UNIT -1 : ELECTROSTATICS
TWO MARK QUESTION AND ANSWERS

1. State law of conservation of electric charge.
   * The total electric charge in the universe is constant and charge can neither be created nor be destroyed.
   * In any physical process, the net change in charge will always be zero.

2. What is meant by quantization of charges?
   * The charge q on any object is equal to an integral multiple of this fundamental unit of charge e.
   * $q = ne$
   * Here n is any integer $(0, \pm 1, \pm 2, \dots)$
   * This is called quantization of electric charge.

3. State Coulomb's law.

   Coulomb's law states that the electrostatic force is directly proportional to the product of the magnitude of the two point charges and inversely proportional to the square of the distance between the two point charges.

4. Write down Coulomb's law in vector form and mention what each term represents.
   - $\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$
   - $\vec{F}_{21}$ - force on the charge q_2 exerted by the charge q_1.
   - q_1 and q_2 - two point charges.
   - r – distance between the two point charges
   - $\hat{r}_{12}$ - unit vector from q_1 to q_2.
   - ε_0 – permittivity of free space.
   - $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$

5. Define relative permittivity.
   - Relative permittivity is defined as the ratio of permittivity of the medium to the permittivity of free space.
   - $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$
   - For vacuum or air, $\varepsilon_r = 1$ and
   - For all other media $\varepsilon_r > 1$.

6. State superposition principle of electric charge.

   The total charge acting on a given charge is equal to the vector sum of forces exerted on it by all the other charges.

7. Define Electric field.
   - Electric field at a point P at a distance r from the point charge q is the force experienced by a unit charge.
   - $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$
   - It is a vector quantity.
   - Its SI unit is NC^{-1}

8. Write Coulomb's law in terms of electric field.
   - If the electric field at a point P is $\vec{E}$, then the force experienced by the test charge q_0 placed at the point P is $\vec{F} = q_0\vec{E}$
   - This is Coulomb's law in terms of electric field.

9. What is meant by Electric field lines?

   Electric field lines are the imaginary curved path along which a unit charge tends to move in an electric field.

10. The electric field lines never intersect. Justify.
    - No two electric field lines intersect each other.
    - If two lines cross at a point, then there will be two different electric field vectors at the same point.
    - As a consequence, if some charge is placed in the intersection point, then it has to move in two different directions at the same time, which is physically impossible.
    - Hence, electric field lines do not intersect.

11. Define Electric dipole.
    - Two equal and opposite charges separated by a small distance constitute an electric dipole.
    - Examples: CO, H_2O, NH_3, HCl

12. What is general definition of electric dipole moment?
    - The magnitude of the electric dipole moment is equal to the product of one of the charges and the distance between them.
    - p = q2a
    - It is a vector quantity.
    - Its SI unit is C m

13. Define electric potential difference.

    The electric potential difference is defined as the work done by an external force to bring unit positive charge form one point to another point.

14. Define electrostatic potential.

    The electric potential at a point P is equal to the work done by an external force to bring a unit positive charge with constant velocity from infinity to the point P in the region of the external electric field $\vec{E}$.

15. What is equipotential surface?

An equipotential surface is a surface on which all the points are at the same potential.

16. Give the relation between electric field and electric potential.

- The electric field is the negative gradient of the electric potential.
- $E = -\frac{dV}{dx}$
- $\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial V}{\partial z}\hat{k}\right)$

17. Define electrostatic potential energy.

The electrostatic potential energy is defined as the work done to bring a test charge from one point to another point in an electrostatic field.

18. Define electric flux.

- The number of electric field lines crossing a given area kept normal to the electric field lines is called electric flux.
- It is a scalar quantity.
- Its unit is $N\ m^2\ C^{-1}$

19. What is meant by electrostatic energy density?

- The energy stored per unit volume of space is defined as energy density.
- Energy stored is $U_E = \frac{1}{2}\varepsilon_0 (Ad)E^2$
- Energy density is $u_E = \frac{1}{2}\varepsilon_0 E^2$

20. What is dielectric or an insulator?

- A dielectric is a non-conducting material has no free electrons.
- The electrons in a dielectric are bound within the atoms.
- Examples: Ebonite, Glass, and Mica

21. What is Polarisation?

- Polarisation $\vec{P}$ is defined as the total dipole moment per unit volume of the dielectric.
- $\vec{P} = \chi_e \vec{E}_{ext}$

22. What is dielectric breakdown?

- When the external electric field applied to a dielectric is very large, it tears the atoms apart so that the bound charges become free charges.
- Then the dielectric starts to conduct electricity.
- This is called dielectric breakdown.

23. What is dielectric strength?

The maximum electric field the dielectric can withstand before it breakdowns is called dielectric strength.

24. Define capacitance. Give its unit.

- ➢ The capacitance C of a capacitor is defined as the ratio of the magnitude of charge on either of the conductor plates to the potential difference existing between the conductors.
- ➢ $C = \frac{Q}{V}$
- ➢ Its SI unit is coulomb per volt or *farad* (F)

25. What is action at points or corona discharge?

The leakage of electric charges from the sharp points of the charged conductor is called as action at points or corona discharge.

THREE MARKS

1. Discuss the basic properties of electric charges.

Electric charge:

- The electric charge is an intrinsic and fundamental property of particles.
- The SI unit of charge is coulomb.

Conservation of charges:

- The total electric charge in the universe is constant and charge can neither be created nor be destroyed.
- In any physical process, the net change in the charge will always be zero.

Quantization of charges:

- The charge q on any object is equal to an integral multiple of this fundamental unit of charge e.
- $q = ne$
- Here n is any integer $(0, \pm1, \pm2, \ldots.)$
- This is called quantization of electric charge.

2. What are the differences between Coulomb force and gravitational force?

Gravitational force	Coulomb force
It is always attractive between two masses.	It can be attractive or repulsive, depending upon the nature of charges
$G = 6.626 \times 10^{-11}\ N\ m^2 kg^{-2}$	$k = 9 \times 10^9\ N\ m^2 C^{-2}$
It is independent of the medium	It depends on nature of the medium in which the two charges are kept at rest.
It is same whether two point masses are at rest or in motion	If the charges are in motion, yet another force (Lorentz force) comes into play in addition to coulomb force.

3. Write a short note on superposition principle of electric charge.

Superposition principle of electric charge:

The total force acting on a give charge is equal to the vector sum of forces exerted on it by all the other charges.

* Consider a system of n charges, namely $q_1, q_2, q_3, \dots \dots q_n$.
* $\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r}_{21}$
* $\vec{F}_{13} = k\frac{q_1 q_3}{r^2}\hat{r}_{31}$
* By continuing this, the total force acting on the charge due to all other charges is given by
* $\vec{F}_1^{tot} = \vec{F}_{12} + \vec{F}_{13} + \dots + \vec{F}_{1n}$
* $\vec{F}_1^{tot} = \left[\frac{q_1 q_2}{r^2}\hat{r}_{21} + \frac{q_1 q_3}{r^2}\hat{r}_{31} + \dots \frac{q_1 q_n}{r^2}\hat{r}_{n1}\right]$

4. What are the properties of electric field lines?

* They start form a positive charge and end at negative charge or at infinity.
* For a positive point charge, the electric field lines pint radially outward
* For a negative point charge, the electric field lines point radially inward.
* The electric field vector at a point in space is tangential to the electric field line at that point.
* The electric field lines are denser in a region where the electric field has large magnitude and less dense in a region where the electric field is of smaller magnitude.
* No twos electric field lines intersect each other.
* The number of electric field lines that emanate from the positive charge or end at a negative charge is directly proportional to the magnitude of the charges.

5. What are the properties of an equipotential surface?

➢ The work done to move a charge q between any two points A and B, $W = q(V_B - V_A)$

➢ If the points A and B lie on the same equipotential surface, work done is zero because $V_A = V_B$

➢ The electric field is normal to an equipotential surface.

6. Discuss the various properties of conductors in electrostatic equilibrium.

* The electric field is zero everywhere inside the conductor. This is true regardless of whether the conductor is solid or hollow.
* There is no net charge inside the conductors. The charges must reside only on the surface of the conductors.
* The electric field outside the conductor is perpendicular to the surface of the conductor and has a magnitude of $\frac{\sigma}{\varepsilon_0}$
* The electrostatic potential has the same value on the surface and inside of the conductor.

7. Write a short note on electrostatic shielding.

* Consider a cavity inside the conductor
* Whatever the charges at the surfaces and whatever the electrical disturbances outside, the electric field inside the cavity is zero.
* A sensitive electrical instrument which is to be protected from external electrical disturbance is kept inside this cavity.
* This is called electrostatic shielding.
* Faraday cage is an instrument used to demonstrate this effect.

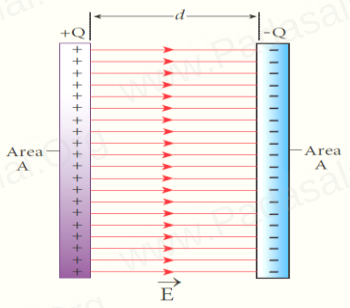
8. Differentiate between non-polar and polar molecules.

Non – polar molecules	Polar molecules
In polar molecules, the centers of positive and negative charges coincide.	In polar molecules, the centers of the positive and negative charges are separated even in the absence of external electric field.
They have no permanent dipole moment.	They have a permanent dipole moment.
Ex: H_2, O_2, CO_2	Ex: H_2O, N_2O, HCl

9. Obtain an expression for the capacitance of a parallel plate capacitor.

- Consider a capacitor with two parallel plates each of cross-sectional area A and separated by a distance d.
- The electric field between two infinite parallel plates is uniform and is given by $E = \frac{\sigma}{\varepsilon_0}$
- Where σ is the surface charge density, $\sigma = \frac{Q}{A}$
- If d^2 << A, then the above result is used even for finite – sized parallel plate capacitor.
- The electric field between the plates is $E = \frac{Q}{A\varepsilon_0}$
- The electric potential between the plates is $V = Ed = \frac{Qd}{A\varepsilon_0}$

- The capacitance , $C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$
- $C \propto A$
- $C \propto \frac{1}{d}$



10. Obtain the expressions for the energy stored in the capacitor and energy density.
   - Capacitor not only stores the charge but also it stores energy.
   - When a battery is connected to the capacitor, electrons fo total charge –Q are transferred from one plate to the other plate.
   - To transfer the charge, work is done by the battery.
   - This work done is stored as electrostatic potential energy in the capacitor.
   - To transfer charge dQ for a potential difference V, dW = V dQ
   - Where V = Q / C
   - $W = \int_0^Q \frac{Q}{C} dQ = \frac{Q^2}{2C}$
   - This work done is stored as electrostatic potential energy (U_E) in the capacitor.
   - $U_E = \frac{Q^2}{2C} = \frac{1}{2} CV^2$
   - This stored energy is directly proportional to the capacitance of the capacitor and the square of the voltage between the plates of the capacitor.
   - As $C = \frac{\varepsilon_0 A}{d}$ and V = Ed
   - $U_E = \frac{1}{2}\varepsilon_0 (Ad) E^2$
   - $Ad = volume$ of the space between the capacitor plates.
   - The energy stored per unit volume of space is defined as energy density, $u_E = \frac{U_E}{Volume}$
   - The energy density, $u_E = \frac{1}{2}\varepsilon_0 E^2$
   - From the above equation, the energy is stored in the electric field existing between the plates of the capacitor.
   - The energy density depends only on the electric field and not on the size of the plates of the capacitor.

11. What are the applications of capacitor?
   - Capacitors are used as flash capacitors in digital camera to release the energy as flash.
   - Capacitors are used in heart defibrillator device to give a sudden surge of a large amount electrical energy to the patient’s chest to retrieve the normal heart function.
   - Capacitors are used in the ignition system of automobile engines to eliminate sparking.
   - Capacitors are used to reduce power fluctuations in power supplies and to increase the efficiency of power transmission.

12. Is it safer to be sit inside a bus than standing under a tree during lightning accompanied by thundering?
   - During lightning accompanied by a thunderstorm, it is always safer to sit inside a bus than in open ground or under a tree.
   - The metal body of the bus provides electrostatic shielding.
   - Since, the electric field inside is zero.
   - During lightning, the charges flow through the body of the conductor to the ground with no effect on the person inside that bus.

13. Write a short note on lightning arrester or lightning conductor.
   - Lightning arrester is a device used to protect tall buildings from lightning strikes.
   - It works on the principle of action at points or corona discharge.
   - This device consists of a long thick copper rod passing from top of the building to the ground.
   - The upper end of the rod has a sharp spike or a sharp needle.
   - The lower ned of the rod is connected to the copper plate which is buried deep into the ground.
   - When a negatively charged cloud is passing above the building, it induces a positive charge on the spike.
   - Since the induced charge density on thin sharp spike is large, it results in a corona discharge.
   - This positive charge ionizes the surrounding air which in turn neutralizes the negative charge in the cloud.
   - The negative charge pushed to the spikes passes through the copper rod and is sagely diverted to the Earth.
   - The lightning arrester does not stop the lightning: rather it diverts the lightning to the ground safely.

FIVE MARKS

1. Explain in detail Coulomb's law and its various aspects.

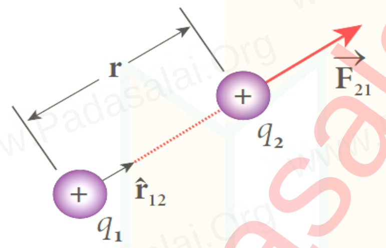
Coulomb's law:

Coulomb's law states that the electrostatic force is directly proportional to the product of the magnitude of the two point charges and is inversely proportional to the square of the distance between the two point charges.

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

Important aspects of Coulomb's law:

- The force on the charge q_2 exerted by the charge q_1 always lies along the line joining the two charges.
- $\hat{r}_{12}$ is the unit vector pointing from charge q_1 to q_2.
- Likewise, the force on the charge q_1 exerted by q_2 is along $-\hat{r}_{12}$
- In SI units, $k = \frac{1}{4\pi\varepsilon_0}$ and its value is $9 \times 10^9\ \mathrm{N\,m^2C^{-2}}$.
- Here ε_0 is the permittivity of fee space or vacuum and the value of $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$
- The magnitude of the electrostatic force between two charges each of one coulomb and separated by a distance of 1 m is calculated as follows:
- $F = \frac{9 \times 10^9 \times 1 \times 1}{1^2} = 9 \times 10^9$ N
- This is a huge quantity, almost equivalent to the weight of one million ton.
- We never come across 1 coulomb of charge in practice.
- Most of the electrical phenomena in day – to – day life involve electrical charges of the order of μC or nC.
- In SI units, Coulomb's law in vacuum takes the form $\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$.
- In a medium of permittivity ε, the force between two point charges is given by $\vec{F}_{21} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}\hat{r}_{12}$



- Since $\varepsilon > \varepsilon_0$, the force between two point charges in a medium other than vacuum is always less than that in vacuum.
- Relative permittivity, $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$
- For vacuum or air, $\varepsilon_r = 1$
- For all other media, $\varepsilon_r > 1$
- Coulomb's law has same structure as Newton's law of gravitation.
- Both are inversely proportional to the square of the distance between the particles.
- The electrostatic force is directly proportional to the product of the magnitude of two point charges and gravitational force is directly proportional to the product of two masses.
- The electrostatic force obeys Newton's third law. $\vec{F}_{12} = -\vec{F}_{21}$
- The expression for Coulomb force is true only for point charges.
- But the point charge is an ideal concept.
- However we can apply Coulomb's law for two charged objects whose sizes are very much smaller than the distance between them.
- In fact, Coulomb discovered his law by considering the charged spheres in the torsion balance as point charges.
- The distance between the two charged spheres is much greater than the radii of the spheres.

2. Define electric field and discuss its various aspects.

Electric field:

- The electric field at the point P at a distance r form the point charge q is the force experienced by a unit charge and is given by $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$
  - It is a vector quantity
  - Its SI unit is NC^{-1}

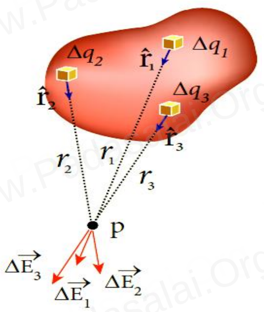
Important aspects of Electric field:

* If the charge q is positive then the electric filed points away from the source charge
* If q is negative, the electric filed points towards the source charge q.
* If the electric field at a point P is $\vec{E}$, then the force experienced by the test charge q_0 placed at the point P is $\vec{F} = q_0\,\vec{E}$
* This is the Coulomb's law in terms of electric filed.
* The electric field is independent of the test charge q_0 and it depends only on the source charge q.

* As distance increases, the electric filed decreases in magnitude.
* Test charge is made sufficiently small so such that it will not modify the electric field of the source charge.
* For continuous and finite charge distributions, integration techniques must be used.
* There are two kinds of electric field: uniform electric field and non – uniform electric field.
* Uniform electric filed will have the same direction and constant magnitude at all points in space.
* Non – uniform electric field will have different directions or different magnitudes or both at different points in space.
* This non – uniformity arises, both in direction and magnitude, with the direction being radially outward or inward and the magnitude changes as distance increases.

3. How do we determine the electric field due to a continuous charge distribution? Explain.

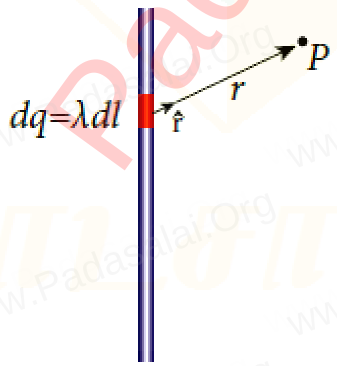
- Consider the following charged object of irregular shape as shown in figure.



- The entire charged object is divided into a large number of charge elements $\Delta q_1, \Delta q_2, \Delta q_3 \ldots\ldots \Delta q_n$, and each charge element Δq is taken as point charge.
- The electric field at a point P is given by $\vec{E} \approx \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n} \frac{\Delta q_i}{r_{iP}^2}\,\hat{r}_{iP}$ ------------- (1)
- However the above equation is only an approximation.
- To incorporate the continuous distribution of charge, we take the limit $\Delta q \rightarrow 0$.
- Equation (1) now becomes, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\,\hat{r}$

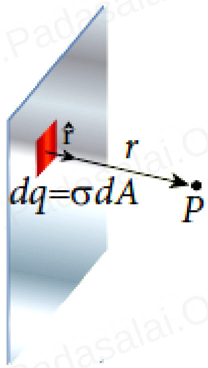
Linear charge distribution:

- Linear charge density is defined as the charge per unit length.
- $\lambda = \frac{Q}{L}$
- Its unit is C m^{-1}
- The charge present in the infinitesimal length dl is $dq = \lambda dl$
- The electric field due to the line of total charge Q is given by $\vec{E} = \frac{\lambda}{4\pi\varepsilon_0}\int \frac{dl}{r^2}\,\hat{r}$



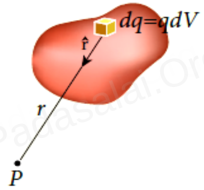
Surface charge distribution:

- Surface charge density is defined as the charge per unit area.
- $\sigma = \frac{Q}{A}$
- Its unit is C m^{-2}
- The charge present in the infinitesimal area dA is $dq = \sigma dA$
- The electric field due to the line of total charge Q is given by $\vec{E} = \frac{\sigma}{4\pi\varepsilon_0}\int \frac{dA}{r^2}\,\hat{r}$



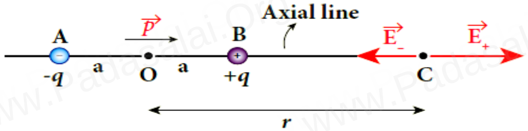
Volume charge distribution:

- Volume charge density is defined as the charge per unit volume.
- $\rho = \frac{Q}{V}$
- Its unit is C m^{-3}
- The charge present in the infinitesimal volume element dV is $dq = \rho dV$
- The electric field due to the line of total charge Q is given by $\vec{E} = \frac{\rho}{4\pi\varepsilon_0}\int \frac{dV}{r^2}\,\hat{r}$

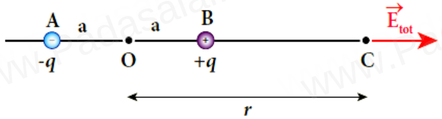


4. Calculate the electric field due to an electric dipole at points on its axial line.

- AB is an electric dipole.
- A point C is located at a distance r from the midpoint O of the dipole along the axial line.



- $\vec{E}_+ = k\,\dfrac{q}{(r-a)^2}\,\hat{p}$ ($\hat{p}$ acts along BC)
- $\vec{E}_- = -k\,\dfrac{q}{(r+a)^2}\,\hat{p}$
- $\vec{E}_{tot} = \vec{E}_+ + \vec{E}_-$
- $\vec{E}_{tot} = kq\left[\dfrac{1}{(r-a)^2} - \dfrac{1}{(r+a)^2}\right]\hat{p}$
- $\vec{E}_{tot} = kq\left[\dfrac{4ra}{(r^2-a^2)^2}\right]\hat{p}$
- The total electric field is along $\vec{E}_+$, since +q is closer to C than –q.



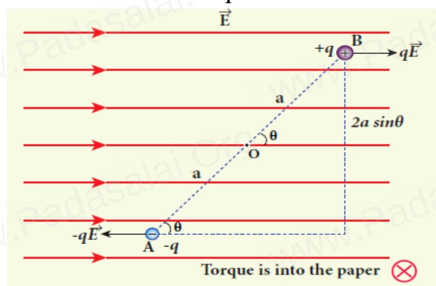
- If r >> a, then $(r^2 - a^2)^2 = r^4$
- $\vec{E}_{tot} = k\left(\dfrac{4aq}{r^3}\right)\hat{p} \qquad (r \gg a)$
-
- $2aq\,\hat{p} = \vec{p}$ and $k = \dfrac{1}{4\pi\varepsilon_0}$
- $\vec{E}_{tot} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2\vec{p}}{r^3} \qquad (r \gg a)$

5. Calculate the electric field due to an electric dipole at points on its axial line.

- AB is an electric dipole.
- A point C is located at a distance r from the midpoint O of the dipole on the equatorial plane.
- $\vec{E}_+ = k\,\dfrac{q}{(r^2+a^2)}$ (along BC)
- $\vec{E}_- = k\,\dfrac{q}{(r^2+a^2)}$ (along CA)
- $\vec{E}_+$ and $\vec{E}_-$ are resolved into two components:
- Perpendicular components $|\vec{E}_+|\sin\theta$ and $|\vec{E}_-|\sin\theta$ are oppositely directed and cancel each other.
- The total electric field is due to the sum of the parallel components $|\vec{E}_+|\cos\theta$ and $|\vec{E}_-|\cos\theta$
- The direction of total electric field is along $-\hat{p}$
- $\vec{E}_{tot} = -|\vec{E}_+|\cos\theta\;\hat{p} - |\vec{E}_-|\cos\theta\;\hat{p}$
- $\vec{E}_{tot} = -2|\vec{E}_+|\cos\theta\;\hat{p} \quad (\because |\vec{E}_+| = |\vec{E}_-|)$
- $\vec{E}_{tot} = -k\,\dfrac{2q\cos\theta}{(r^2+a^2)}\,\hat{p}$
- $\vec{E}_{tot} = -k\,\dfrac{2qa}{(r^2+a^2)^{\frac{3}{2}}}\,\hat{p}\left(\because \cos\theta = \dfrac{a}{\sqrt{r^2+a^2}}\right)$
- $\vec{E}_{tot} = -k\,\dfrac{\vec{p}}{(r^2+a^2)^{\frac{3}{2}}} \quad (\because \vec{p} = 2qa\hat{p})$
- At very large distances, r >> a
- $\vec{E}_{tot} = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{\vec{p}}{r^3} \quad (r \gg a)\left(k = \dfrac{1}{4\pi\varepsilon_0}\right)$

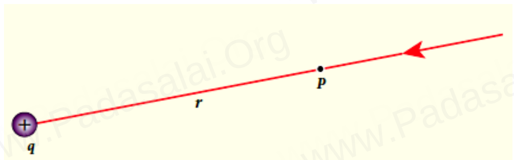
6. Derive an expression for the torque experienced by a dipole due to a uniform electric field.

* Consider an electric dipole of dipole moment $\vec{p}$ placed in a uniform electric field $\vec{E}$.
* The charge +q will experience a force $q\vec{E}$ in the direction of the field.
* The charge -q will experience a force $q\vec{E}$ in the direction opposite to the field.
* Since the external field is uniform, the total force acting on the dipole is zero.
* These two forces acting at different points will constitute a couple and the dipole experience a torque.
* This torque tends to rotate the dipole.
* The total torque is perpendicular to the plane of the paper and is directed into it.
* $\vec{\tau} = \overrightarrow{OA}$ x $(-q\vec{E}) + \overrightarrow{OB}$ x $(q\vec{E})$
* $\tau = (OA)(qE)\sin\theta + (OA)(qE)\sin\theta$
* $\tau = 2qaE\sin\theta$
* $\tau = pE\sin\theta$
* $\vec{\tau} = \vec{p}$ x $\vec{E}$
* Torque is maximum when $\theta = 90°$
* Torque is zero when $\theta = 0°$
* If the electric field is not uniform, then there will be net force acting on the dipole in addition to the torque.

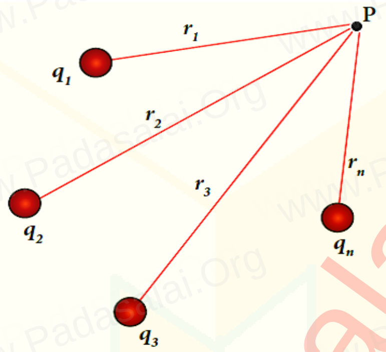


7. Derive an expression for electrostatic potential due to a point charge.

* Consider a positive charge q kept fixed at the origin.
* Let P be a point at distance r from the charge q.
* The electric potential at P is $V = -\int_{\infty}^{r} \vec{E}.d\vec{r}$
* $\vec{E} = k\frac{q}{r^2}\hat{r}$
* $V = -k\int_{\infty}^{r} \frac{q}{r^2}\,\hat{r}.d\vec{r}$
* $V = -k\int_{\infty}^{r} \frac{q}{r^2}\,dr \quad (\because \hat{r}.d\vec{r} = dr)$
* After integration, $V = k\frac{q}{r}$
* $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad \left(\because k = \frac{1}{4\pi\varepsilon_0}\right)$
* For positive charge, V > 0
* For negative charge, V < 0
* For negative charge, $V = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$
* The potential due to positive charge decreases as the distance increases.
* The potential due to negative charge increases as the distance is increased.
* At infinity, electrostatic potential is zero.
* A positive charge moves from a point of higher electrostatic potential to lower electrostatic potential.
* A negative charge moves from a point of lower electrostatic potential to higher electrostatic potential.



* Potential due to collection of point charges is
* $V_{tot} = V_1 + V_2 + V_3 + \cdots + V_4$
* $V_{tot} = k\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} \ldots\ldots + \frac{q_n}{r_n}\right]$
* $V_{tot} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$



8. Derive an expression for electrostatic potential due to an electric dipole.

* Consider two equal and opposite charges separated by a small distance $2a$.
* The point P is located at a distance r from the midpoint of the dipole.
* Let θ be the angle between the line OP and dipole axis AB.
* Potential at P due to +q, $V_1 = k\frac{q}{r_1}$
* Potential at P due to - q, $V_2 = -k\frac{q}{r_2}$
* Total potential at P, $V = kq\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$
* Suppose if the point P is far away from the dipole, such that r >> a, then equation can be expressed in terms of r.

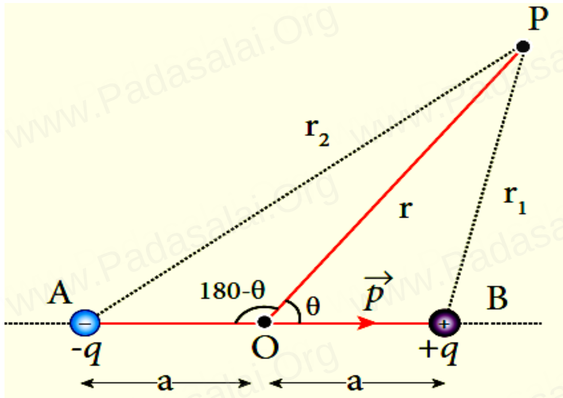
calculation for $\frac{1}{r_1}$	calculation for $\frac{1}{r_2}$
By the cosine law for triangle BOP	By the cosine law for triangle AOP
$r_1^2 = r^2 + a^2 - 2ra\cos\theta$	$r_2^2 = r^2 + a^2 + 2ra\cos\theta$
$r_1^2 = r^2\left(1 + \frac{a^2}{r^2} - \frac{2a}{r}\cos\theta\right)$	$r_2^2 = r^2\left(1 + \frac{a^2}{r^2} + \frac{2a}{r}\cos\theta\right)$
Neglecting $\frac{a^2}{r^2}$ since r >> a	Neglecting $\frac{a^2}{r^2}$ since r >> a
$r_1^2 = r^2\left(1 - \frac{2a}{r}\cos\theta\right)$	$r_2^2 = r^2\left(1 + \frac{2a}{r}\cos\theta\right)$
$r_1 = r\left(1 - \frac{2a}{r}\cos\theta\right)^{\frac{1}{2}}$	$r_2 = r\left(1 + \frac{2a}{r}\cos\theta\right)^{\frac{1}{2}}$
$\frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{2a}{r}\cos\theta\right)^{-\frac{1}{2}}$	$\frac{1}{r_2} = \frac{1}{r}\left(1 + \frac{2a}{r}\cos\theta\right)^{-\frac{1}{2}}$
Using Binomial theorem	Using Binomial theorem
$\frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{a}{r}\cos\theta\right)$	$\frac{1}{r_2} = \frac{1}{r}\left(1 - \frac{a}{r}\cos\theta\right)$

* $V = kq\left[\frac{1}{r}\left(1 + \frac{a}{r}\cos\theta\right) - \frac{1}{r}\left(1 - \frac{a}{r}\cos\theta\right)\right]$
* $V = k\frac{2aq\cos\theta}{r^2}$
* $V = k\frac{p\cos\theta}{r^2} \qquad (\because p = q2a)$

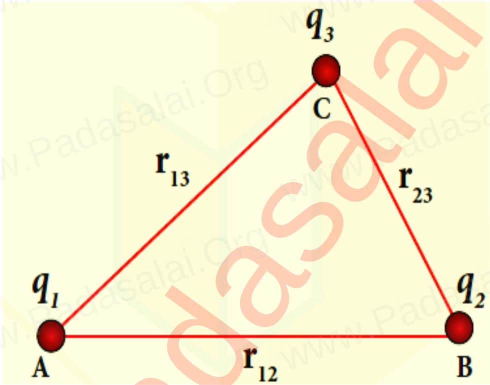
- $V = \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{r^2}$ $\left(\because k = \frac{1}{4\pi\varepsilon_0}\right)$
- $V = \frac{1}{4\pi\varepsilon_0} \frac{\vec{p}.\hat{r}}{r^2}$ $(r >> a)$ $(\because p\cos\theta = \vec{p}.\hat{r})$

Special cases:

- If $\theta = 0°$, then $V = \frac{1}{4\pi\varepsilon_0} \frac{p}{r^2}$
- If $\theta = 180°$, then $V = -\frac{1}{4\pi\varepsilon_0} \frac{p}{r^2}$
- If $\theta = 90°$, then $V = 0$



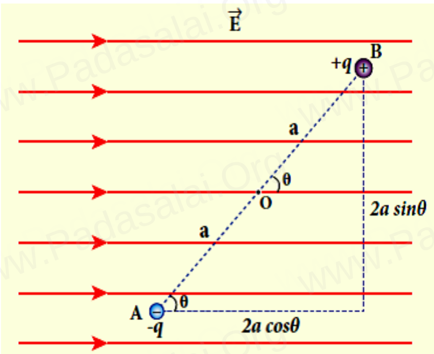
9. Obtain an expression for potential energy due to a collection of three point charges which are separated by finite distances.
   - The electric potential at a point at a distance r from point charge q_1 is given by $V = k\frac{q_1}{r}$
   - This potential V is the work done to bring a unit positive charge from infinity to the point.
   - Now if the charge q_2 is brought from infinity to that point at a distance r from q_1, then $W = q_2V$
   - This work done is stored as the electrostatic potential energy U of a system of charges q_1 *and* q_2 separated by a distance r.
   - $U = q_2V = k\frac{q_1q_2}{r^2}$
   - The electrostatic potential energy depends only on the distance between the two point charges.
   - Three charges are arranged in the following configuration as shown in Figure.



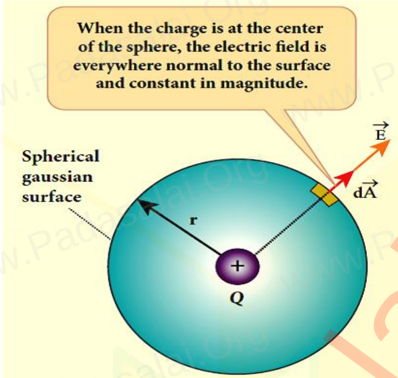
   - To calculate the total electrostatic potential energy, we use the following procedure.
   - Bringing a charge q_1 from infinity to the point A requires no work, because there are no other charges already present in the vicinity of charge q_1.
   - To bring the second charge q_2 to the point B, work must be done against the electric field created by the charge q_1.
   - $W = q_2 \ V_{1B}$
   - $U = k\frac{q_1q_2}{r_{12}}$
   - Similarly to bring the charge q_3 to the point C, work has to be done against the total electric field due to both charges q_1 *and* q_2.
   - $W = q_3(V_{1C} + V_{2C})$
   - $U = k\left(\frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$
   - The total electrostatic potential energy for the system of three charges is
   - $U = k\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$
   - This total potential energy U is equal to the total external work done to assemble the three charges at the given locations.

10. Derive an expression for electrostatic potential energy of the dipole in a uniform electric field.
   - Consider a dipole placed in the uniform electric field $\vec{E}$.
   - A dipole experiences a torque when kept in an uniform electric field $\vec{E}$.
   - This torque rotates the dipole to align it with the direction of the electric field.
   - To rotate the dipole form its initial angle θ' to another angle θ against the torque exerted by the electric field, an equal and opposite external torque must be applied on the dipole.
   - The work done by the external torque to rotate the dipole from angle θ' to θ to at constant angular velocity is $W = \int_{\theta'}^{\theta} \tau_{ext} \, d\theta$
   - $\vec{\tau}_E = \vec{p} \text{ x } \vec{E}$
   - $\tau_{ext} = \tau_E = |\vec{p} \text{ x } \vec{E}| = pE\sin\theta$

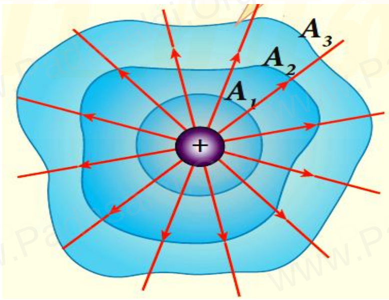
- $W = \int_{\theta'}^{\theta} pE\sin\theta \, d\theta$
- $W = pE(\cos\theta' - \cos\theta) d\theta$
- This work done is equal to the potential energy difference between the angular positions θ and θ' .
- $U(\theta) - U(\theta') = \Delta U = -pE\cos\theta + pE\cos\theta'$
- $If \theta' = 90°, then\ U(\theta') = 0$
- The potential energy stored in the system of dipole kept in the uniform electric field is given by $U = -pE\cos\theta = -\vec{p}\cdot\vec{E}$
- $If \theta = 180°$, dipole is aligned anti – parallel to the external field and the potential energy is maximum.
- $If \theta = 0°$, dipole is aligned parallel to the external field and the potential energy is minimum.



11. Obtain Gauss's law form Coulomb's law.
   - A positive point charge Q is surrounded by an imaginary sphere of radius r.
   - Total electric flux through the closed surface of the sphere is $\phi_E = \oint \vec{E}.d\vec{A} = \oint E\ dA\cos\theta$
   - The electric field of the point charge is directed radially outward at all points on the surface of the sphere.
   - Therefore , the direction of the area element $d\vec{A}$ is along the electric field $\vec{E}$ and $\theta = 0°$



- $\phi_E = \oint E\ dA \quad (\because \cos 0° = 1)$
- $\phi_E = E\oint dA$
- $\oint dA = 4\pi r^2$ and $E = k\dfrac{Q}{r^2}$
- $\phi_E = \dfrac{Q}{\varepsilon_0}$ ------------ (1)
- The above equation is called Gauss's law.
- The equation (1) is equally true for any arbitrary shaped surface which encloses the charge Q.



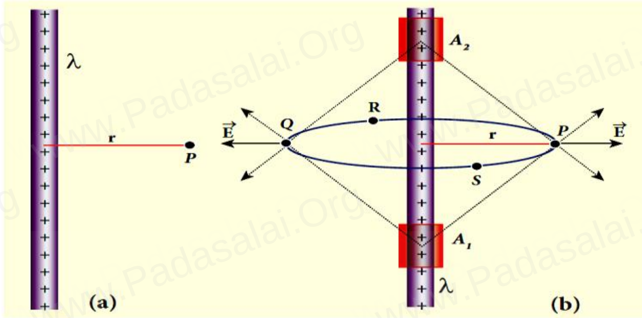
- It is seen that the total electric flux is the same for closed surfaces A_1, A_2 and A_3.
- Gauss's law states that if a charge Q is enclosed by an arbitrary closed surface, then the total electric flux ϕ_E through the closed surface is

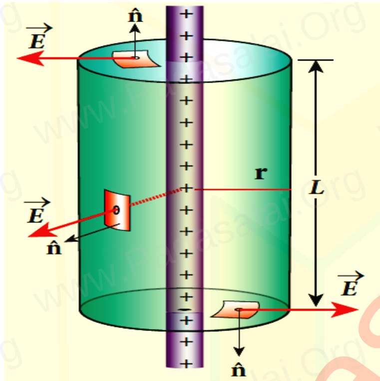
$$\phi_E = \frac{Q_{encl}}{\varepsilon_0}$$

- Q_{encl} denotes the charges inside the closed surface.

12. Obtain the expression for electric field due to an infinitely long charged wire.
   * Consider an infinitely long straight wire having uniform linear charge density λ.
   * Let P be a point located at a perpendicular distance r from the wire.
   * The electric field at the point P can be found using Gauss's law.
   * A_1 and A_2 are the two small charge elements on the wire which are at equal distances from the point P.
   * The resultant electric field due to these two charge elements points radially away from the charged wire and the magnitude of electric field is same at all points on the circle of radius r.
   * Let us choose a cylindrical Gaussian surface of radius r and length L.
   * The total electric flux in this closed surface is $\phi_E = \oint \vec{E}.d\vec{A}$
   * $\phi_E = \int_{\text{Curved surface}} \vec{E}.d\vec{A} + \int_{\text{Top surface}} \vec{E}.d\vec{A} + \int_{\text{Bottom surface}} \vec{E}.d\vec{A}$

* For the curved surface, $\vec{E}.d\vec{A} = E.dA$ $(\because \theta = 0°, \cos 0° = 1)$
* For the top and bottom surfaces, $\vec{E}.d\vec{A} = 0$ $(\because \theta = 90°, \cos 90° = 0)$
* Therefore, the total electric flux through the curved surface is
* $\phi_E = \int_{\substack{\text{Curved}\\\text{surface}}} \vec{E}.d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$
* $Q_{encl} = \lambda L$
* $E \int_{\substack{\text{Curved}\\\text{surface}}} dA = \frac{\lambda L}{\varepsilon_0}$
* $\int_{\substack{\text{Curved}\\\text{surface}}} dA = 2\pi r L$
* $E.2\pi r L = \frac{\lambda L}{\varepsilon_0}$
* $E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$
* $\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}\hat{r}$
* The electric field due to the infinite charged wire depends on $\frac{1}{r}$ rather than $\frac{1}{r^2}$ for a point charge.
* The electric field is always along the perpendicular direction to wire.
* If $\lambda > 0$ then $\vec{E}$ points perpendicular outward from the wire.
* If $\lambda < 0$ then $\vec{E}$ points perpendicular inward.



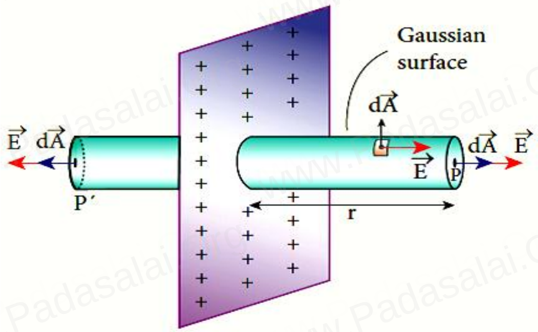


13. Obtain the expression for electric field due to a charged infinite plane sheet.
   - Consider an infinite plane sheet of charges with uniform surface charge density σ.
   - Let P be a point at a distance of r from the sheet.
   - Since the plane is infinitely large, the electric field should be same at all points equidistant from the plane and radially directed at all points.
   - A cylindrical shaped Gaussian surface of length 2r and area A of the flat surfaces is chosen such that the infinite plane sheet passes perpendicular through the middle part of the Gaussian surface.

$$\phi_E = \oint \vec{E}.d\vec{A}$$

$$\int_{\substack{\text{Curved}\\\text{surface}}} \vec{E}.d\vec{A} + \int_{P} \vec{E}.d\vec{A} + \int_{P'} \vec{E}.d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

- At curved surface, E and dA are perpendicular to each other.
- At P and P', E and dA are parallel to each other.
- $\int_P E.dA + \int_{P'} E.dA = \frac{Q_{encl}}{\varepsilon_0}$
- $Q_{encl} = \sigma A$
- $2E \int_P dA = \frac{\sigma A}{\varepsilon_0}$
- $\int_P dA = A$
- $2EA = \frac{\sigma A}{\varepsilon_0}$
- $E = \frac{\sigma}{2\varepsilon_0}$
- $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$

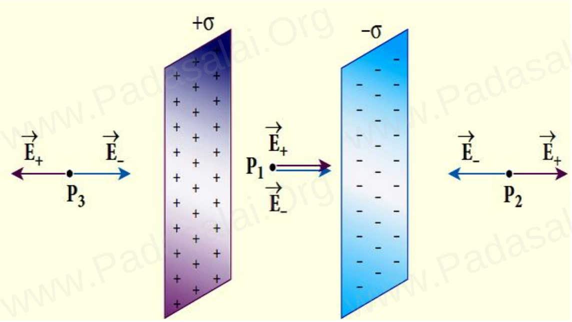


14. Obtain the expression for electric field due to two parallel charged plane sheets.

* Consider two infinitely large charged plane sheets with equal and opposite charge densities $+\sigma$ and $-\sigma$ which are placed parallel to each other.
* The magnitude of the electric field due to an infinite charged plane sheet is $\frac{\sigma}{2\varepsilon_0}$ and it points perpendicularly outward if $\sigma > 0$ and points inward if $\sigma < 0$.
* At the points P_2 and P_3, the electric fields due to both plates are equal in magnitude an opposite in direction.
* As a result, electric field at a point outside the plates is zero.
* But inside the plate, electric fields are in same direction.
* The total electric field at P_1 is

$$E_{inside} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

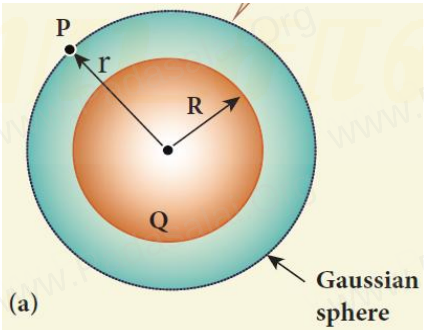
* The direction of the electric field inside the plates is directed from positively charged plate to negatively charged plate and is uniform everywhere inside the plate.



15. Obtain the expression for electric field due to a uniformly charged spherical shell.

At a point outside the spherical shell (r >R):

* Let us choose a point P outside the shell at a distance r from the center.
* The charge is uniformly distributed on the surface of the sphere.
* Hence the electric field must point radially outward if Q > 0 and point radially inward if Q < 0.
* $\oint_{\text{Gaussian surface}} \vec{E}.d\vec{A} = \frac{Q}{\varepsilon_0}$
* $E \oint_{\text{Gaussian surface}} .dA = \frac{Q}{\varepsilon_0}$
* $\oint_{\text{Gaussian surface}} dA = 4\pi r^2$
* $E.4\pi r^2 = \frac{Q}{\varepsilon_0}$
* $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$
* $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$

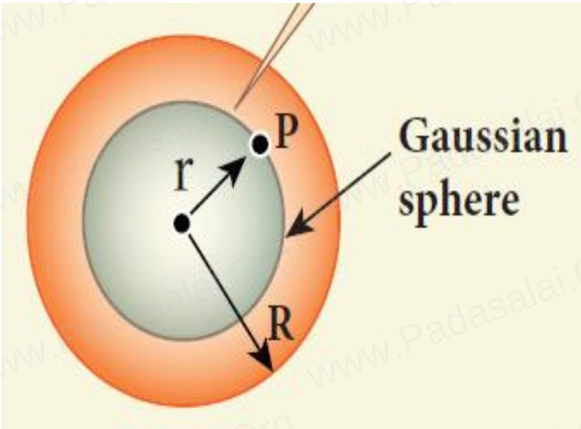


At a point on the surface of the spherical shell (r = R):

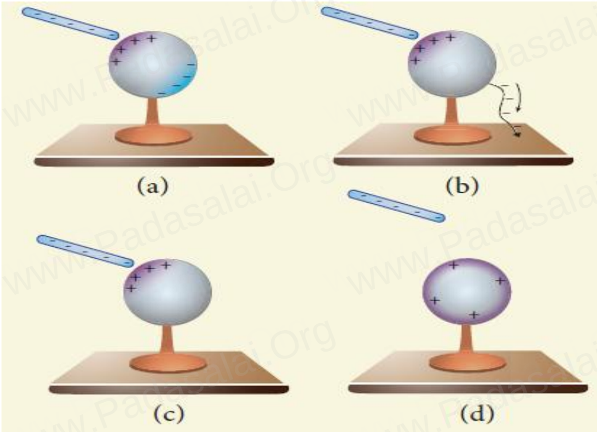
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \quad (r = R)$$

At a point outside the spherical shell (r <R):

* Consider a point P inside the shell at a distance r form the center.
* A Gaussian sphere of radius r is constructed.
* $\oint_{\text{Gaussian surface}} \vec{E}.d\vec{A} = \frac{Q}{\varepsilon_0}$
* $E.4\pi r^2 = \frac{Q}{\varepsilon_0}$
* Since Gaussian surface encloses no charge, Q = 0.
* E = 0.
* The electric field due to the uniformly charged spherical shell is zero at all points inside the shell.



16. Explain the process of electrostatic induction.



* Consider an uncharged conducting sphere at rest on an insulating stand.
* Suppose a negatively charged rod is brought near the conductor without touching it.
* The negative charge of the rod repels the electrons in the conductor to the opposite side.
* As a result, positive charges are induced near the region of the charged rod while negative charges on the farther side.
* Before introducing the charged rod, the free electrons were distributed uniformly on the surface of the conductor and the net charge is zero.
* Once the charged rod is brought near the conductor, the distribution is no longer uniform with more electrons located on the farther side of the rod and positive charges are located closer to the rod.
* But the total charge is zero.
* Now the conducting sphere is connected to the ground through a conducting wire.
* This is called grounding.
* Since the ground can always receive any amount of electrons, grounding removes the electron from the conducting sphere.
* When the grounding wire is removed from the conductor, the positive charges remain near the charged rod.
* Now the charged rod is taken away from the conductor.
* As soon as the charged rod is removed, the positive charge gets distributed uniformly on the surface of the conductor.

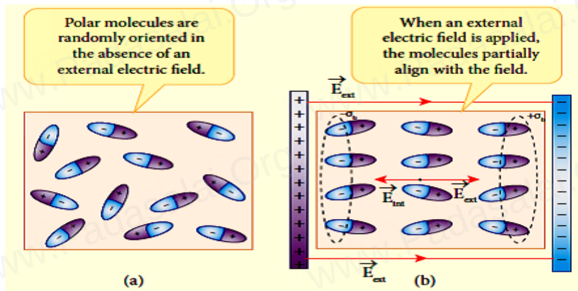
17. Explain dielectrics in detail and how an electric field is induced inside a dielectric.

Dielectrics:

* A dielectric is a non – conducting material and has no free electrons.
* The electrons in a dielectric are bound within the atoms.
* Ebonite, glass and mica are some examples.
* When an external electric field is applied, the electrons are not free to move anywhere but they are realigned in a specific way.
* A dielectric is made up of either polar molecules or non – polar molecules.

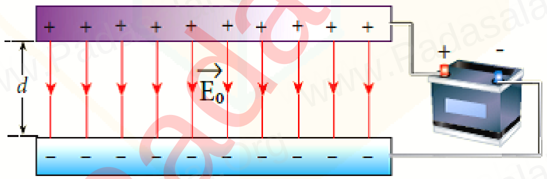
Induced electric field inside the dielectric:

* When an external electric field is applied on a conductor, the charges are aligned in such a way that an internal electric field is created which cancels the external electric field.
* But in the case of a dielectric, which has no free electrons, the external electric field only realings the charges so that an internal electric field is produced.
* The magnitude of the internal electric field is smaller than that of external electric field.
* Therefore the net electric field inside the dielectric is no t zero but is parallel to an external electric field with magnitude less than that of the external electric field.
* For example, let us consider a rectangular dielectric slab placed between two oppositely charged plates.
* The uniform electric field between the plates acts as an external electric field which polarizes the dielectric placed between plates.
* The positive charges are induced on one side surface and negative charges are induced on the other side of surface.
* But inside the dielectric, the net charge is zero even in a small volume.
* So the dielectric in the external field is equivalent to two oppositely charged sheets with the surface charge densities $+\sigma_b$ and $-\sigma_b$
* These charges are called bound charges.
* They are not free to move like free electrons in conductors.

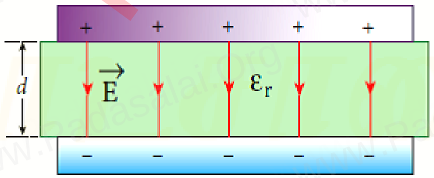


18. Explain in detail the effect of a dielectric placed in a parallel plate capacitor When the capacitor is disconnected from the battery:

* Consider a capacitor with two parallel plates each of cross –sectional area A and are separated by a distance d.
* The capacitor is charged by a battery of voltage V_o and the charge stored si Q_o.
* The capacitance of the capacitor without the dielectric is $C_o = V_o / Q_o$
* The battery is then disconnected from the capacitor and the dielectric is inserted between the plates.
* The introduction of dielectric between the plates will decrease the electric field.
* $E = \frac{E_0}{\varepsilon_r}$
* E_0 is the electric field inside the capacitors when there is no dielectric.
* ε_r is the relative permeability of the dielectric.
* Since $\varepsilon_r > 1, \quad E < E_0$
* As a result, the electrostatic potential difference between the plates is also reduced.
* But at the same time, the charge will remain constant once the battery is disconnected.
* $V = Ed = \frac{E_0}{\varepsilon_r} d = \frac{V_0}{\varepsilon_r}$
* $C = \frac{Q_0}{V}$
* $C = \varepsilon_r \frac{Q_0}{V_0} = \varepsilon_r C_0$
* $C = \varepsilon_r C_0$
* Since $\varepsilon_r > 1, \quad C > C_0$
* Thus insertion of the dielectric increases the capacitance.
* $C = \frac{\varepsilon_r \varepsilon_0 A}{d} = \frac{\varepsilon A}{d}$
* The energy stored in the capacitor before the insertion of dielectric is $U_0 = \frac{1}{2}\frac{Q_0^2}{C_0}$
* The energy stored in the capacitor after the insertion of dielectric is $U = \frac{U_o}{\varepsilon_r}$
* Since $\varepsilon_r > 1, \quad U < U_0$
* There is a decrease in energy because, when the dielectric is inserted, the capacitor spends some energy in pulling the dielectric inside.



(a)

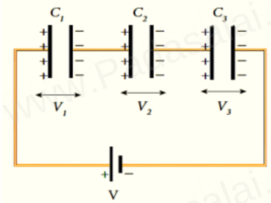
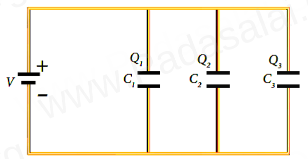
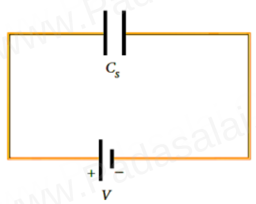
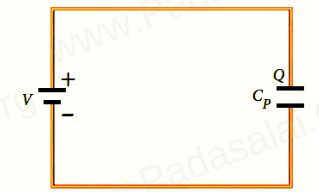


(b)

19. Explain in detail the effect of a dielectric placed in a parallel plate capacitor when the battery remains connected to the capacitor.

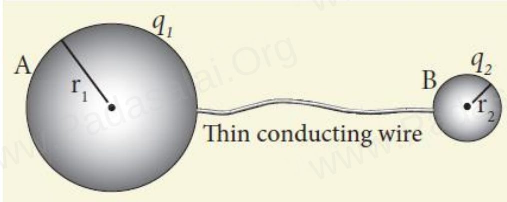
* The potential difference V_0 across the plates remains constant.
* But it is found experimentally that when dielectric is inserted, the charge stored in the capacitor is increased by a factor ε_r.
* $Q = \varepsilon_r Q_0$
* $C = \varepsilon_r C_0$
* $C_0 = \frac{\varepsilon_0 A}{d}$
* $C = \frac{\varepsilon A}{d}$
* The energy stored in the capacitor before the insertion of dielectric is $U_0 = \frac{1}{2} C_0 V_0^2$
* The energy stored in the capacitor after the insertion of dielectric is $U = \varepsilon_r U_o$

20. Derive the expression for the resultant capacitance, when capacitors are connected in series and in parallel.

Capacitors in series	Capacitors in parallel
Three capacitors of capacitance C_1, C_2 and C_3 are connected in series.	Three capacitors of capacitance C_1, C_2 and C_3 are connected in parallel.
Charge Q across each capacitor is same	Potential difference across each capacitor is same.
$V = V_1 + V_2 + V_3$	$Q = Q_1 + Q_2 + Q_3$
$Q = CV$	$Q = CV$
$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$	$Q = C_1V + C_2V + C_3V$
$V = \frac{Q}{C_S}$	$Q = C_PV$
$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$	$C_PV = C_1V + C_2V + C_3V$
$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$	$C_P = C_1 + C_2 + C_3$
	
	

21. Explain in detail how charges are distributed in a conductor.

* Consider two conducting spheres A and B of radii r1 and r2 respectively connected to each other by a thin conducting wire.
* The distance between the spheres is much greater than the radii of either sphere.
* If a charge Q is introduced into any one of the spheres, this charge Q is redistributed into both the spheres such that the electrostatic potential is same in both the spheres.
* They are now uniformly charged and attain electrostatic equilibrium.
* Let q_1 be the charge residing on the surface of sphere A.
* Let q_2 be the charge residing on the surface of the sphere B.
* $Q = q_1 + q_2$
* The charges are distributed only on the surface and there is no net charge inside the conductor.
* The electrostatic potential at the surface of the sphere A is given by $V_A = k\frac{q_1}{r_1}$
* The electrostatic potential at the surface of the sphere A is given by $V_B = k\frac{q_2}{r_2}$
* The surface of the conductor is an equipotential.
* $V_A = V_B$
* $\frac{q_1}{r_1} = \frac{q_2}{r_2}$
* $q_1 = 4\pi r_1^2 \sigma_1$
* $q_2 = 4\pi r_2^2 \sigma_2$
* $\sigma_1 r_1 = \sigma_2 r_2$
* $\sigma\, r = \textit{constant}$



22. Explain in detail the construction and working of a Van de Graff generator.

Principle: Action of points and Electrostatic induction.

Construction:

- A hollow metallic sphere A is mounted on insulating pillars.
- A pulley B is mounted at the centre of the sphere.
- Another pulley C is mounted near the bottom.
- A silk belt moves over the pulleys.
- The pulley C is driven continuously by an electric motor.
- The comb shaped conductors D and E are mounted near the pulleys.
- A positive potential of **10^4 V** is given to D by a power supply.
- E is connected to the inner side of the hollow sphere.

WORKING:

ACTION OF POINTS:

- Because of high electric field near D, the air gets ionized due to action of points.

- The negative charges in air move towards the needles and positive charges are repelled on towards the belt.
- These positive charges stick to the belt, move up and reach near E.

ELECTROSTATIC INDUCTION:

- As a result of electrostatic induction, the comb E acquires negative charge and the sphere acquires positive charge.
- The acquired positive charge is distributed on the outer surface of the sphere.
- The high electric field at E ionizes the air.
- The negative charges are repelled to the belt, neutralizes the positive charge on the belt before the belt passes over the pulley.
- Hence the descending belt will be left uncharged.

Leakage of electric charge:

- Thus the machine continuously transfers the positive charge to the sphere.
- As a result, the potential of the sphere keeps increasing till it attains a limiting value.
- After this stage leakage of charge to the surrounding starts due to the ionization of the air.

Prevention of leaking electric charge:

- **The leakage of charge** from the sphere can be **reduced by enclosing it in a gas filled steel chamber** at a very high temperature.

Uses:

- It is used to produce large electrostatic potential difference of the order of **10^7 V.**
- This high voltage is used to **accelerate positive ions (protons, deuterons) for the purpose of nuclear disintegration.**