

EXERCISE 1.8

Choose the Correct answer:

1. If $|adj(adj(A))| = |A|^9$, then the order of the square matrix A is

- (a) 3 (b) 4 (c) 2 (d) 5

2. If A is a 3×3 non-singular matrix such that $AA^T = A^T A$ and $B = A^{-1}A^T$, then $BB^T =$

- (a) A (b) B (c) I (d) B^T

3. If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, $B = adj A$ and $C = 3A$, then $\frac{|adj B|}{|C|} =$

- (a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{1}{4}$ (d) 1

4. If $A \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$, then A

- (a) $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$

5. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ then $9I - A =$

- (a) A^{-1} (b) $\frac{A^{-1}}{2}$ (c) $3A^{-1}$ (d) $2A^{-1}$

6. If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then $|adj(AB)|$

- (a) -40 (b) -80 (c) -60 (d) -20

7. If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then x is

- (a) 15 (b) 12 (c) 14 (d) 11

8. If $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the value of a_{23} is

- (a) 0 (b) -2 (c) -3 (d) -1

9. If A, B and C are invertible matrices of some order, then which one of the following is not true?

- (a) $adj A = |A|A^{-1}$ (b) $adj(AB) = (adj A)(adj B)$ (c) $\det A^{-1} = (\det A)^{-1}$ (d) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

10. If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then B^{-1}

- (a) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

11. If $A^T A^{-1}$ is symmetric, then $A^2 =$

- (a) A^{-1} (b) $(A^T)^2$ (c) A^T (d) $(A^{-1})^2$

12. If A is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ then $(A^{-1})^2$

- (a) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

13. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 5 \\ x & 3 \\ 5 & 5 \end{bmatrix}$ and $A^T = A^{-1}$, then the value of x is

- (a) $-\frac{4}{5}$ (b) $-\frac{3}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

14. If $A = \begin{bmatrix} 1 & \tan \frac{\theta}{2} \\ -\tan \frac{\theta}{2} & 1 \end{bmatrix}$ and $AB = I$, then $B =$

- (a) $(\cos^2 \frac{\theta}{2}) A$ (b) $(\cos^2 \frac{\theta}{2}) A^T$ (c) $(\cos^2 \theta) I$ (d) $(\sin^2 \frac{\theta}{2}) A$

15. If $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ and $A(adj A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then $k =$

- (a) 0 (b) $\sin \theta$ (c) $\cos \theta$ (d) 1

16. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

- (a) 17 (b) 14 (c) 19 (d) 21

17. If $adj A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and $adj B = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ then $adj(AB)$ is

- (a) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ (c) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ (d) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

18. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

- (a) 1 (b) 2 (c) 4 (d) 3

19. If $x^a y^b = e^m$, $x^c y^d = e^n$, $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ then the values of x and y are respectively,

- (a) $e^{(\Delta_2/\Delta_1)}$, $e^{(\Delta_3/\Delta_1)}$ (b) $\log(\Delta_1/\Delta_3)$, $\log(\Delta_2/\Delta_3)$ (c) $\log(\Delta_2/\Delta_1)$, $\log(\Delta_3/\Delta_1)$ (d) $e^{(\Delta_1/\Delta_3)}$, $e^{(\Delta_2/\Delta_3)}$

20. Which of the following is/are correct?

(i) Adjoint of a symmetric matrix is also a symmetric matrix.

(ii) Adjoint of a diagonal matrix is also a diagonal matrix.

(iii) If A is a square matrix of order n and λ is a scalar, then $adj(\lambda A) = \lambda^n adj(A)$

(iv) $A(adj A) = (adj A)A = |A|I$

- (a) Only (i) (b) (ii) and (iii) (c) (iii) and (iv) (d) (i), (ii) and (iv)

21. If $\rho(A) = \rho(A|B)$ then the system $AX = B$ of linear equations is

- (a) consistent and has a unique solution (b) consistent
(c) consistent and has infinitely many solution (d) inconsistent

22. If $0 \leq \theta \leq \pi$ and the system of equations $x + (\sin \theta)y - (\cos \theta)z = 0$, $(\cos \theta)x - y + z = 0$, $(\sin \theta)x + y - z = 0$ has a non-trivial solution then θ is

- (a) $\frac{2\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{4}$

23. The augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda - 7 & \mu + 5 \end{bmatrix}$. The system has infinitely many solutions if

- (a) $\lambda = 7, \mu \neq -5$ (b) $\lambda = -7, \mu = 5$ (c) $\lambda \neq 7, \mu \neq -5$ (d) $\lambda = 7, \mu = -5$

24. Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $4B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$. If B is the inverse of A , then the value of x is

- (a) 2 (b) 4 (c) 3 (d) 1

25. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then $adj(adj A)$ is

- (a) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{bmatrix}$

EXERCISE 2.9

Choose the correct or the most suitable answer from the given four alternatives :

1. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is
 (a) 0 (b) 1 (c) -1 (d) i
2. The value of $\sum_{i=1}^{13} (i^n + i^{n-1})$ is
 (a) $1+i$ (b) i (c) 1 (d) 0
3. The area of the triangle formed by the complex numbers z , iz and $z + iz$ in the Argand's diagram is
 (a) $\frac{1}{2}|z|^2$ (b) $|z|^2$ (c) $\frac{3}{2}|z|^2$ (d) $2|z|^2$
4. The conjugate of a complex number is $\frac{1}{i-2}$. Then, the complex number is
 (a) $\frac{1}{i+2}$ (b) $\frac{-1}{i+2}$ (c) $\frac{-1}{i-2}$ (d) $\frac{1}{i-2}$
5. If $z = \frac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$ then $|z|$ is equal to
 (a) 0 (b) 1 (c) 2 (d) 3
6. If z is a non-zero complex number, such that $2iz^2 = \bar{z}$ then $|z|$ is
 (a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3
7. If $|z - 2 + i| \leq 2$, then the greatest value of $|z|$ is
 (a) $\sqrt{3} - 2$ (b) $\sqrt{3} + 2$ (c) $\sqrt{5} - 2$ (d) $\sqrt{5} + 2$
8. If $\left|z - \frac{3}{z}\right| = 2$, then the least value of $|z|$ is
 (a) 1 (b) 2 (c) 3 (d) 5
9. If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is
 (a) z (b) $\bar{z}$ (c) $\frac{1}{z}$ (d) 1
10. The solution of the equation $|z| - z = 1 + 2i$ is
 (a) $\frac{3}{2} - 2i$ (b) $-\frac{3}{2} + 2i$ (c) $2 - \frac{3}{2}i$ (d) $2 + \frac{3}{2}i$
11. If $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$, and $|9z_1 z_2 + 4z_1 z_3 + z_2 z_3| = 12$, then the value of $|z_1 + z_2 + z_3|$ is
 (a) 1 (b) 2 (c) 3 (d) 4
12. If z is a complex number such that $z \in C/R$ and $z + \frac{1}{z} \in R$, then $|z|$ is
 (a) 0 (b) 1 (c) 2 (d) 3
13. z_1, z_2 and z_3 be three complex numbers such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$, then $z_1^2 + z_2^2 + z_3^2$ is
 (a) 3 (b) 2 (c) 1 (d) 0
14. If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is
 (a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3
15. If $z = x + iy$ is a complex number such that $|z + 2| = |z - 2|$, then the locus of z is
 (a) real axis (b) imaginary axis (c) ellipse (d) circle

16. The principal argument of $\frac{3}{-1+i}$ is
 (a) $\frac{-5\pi}{6}$ (b) $\frac{-2\pi}{3}$ (c) $\frac{-3\pi}{4}$ (d) $\frac{-\pi}{2}$
17. The principal argument of $(\sin 40^\circ + i \cos 40^\circ)^5$ is
 (a) -110° (b) -70° (c) 70° (d) 110°
18. If $(1+i)(1+2i)(1+3i) \dots (1+ni) = x + iy$, then $2 \cdot 5 \cdot 10 \dots (1+n^2)$ is
 (a) 1 (b) i (c) $x^2 + y^2$ (d) $1 + n^2$
19. If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7 = A + B\omega$, then (A, B) equals
 (a) (1,0) (b) (-1,1) (c) (0,1) (d) (1,1)
20. The principal argument of the complex number $\frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$ is
 (a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{2}$
21. If α and β are the roots of $x^2 + x + 1 = 0$, then $\alpha^{2020} + \beta^{2020}$ is
 (a) -2 (b) -1 (c) 1 (d) 2
22. The product of all four values of $\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)^{3/4}$ is
 (a) -2 (b) -1 (c) 1 (d) 2
23. If $\omega \neq 1$ is a cubic root of unity and $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to
 (a) 1 (b) -1 (c) $\sqrt{3}i$ (d) $-\sqrt{3}i$
24. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is
 (a) $\text{cis } \frac{2\pi}{3}$ (b) $\text{cis } \frac{4\pi}{3}$ (c) $-\text{cis } \frac{2\pi}{3}$ (d) $-\text{cis } \frac{4\pi}{3}$
25. If $\omega = \text{cis } \frac{2\pi}{3}$, then the number of distinct roots of $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$
 (a) 1 (b) 2 (c) 3 (d) 4

EXERCISE 3.6

1. Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$.
2. Discuss the maximum possible number of positive and negative roots of the polynomial equations $x^2 - 5x + 6$ and $x^2 - 5x + 16$. Also draw rough sketch of the graphs.
3. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions.
4. Determine the number of positive and negative roots of the equation $x^9 - 5x^8 - 14x^7 = 0$.
5. Find the exact number of real roots and imaginary of the equation $x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$.

EXERCISE 3.7

Choose the most suitable answer:

1. A zero of $x^3 + 64$ is

- (a) 0 (b) 4 (c) $4i$ (d) -4

2. If f and g are polynomials of degrees m and n respectively, and if $h(x) = (f \circ g)(x)$, then the degree of h is

- (a) mn (b) $m + n$ (c) m^n (d) n^m

3. A polynomial equation in x of degree n always has

- (a) n distinct roots (b) n real roots (c) n imaginary roots (d) at most one root.

4. If α, β and γ are the roots of $x^3 + px^2 + qx + r$, then $\sum \frac{1}{\alpha}$ is

- (a) $-\frac{q}{r}$ (b) $-\frac{p}{r}$ (c) $\frac{q}{r}$ (d) $\frac{-q}{p}$

5. According to the rational root theorem, which number is not possible rational root of $4x^7 + 2x^4 - 10x^3 - 5$?

- (a) -1 (b) $\frac{5}{4}$ (c) $\frac{4}{5}$ (d) 5

6. The polynomial $x^3 - kx^2 + 9x$ has three real roots if and only if, k satisfies

- (a) $|k| \leq 6$ (b) $k = 0$ (c) $|k| > 6$ (d) $|k| \geq 6$

7. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

- (a) 2 (b) 4 (c) 1 (d) ∞

8. If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive root, if and only if

- (a) $a \geq 0$ (b) $a > 0$ (c) $a < 0$ (d) $a \leq 0$

9. The polynomial $x^3 + 2x + 3$ has

- (a) one negative and two real roots (b) one positive and two imaginary roots
(c) three real roots (d) no solution

10. The number of positive roots of the polynomial $\sum_{j=0}^n nC_r (-1)^r x^r$ is

- (a) 0 (b) n (c) $< n$ (d) r

EXERCISE 4.6

Choose the correct or the most suitable answer from the given four alternatives.

- The value of $\sin^{-1}(\cos x)$, $0 \leq x \leq \pi$ is
 (a) $\pi - x$ (b) $x - \frac{\pi}{2}$ (c) $\frac{\pi}{2} - x$ (d) $\pi + x$
- If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$; then $\cos^{-1}x + \cos^{-1}y$ is equal to
 (a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) π
- $\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{12}{13} + \sec^{-1}\frac{5}{3} - \operatorname{cosec}^{-1}\frac{13}{12}$ is equal to
 (a) 2π (b) π (c) 0 (d) $\tan^{-1}\frac{12}{65}$
- If $\sin^{-1}x = 2\sin^{-1}\alpha$ has a solution, then
 (a) $|\alpha| \leq \frac{1}{\sqrt{2}}$ (b) $|\alpha| \geq \frac{1}{\sqrt{2}}$ (c) $|\alpha| < \frac{1}{\sqrt{2}}$ (d) $|\alpha| > \frac{1}{\sqrt{2}}$
- $\sin^{-1}(\cos x) = \frac{\pi}{2} - x$ is valid for
 (a) $-\pi \leq x \leq 0$ (b) $0 \leq x \leq \pi$ (c) $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ (d) $-\frac{\pi}{4} \leq x \leq \frac{3\pi}{4}$
- If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$, the value of $x^{2017} + y^{2018} + z^{2019} - \frac{9}{x^{101} + y^{101} + z^{101}}$ is
 (a) 0 (b) 1 (c) 2 (d) 3
- If $\cot^{-1}x = \frac{2\pi}{5}$ for some $x \in R$, the value of $\tan^{-1}x$ is
 (a) $-\frac{\pi}{10}$ (b) $\frac{\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $-\frac{\pi}{5}$
- The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is
 (a) $[1, 2]$ (b) $[-1, 1]$ (c) $[0, 1]$ (d) $[-1, 0]$
- If $x = \frac{1}{5}$, the value of $\cos(\cos^{-1}x + 2\sin^{-1}x)$ is
 (a) $-\sqrt{\frac{24}{25}}$ (b) $\sqrt{\frac{24}{25}}$ (c) $\frac{1}{5}$ (d) $-\frac{1}{5}$
- $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to
 (a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ (b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ (c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ (d) $\tan^{-1}\left(\frac{1}{2}\right)$
- If the function $f(x) = \sin^{-1}(x^2 - 3)$, then x belongs to
 (a) $[-1, 1]$ (b) $[\sqrt{2}, 2]$ (c) $[-2, -\sqrt{2}] \cup [\sqrt{2}, 2]$ (d) $[-2, -\sqrt{2}] \cap [\sqrt{2}, 2]$
- If $\cot^{-1}2$ and $\cot^{-1}3$ are two angles of a triangle, then the third angle is
 (a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$
- $\sin^{-1}\left(\tan\frac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) = \frac{\pi}{6}$. Then x is a root of the equation
 (a) $x^2 - x - 6 = 0$ (b) $x^2 - x - 12 = 0$ (c) $x^2 + x - 12 = 0$ (d) $x^2 + x - 6 = 0$
- $\sin^{-1}(2\cos^2x - 1) + \cos^{-1}(1 - 2\sin^2x) =$
 (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$
- If $\cot^{-1}(\sqrt{\sin\alpha}) + \tan^{-1}(\sqrt{\sin\alpha}) = u$, then $\cos 2u$ is equal to
 (a) $\tan^2\alpha$ (b) 0 (c) -1 (d) $\tan 2\alpha$
- If $|x| \leq 1$, then $2\tan^{-1}x - \sin^{-1}\frac{2x}{1+x^2}$ is equal to
 (a) $\tan^{-1}x$ (b) $\sin^{-1}x$ (c) 0 (d) π
- The equation $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has
 (a) no solution (b) unique solution (c) two solutions (d) infinite number of solutions
- If $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then x is equal to
 (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{5}}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{\sqrt{3}}{2}$
- If $\sin^{-1}\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, then the value of x is
 (a) 4 (b) 5 (c) 2 (d) 3
- $\sin(\tan^{-1}x)$, $|x| < 1$ is equal to
 (a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

EXERCISE 5.6

Choose the most appropriate answer.

- The equation of the circle passing through (1,5) and (4,1) and touching y -axis is $x^2 + y^2 - 5x - 6y + 9 + \lambda(4x + 3y \pm 19) = 0$ where λ is equal to
 (a) $0, -\frac{40}{9}$ (b) 0 (c) $\frac{40}{9}$ (d) $-\frac{40}{9}$
- The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is
 (a) $\frac{4}{3}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{3}{2}$
- The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if
 (a) $15 < m < 65$ (b) $35 < m < 85$ (c) $-85 < m < -35$ (d) $-35 < m < 15$
- The length of the diameter of the circle which touches the x -axis at the point (1,0) and passes through the point (2,3).
 (a) $\frac{6}{5}$ (b) $\frac{5}{3}$ (c) $\frac{10}{3}$ (d) $\frac{3}{5}$
- The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is
 (a) 1 (b) 3 (c) $\sqrt{10}$ (d) $\sqrt{11}$
- The centre of the circle inscribed in a square formed by the lines $x^2 - 8x - 12 = 0$ and $y^2 - 14y + 45 = 0$ is
 (a) (4,7) (b) (7,4) (c) (9,4) (d) (4,9)
- The equation of the normal to the circle $x^2 + y^2 - 2x - 2y + 1 = 0$ which is parallel to the line $2x + 4y = 3$ is
 (a) $x + 2y = 3$ (b) $x + 2y + 3 = 0$ (c) $2x + 4y + 3 = 0$ (d) $x - 2y + 3 = 0$
- If $P(x, y)$ be any point on $16x^2 + 25y^2 = 400$ with foci $F_1(3,0)$ and $F_2(-3,0)$ then $PF_1 + PF_2$ is
 (a) 8 (b) 6 (c) 10 (d) 12
- The radius of the circle passing through the point (6,2) two of whose diameter are $x + y = 6$ and $x + y = 24$ is
 (a) 10 (b) $2\sqrt{5}$ (c) 6 (d) 4
- The area of quadrilateral formed with foci of the hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ is
 (a) $4(a^2 + b^2)$ (b) $(a^2 + b^2)$ (c) $a^2 + b^2$ (d) $\frac{1}{2}(a^2 + b^2)$
- If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x - 3)^2 + (y + 2)^2 = r^2$, then the value of r^2 is
 (a) 2 (b) 3 (c) 1 (d) 4
- If $x + y = k$ is a normal to the parabola $y^2 = 12x$, then the value of k is
 (a) 3 (b) -1 (c) 1 (d) 9
- The ellipse $E_1: \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0,4) circumscribes the rectangle R . The eccentricity of the ellipse is
 (a) $\frac{\sqrt{2}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$
- Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ parallel to the straight line $2x - y = 1$. One of the points of contact of tangents on the hyperbola is

- (a) $\left(\frac{9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ (b) $\left(\frac{-9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (c) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (d) $(3\sqrt{3}, -2\sqrt{2})$

15. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ having centre at (0,3) is

- (a) $x^2 + y^2 - 6y - 7 = 0$ (b) $x^2 + y^2 - 6y + 7 = 0$ (c) $x^2 + y^2 - 6y - 5 = 0$ (d) $x^2 + y^2 - 6y + 5 = 0$

16. Let C be the circle with centre at (1,1) and radius = 1. If T is the circle centered at (0,y) passing through the origin and touching the circle C externally, then the radius of T is equal to

- (a) $\frac{\sqrt{3}}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

17. Consider an ellipse whose centre is of the origin and its major axis is along x -axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6, then the area of the quadrilateral inscribed in the ellipse with diagonals as major and minor axis of the ellipse is

- (a) 8 (b) 32 (c) 80 (d) 40

18. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

- (a) $2ab$ (b) ab (c) $\sqrt{ab}$ (d) $\frac{a}{b}$

19. An ellipse has OB as semi minor axes, F and F' its foci and the angle $\angle FBF'$ is a right angle. Then the eccentricity of the ellipse is

- (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{\sqrt{3}}$

20. The eccentricity of the ellipse $(x - 3)^2 + (y - 4)^2 = \frac{y^2}{9}$ is

- (a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{3\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

21. If the two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles then the locus of P is

- (a) $2x + 1 = 0$ (b) $x = -1$ (c) $2x - 1 = 0$ (d) $x = 1$

22. The circle passing through (-1,2) and touching the axis of x at (3,0) passing through the point

- (a) (-5, 2) (b) (2,-5) (c) (5, -2) (d) (-2,5)

23. The locus of a point whose distance from (-2,0) is $\frac{2}{3}$ times its distance from the line $x = \frac{-9}{2}$

- (a) a parabola (b) a hyperbola (c) an ellipse (d) a circle

24. The values of m for which the line $y = mx + 2\sqrt{5}$ touches the hyperbola $16x^2 - 9y^2 = 144$ are the roots of , then the value of (a+b) is

- (a) 2 (b) 4 (c) 0 (d) -2

25. If the coordinates at one end of a diameter of the circle $x^2 + y^2 - 8x - 4y + c = 0$ are (11,2) the coordinates of the other end are

- (a) (-5, 2) (b) (2,-5) (c) (5, -2) (d) (-2,5)

EXERCISE 6.10

Choose the correct or most suitable answer :

- If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $[\vec{a}, \vec{b}, \vec{c}]$ is equal to
 (a) 2 (b) -1 (c) 1 (d) 0
- If a vector $\vec{a}$ lies in the plane of $\vec{\beta}$ and $\vec{\gamma}$, then
 (a) $[\vec{a}, \vec{\beta}, \vec{\gamma}] = 1$ (b) $[\vec{a}, \vec{\beta}, \vec{\gamma}] = -1$ (c) $[\vec{a}, \vec{\beta}, \vec{\gamma}] = 0$ (d) $[\vec{a}, \vec{\beta}, \vec{\gamma}] = 2$
- If $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$ then the value of $[\vec{a}, \vec{b}, \vec{c}]$ is
 (a) $|\vec{a}| |\vec{b}| |\vec{c}|$ (b) $\frac{1}{3} |\vec{a}| |\vec{b}| |\vec{c}|$ (c) 1 (d) -1
- If $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that $\vec{a}$ is perpendicular to $\vec{b}$, and is parallel to $\vec{c}$ then $\vec{a} \times (\vec{b} \times \vec{c})$ is equal to
 (a) $\vec{a}$ (b) $\vec{b}$ (c) $\vec{c}$ (d) $\vec{0}$
- If $[\vec{a}, \vec{b}, \vec{c}] = 1$ then the value of $\frac{\vec{a} \cdot (\vec{b} \times \vec{c})}{(\vec{c} \times \vec{a}) \cdot \vec{b}} + \frac{\vec{b} \cdot (\vec{c} \times \vec{a})}{(\vec{a} \times \vec{b}) \cdot \vec{c}} + \frac{\vec{c} \cdot (\vec{a} \times \vec{b})}{(\vec{b} \times \vec{c}) \cdot \vec{a}}$ is
 (a) 1 (b) -1 (c) 2 (d) 3
- The volume of the parallelepiped with its edges represented by the vectors $\hat{i} + \hat{j}, \hat{i} + 2\hat{j}, \hat{i} + \hat{j} + \pi\hat{k}$ is
 (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{\pi}{4}$
- If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a}, \vec{b}, \vec{a} \times \vec{b}] = \frac{\pi}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is
 (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$
- If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j}, \vec{c} = \hat{i}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda \vec{a} + \mu \vec{b}$ then the value of $\lambda + \mu$ is
 (a) 0 (b) 1 (c) 6 (d) 3
- If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non-zero vectors such that $[\vec{a}, \vec{b}, \vec{c}] = 3$ then $\{[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}]\}^2$ is equal to
 (a) 81 (b) 9 (c) 27 (d) 18
- If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b} + \vec{c}}{\sqrt{2}}$ then the angle between $\vec{a}$ and $\vec{b}$ is
 (a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{4}$ (d) π
- If the volume of the parallelepiped with $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}$ as coterminal edges is 8 cubic units, then the volume of the parallelepiped with $(\vec{a} \times \vec{b}) \times (\vec{b} \times \vec{c}), (\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})$ and $(\vec{c} \times \vec{a}) \times (\vec{a} \times \vec{b})$ as coterminal edges is,
 (a) 8 cubic units (b) 512 cubic units (c) 64 cubic units (d) 24 cubic units
- Consider the vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ such that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$. Let P_1 and P_2 be the planes determined by the pairs of vectors $\vec{a}, \vec{b}$ and $\vec{c}, \vec{d}$ respectively. Then the angle between P_1 and P_2 is
 (a) 0° (b) 45° (c) 60° (d) 90°

13. If $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$ where $\vec{a}, \vec{b}, \vec{c}$ are any three vectors such that $\vec{b} \cdot \vec{c} \neq 0$ and $\vec{a} \cdot \vec{b} \neq 0$ then $\vec{a}$ and $\vec{c}$ are

- (a) perpendicular (b) parallel (c) inclined at an angle $\frac{\pi}{3}$ (d) inclined at an angle $\frac{\pi}{6}$

14. If $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{c} = 3\hat{i} + 5\hat{j} - \hat{k}$ then a vector perpendicular to $\vec{a}$ and lies in the plane containing $\vec{b}$ and $\vec{c}$ is

- (a) $-17\hat{i} + 21\hat{j} - 97\hat{k}$ (b) $17\hat{i} + 21\hat{j} - 123\hat{k}$ (c) $-17\hat{i} - 21\hat{j} + 97\hat{k}$ (d) $-17\hat{i} - 21\hat{j} - 97\hat{k}$

15. The angle between the lines $\frac{x-2}{3} = \frac{y+1}{-2}, z = 2$ and $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$ is

- (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

16. If the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x + 3y - \alpha z + \beta = 0$ then (α, β) is

- (a) (-5, 5) (b) (-6, 7) (c) (5, -5) (d) (6, -7)

17. The angle between the line $\vec{r} = (\hat{i} + 2\hat{j} - 3\hat{k}) + t(2\hat{i} + \hat{j} - 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} + \hat{j}) + 4 = 0$ is

- (a) 0° (b) 30° (c) 45° (d) 90°

18. The coordinates of the point where the line $\vec{r} = (6\hat{i} - \hat{j} - 3\hat{k}) + t(-\hat{i} + 4\hat{k})$ meets the plane $\vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) = 3$ are

- (a) (2, 1, 0) (b) (7, -1, -7) (c) (1, 2, -6) (d) (5, -1, 1)

19. Distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is

- (a) 0 (b) 1 (c) 2 (d) 3

20. The distance between the planes $x + 2y + 3z + 7 = 0$ and $2x + 4y + 6z + 7 = 0$ is

- (a) $\frac{\sqrt{7}}{2\sqrt{2}}$ (b) $\frac{7}{2}$ (c) $\frac{\sqrt{7}}{2}$ (d) $\frac{7}{2\sqrt{2}}$

21. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$ then

- (a) $c = \pm 3$ (b) $c = \pm \sqrt{3}$ (c) $c > 0$ (d) $0 < c < 1$

22. The vector equation $\vec{r} = (\hat{i} - 2\hat{j} - \hat{k}) + t(6\hat{i} - \hat{k})$ represents a straight line passing through the points

- (a) (0, 6, -1) and (1, -2, -1) (b) (0, 6, -1) and (1, -4, -2) (c) (1, -2, -1) and (1, 4, -2) (d) (1, -2, -1) and (0, -6, 1)

23. If the distance of the point (1, 1, 1) from the origin is half of its distance from the plane $x + y + z + k = 0$, then the values of k are

- (a) ± 3 (b) ± 6 (c) -3, 9 (d) 3, -9

24. If the planes $\vec{r} \cdot (2\hat{i} - \lambda\hat{j} + \hat{k}) = 3$ and $\vec{r} \cdot (4\hat{i} + \hat{j} - \mu\hat{k}) = 3$ are parallel, then the value of λ and μ are

- (a) $\frac{1}{2}, -2$ (b) $-\frac{1}{2}, 2$ (c) $-\frac{1}{2}, -2$ (d) $\frac{1}{2}, 2$

25. If the length of the perpendicular from the origin to the plane $2x + 3y + \lambda z = 1$, $\lambda > 0$ is $\frac{1}{5}$, then the value of λ is

- (a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) 0 (d) 1