} = (0, 1, 8, 27)$$

65) If $f: \mathbb{R} \rightarrow \mathbb{R}$ is given by $f(x) = x^3 + 3$ then $f^{-1}(x)$ is equal to

- 1) $x^{\sqrt[3]{3}} - 3$
- 2) $x^{\sqrt[3]{3}} + 3$
- 3) $(x-3)^{\sqrt[3]{3}}$
- 4) $x + x^{\sqrt[3]{3}}$

$$y = f(x) = x^3 + 3 \Rightarrow y - 3 = x^3 \Rightarrow x = (y-3)^{\sqrt[3]{3}}$$

$$y = f(x) = f((y-3)^{\sqrt[3]{3}}) = ((y-3)^{\sqrt[3]{3}})^3 + 3 = y \quad (\text{onto})$$

$$\Rightarrow f^{-1}(y) = (y-3)^{\sqrt[3]{3}} \Rightarrow f^{-1}(x) = (x-3)^{\sqrt[3]{3}}$$

66) let $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^2 - 3$ then f^{-1} is given by

- 1) $\sqrt{x+3}$
- 2) $\sqrt{x}+3$
- 3) $x+\sqrt{3}$
- 4) none of these

$$y = f(x) = x^2 - 3 \Rightarrow x^2 = y + 3$$

$$x = \sqrt{y+3}$$

$$f^{-1}(y) = \sqrt{y+3} \quad \therefore f^{-1}(x) = \sqrt{x+3}$$

67) If $A = \{1, 2, 4\}$ $B = \{2, 4, 5\}$ $C = \{2, 5\}$ then $(A-B) \times (B-C)$
 1) $\{(1, 2), (1, 5), (2, 5)\}$ 2) $\{(1, 4)\}$ 3) $(1, 4)$ 4) none of these

$$A-B = \{1\} \quad (A-B) \times (B-C) = \{(1, 4)\}$$

$$B-C = \{4\}$$

68) If $A = \{1, 2, 3\}$ and $B = \{1, 4, 6, 9\}$ and R is a relation from A to B defined by x is greater than y . The range of R is
 1) $\{1, 4, 6, 9\}$ 2) $\{4, 6, 9\}$ 3) $\{1\}$ 4) none of these

$$R: (2, 1) (3, 1) \quad \therefore \text{The range} = \{1\}.$$

69) Let R be the relation on N defined by $x+2y=8$ The domain of R is
 1) $\{2, 4, 8\}$ 2) $\{2, 4, 6, 8\}$ 3) $\{2, 4, 6\}$ 4) $\{1, 2, 3, 4\}$.

$$y = \frac{8-x}{2} \quad \text{when we sub. } \{2, 4, 6\} \text{ for } x \text{ we will get}$$

$$y \in N. \therefore \text{the domain is } \{2, 4, 6\}.$$

70) If the set A has p elements and B has q elements then The number of elements in $A \times B$ is
 a) $p+q$ 2) $p+q+1$ 3) pq 4) p^2

71) Let R be a relation from a finite set A to the set B then
 1) $R = A \cup B$ 2) $R = A \cap B$ 3) $R \subseteq A \times B$ 4) $R \subseteq B \times A$

72) A set contains n elements then the number of elements in the power set is 2^n

73) The number of elements in the power set of a null set is $2^0 = 1$

74) If A and B are two sets s.t $n(A) = 20$, $n(B) = 25$, $n(A \cup B) = 40$
 Then write $n(A \cap B)$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

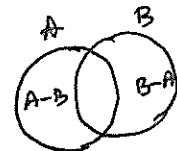
$$= 20 + 25 - 40 = 5$$

75) If A and B are two sets s.t $n(A) = 15$ $n(B) = 32$ $n(A \cap B) = 47$
 then $n(A \cup B)$ is

$$(A \cup B) = (A - B) \cup B$$

$$n(A \cup B) = n(A - B) + n(B)$$

$$= 47 + 32 = 79$$



76) For any set A , $(A')'$ is equal to

1) A' 2) A 3) $\emptyset$ 4) none of these

77) Let A and B be two sets in the same universal set then $A-B$

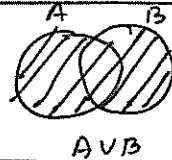
- 1) $A \cap B$ 2) $A' \cap B$ 3) $A \cap B'$ 4) none of these

78) The number of sub sets of a set containing elements is

- 1) n 2) $2^n - 1$ 3) n^2 4) 2^n

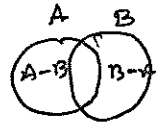
79) For any two sets A and B $A \cap (A \cup B)$

- 1) A 2) B 3) $\emptyset$ 4) none of these



80) The symmetric difference of A and B is not equal to

- 1) $(A-B) \cap (B-A)$ 2) $(A-B) \cup (B-A)$ 3) $(A \cup B) - (A \cap B)$ 4) $\{(A \cup B) - A\} \cup \{(A \cup B) - B\}$



81) The symmetric difference of $A = \{1, 2, 3\}$ $B = \{3, 4, 5\}$ is

- 1) $\{1, 2\}$ 2) $\{1, 2, 4, 5\}$ 3) $\{4, 3\}$ 4) $\{2, 5, 1, 4, 3\}$

$$A-B = \{1, 2\} \quad B-A = \{4, 5\} \quad (A-B) \cup (B-A) = \{1, 2, 4, 5\}$$

82) For any two sets A and B $(A-B) \cup (B-A)$ is

- 1) $(A-B) \cup A$ 2) $(B-A) \cup B$ 3) $(A \cup B) - (A \cap B)$ 4) $(A \cup B) \cap (A \cap B)$

Draw the diagram and verify.

83) Which of the following statement is false.

- 1) $A-B = A \cap B'$ 2) $A-B = A - (A \cap B)$ 3) $A-B = A \cap B'$ 4) $A-B = (A \cup B) - B$

Draw the diagram and verify.

84) For any three sets A, B and C.

- 1) $A \cap (B-C) = (A \cap B) - (A \cap C)$ 2) $A \cap (B-C) = (A \cap B) - C$
3) $A \cup (B-C) = (A \cup B) \cap (A \cup C')$ 4) $A \cup (B-C) = (A \cup B) \cap (A \cap C)$

85) Let U be the universal set containing 700 elements. If A, B are the subsets of U s.t. $n(A) = 200$, $n(B) = 300$, $n(A \cap B) = 100$, $n(A' \cap B') = \dots$

- 1) 400 2) 600 3) 300 4) none of these

$$\begin{aligned} n(A' \cap B') &= n(A \cup B)' = n(U) - n(A \cup B) & n(A \cup B) &= 200 + 300 - 100 \\ & & &= 400 \\ & & &= 700 - 400 \\ & & &= 300 \end{aligned}$$

86) $A = \{1, 2, 3, 4, 5\}$ Then the number of proper subsets of A is

- a) 120 b) 30 c) 31 d) 32

$$\text{Number of proper sub sets} = 2^n - 1 = 2^5 - 1 = 31$$

87)

Book one Mark.

1. If $A = \{(x, y) : y = e^x, x \in \mathbb{R}\}$ and $B = \{(x, y) : y = e^{-x}, x \in \mathbb{R}\}$ then $n(A \cap B)$ is

TBP 1) infinity 2) 0 3) 1 4) 2

$\because f(-x)$ is symmetric about y axis e^x and e^{-x} cuts at one point $\therefore n(A \cap B) = 1$



2) If $A = \{(x, y) : y = \sin x, x \in \mathbb{R}\}$ $B = \{(x, y) : y = \cos x, x \in \mathbb{R}\}$ then $A \cap B$ contains

TBP 1) no elements 2) infinitely many elements 3) only one point 4) cannot be determined.

3) The relation R defined on a set $A = \{0, -1, 1, 2\}$ by xRy if $|x^2 + y^2| \leq 2$ then which one of the following is true?

1) $R = \{(0, 0) (0, -1) (0, 1) (-1, 0) (1, 0) (1, 0)\}$

TBP 2) $R^{-1} = \{(0, 0) (0, -1) (0, 1) (-1, 0) (1, 0)\}$

3) Domain of $R = \{0, -1, 1, 2\}$

4) Range of R is $\{0, -1, 1\}$

In one the range is 2 it is not possible $|4| \neq 2$
In Domain 2 should not be there

4) $f(x) = |x-2| + |x+2|$ if $x \in \mathbb{R}$ then

TBP 1) $f(x) = \begin{cases} -2x & \text{if } x \in (-\infty, -2] \\ 4 & \text{if } x \in (-2, 2) \\ 2x & \text{if } x \in (2, \infty) \end{cases}$

2) $f(x) = \begin{cases} -2x & x \in (-\infty, -2] \\ +4x & x \in (-2, \infty) \\ -2x & x \in (2, \infty) \end{cases}$

3) $f(x) = \begin{cases} -2x & x \in (-\infty, -2) \\ -4x & x \in (-2, 2] \\ 2x & x \in (2, \infty) \end{cases}$

4) $f(x) = \begin{cases} -2x & x \in (-\infty, -2) \\ 2x & \text{if } x \in (-2, 2] \\ 2x & x \in (2, \infty) \end{cases}$

in $(-\infty, -2]$

$$f(x) = -(x-2) - (x+2) = -2x$$

in $(-2, 2]$

$$f(x) = (x+2) - (x-2) = 4$$

in $(2, \infty)$

$$f(x) = (x-2) + (x+2) = 2x$$

- 5) Let R be the set of all real numbers. Consider the following rule sets of the plane $R \times R$ $S = \{(x, y) : x + 1 \text{ and } 0 < x < 2\}$ and $T = \{(x, y) : x - y \text{ is an integer}\}$ then which of the following is true
- 1) T is an equivalence relation but S is not an equivalence relation
 2) Neither S nor T is an equivalence relation
 3) Both S and T are equivalence relation
 4) S is equivalence but T is not an equivalence

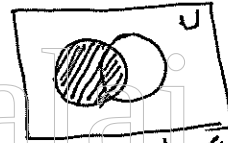
S is not reflexive $\therefore$ it is not equivalence.

T is equivalence $\because a - a = 0 \in \mathbb{Z}$. Reflexive

$a - b = +b - a \in \mathbb{Z}$ symmetry

$a - b \in \mathbb{Z}, b - c \in \mathbb{Z}, a - c \in \mathbb{Z}$ transitive

- 6) Let A and B be subsets of the universal set N , the set of all natural numbers then $A' \cup ((A \cap B) \cup B')$ is
- 1) A 2) A' 3) B 4) N .



$$A' \cup ((A \cap B) \cup B') = U.$$

- 7) The number of students who take both the subjects Maths and chemistry is 70. This represents 10% the enrollment in Maths and 14% of the enrollment in chemistry. The number of students take at least one of these two subjects is
- 1) 1120 2) 1130 3) 1000 4) insufficient data

- 8) If $n((A \times B) \cap (A \cap C)) = 8$ and $n(B \cap C) = 2$ then $n(A)$ is
- 1) 6 2) 4 3) 6 4) 5

We know $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$$n(A) \cdot n(B \cap C) = n[(A \times B) \cap (A \times C)]$$

$$n(A) \cdot 2 = 8$$

$$n(A) = 4.$$

- 9) If $n(A) = 2$ $n(B \cup C) = 3$ then $n[(A \times B) \cup (A \times C)]$ is
- 1) 2^3 2) 3^2 3) 6 4) 5

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$

$$n(A) \cdot n(B \cup C) = n[(A \times B) \cup (A \times C)]$$

$$2 \times 3 = n[(A \times B) \cup (A \times C)]$$

$$6 \Rightarrow n[(A \times B) \cup (A \times C)]$$

10) If two sets A and B have elements in common, then the number of elements common to $A \times B$ and $B \times A$ is
 TBP 1) 2^{17} 2) 17^2 3) 34 4) insufficient data.

If A and B any two non empty sets having n elements in common then $A \times B$ and $B \times A$ have n^2 elements in common

11) For non empty sets A and B, $A \cap B$ then $(A \times B) \cap (B \times A)$ is equal to
 TBP 1) $A \cap B$ 2) $A \times A$ 3) $B \times B$ 4) none of these

Do it yourself by taking suitable sets.

12) The number of relations on a set containing 3 elements is
 TBP 1) 9 2) 81 3) 512 3) 1024.

The number of elements in set A is n then the number of relation on the set itself is 2^{n^2} . Here $2^{3^2} = 2^9 = 512$.

13) Let R be the universal relation on a set x with more than one element. Then R is
 TBP 1) not reflexive 2) not symmetric 3) transitive 4) none of these

Let $A = \{1, 2, 3\}$ $R = \{(1,1) (1,2) (1,3) (2,1) (2,2) (2,3) (3,1) (3,2) (3,3)\}$ is universal relation.
 It is symmetric, reflexive and transitive.

14) Let $X = \{1, 2, 3, 4\}$ and $R = \{(1,1) (1,2) (1,3) (2,2) (3,3) (2,1) (3,1) (1,4) (4,1)\}$ then
 TBP 1) reflexive 2) symmetric 3) transitive 4) equivalence.

There is $(4,4)$ it is not reflexive. $(2,1) \in R$ $(1,3) \in R$ but $(2,3) \notin R$
 $\therefore$ not transitive $\therefore$ not equivalence Hence symmetric.

15) The range of the function $\frac{1}{1-2\sin x}$ is
 TBP 1) $(-\infty, -1) \cup (1/3, \infty)$ 2) $(-1, 1/3)$ 3) $[-1, 1/3]$ 4) $(-\infty, -1] \cup [1/3, \infty)$

clearly $-1 \leq \sin x \leq 1$ $\Rightarrow \frac{1}{3} \leq \frac{1}{1-2\sin x} \leq -1$

$-2 \leq 2\sin x \leq 2$

$2 \geq -2\sin x \geq -2$ $\therefore (-\infty, -1] \cup [1/3, \infty)$

$3 \geq 1-2\sin x \geq -1$

16) The range of the function $f(x) = |\lfloor x \rfloor - x|$ $x \in \mathbb{R}$ is
 TBP 1) $[0, 1]$ 2) $[0, \infty)$ 3) $\checkmark [0, 1)$ 4) $(0, 1)$

$$\lfloor 2 \rfloor = 2 \quad \lfloor 2.3 \rfloor = 2 \quad \therefore f(x) = |2 - 2| = 0$$

$$|2 - 2.3| = 0.3 < 1$$

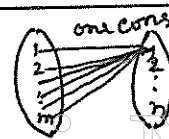
At the most we can put the value of $x = 2.9 \dots$

$$\lfloor 2.9 \rfloor = 2 \quad \therefore |2 - 2.9| = 0.9 < 1$$

$\therefore$ The range is $[0, 1)$

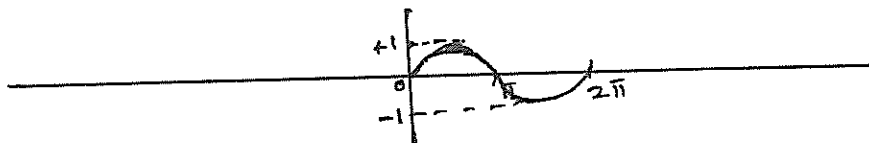
17) The rule $f(x) = x^2$ is a bijection if the domain and co-domain are given by
 TBP 1) $\mathbb{R}, \mathbb{R}$ 2) $\mathbb{R}, (0, \infty)$ 3) $(0, \infty), \mathbb{R}$ 4) $\checkmark [0, \infty), [0, \infty)$

18) The number of constant functions from a set containing m elements to a set containing n elements.
 TBP 1) mn 2) m 3) $\checkmark n$ 4) $m+n$.



one constant. Similarly we can find n constant functions.

19) The function $f: [0, 2\pi] \rightarrow [-1, 1]$ defined by $f(x) = \sin x$.
 TBP 1) one-to-one 2) $\checkmark$ onto 3) bijection 4) cannot be defined.



$\because$ at $x=0$ $\sin 0 = 0$ for two values of x same value of y
 $x=\pi$ $\sin \pi = 0$

$\therefore$ it is not one-to-one.

But every element of y it has unique pre-image of x .
 $\therefore$ onto

20) If the function $f: [-3, 3] \rightarrow S$ defined by $f(x) = x^2$ is onto then
 TBP S is ... 1) $[-9, 9]$ 2) $\mathbb{R}$ 3) $[-3, 3]$ 4) $\checkmark [0, 9]$.

$\because$ Domain is $[-3, 3]$ the range is $[0, 9]$. This f is many to one.

21) Let $X = \{1, 2, 3, 4\}$ $Y = \{a, b, c, d\}$ $f = \{(1, a) (4, b) (2, c) (3, d) (2, d)\}$ Then f is
 TBP 1) one-to-one function 2) an onto function
 3) not one-to-one 4) $\checkmark$ not a function

∴ 2 ∈ Domain has different images in the Range
 ∴ it is not at all a function.

22) The inverse of $f(x) = \begin{cases} x & x < 1 \\ x^2 & 1 \leq x \leq 4 \\ 8\sqrt{x} & x > 4 \end{cases}$ is

1) $f^{-1}(x) = \begin{cases} x & x < 1 \\ \sqrt{x} & 1 \leq x \leq 16 \\ \frac{x^2}{64} & x > 16 \end{cases}$

2) $f^{-1}(x) = \begin{cases} -x & x \leq 1 \\ \sqrt{x} & 1 \leq x \leq 16 \\ \frac{x^2}{64} & x > 16 \end{cases}$

3) $f^{-1}(x) = \begin{cases} x^2 & x < 1 \\ \sqrt{x^2} & 1 \leq x \leq 16 \\ \frac{x^2}{64} & x > 16 \end{cases}$

4) $f^{-1}(x) = \begin{cases} 2x & x < 1 \\ \sqrt{x} & 1 \leq x \leq 16 \\ \frac{x^2}{8} & x > 16 \end{cases}$

let $y = f(x) = x = y$
 $f^{-1}(y) = y \Rightarrow f^{-1}(x) = x$

$y = f(x) = x^2 \Rightarrow x = \sqrt{y}$

$f^{-1}(y) = \sqrt{y} \Rightarrow f^{-1}(x) = \sqrt{x}$

$y = f(x) = 8\sqrt{x} \Rightarrow \sqrt{x} = \frac{y}{8}$

$x = \frac{y^2}{64}$
 $f^{-1}(y) = \frac{y^2}{64} \Rightarrow f^{-1}(x) = \frac{x^2}{64}$

23) TBP Let $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = \sin x + \cos x$.
 1) an odd function 2) neither an odd nor an even fn.
 3) an even function 4) both odd and even fn.

$f(-x) = -\sin x + \cos x$
 $\neq f(x)$
 $\neq -f(x)$

24) TBP Let $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 1 - |x|$ then the range of f is
 1) $\mathbb{R}$ 2) $(1, \infty)$ 3) $(-1, \infty)$ 4) $(-1, 1]$

$f(x) = 1 - x \quad x \geq 0 \quad \forall x \in (0, \infty) \quad f(x) = (-\infty, 1]$
 $1 + x \quad x < 0 \quad x \in (-\infty, 0) \quad (-\infty, 0]$

25) The function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = \frac{(x^2 + \cos x)(1 + x^4)}{(x - \sin x)(2x - x^3)} + e^{-1|x|}$

TBP 1) an odd function 2) neither odd nor even fn. 3) even fn.
 4) both odd and even fn.

$\frac{(E+E)(E+E)}{(0-0)(0-0)} + E = \frac{E}{E} + E = E + E = E$



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