EXERCISE 1.8

Choose the Correct answer:

1. If $|adj(adj\ (A))| = |A|^9$, then the order of the square matrix A is

(a) 3 (b) 4 (c) 2 (d) 5

2. If A is a 3×3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then $BB^T =$

(a) A (b) B (c) I (d) B^T

3. If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, $B = adj\ A$ and $C = 3A$, then $\frac{|adj\ B|}{|C|} =$

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{1}{4}$ (d) 1

4. If $A\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$, then A

(a) $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$

5. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ then $9I - A =$

(a) A^{-1} (b) $\frac{A^{-1}}{2}$ (c) $3A^{-1}$ (d) $2A^{-1}$

6. If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ $and\ B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ $then\ |adj(AB)|$

(a) −40 (b) −80 (c) −60 (d) −20

7. If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then x is

(a) 15 (b) 12 (c) 14 (d) 11

8. If $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ $and\ A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the value of a_{23} is

(a) 0 (b) −2 (c) −3 (d) −1

9. If A, B and C are invertible matrices of some order, then which one of the following is not true?

(a) $adjA = |A|A^{-1}$ (b) $adj(AB) = (adj\ A)(adj\ B)$ (c) $\det A^{-1} = (\det A)^{-1}$ (d) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

10. If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then B^{-1}

(a) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

11. If A^TA^{-1} is symmetric, then $A^2 =$

(a) A^{-1} (b) $(A^T)^2$ (c) A^T (d) $(A^{-1})^2$

12. If A is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ then $(A^{-1})^2$

(a) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

13. If $A = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$ $and\ A^T = A^{-1}$, then the value of x is

(a) $-\frac{4}{5}$ (b) $-\frac{3}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

14. If $A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $AB = I$, then $B =$

(a) $\left(cos^2\frac{\theta}{2}\right)A$ (b) $\left(cos^2\frac{\theta}{2}\right)A^T$ (c) $(cos^2\theta)\,I$ (d) $\left(sin^2\frac{\theta}{2}\right)A$

15. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A\,(adjA) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then $k =$

(a) 0 (b) sinθ (c) cosθ (d) 1

16. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

(a) 17 (b) 14 (c) 19 (d) 21

17. If $adjA = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and $adjB = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ $then\ adj(AB)$ is

(a) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ (c) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ (d) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

18. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

(a) 1 (b) 2 (c) 4 (d) 3

19. If $x^a y^b = e^m,\ \ x^c y^d = e^n, \Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}\ \Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}\ \Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ then the values of x and y are respectively,

(a) $e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$ (b) $\log(\Delta_1/\Delta_3),\ \log(\Delta_2/\Delta_3)$ (c) $\log(\Delta_2/\Delta_1),\ \log(\Delta_3/\Delta_1)$ (d) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$

20. Which of the following is/are correct?

(i) Adjoint of a symmetric matrix is also a symmetric matrix.
(ii) Adjoint of a diagonal matrix is also a diagonal matrix.
(iii) If A is a square matrix of order n and λ is a scalar, then $adj(\lambda A) = \lambda^n adj(A)$
(iv) $\boldsymbol{A(adj\ A) = (adj\ A)A = |A|I}$
(a) Only (i) (b) (ii) and (iii) (c) (iii) and (iv) (d) (i), (ii) and (iv)

21. If $\rho(A) = \rho\big((A|B)\big)$ then the system AX B = of linear equations is

(a) consistent and has a unique solution (b) consistent
(c) consistent and has infinitely many solution (d) inconsistent

22. If $0 \le \theta \le \pi$ and the system of equations $x + (sin\theta)y - (cos\theta)z = 0,\ \ (cos\theta)x - y + z = 0,\ \ (sin\theta)x + y - z = 0$ has a non-trivial solution then θ is

(a) $\frac{2\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{4}$

23. The augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda - 7 & \mu + 5 \end{bmatrix}$. The system has infinitely many solutions if

(a) $\lambda = 7, \mu \ne -5$ (b) $\lambda = -7, \mu = 5$ (c) $\lambda \ne 7, \mu \ne -5$ (d) $\lambda = 7, \mu = -5$

24. Let $= \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ $and\ 4B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$. If B is the inverse of A, then the value of x is

(a) 2 (b) 4 (c) 3 (d) 1

25. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then $adj(adj\ A)$ is

(a) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{bmatrix}$

EXERCISE 2.9

Choose the correct or the most suitable answer from the given four alternatives :

1. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is

(a) 0 (b) 1 (c) -1 (d) i

2. The value of $\sum_{i=1}^{13}\left(i^n + i^{n-1}\right)$ is

(a) $1+i$ (b) i (c) 1 (d) 0

3. The area of the triangle formed by the complex numbers z, iz and $z + iz$ in the Argand's diagram is

(a) $\frac{1}{2}|z|^2$ (b) $|z|^2$ (c) $\frac{3}{2}|z|^2$ (d) $2\,|z|^2$

4. The conjugate of a complex number is $\frac{1}{i-2}$. Then, the complex number is

(a) $\frac{1}{i+2}$ (b) $\frac{-1}{i+2}$ (c) $\frac{-1}{i-2}$ (d) $\frac{1}{i-2}$

5. If $z = \frac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$ then $|z|$ is equal to

(a) 0 (b) 1 (c) 2 (d) 3

6. If z is a non-zero complex number, such that $2iz^2 = \bar{z}$ *then* $|z|$ is

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

7. If $|z - 2 + i| \leq 2$, then the greatest value of $|z|$ is

(a) $\sqrt{3} - 2$ (b) $\sqrt{3} + 2$ (c) $\sqrt{5} - 2$ (d) $\sqrt{5} + 2$

8. If $\left|z - \frac{3}{z}\right| = 2$, then the least value of $|z|$ is

(a) 1 (b) 2 (c) 3 (d) 5

9. If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is

(a) z (b) $\bar{z}$ (c) $\frac{1}{z}$ (d) 1

10. The solution of the equation $|z| - z = 1 + 2i$ is

(a) $\frac{3}{2} - 2i$ (b) $-\frac{3}{2} + 2i$ (c) $2 - \frac{3}{2}i$ (d) $2 + \frac{3}{2}i$

11. If $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$, and $|9z_1 z_2 + 4z_1 z_3 + z_2 z_3| = 12$, then the value of $|z_1 + z_2 + z_3|$ is

(a) 1 (b) 2 (c) 3 (d) 4

12. If z is a complex number such that $z \in C/R$ and $z + \frac{1}{z} \in R$, then $|z|$ is

(a) 0 (b) 1 (c) 2 (d) 3

13. z_1, z_2 and z_3 be three complex numbers such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$, then ${z_1}^2 + {z_2}^2 + {z_3}^2$ is

(a) 3 (b) 2 (c) 1 (d) 0

14. If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

15. If $z = x + iy$ is a complex number such that $|z + 2| = |z - 2|$, then the locus of z is

(a) real axis (b) imaginary axis (c) ellipse (d) circle

16. The principal argument of $\frac{3}{-1+i}$ is

(a) $\frac{-5\pi}{6}$ (b) $\frac{-2\pi}{3}$ (c) $\frac{-3\pi}{4}$ (d) $\frac{-\pi}{2}$

17. The principal argument of $(\sin 40^0 + i\cos 40^0)^5$ is

(a) $-110°$ (b) $-70°$ (c) $70°$ (d) $110°$

18. If $(1+i)(1+2i)(1+3i)\ldots\ldots(1+ni) = x+iy$,then $2\cdot 5\cdot 10\cdot\cdots\cdots(1+n^2)$ is

(a) 1 (b) i (c) x^2+y^2 (d) $1+n^2$

19. If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7 = A + B\omega$, then (A, B) equals

(a) (1,0) (b) (-1,1) (c) (0,1) (d) (1,1)

20. The principal argument of the complex number $\frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$ is

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{2}$

21. If α and β are the roots of $x^2+x+1=0$, then $\alpha^{2020}+\beta^{2020}$ is

(a) -2 (b) -1 (c) 1 (d) 2

22. The product of all four values of $\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{3/4}$ is

(a) -2 (b) -1 (c) 1 (d) 2

23. If $\omega \neq 1$ is a cubic root of unity and $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to

(a) 1 (b) -1 (c) $\sqrt{3}i$ (d) $-\sqrt{3}i$

24. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

(a) $cis\frac{2\pi}{3}$ (b) $cis\frac{4\pi}{3}$ (c) $-cis\frac{2\pi}{3}$ (d) $-cis\frac{4\pi}{3}$

25. If $\omega = cis\frac{2\pi}{3}$, then the number of distinct roots of $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$

(a) 1 (b) 2 (c) 3 (d) 4

EXERCISE 3.6

1. Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$.

2. Discuss the maximum possible number of positive and negative roots of the polynomial equations $x^2 - 5x + 6$ and $x^2 - 5x + 16$. Also draw rough sketch of the graphs.

3. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions.

4. Determine the number of positive and negative roots of the equation $x^9 - 5x^8 - 14x^7 = 0$.

5. Find the exact number of real roots and imaginary of the equation $x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$.

EXERCISE 3.7

Choose the most suitable answer:

1. A zero of $x^3 + 64$ is

(a) 0 (b) 4 (c) $4i$ (d) -4

2. If *f* and *g* are polynomials of degrees *m* and *n* respectively, and if $h(x) = (f \circ g)(x)$, then the degree of *h* is

(a) mn (b) $m + n$ (c) m^n (d) n^m

3. A polynomial equation in *x* of degree *n* always has

(a) *n* distinct roots (b) *n* real roots (c) *n* imaginary roots (d) at most one root.

4. If α, β and γ are the roots of $x^3 + px^2 + qx + r$, then $\sum \frac{1}{\alpha}$ is

(a) $\frac{-q}{r}$ (b) $\frac{-p}{r}$ (c) $\frac{q}{r}$ (d) $\frac{-q}{p}$

5. According to the rational root theorem, which number is not possible rational root of $4x^7 + 2x^4 - 10x^3 - 5$?

(a) -1 (b) $\frac{5}{4}$ (c) $\frac{4}{5}$ (d) 5

6. The polynomial $x^3 - kx^2 + 9x$ has three real roots if and only if, *k* satisfies

(a) $|k| \leq 6$ (b) $k = 0$ (c) $|k| > 6$ (d) $|k| \geq 6$

7. The number of real numbers in $[0, 2\pi]$ satisfying $sin^4 x - 2sin^2 x + 1$ is

(a) 2 (b) 4 (c) 1 (d) ∞

8. If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive root, if and only if

(a) $a \geq 0$ (b) $a > 0$ (c) $a < 0$ (d) $a \leq 0$

9. The polynomial $x^3 + 2x + 3$ has

(a) one negative and two real roots (b) one positive and two imaginary roots

(c) three real roots (d) no solution

10. The number of positive roots of the polynomial $\sum_{j=0}^{n} nC_r (-1)^r x^r$ is

(a) 0 (b) n (c) $< n$ (d) r

EXERCISE 4.6

Choose the correct or the most suitable answer from the given four alternatives.

1. The value of $sin^{1}(\cos x), 0 \leq x \leq \pi$ is

(a) $\pi - x$ (b)$x - \frac{\pi}{2}$ (c) $\frac{\pi}{2} - x$ (d) $\pi + x$

2. If $sin^{-1}x + sin^{-1}y = \frac{2\pi}{3}$; then $cos^{-1}x + cos^{-1}y$ is equal to

(a)$\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) π

3. $sin^{-1}\frac{3}{5} - cos^{-1}\frac{12}{13} + sec^{-1}\frac{5}{3} - cosec^{-1}\frac{13}{12}$ is equal to

(a) 2π (b) π (c) 0 (d) $tan^{-1}\frac{12}{65}$

4. If $sin^{-1}x = 2sin^{-1}\alpha$ has a solution, then

(a) $|\alpha| \leq \frac{1}{\sqrt{2}}$ (b) $|\alpha| \geq \frac{1}{\sqrt{2}}$ (c) $|\alpha| < \frac{1}{\sqrt{2}}$ (d) $|\alpha| > \frac{1}{\sqrt{2}}$

5. $sin^{-1}(\cos x) = \frac{\pi}{2} - x$ is valid for

(a) $-\pi \leq x \leq 0$ (b)$0 \leq x \leq \pi$ (c)$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ (d)$-\frac{\pi}{4} \leq x \leq \frac{3\pi}{4}$

6. If $sin^{-1}x + sin^{-1}y + sin^{-1}z = \frac{3\pi}{2}$, the value of $x^{2017} + y^{2018} + z^{2019} - \frac{9}{x^{101}+y^{101}+z^{101}}$ is

(a) 0 (b) 1 (c) 2 (d) 3

7. If $cot^{-1}x = \frac{2\pi}{5}$ for some $x \in R$, the value of $tan^{-1}x$ is

(a)$-\frac{\pi}{10}$ (b) $\frac{\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $-\frac{\pi}{5}$

8. The domain of the function defined by $f\ (x) = sin^{-1}\sqrt{x-1}$ is

(a) $[1,2]$ (b) $[-1,1]$ (c) $[0,1]$ (d) $[-1,0]$

9 If $x = \frac{1}{5}$, the value of $\cos(cos^{-1}x + 2\ sin^{-1}x)$ is

(a) $-\sqrt{\frac{24}{25}}$ (b) $\sqrt{\frac{24}{25}}$ (c) $\frac{1}{5}$ (d)$-\frac{1}{5}$

10. $tan^{-1}\left(\frac{1}{4}\right) + tan^{-1}\left(\frac{2}{9}\right)$ is equal to

(a)$\frac{1}{2}cos^{-1}\left(\frac{3}{5}\right)$ (b) $\frac{1}{2}sin^{-1}\left(\frac{3}{5}\right)$ (c)$\frac{1}{2}tan^{-1}\left(\frac{3}{5}\right)$ (d) $tan^{-1}\left(\frac{1}{2}\right)$

11. If the function $f(x) = sin^{-1}(x^2 - 3)$, then x belongs to

(a) $[-1,1]$ (b) $[\sqrt{2},2]$ (c) $[-2,-\sqrt{2}] \cup [\sqrt{2},2]$ (d) $[-2,-\sqrt{2}] \cap [\sqrt{2},2]$

12. If $cot^{-1}2$ and $cot^{-1}3$ are two angles of a triangle, then the third angle is

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

13. $sin^{-1}\left(\tan\frac{\pi}{4}\right) - sin^{-1}\left(\sqrt{\frac{3}{x}}\right) = \frac{\pi}{6}$. Then x is a root of the equation

(a) $x^2 - x - 6 = 0$ (b) $x^2 - x - 12 = 0$ (c) $x^2 + x - 12 = 0$ (d) $x^2 + x - 6 = 0$

14. $sin^{-1}(2cos^2x - 1) + cos^{-1}(1 - 2sin^2x) =$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

15. If $cot^{-1}(\sqrt{\sin\alpha}) + tan^{-1}(\sqrt{\sin\alpha}) = u$, then $\cos 2u$ is equal to

(a) $tan^2\alpha$ (b)0 (c)-1 (d) $\tan 2\alpha$

16. If $|x| \leq 1$, then $2tan^{-1}x - sin^{-1}\frac{2x}{1+x^2}$ is equal to

(a) $tan^{-1}x$ (b) $sin^{-1}x$ (c) 0 (d) π

17. The equation $tan^{-1}x - cot^{-1}x = tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has

(a) no solution (b) unique solution (c) two solutions (d) infinite number of solutions

18. If $sin^{-1}x + cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then x is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{5}}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{\sqrt{3}}{2}$

19. If $sin^{-1}\left(\frac{x}{5}\right) + cosec^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, then the value of x is

(a) 4 (b) 5 (c) 2 (d) 3

20. $\sin(tan^{-1}x), |x| < 1$ is equal to

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

EXERCISE 5.6

Choose the most appropriate answer.

1. The equation of the circle passing through (1,5) and (4,1) and touching y -axis is

$x^2+y^2-5x-6y+9+\lambda(4x+3y\pm 19)=0$ where λ is equal to

(a) $0,-\frac{40}{9}$ (b) 0 (c) $\frac{40}{9}$ (d) $-\frac{40}{9}$

2. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is

(a) $\frac{4}{3}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{3}{2}$

3. The circle $x^2+y^2=4x+8y+5$ intersects the line $3x-4y=m$ at two distinct points if

(a) $15<m<65$ (b) $35<m<85$ (c) $-85<m<-35$ (d) $-35<m<15$

4. The length of the diameter of the circle which touches the x -axis at the point (1,0) and passes through the point (2,3).

(a) $\frac{6}{5}$ (b) $\frac{5}{3}$ (c) $\frac{10}{3}$ (d) $\frac{3}{5}$

5. The radius of the circle $3x^2+by^2+4bx-6by+b^2=0$ is

(a) 1 (b) 3 (c) $\sqrt{10}$ (d) $\sqrt{11}$

6. The centre of the circle inscribed in a square formed by the lines $x^2-8x-12=0$ and $y^2-14y+45=0$ is

(a) (4,7) (b) (7,4) (c) (9,4) (d) (4,9)

7. The equation of the normal to the circle $x^2+y^2-2x-2y+1=0$ which is parallel to the line $2x+4y=3$ is

(a) $x+2y=3$ (b) $x+2y+3=0$ (c) $2x+4y+3=0$ (d) $x-2y+3=0$

8. If $P(x,y)$ be any point on $16x^2+25y^2=400$ with foci $F_1(3,0)$ and $F_2(-3,0)$ then PF_1+PF_2 is

(a) 8 (b) 6 (c) 10 (d) 12

9. The radius of the circle passing through the point (6,2) two of whose diameter are $x+y=6$ and $x+y=24$ is

(a) 10 (b) $2\sqrt{5}$ (c) 6 (d) 4

10. The area of quadrilateral formed with foci of the hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ is

(a) $4(a^2+b^2)$ (b) (a^2+b^2) (c) a^2+b^2 (d) $\frac{1}{2}(a^2+b^2)$

11. If the normals of the parabola $y^2=4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2+(y+2)^2=r^2$, then the value of r^2 is

(a) 2 (b) 3 (c) 1 (d) 4

12. If $x+y=k$ is a normal to the parabola $y^2=12x$, then the value of k is

(a) 3 (b) -1 (c) 1 (d) 9

13. The ellipse $E_1: \frac{x^2}{9}+\frac{y^2}{4}=1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0,4) circumscribes the rectangle R . The eccentricity of the ellipse is

(a) $\frac{\sqrt{2}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

14. Tangents are drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ parallel to the straight line $2x-y=1$. One of the points of contact of tangents on the hyperbola is

(a) $\left(\frac{9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ (b) $\left(\frac{-9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (c) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (d) $\left(3\sqrt{3}, -2\sqrt{2}\right)$

15. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ having centre at (0,3)is

(a)$x^2 + y^2 - 6y - 7 = 0$ (b) $x^2 + y^2 - 6y + 7 = 0$ (c) $x^2 + y^2 - 6y - 5 = 0$ (d) $x^2 + y^2 - 6y + 5 = 0$

16. Let *C* be the circle with centre at (1,1) and radius =1. If *T* is the circle centered at (0,y) passing through the origin and touching the circle *C* externally, then the radius of *T* is equal to

(a)$\frac{\sqrt{3}}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

17. Consider an ellipse whose centre is of the origin and its major axis is along *x*-axis. If its eccentrcity is $\frac{3}{5}$ and the distance between its foci is 6, then the area of the quadrilateral inscribed in the ellipse with diagonals as major and minor axis of the ellipse is

(a) 8 (b) 32 (c) 80 (d) 40

18. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

(a) $2ab$ (b) ab (c) $\sqrt{ab}$ (d) $\frac{a}{b}$

19. An ellipse has *OB* as semi minor axes, *F* and *F'* its foci and the angle ***FBF'*** is a right angle. Then the eccentricity of the ellipse is

(a)$\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{\sqrt{3}}$

20. The eccentricity of the ellipse $(x-3)^2 + (y-4)^2 = \frac{y^2}{9}$ is

(a)$\frac{\sqrt{3}}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{3\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

21. If the two tangents drawn from a point *P* to the parabola $y^2 = 4x$ are at right angles then the locus of *P* is

(a) $2x + 1 = 0$ (b) $x = -1$ (c) $2x - 1 = 0$ (d) $x = 1$

22. The circle passing through (-1,2) and touching the axis of *x* at (3,0) passing through the point

(a) (-5, 2) (b) (2,-5) (c) (5, -2) (d) (-2,5)

23. The locus of a point whose distance from (-2,0) is $\frac{2}{3}$ times its distance from the line $x = \frac{-9}{2}$

(a) a parabola (b) a hyperbola (c) an ellipse (d) a circle

24. The values of *m* for which the line $y = mx + 2\sqrt{5}$ touches the hyperbola $16x^2 - 9y^2 = 144$ are the roots of , then the value of (a+b) is

(a) 2 (b) 4 (c) 0 (d) -2

25. If the coordinates at one end of a diameter of the circle $x^2 + y^2 - 8x - 4y + c = 0$ are (11,2) the coordinates of the other end are

(a) (-5, 2) (b) (2,-5) (c) (5, -2) (d) (-2,5)

EXERCISE 6.10

Choose the correct or most suitable answer :

1. If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $[\vec{a}, \vec{b}, \vec{c}]$ is equal to

(a) 2 (b) -1 (c) 1 (d) 0

2. If a vector $\vec{\alpha}$ lies in the plane of $\vec{\beta}$ and $\vec{\gamma}$, then

(a) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 1$ (b) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = -1$ (c) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 0$ (d) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 2$

3. If $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$ then the value of $[\vec{a}, \vec{b}, \vec{c}]$ is

(a) $|\vec{a}\,|\,|\vec{b}\,|\,|\vec{c}\,|$ (b) $\frac{1}{3}|\vec{a}\,|\,|\vec{b}\,|\,|\vec{c}\,|$ (c) 1 (d) -1

4. If $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that $\vec{a}$ is perpendicular to $\vec{b}$, and is parallel to $\vec{c}$ then $\vec{a} \times (\vec{b} \times \vec{c})$ is equal to

(a) $\vec{a}$ (b) $\vec{b}$ (c) $\vec{c}$ (d) $\vec{0}$

5. If $[\vec{a}, \vec{b}, \vec{c}] = 1$ then the value of $\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a})\cdot\vec{b}} + \frac{\vec{b}\cdot(\vec{c}\times\vec{a})}{(\vec{a}\times\vec{b})\cdot\vec{c}} + \frac{\vec{c}\cdot(\vec{a}\times\vec{b})}{(\vec{c}\times\vec{b})\cdot\vec{a}}$ is

(a) 1 (b) -1 (c) 2 (d) 3

6. The volume of the parallelepiped with its edges represented by the vectors $\hat{\imath} + \hat{\jmath}, \hat{\imath} + 2\hat{\jmath}, \hat{\imath} + \hat{\jmath} + \pi\hat{k}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{\pi}{4}$

7. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a}, \vec{b}, \vec{a} \times \vec{b}] = \frac{\pi}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

8. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + \hat{\jmath}$, $\vec{c} = \hat{\imath}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda\vec{a} + \mu\vec{b}$ then the value of $\lambda +$ is

(a) 0 (b) 1 (c) 6 (d) 3

9. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non-zero vectors such that $[\vec{a}, \vec{b}, \vec{c}] = 3$ then $\{[\vec{a} \times \vec{b}, \ \vec{b} \times \vec{c}, \ \vec{c} \times \vec{a}]\}^2$ is equal to

(a) 81 (b) 9 (c) 27 (d)18

10. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b} + \vec{c}}{\sqrt{2}}$ then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{4}$ (d) π

11. If the volume of the parallelepiped with $\vec{a} \times \vec{b}$, $\vec{b} \times \vec{c}$, $\vec{c} \times \vec{a}$ as coterminous edges is 8 cubic units, then the volume of the parallelepiped with $(\vec{a} \times \vec{b}) \times (\vec{b} \times \vec{c})$, $(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})$ and $(\vec{c} \times \vec{a}) \times (\vec{a} \times \vec{b})$ as coterminous edges is,

(a) 8 cubic units (b) 512 cubic units (c) 64 cubic units (d) 24 cubic units

12. Consider the vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ such that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$. Let P_1 and P_2 be the planes determined by the pairs of vectors $\vec{a}, \vec{b}$ and $\vec{c}, \vec{d}$ respectively. Then the angle between P_1 and P_2 is

(a) 0° (b) 45° (c) 60° (d) 90°

13. If $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$ where $\vec{a}, \vec{b}, \vec{c}$ are any three vectors such that $\vec{b} \cdot \vec{c} \neq 0$ and $\vec{a} \cdot \vec{b} \neq 0$ then $\vec{a}$□and $\vec{c}$ are

(a) perpendicular (b) parallel (c) inclined at an angle $\frac{\pi}{3}$ (d) inclined at an angle $\frac{\pi}{6}$

14. If $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{c} = 3\hat{i} + 5\hat{j} - \hat{k}$ then a vector perpendicular to $\vec{a}$ and lies in the plane containing $\vec{b}$ and $\vec{c}$ is

(a) $-17\hat{i} + 21\hat{j} - 97\hat{k}$ (b) $17\hat{i} + 21\hat{j} - 123\hat{k}$ (c) $-17\hat{i} - 21\hat{j} + 97\hat{k}$ (d) $-17\hat{i} - 21\hat{j} - 97\hat{k}$

15. The angle between the lines $\frac{x-2}{3} = \frac{y+1}{-2}, z = 2$ and $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

16. If the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x + 3y - \alpha z + \beta = 0$ then (α, β) is

(a) (-5,5) (b) (-6, 7) (c) (5, -5) (d) (6, -7)

17. The angle between the line $\vec{r} = (\hat{i} + 2\hat{j} - 3\hat{k}) + t(2\hat{i} + \hat{j} - 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} + \hat{j}) + 4 = 0$ is

(a) 0° (b) 30° (c) 45° (d) 90°

18. The coordinates of the point where the line $\vec{r} = (6\hat{i} - \hat{j} - 3\hat{k}) + t(-\hat{i} + 4\hat{k})$ meets the plane $\vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) = 3$ are

(a) (2, 1, 0) (b) (7, -1, -7) (c) (1, 2, -6) (d) (5, -1, 1)

19. Distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is

(a) 0 (b) 1 (c) 2 (d) 3

20. The distance between the planes $x + 2y + 3z + 7 = 0$ and $2x + 4y + 6z + 7 = 0$ is

(a) $\frac{\sqrt{7}}{2\sqrt{2}}$ (b) $\frac{7}{2}$ (c) $\frac{\sqrt{7}}{2}$ (d) $\frac{7}{2\sqrt{2}}$

21. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$ then

(a) $c = \pm 3$ (b) $c = \pm\sqrt{3}$ (c) c > 0 (d) 0 < c < 1

22. The vector equation $\vec{r} = (\hat{i} - 2\hat{j} - \hat{k}) + t(6\hat{i} - \hat{k})$ represents a straight line passing through the points

(a) (0, 6, -1) and (1, -2, -1) (b) (0, 6, -1) and (1, -4, -2) (c) (1, -2, -1) and (1, 4, -2) (d) (1, -2, -1) and (0, -6, 1)

23. If the distance of the point (1, 1, 1) from the origin is half of its distance from the plane $x + y + z + k = 0$, then the values of k are

(a) ± 3 (b) ± 6 (c) -3, 9 (d) 3, -9

24. If the planes $\vec{r} \cdot (2\hat{i} - \lambda\hat{j} + \hat{k}) = 3$ and $\vec{r} \cdot (4\hat{i} + \hat{j} - \mu\hat{k}) = 3$ are parallel, then the value of λ and μ are

(a) $\frac{1}{2}, -2$ (b) $-\frac{1}{2}, 2$ (c) $-\frac{1}{2}, -2$ (d) $\frac{1}{2}, 2$

25. If the length of the perpendicular from the origin to the plane $2x + 3y + \lambda z = 1$, $\lambda > 0$ is $\frac{1}{5}$, then the value of λ is

(a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) 0 (d) 1