Padasalai

UNIT TEST I TIME : 1.30 hrs

I Choose the correct answer $5 \times 1 = 5$

1. If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then $|adj(AB)| =$

1) -40 2) -80 3) -60 4) -20

2. The augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda+7 & \mu-5 \end{bmatrix}$. The system has infinitely many solutions if 1) $\lambda = 7, \mu \neq -5$ 2) $\lambda = -7, \mu = 5$ 3) $\lambda \neq 7, \mu \neq -5$ 4) $\lambda = 7, \mu = -5$

3. If $\rho(A) \neq \rho([A|B])$, then the system $AX = B$ of linear equations is 1) Consistent and has a unique solution 2) Consistent 3) Consistent and has infinitely many solution 4) inconsistent

4. If $A^T A^{-1}$ is symmetric, then $A^2 =$ 1) A^{-1} 2) $(A^T)^2$ 3) A^T 4) $(A^{-1})^2$

5. Every non-singular matrix can be transformed to an ______ matrix, by a sequence of elementary row operations

a) Null b) identity c) Symmetric d) orthogonal.

II i) Answer any three questions from 6 to 9.

ii) question number 10 is compulsory. $4 \times 2 = 8$

6. State Rouche'-Capelli theorem.

7. If A and B are any two non-singular square matrices of order n, then $adj(AB) = (adj B)(adj A)$.

8. Verify the property $(A^T)^{-1} = (A^{-1})^T$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$

9. Find the rank of $\begin{bmatrix} 1 & 2 \\ -1 & -2 \\ 2 & 4 \end{bmatrix}$

10. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(adj A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, the find the value of k.

III i) Answer any three questions from 11 to 14.

ii) Question number 15 is compulsory $4 \times 3 = 12$.

11. Test the consistency of the following system of linear equations $x-y+z=-9$, $2x-y+z=4$, $3x-y+z=6$, $4x-y+2z=7$

12. Solve by Gauss elimination method $2x-2y+3z=2$, $x+2y-z=3$, $3x-y+2z=1$

13. Solve by matrix Inversion method: $5x+2y=3$, $3x+2y=5$

14. Find the inverse of $A=\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss-Jordan method

15. Given $A=\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$, $B=\begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$ and $C=\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, find a matrix X such that $A \times B = C$

IV i) Answer any four questions from 16 to 20.

ii) question number 21 is compulsory. $5 \times 5 = 25$

16. If $A=\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, Show that $A^{-1} = \frac{1}{2}(A^2 - 3I)$

17. If $A=\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$. find the products AB and BA and hence solve the system of equations $x-y+z=4$, $x-2y-2z=9$, $2x+y+3z=1$

18. A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution?

19. Find the value of k for which the equations $kx-2y+z=1$, $x-2ky+z=-2$, $x-2y+kz=1$ have (i) no solution (ii) unique solution (iii) infinitely many soln

20. By using Gaussian elimination method, balance the chemical reaction equation $C_2H_6 + O_2 \longrightarrow H_2O + CO_2$

21. If the system of equations $px+by+cz=0$, $ax+qy+cz=0$, $ax+by+rz=0$ has a non-trivial solution and $p \neq a$, $q \neq b$, $r \neq c$ Prove that $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$.

XII - MATHEMATICS

UNIT TEST - II

MARKS : 50

TIME : 1 hr 30 Mins

I. Choose the correct answer $5 \times 1 = 5$

1. If z is a complex number such that $z \in C \setminus R$ and $z + \frac{1}{z} \in R$, then $|z|$ is
   1) 0 2) 1 3) 2 4) 3
2. If α and β are the roots of $x^2 + x + 1 = 0$, then $\alpha^{2020} + \beta^{2020}$ is
   1) -2 2) -1 3) 1 4) 2
3. If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is
   1) $\frac{1}{2}$ 2) 1 3) 2 4) 3
4. The complex number ______ is a rotation of z by θ radians in the counter clockwise direction about the origin 1) $e^{i\theta}$ 2) $z\theta$ 3) $ze^{i\theta}$ 4) ze^{θ}
5. The product of all the n^{th} root of unity is 1) 1 2) 0 3) $(-1)^n$ 4) $(-1)^{n-1}$

II. i) Answer any four questions ii) Question number 10 is compulsory $4 \times 2 = 8$

6. Which one of the points $i, -2+i$ and 3 is farthest from the origin?
7. If $\frac{z+3}{z-5i} = \frac{1+4i}{2}$, find the complex number z.
8. Find the square root of $-6+8i$
9. If $|z| = 1$, show that $2 \le |z^2 - 3| \le 4$
10. Find the cube roots of unity.

III i) Answer any four questions ii) Question number 15 is compulsory $4 \times 3 = 12$

11. If z_1, z_2 and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = |z_1 + z_2 + z_3| = 1$. Find the value of $\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$.
12. Show that $|3z - 5 + i| = 4$ represents a circle, and find its centre and radi
13. Obtain the Cartesian form of the locus of $z = x + iy$ in $[Re(iz)]^2 = 3$
14. Find the principal argument Arg z, when $z = \frac{-2}{1 + i\sqrt{3}}$
15. Find the value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$.

IV Answer all the questions $5 \times 5 = 25$

16. a) i) Simplify: $\sum_{n=1}^{102} i^n$ ii) Find the value of $i^1 \cdot i^2 \cdot i^3 \cdots i^{2020}$

(or)

b) i) $\sqrt{ab} = \sqrt{a}\sqrt{b}$ - Is it true. Justify your answer.

ii). Find the value of the real numbers x and y, if the complex number $(2+i)x + (1-i)y + 2i - 3$ and $x + (-1+2i)y + 1 + i$ are equal

17. a) State any five properties of Complex number.

(or)

b) i) Prove that $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$, where x_1, x_2, y_1 and $y_2 \in R$

ii) Prove that z is purely imaginary iff $z = -\bar{z}$

18. a) i) The Complex numbers u, v and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$. If $v = 3-4i$ and $w = 4+3i$, find u in rectangular form.

ii) Show that $(2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}$ is purely imaginary.

(or)

b). Show that the points $1, -\frac{1}{2} + i\frac{\sqrt{3}}{2}$ and $-\frac{1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

19) a) If $z = x+iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, then show that $x^2+y^2+3x-3y+2=0$.

(or)

b) Find the value of $(-\sqrt{3}+3i)^{31}$

20) a). Suppose z_1, z_2 and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1+i\sqrt{3}$, then find z_2 and z_3.

(or)

b). Solve: $z^4+1=0$.

IV Answer all the questions $5\times5=25$

16) a) If α, β and γ are the roots of the cubic equation $x^3+2x^2+3x+4=0$, form a cubic equation whose roots are
(i) $2\alpha, 2\beta, 2\gamma$ (ii) $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ iii) $-\alpha, -\beta, -\gamma$

(OR)

b) If $2+i$ and $3-\sqrt{2}$ are roots of the equation $x^6-13x^5+62x^4-126x^3+65x^2+127x-140=0$. find all roots.

17) a) Determine k and solve the equation $2x^3-6x^2+3x+k=0$ if one of its roots is twice the sum of the other two roots.

(OR)

b) Solve: $(x-2)(x-7)(x-3)(x+2)+19=0$.

18) a) Prove that A Polynomial equation $a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+\cdots+a_2x^2+a_1x+a_0=0$, $(a_n\neq0)$ is a reciprocal equation iff one of the following two statements is true.
(i) $a_n=a_0$, $a_{n-1}=a_1$, $a_{n-2}=a_2 \ldots$
(ii) $a_n=-a_0$, $a_{n-1}=-a_1$, $a_{n-2}=-a_2 \ldots$

(OR)

b) If $P(x)=0$ is a polynomial equation such that whenever α is a root, $\frac{1}{\alpha}$ is also a root, then the polynomial equation $p(x)=0$ must be a reciprocal equation" - IS the Statement is true. Justify your answer.

19) a) Solve: $x^4-10x^3+26x^2-10x+1=0$.

(OR)

b) i) Write Descartes Rule
ii) Show that the equation $x^9-5x^5+4x^4+2x^2+1=0$ has atleast 6 imaginary solutions.

20) a). Find the condition that the roots of $ax^3+bx^2+cx+d=0$ are in G.P. Assume $a,b,c,d\neq0$.

(OR)

b) Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5}-\sqrt{3}$ as a root.

XII - MATHEMATICS

UNIT TEST - III

MARKS: 50

TIME: 1 hr 30 Mins

I Choose the Correct answer $5 \times 1 = 5$

1. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is 1) 2 2) 4 3) 1 4) ∞
2. A Polynomial equation in x of degree n always has 1) n distinct roots 2) n real roots 3) n imaginary roots 4) atmost 1 root
3. A zero of $x^3 - 64$ is 1) 0 2) 4 3) $4i$ 4) -4
4. Which of the following is not a root of $x^5 - 2x^4 - x + 2$ 1) 2 2) i 3) $-i$ 4) -2
5. What is the Condition that the roots $x^3 + px^2 + qx + r = 0$ are in A.P 1) $9pq = 2p^3 + 27r$ 2) $ac^3 = db^3$ 3) $9pqr = 27r^3 + 2q^3$ 4) $9pq = 27r^3 + 2q$

II Answer any four questions. Question number 10 is Compulsory $4 \times 2 = 8$

6. Find the Condition that the roots of $x^3 + ax^2 + bx + c = 0$ are in the ratio $p : q : r$.
7. Solve: $x^4 - 9x^2 + 20 = 0$.
8. State rational root theorem.
9. Define Reciprocal polynomial.
10. "For a polynomial equation with rational coefficients, irrational roots occur in pairs" - Is it true. Justify.

III Answer any four questions. Question number 15 is Compulsory $4 \times 3 = 12$

11. Find all real numbers satisfying $4^x - 3(2^{x+2}) + 2^5 = 0$.
12. Solve: $x^9 - 5x^8 - 14x^7 = 0$.
13. Find solution, if any of the equation $2\cos^2 x - 9\cos x + 4 = 0$.
14. Solve the equation $9x^3 - 36x^2 + 44x - 16 = 0$ if the roots form an A.P.
15. Let p and q be rational numbers such that $\sqrt{q}$ is irrational. If $p + \sqrt{q}$ is a root of a quadratic equation with rational coefficients, then Prove that $p - \sqrt{q}$ is also a root of the same equation.

XII - MATHEMATICS MARKS : 50
UNIT TEST - IV TIME : 1 hr. 30 Mins

I. Choose the correct answer $5\times1=5$

1. $\sin^{-1}(2x\sqrt{1-x^2}) = 2\cos^{-1}x$ if
   1) $x<1$ 2) $x>\frac{1}{2}$ 3) $x>\frac{1}{\sqrt{2}}$ 4) $\frac{1}{\sqrt{2}}\le x\le 1$

2. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$, the value of
   $x^{2017}+y^{2018}+z^{2019}-\frac{6}{x^{101}+y^{101}+z^{101}}$ is 1) 0 2) 1 3) 2 4) 3

3. If $\cot^{-1}2$ and $\cot^{-1}3$ are two angles of a triangle, then the third angle is 1) $\frac{\pi}{4}$ 2) $\frac{3\pi}{4}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{3}$

4. If $\sin^{-1}x=2\sin^{-1}\alpha$ has a solution, then
   1) $|\alpha|\le\frac{1}{\sqrt{2}}$ 2) $|\alpha|\ge\frac{1}{\sqrt{2}}$ 3) $|\alpha|<\frac{1}{\sqrt{2}}$ 4) $|\alpha|>\frac{1}{\sqrt{2}}$

5. $\pi-\sec^{-1}x=$ ______ if $|x|\ge1$ 1) $\operatorname{cosec}^{-1}x$ 2) $\cos^{-1}x$ 3) $\sec^{-1}(-x)$ 4) $\sec^{-1}x$

II. Answer any four questions. Question no. 10 is compulsory $4\times2=8$

6. Sketch the graph of $y=\sin\left(\frac{1}{3}x\right)$ for $0\le x<6\pi$.
7. Is $\cos^{-1}(-x)=\pi-\cos^{-1}(x)$ true? Justify your answer.
8. Draw the graph of inverse tangent function.
9. Find the value of $\cos\left(\sin^{-1}\left(\frac{4}{5}\right)-\tan^{-1}\left(\frac{3}{4}\right)\right)$.
10. Prove that $\sin^{-1}(2x\sqrt{1-x^2})=2\sin^{-1}x$ if $|x|\le\frac{1}{\sqrt{2}}$

III. Answer any four questions. Question no. 15 is compulsory. $4\times3=12$

11. For what value of x, the inequality $\frac{\pi}{2}<\cos^{-1}(3x-1)<\pi$ holds?
12. Find the principal value of $\sec^{-1}(-2)$.
13. Prove that $\sin^{-1}\left(\frac{1}{x}\right)=\operatorname{cosec}^{-1}x$, if $x\in\mathbb{R}\setminus(-1,1)$
14. Write the cofunction inverse identities.
15. Solve: $\sin^{-1}\left(\tan\frac{\pi}{4}\right)-\sin^{-1}\left(\sqrt{\frac{3}{x}}\right)=\frac{\pi}{6}$

IV Answer all the questions $5 \times 5 = 25$

16. a). Find the value of i) $\cos\left(\cos^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{4}{5}\right)\right)$

ii) $\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right)+\cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$

(OR)

b) Find the value of i) $\sin^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)+\cot^{-1}(2)$

ii) $\cot^{-1}(1)+\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)-\sec^{-1}(-\sqrt{2})$

17. a) Prove that $\sin^{-1}x+\sin^{-1}y=\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right)$, where either $x^2+y^2\leq 1$ or $xy<0$

(OR)

b) Evaluate: $\sin\left[\sin^{-1}\left(\frac{3}{5}\right)+\sec^{-1}\left(\frac{5}{4}\right)\right]$

18. a) If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$ and $0<x,y,z<1$, Show that $x^2+y^2+z^2+2xyz=1$ (OR)

b) Solve: i) $\sin^{-1}x > \cos^{-1}x$ ii) $\sin^{-1}\frac{5}{x}+\sin^{-1}\frac{12}{x}=\frac{\pi}{2}$.

19. a) Solve: $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$

(OR)

b) Solve: $\cot^{-1}x-\cot^{-1}(x+2)=\frac{\pi}{12}$, $x>0$.

20. a) Prove that $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

(OR)

b) If $a_1, a_2, a_3, \ldots \ldots a_n$ is an A.P with common differenced

Prove that $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\cdots+\tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right]$

$$=\frac{a_n-a_1}{1+a_1a_n}$$