2019-20 EDITION

SURA'S SUPER GUIDE

SURA'S

MATHEMATICS

As per the Updated New Textbook for 2019-20

FULL-YEAR GUIDE

100% SUCCESS

- Complete Solutions to Textbook Exercises.
- Exhaustive Additional Question for all Units.
- Chapter-wise Unit Tests with Answers.
- Summative Assessment Question Paper I, II & III Term (2018-19) Answers are given for Practice.
- I & II Term SA (2018-19) Questions are Incorporated with ⊗ Mark at Appropriate Sections.

9th Std.

SURA'S

Mathematics

IX - Standard

Based on the latest Syllabus and updated New Textbook for 2019-20

Salient Features :

- Guide as per the latest Textbook for 2019 - 20
- Complete Solutions to Textbook Exercises.
- Exhaustive Additional Question in all Units.
- Chapter-wise Unit Tests with Answers.
- Summative Question Paper with Answers for 2018-19.

SURA PUBLICATIONS
Chennai

orders@surabooks.com Ph : 9600175757 / 8124201000 / 8124301000

2019 - 20 Edition

ISBN : 978-81-8449-559-1
Code No. : SG 56

Author :

Mrs. S. Thamarai, M.Sc., M.A., M.Ed., M.Phil.
Chennai

Head Office:

1620, 'J' Block, 16th Main Road,
Anna Nagar, **Chennai - 600 040.**
Phones: 044-26162173, 26161099.
Mob : 81242 01000/ 81243 01000
Fax : (91) 44-26162173
e-mail : orders @surabooks.com
website : www.surabooks.com

NOTE FROM PUBLISHER

It gives me great pride and pleasure in bringing to you **Sura's Mathematics Guide** for for **9th Standard**. It is prepared as per the New Syllabus and New Textbook for full year for the year 2019-20.

This guide encompasses all the requirements of the students to comprehend the text and the evaluation of the textbook.

- Additional questions have been provided exhaustively for clear understanding of the units under study.
- Chapter-wise Unit Test are given.

In order to learn effectively, I advise students to learn the subject section-wise and practice the exercises given. It will be a teaching companion to teachers and a learning companion to students.

Though these salient features are available in this Guide, I cannot negate the indispensable role of the teachers in assisting the student to understand the subject thoroughly.

I sincerely believe this guide satisfies the needs of the students and bolsters the teaching methodologies of the teachers.

I pray the almighty to bless the students for consummate success in their examinations.

Subash Raj, B.E., M.S.
- Publisher
Sura Publications

All the Best

For More Information - Contact

Doubts in Our Guides	:	enquiry@surabooks.com
For Order	:	orders@surabooks.com
Contact	:	96001 75757 / 8124301000
Whatsapp	:	8124201000 / 9840926027
Online Site	:	**www.surabooks.com**

For Free Study Materials Visit **http://tnkalvi.in**

CONTENTS

TO ORDER WITH US

SCHOOLS and TEACHERS:

We are grateful for your support and patronage to **'SURA PUBLICATIONS'**

Kindly prepare your order in your School letterhead and send it to us.

For Orders contact: 81242 01000 / 81243 01000

DIRECT DEPOSIT

A/c Name	: **Sura Publications**	A/c Name	: **Sura Publications**
Our A/c No.	: **36550290536**	Our A/c No.	: **21000210001240**
Bank Name	: **STATE BANK OF INDIA**	Bank Name	: **UCO BANK**
Bank Branch	: PADI	Bank Branch	: Anna Nagar West
IFSC	: SBIN0005083	IFSC	: UCBA0002100
A/c Name	: **Sura Publications**	A/c Name	: **Sura Publications**
Our A/c No.	: **6502699356**	Our A/c No.	: **1154135000017684**
Bank Name	: **INDIAN BANK**	Bank Name	: **KVB BANK**
Bank Branch	: ASIAD COLONY	Bank Branch	: Anna Nagar
IFSC	: IDIB000A098	IFSC	: KVBL0001154

After Deposit, please send challan and order to our address.
email : orders@surabooks.com / Whatsapp : 81242 01000.

DEMAND DRAFT / CHEQUE

Please send Demand Draft / cheque in favour of **'SURA PUBLICATIONS'** payable at **Chennai**.

The Demand Draft / cheque should be sent with your order in School letterhead.

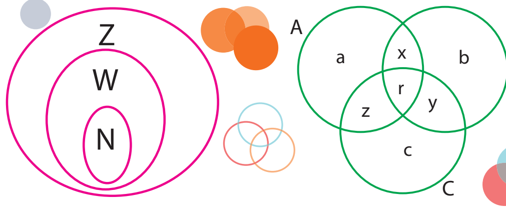
STUDENTS :

Order via Money Order (M/O) to

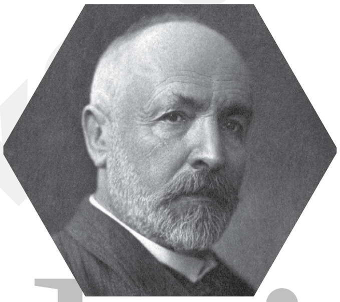
SURA PUBLICATIONS
1620, 'J' Block, 16th Main Road, Anna Nagar,
Chennai - 600 040.
Phones : 044-26162173, 26161099.
Mobile : 96001 75757/ 81242 01000/81243 01000.
email : orders@surabooks.com Website : www.surabooks.com


40 YEARS SERVICE TO EDUCATION
SURA'S
2019-20 EDITION
SCHOOL GUIDES
For Class
9th Standard
New Syllabus
English & Tamil Medium
SURA PUBLICATIONS
1620, 'J' Block, 16th Main Road, Anna Nagar,
Chennai - 600 040. INDIA. Phones : 044-26162173, 26161099.
Mobile : 96001 75757, 81242 01000, 81243 01000
email : enquiry@surabooks.com
orders@surabooks.com
Buy online @
surabooks.com

1 SET LANGUAGE



1.1 Introduction



In our daily life, we often deal with collection of objects like books, stamps, coins, etc. Set language is a mathematical way of representing a collection of objects.

1.2 Set :

(i) A set is a well - defined collection of objects.

(ii) The objects of a set are called its members or elements.

For example,

1. The collection of all books in a District Central Library.
2. The collection of all colours in a rainbow.

1.3 Representation of a Set :

The collection of odd numbers can be described in many ways:

(1) "The set of odd numbers" is a fine description, we understand it well.

(2) It can be written as {1, 3, 5, ...} and you know what I mean.

(3) Also, it can be said as the collection of all numbers x where x is an odd number.

1.3.1 Descriptive Form :

In descriptive form, a set is described in words.

For Example,

(i) The set of all vowels in English alphabets.

(ii) The set of whole numbers.

1.3.2 Set Builder Form or Rule Form :

In set builder form, all the elements are described by a rule.

For example,

(i) A = $\{x : x$ is a vowel in English alphabets$\}$

(ii) B = $\{x \mid x$ is a whole number$\}$

1.3.3 Roster Form or Tabular Form

A set can be described by listing all the elements of the set.

For example,

(i) A = {a, e, i, o, u}

(ii) B = {0,1,2,3,...}

orders@surabooks.com Ph : 9600175757 / 8124201000 / 8124301000

Exercise 1.1

1. Which of the following are sets?

(i) The Collection of prime numbers upto 100.

(ii) The Collection of rich people in India.

(iii) The Collection of all rivers in India.

(iv) The Collection of good Hockey players.

Sol. (i) A = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97}

As the collection of prime numbers upto 100 is known and can be counted (well defined). Hence this is a set.

(ii) The collection of rich people in India. Rich people has no definition.

Hence, it is not a set.

(iii) A = {Cauvery, Sindhu, Ganga, ...}

Hence, it is a set.

(iv) The collection of good hockey players is not a well - defied collection because the criteria for determining a hockey player's talent may vary from person to person.

Hence, this collection is not a set.

2. List the set of letters of the following words in Roster form.

(i) INDIA **(ii) PARALLELOGRAM**

(iii) MISSISSIPPI **(iv) CZECHOSLOVAKIA**

Sol. (i) A = {I, N, D, A}

(ii) B = {P, A, R, L, E, O, G, M}

(iii) C = {M, I, S, P}

(iv) D = {C, Z, E, H, O, S, L, V, A, K, I}.

3. Consider the following sets A = {0, 3, 5, 8} B = {2, 4, 6, 10} C = {12, 14, 18, 20}

(a) State whether True or false.

(i) $18 \in C$ (ii) $6 \notin A$ (iii) $14 \notin C$ (iv) $10 \in B$ (v) $5 \in B$ (vi) $0 \in B$

(b) Fill in the blanks?

(i) $3 \in$ ____ (ii) $14 \in$ ____ (iii) 18 ____ B (iv) 4 ____ B

Sol. (a) (i) True (ii) True (iii) False (iv) True (v) False (vi) False.

(b) (i) A (ii) C (iii) $\notin$ (iv) $\in$

4. Represent the following sets in Roster form.

(i) A = The set of all even natural numbers less than 20.

(ii) $B = \{y : y = \frac{1}{2n}, n \in \mathbb{N}, n \leq 5\}$

(iii) $C = \{x : x$ is perfect cube, $27 < x < 216\}$

(iv) $D = \{x : x \in \mathbb{Z}, -5 < x \leq 2\}$

Sol. (i) A = {2, 4, 6, 8, 10, 12, 14, 16, 18}

(ii) N = { 1, 2, 3, 4, 5}

$$\text{if, } n = 1, y = \frac{1}{2n} = \frac{1}{2\times1} = \frac{1}{2}$$

$$n = 2, y = \frac{1}{2\times2} = \frac{1}{4}$$

$$n = 3, y = \frac{1}{2\times3} = \frac{1}{6}$$

$$n = 4, y = \frac{1}{2\times4} = \frac{1}{8}$$

$$n = 5, y = \frac{1}{2\times5} = \frac{1}{10}$$

$$\therefore B = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}\right\}$$

(iii) C = {64, 125}

(iv) D = {–4, –3, –2, –1, 0, 1, 2}

5. Represent the following sets in set builder form.

(i) B = The set of all Cricket players in India who scored double centuries in One Day Internationals.

(ii) $C = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\right\}$.

(iii) D = The set of all tamil months in a year.

(iv) E = The set of odd Whole numbers less than 9.

Sol. (i) B = {x : x is an Indian player who scored double centuries in one day internationals}

(ii) $C = \{x : x = \frac{n}{n+1}, n \in \mathbb{N}\}$

(iii) D = {x : x is a tamil month in a year}

iv) E = {x : x is odd number, $x \in \mathbb{W}$, $x < 9$, where W is the set of whole numbers}.

6. Represent the following sets in descriptive form.

(i) P = { January, June, July}

(ii) Q = {7,11,13,17,19,23,29}

(iii) R = {x : $x \in \mathbb{N}$, $x < 5$}

(iv) S = {x : x is a consonant in English alphabets}

Sol. (i) P is the set of English Months begining with J.

(ii) Q is the set of all prime numbers between 5 and 31.

(iii) R is the set of all natural numbers less than 5.

(iv) S is the set of all English consonants.

1.4. Types of sets

1.4.1 Empty Set or Null Set :

A set consisting of no element is called the empty set or null set or void set.

For example,

A=$\{x : x$ is an odd integer and divisible by 2$\}$

$\therefore$ A=$\{ \}$ or $\varnothing$

1.4.2 Singleton Set :

A set which has only one element is called a singleton set.

For example,

A = $\{x : 3 < x < 5, x \in \mathbb{N}\}$

1.4.3 Finite Set :

A set with finite number of elements is called a finite set.

For example,

1. The set of family members.
2. The set of indoor/outdoor games you play.

1.4.4 Infinite Set :

A set which is not finite is called an infinite set.

For example,

(i) $\{5,10,15,...\}$ (ii) The set of all points on a line.

1.4.5 Equivalent Sets :

Two finite sets A and B are said to be equivalent if they contain the same number of elements. It is written as A ≈ B.

If A and B are equivalent sets, then $n(A) = n(B)$.

1.4.6 Equal Sets :

Two sets are said to be equal if they contain exactly the same elements, otherwise they are said to be unequal.

In other words, two sets A and B are said to be equal, if

(i) every element of A is also an element of B

(ii) every element of B is also an element of A.

1.4.7 Subset :

Let A and B be two sets. If every element of A is also an element of B, then A is called a subset of B. We write A ⊆ B.

1.4.8 Proper Subset :

Let A and B be two sets. If A is a subset of B and A≠B, then A is called a proper subset of B and we write A ⊂ B.

For example,

If A=$\{1,2,5\}$ and B=$\{1,2,3,4,5\}$ then A is a proper subset of B i.e. A ⊂ B.

1.4.9 Power set :

The set of all subsets of A is said to be the power set of the set A and is denoted as P(A)

For example,

Let A = $\{-3, 4\}$

The subsets of A are , $\varnothing$, $\{-3\}$, $\{4\}$, $\{-3, 4\}$

Then the power set of A is P(A) = $\{\varnothing, \{-3\}, \{4\}, \{-3, 4\}\}$

Exercise 1.2

1. Find the cardinal number of the following sets.

(i) $M = \{p, q, r, s, t, u\}$

(ii) $P = \{x : x = 3n + 2, n \in \mathbb{W} \text{ and } x < 15\}$

(iii) $Q = \{y : y = \frac{4}{3n}, n \in \mathbb{N} \text{ and } 2 < n \leq 5\}$

(iv) $R = \{x : x$ **is an integers,** $x \in \mathbb{Z}$ **and** $-5 \leq x < 5\}$

(v) **S = The set of all leap years between 1882 and 1906.**

Sol. (i) $n(M) = 6$

(ii) $W = \{0, 1, 2, 3, ...\}$

if $n = 0$, $x = 3(0) + 2 = 2$

if $n = 1$, $x = 3(1) + 2 = 5$

if $n = 2$, $x = 3(2) + 2 = 8$

if $n = 3$, $x = 3(3) + 2 = 11$

if $n = 4$, $x = 3(4) + 2 = 14$

$\therefore P = \{2, 5, 8, 11, 14\}$

$n(P) = 5$

(iii) $N = \{1, 2, 3, 4, ...\}$

$n \in \{3, 4, 5\}$

if $n = 3$, $y = \frac{4}{3(3)} = \frac{4}{9}$

if $n = 4$, $y = \frac{4}{3(4)} = \frac{4}{12}$

if $n = 5$, $y = \frac{4}{3(5)} = \frac{4}{15}$

$Q = \left\{\frac{4}{9}, \frac{4}{12}, \frac{4}{15}\right\}$

$n(Q) = 3$

(iv) $x \in z$

$R = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$n(R) = 10.$

(v) $S = \{1884, 1888, 1892, 1896, 1904\}$

$n(S) = 5.$

2. Identify the following sets as finite or infinite.

(i) **X = The set of all districts in Tamilnadu.**

(ii) **Y = The set of all straight lines passing through a point.**

(iii) $A = \{x : x \in \mathbb{Z} \text{ and } x < 5\}$

(iv) $B = \{x : x^2 - 5x + 6 = 0, x \in \mathbb{N}\}$

Sol. (i) Finite set

(ii) Infinite set

(iii) A = { ...., –2, –1, 0, 1, 2, 3,4} $\therefore$ Infinite set

(iv) $x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

B = {3, 2} $\therefore$ Finite set.

3. Which of the following sets are equivalent or unequal or equal sets?

(i) A = The set of vowels in the English alphabets.
B = The set of all letters in the word "VOWEL"

(ii) C = {2, 3, 4, 5}
D = $\{x : x \in \mathbb{W}, 1 < x < 5\}$

(iii) X = A={ x : x is a letter in the word "LIFE"}
Y = {F, I, L, E}

(iv) G = {x : x is a prime number and $3 < x < 23$}
H = {x : x is a divisor of 18}

Sol. (i) A = {a, e, i, o, u}
B = {V, O,W, E, L}
The sets A and B contain the same number of elements. $\therefore$ Equivalent sets

(ii) C = {2, 3, 4, 5}
D = {2, 3, 4} $\therefore$ Unequal sets

(iii) X = {L, I, F, E}
Y = {F, I, L, E}
The sets X and Y contain the exactly the same elements. $\therefore$ Equal sets.

(iv) G = {5, 7, 11, 13, 17, 19}
H = {1, 2, 3, 6, 9, 18} $\therefore$ Equivalent sets.

4. Identify the following sets as null set or singleton set.

(i) A = $\{x : x \in \mathbb{N}, 1 < x < 2\}$

(ii) B = The set of all even natural numbers which are not divisible by 2.

(iii) C = {0}.

(iv) D = The set of all triangles having four sides.

Sol. (i) A = { } $\because$ There is no element in between 1 and 2 in Natural numbers. $\therefore$ Null set

(ii) B = { } $\because$ All even natural numbers are divisible by 2. $\therefore$ B is Null set

(iii) C = {0} $\therefore$ Singleton set

(iv) D = { } $\because$ No triangle has four sides. $\therefore$ D is a Null set.

5. State which pairs of sets are disjoint or overlapping?

(i) A={f,i,a,s} and B={a,n,f,h,s}

(ii) C ={x : x is a prime number, x >2} and D ={x : x is an even prime number}

(iii) E={x : x is a factor of 24} and F={x : x is a multiple of 3, $x <$ 30}

Sol. (i)

A = {f, i, a, s}

B = {a, n, f, h, s}

$A \cap B = \{f, i, a, s\} \cap \{a, n, f, h, s\} = \{f, a, s\}$

Since $A \cap B \neq \phi$, A and B are overlapping sets.

(ii)

C = {3, 5, 7,11, ...}

D = {2}

$C \cap D = \{3, 5, 7, 11, ...\} \cap \{2\} = \{\ \}$

Since $C \cap D = \varnothing$, C and D are disjoint sets.

(iii) $E = \{1, 2, 3, 4, 6, 8, 12, 24\}$

$F = \{3, 6, 9, 12, 15, 18, 21, 24, 27\}$

$E \cap F = \{1, 2, 3, 4, 6, 8, 12, 24\} \cap \{3, 6, 9, 12, 15, 18, 21, 24, 27\}$

$= \{3, 6, 12, 24\}$

Since $E \cap F \neq \phi$, E and F are overlapping sets.

6. If S = {square,rectangle,circle,rhombus,triangle}, list the elements of the following subset of S.

(i) The set of shapes which have 4 equal sides.

(ii) The set of shapes which have radius.

(iii) The set of shapes in which the sum of all interior angles is 180°

(iv) The set of shapes which have 5 sides.

Sol. (i) {Square, Rhombus} (ii) {Circle}

(iii) {Triangle} (iv) Null set.

7. If A={a,{a,b}}, write all the subsets of A.

Sol. $A = \{a, \{a, b\}\}$ subsets of A are $\{\ \}$ $\{a\}$, $\{a, b\}$, $\{a, \{a, b\}\}$.

8. Write down the power set of the following sets.

(i) A={a,b} (ii) B={1,2,3} (iii) D={p,q,r,s} (iv) E = $\varnothing$

Sol. (i) The subsets of A are $\varnothing$, $\{a\}$, $\{b\}$, $\{a, b\}$

The power set of A

$P(A) = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$

(ii) The subsets of B are ϕ, {1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}, {1, 2, 3}

The power set of B

$P(B) = \{\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$

(iii) The subset of D are $\varnothing$, $\{p\}$, $\{q\}$, $\{r\}$, $\{s\}$, $\{p, q\}$, $\{p, r\}$, $\{p, s\}$, $\{q, r\}$, $\{q, s\}$, $\{r, s\}$,$\{p, q, r\}$, $\{q, r, s\}$, $\{p, r, s\}$, $\{p, q, s\}$, $\{p, q, r, s\}\}$

The power set of D

$P(D) = \{\varnothing, \{p\}, \{q\}, \{r\}, \{s\}, \{p, q\}, \{p, r\}, \{p, s\}, \{q, r\}, \{q, s\}, \{r, s\}, \{p, q, r\}, \{q, r, s\}, \{p, r, s\}, \{p, q, s\}, \{p, q, r, s\}$

(iv) The power set of E

$P(E) = \{\ \}$.

9. Find the number of subsets and the number of proper subsets of the following sets.

(i) W={red,blue,yellow} **(ii) X ={ x^2 : $x \in \mathbb{N}$, $x^2 \leq 100$}.**

Sol. (i) Given W = {red, blue, yellow}

Then $n(W) = 3$

The number of subsets $= n[P(W)] = 2^3 = 8$

The number of proper subsets $= n[P(W)] - 1 = 2^3 - 1 = 8 - 1 = 7$

(ii) Given X = {1, 2, 3, ...}

$X^2 = \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$

$n(X) = 10$

The Number of subsets $= n[P(X)] = 2^{10} = 1024$

The Number of proper subsets $= n[P(X)] - 1 = 2^{10} - 1 = 1024 - 1 = 1023$.

10. (i) If $n(A) = 4$, find $n[P(A)]$. (ii) If $n(A) = 0$, find $n[P(A)]$.
(iii) If $n[P(A)] = 256$, find $n(A)$.

Sol. (i)
$$n(A) = 4$$
$$n[P(A)] = 2^n = 2^4 = 16$$

(ii)
$$n(A) = 0$$
$$n[P(A)] = 2^0 = 1$$

(iii)
$$n[P(A)] = 256$$

$$\begin{array}{r|l} 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$$n[P(A)] = 2^8$$
$$\therefore n(A) = 8.$$

1.5 Set operations :

1.5.1 Complement of a Set

The Complement of a set A is the set of all elements of U (the universal set) that are not in A.

It is denoted by A′ or A^C. In symbols $A' = \{x : x \in U, x \notin A\}$

1.5.2 Union of Two Sets

The union of two sets A and B is the set of all elements which are either in A or in B or in both. It is denoted by A∪B and read as A union B.

In symbol, $A \cup B = \{x : x \in A \text{ or } x \in B\}$

1.5.3 Intersection of Two Sets

The intersection of two sets A and B is the set of all elements common to both A and B. It is denoted by A∩B and read as A intersection B.

In symbol , $A \cap B = \{x : x \in A \text{ and } x \in B\}$

1.5.4 Difference of Two Sets

Let A and B be two sets, the difference of sets A and B is the set of all elements which are in A, but not in B. It is denoted by A–B or A\B and read as A difference B.

In symbol, $A - B = \{ x : x \in A \text{ and } x \notin B\}$; $B - A = \{ y : y \in B \text{ and } y \notin A\}$.

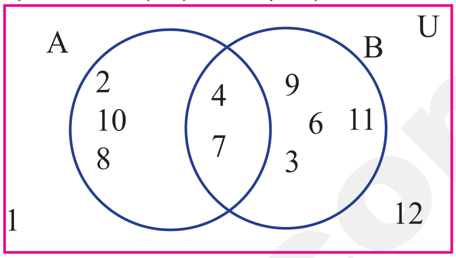
1.5.5 Symmetric Difference of Sets

The symmetric difference of two sets A and B is the set $(A - B) \cup (B - A)$. It is denoted by $A \Delta B$.

$A \Delta B = \{ x : x \in A - B \text{ or } x \in B - A\}$

Exercise 1.3

1. Using the given venn diagram, write the elements of
(i) A (ii) B (iii) A∪B (iv) A∩B (v) A – B (vi) B – A (vii) A′ (viii) B′
(ix) U



Sol.
(i) A = {2, 4, 7, 8, 10}
(ii) B = {3, 4, 6, 7, 9, 11}
(iii) A∪B = {2, 3, 4, 6, 7, 8, 9, 10, 11}
(iv) A∩B = {4, 7}
(v) A – B = {2, 8, 10}
(vi) B – A = {3, 6, 9, 11}
(vii) A′ = {1, 3, 6, 9, 11, 12}
(viii) B′ = {1, 2, 8, 10, 12}
(ix) U = {1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12}.

2. Find A∪B, A∩B, A – B and B – A for the following sets.
(i) A = {2, 6, 10, 14} and B = {2, 5, 14, 16}
(ii) A = {*a, b, c, e, u*} and B = {*a, e, i, o, u*}
(iii) A={$x : x \in \mathbb{N}, x \leq 10$} and B={$x : x \in \mathbb{W}, x < 6$}
(iv) A = Set of all letters in the word "mathematics" and
B = Set of all letters in the word "geometry"

Sol. (i) A = {2, 6, 10, 14} and B = {2, 5, 14, 16}

$A \cup B = \{2, 6, 10, 14\} \cup \{2, 5, 14, 16\} = \{2, 5, 6, 10, 14, 16\}$
$A \cap B = \{2, 6, 10, 14\} \cap \{2, 5, 14, 16\} = \{2, 14\}$
$A - B = \{\cancel{2}, 6, 10, \cancel{14}\} - \{2, 5, 14, 16\} = \{6, 10\}$
$B - A = \{\cancel{2}, 5, \cancel{14}, 16\} - \{2, 6, 10, 14\} = \{5, 16\}$

(ii) A = {*a, b, c, e, u*} and B = {*a, e, i, o, u*}

$A \cup B = \{a, b, c, e, u\} \cup \{a, e, i, o, u\} = \{a, b, c, e, i, o, u\}$
$A \cap B = \{a, b, c, e, u\} \cap \{a, e, i, o, u\} \{a, e, u\}$
$A - B = \{\cancel{a}, b, c, \cancel{e}, \cancel{u}\} - \{a, e, i, o, u\} = \{b, c\}$
$B - A = \{\cancel{a}, \cancel{e}, i, o, \cancel{u}\} - \{a, b, c, e, u\} = \{i, o\}$

(iii) $x \in$ {1, 2, 3, ...} ; $x \in$ {0, 1, 2, 3, 4, 5, ...}

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$B = \{0, 1, 2, 3, 4, 5\}$
$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \cup \{0, 1, 2, 3, 4, 5\}$
$= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A \cap B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \cap \{0, 1, 2, 3, 4, 5\} = \{1, 2, 3, 4, 5\}$
$A - B = \{\cancel{1}, \cancel{2}, \cancel{3}, \cancel{4}, \cancel{5}, 6, 7, 8, 9, 10\} - \{0, 1, 2, 3, 4, 5\} = \{6, 7, 8, 9, 10\}$
$B - A = \{0, \cancel{1}, \cancel{2}, \cancel{3}, \cancel{4}, \cancel{5}\} - \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} = \{0\}$

(iv) A = {*m, a, t, h, e, i, c, s*}, B = {*g, e, o, m, t, r, y*}

$A \cup B = \{m, a\ t, h, e, i, c, s\} \cup \{g, e, o, m, t, r, y\} = \{m, a, t, h, e, i, c, s, g, o, r, y\}$
$A \cap B = \{m, a, t, h, e, i, c, s\} \cap \{g, e, o, m, t, r, y\} = \{m, t, e\}$
$A - B = \{\cancel{m}, a, \cancel{t}, h, \cancel{e}, i, c, s\} - \{g, e, o, m, t, r, y\} = \{a, h, i, c, s\}$
$B - A = \{g, \cancel{e}, o, \cancel{m}, \cancel{t}, r, y\} - \{m, a, t, h, e, i, c, s\} = \{g, o, r, y\}$

3. If U = {*a, b, c, d, e, f, g ,h*}, A = {*b, d, f, h*} and B = {*a, d, e, h*}, find the following sets.

(i) A' (ii) B' (iii) $A' \cup B'$ (iv) $A' \cap B'$
(v) $(A \cup B)'$ (vi) $(A \cap B)'$ (vii) $(A')'$ (viii) $(B')'$

Sol. (i) $A' = U - A = \{a, \cancel{b}, c, \cancel{d}, e, \cancel{f}, g, \cancel{h}\} - \{b, d, f, h\} = \{a, c, e, g\}$

(ii) $B' = U - B = \{\cancel{a}, b, c, \cancel{d}, \cancel{e}, f, g, \cancel{h}\} - \{a, d, e, h\} = \{b, c, f, g\}$

(iii) $A' \cup B' = \{a, c, e, g\} \cup \{b, c, f, g\} = \{a, b, c, e, f, g\}$

(iv) $A' \cap B' = \{a, c, e, g\} \cap \{b, c, f, g\} = \{c, g\}$

(v) $(A \cup B)' = U - (A \cup B) = \{\cancel{a}, \cancel{b}, c, \cancel{d}, \cancel{e}, \cancel{f}, g, \cancel{h}\} - \{a, b, d, e, f, h\} = \{c, g\}$

(vi) $(A \cap B)' = U - (A \cap B)$
$= \{a, b, c, \cancel{d}, e, f, g, \cancel{h}\} - \{d, h\} = \{a, b, c, e, f, g\}$

(vii) $(A')' = U - A' = \{\cancel{a}, b, \cancel{c}, d, \cancel{e}, f, \cancel{g}, h\} - \{a, c, e, g\} = \{b, d, f, h\}$

(viii) $(B')' = U - B' = \{a, \cancel{b}, \cancel{c}, d, e, \cancel{f}, \cancel{g}, h\} - \{b, c, f, g\} = \{a, d, e, h\}$

4. Let U = {0, 1, 2, 3, 4, 5, 6, 7}, A = {1, 3, 5, 7} and B = {0, 2, 3, 5, 7}, find the following sets.

(i) A' (ii) B' (iii) $A' \cup B'$ (iv) $A' \cap B'$ (v) $(A \cup B)'$ (vi) $(A \cap B)'$ (vii) $(A')'$ (viii) $(B')'$

Sol. (i) $A' = U - A = \{0, \cancel{1}, 2, \cancel{3}, 4, \cancel{5}, 6, \cancel{7}\} - \{1, 3, 5, 7\} = \{0, 2, 4, 6\}$

(ii) $B' = U - B = \{\cancel{0}, 1, \cancel{2}, \cancel{3}, 4, \cancel{5}, 6, \cancel{7}\} - \{0, 2, 3, 5, 7\} = \{1, 4, 6\}$

(iii) $A' \cup B' = \{0, 2, 4, 6\} \cup \{1, 4, 6\} = \{0, 1, 2, 4, 6\}$

(iv) $A' \cap B' = \{0, 2, 4, 6\} \cap \{1, 4, 6\} = \{4, 6\}$

(v) $(A \cup B)' = U - (A \cup B) = \{\cancel{0}, \cancel{1}, \cancel{2}, \cancel{3}, 4, \cancel{5}, 6, \cancel{7}\} - \{0, 1, 2, 3, 5, 7\}$
$= \{4, 6\}$

(vi) $(A \cap B)' = U - (A \cap B) = \{0, 1, 2, \cancel{3}, 4, \cancel{5}, 6, \cancel{7}\} - \{3, 5, 7\} = \{0, 1, 2, 4, 6\}$

(vii) $(A')' = U - A' = \{\cancel{0}, 1, \cancel{2}, 3, \cancel{4}, 5, \cancel{6}, 7\} - \{0, 2, 4, 6\} = \{1, 3, 5, 7\}$

(viii) $(B')' = U - B' = \{0, \cancel{1}, 2, 3, \cancel{4}, 5, \cancel{6}, 7\} - \{1, 4, 6\} = \{0, 2, 3, 5, 7\}$.

5. Find the symmetric difference between the following sets.

(i) P = {2, 3, 5, 7, 11} and Q = {1, 3, 5, 11}
(ii) R = {*l, m, n, o, p*} and S = {*j, l, n, q*}
(iii) X = {5, 6, 7} and Y = {5, 7, 9, 10}

Sol. (i)
$P = \{2, 3, 5, 7, 11\}$
$Q = \{1, 3, 5, 11\}$
$P - Q = \{2, \cancel{3}, \cancel{5}, 7, \cancel{11}\} - \{1, 3, 5, 11\} = \{2, 7\}$
$Q - P = \{1, \cancel{3}, \cancel{5}, \cancel{11}\} - \{2, 3, 5, 7, 11\} = \{1\}$
$P \Delta Q = (P - Q) \cup (Q - P) = \{2, 7\} \cup \{1\} = \{1, 2, 7\}$

(ii)
$R = \{l, m, n, o, p\}$
$S = \{j, l, n, q\}$
$R - S = \{l, m, n, o, p\} - \{j, l, n, q\} = \{m, o, p\}$
$S - R = \{j, \cancel{l}, \cancel{n}, q\} - \{l, m, n, o, p\} = \{j, q\}$
$R \Delta S = (R - S) \cup (S - R) = \{m, o, p\} \cup \{j, q\} = \{j, m, o, p, q\}$

(iii) $X = \{5, 6, 7\}$

$Y = \{5, 7, 9, 10\}$

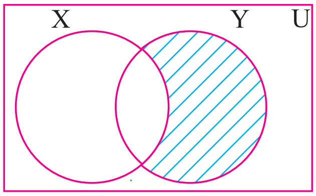
$X - Y = \{\cancel{5}, 6, \cancel{7}\} - \{5, 7, 9, 10\} = \{6\}$

$Y - X = \{\cancel{5}, \cancel{7}, 9, 10\} - \{5, 6, 7\} = \{9, 10\}$

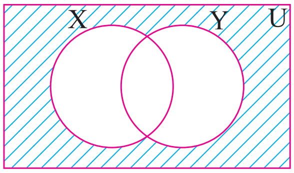
$X \Delta Y = (X - Y) \cup (Y - X) = \{6\} \cup \{9, 10\} = \{6, 9, 10\}.$

6. Using the set symbols, write down the expressions for the shaded region in the following

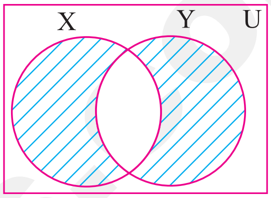
(i)



(ii)



(iii)



Sol. (i) $X - Y$ (ii) $(X \cup Y)'$ (iii) $(X - Y) \cup (X - Y)$.

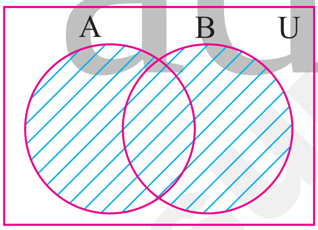
7. Let A and B be two overlapping sets and the universal set U. Draw appropriate Venn diagram for each of the following,

(i) $A \cup B$ **(ii)** $A \cap B$ **(iii)** $(A \cap B)'$

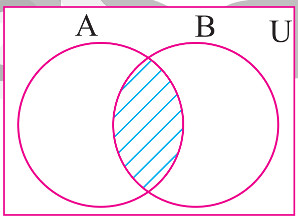
(iv) $(B - A)'$ **(v)** $A' \cup B'$ **(vi)** $A' \cap B'$

(vii) What do you observe from the diagram (iii) and (v)?

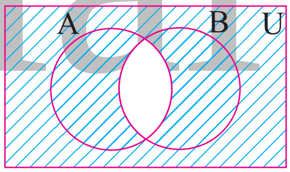
Sol. **(i)** **$A \cup B$**



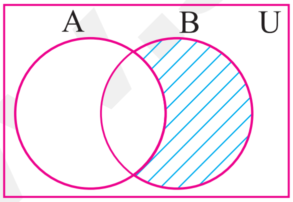
(ii) **$A \cap B$**



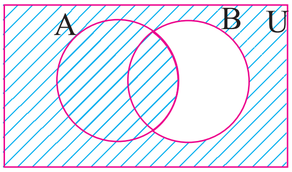
(iii) **$(A \cap B)'$**



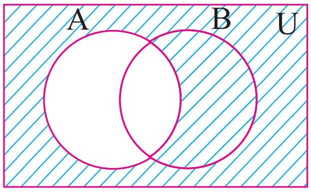
(iv) **$B - A$**



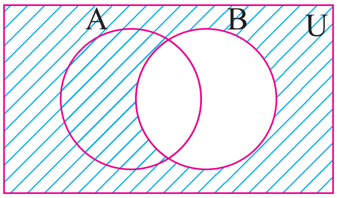
$(B - A)'$



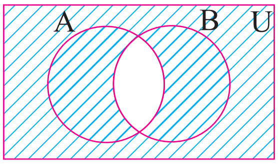
(v) **A'**



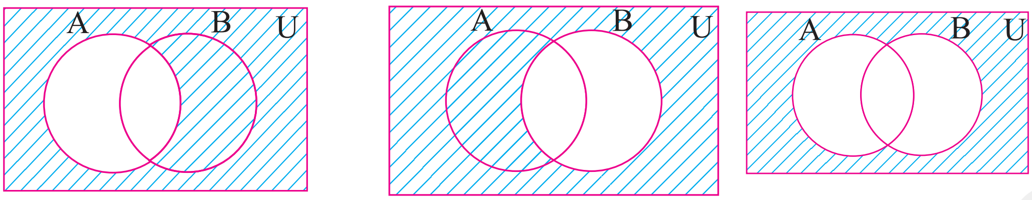
B'



$A' \cup B'$



(vi) A′ B′ A′ ∩ B′



(vii) From the diagrams (iii) and (v) we observe that $(A\cap B)' = A'\cup B'$.

1.6 Properties of Set Operations :

It is an interesting investigation to find out if operations among sets (like union, intersection, etc) follow mathematics properties such as commutativity, Associativity, etc.

Exercise 1.4

1. If P = {1, 2, 5, 7, 9}, Q = {2, 3, 5, 9, 11}, R = {3, 4, 5, 7, 9} and S = {2, 3, 4, 5, 8}, then find

(i) $(P\cup Q)\cup R$ (ii) $(P\cap Q)\cap S$ (iii) $(Q\cap S)\cap R$

Sol. **(i) $(P\cup Q)\cup R$**

$$(P\cup Q) = \{1, 2, 5, 7, 9\} \cup \{2, 3, 5, 9, 11\} = \{1, 2, 3, 5, 7, 9, 11\}$$
$$(P\cup Q)\cup R = \{1, 2, 3, 5, 7, 9, 11\} \cup \{3, 4, 5, 7, 9\} = \{1, 2, 3, 4, 5, 7, 9, 11\}$$

(ii) $(P\cap Q)\cap S$

$$(P\cap Q) = \{1, \underline{2}, \underline{5}, 7, \underline{9}\} \cap \{\underline{2}, 3, \underline{5}, \underline{9}, 11\} = \{2, 5, 9\}$$
$$(P\cap Q)\cap S = \{\underline{2}, \underline{5}, 9\} \cap \{\underline{2}, 3, 4, \underline{5}, 8\} = \{2, 5\}$$

(iii) $(Q\cap S)\cap R$

$$(Q\cap S) = \{\underline{2}, \underline{3}, \underline{5}, 9, 11\} \cap \{\underline{2}, \underline{3}, 4, \underline{5}, 8\} = \{2, 3, 5\}$$
$$(Q\cap S)\cap R = \{2, \underline{3}, \underline{5}\} \cap \{\underline{3}, 4, \underline{5}, 7, 9\} = \{3, 5\}$$

2. Test for the commutative property of union and intersection of the sets
P = { *x* : *x* is a real number between 2 and 7} and
Q = { *x* : *x* is an irrational number between 2 and 7}

Sol. **Commulative Property of union of sets**

$\boxed{(A\cup B) = (B\cup A)}$

$$\text{Here } P = \{3, 4, 5, 6\},\ Q = \{\sqrt{3}, \sqrt{5}, \sqrt{6}\}$$
$$P\cup Q = \{3, 4, 5, 6\} \cup \{\sqrt{3}, \sqrt{5}, \sqrt{6}\} = \{3, 4, 5, 6, \sqrt{3}, \sqrt{5}, \sqrt{6}\} \quad \text{... (1)}$$
$$Q\cup P = \{\sqrt{3}, \sqrt{5}, \sqrt{6}\} \cup \{3, 4, 5, 6\} = \{\sqrt{3}, \sqrt{5}, \sqrt{6}, 3, 4, 5, 6\} \quad \text{... (2)}$$
$$(1) = (2)$$
$$\therefore P\cup Q = Q\cup P$$

∴ It is verified that union of sets is commutative.

Commutative Property of intersection of sets

$\boxed{(P\cap Q) = (Q\cap P)}$

$$P\cap Q = \{3, 4, 5, 6\}\cap\{\sqrt{3},\sqrt{5},\sqrt{6}\} = \{\ \ \} \quad \text{... (1)}$$
$$Q\cap P = \{\sqrt{3},\sqrt{5},\sqrt{6}\} \cap \{3, 4, 5, 6\} = \{\ \ \} \quad \text{... (2)}$$

From (1) and (2)

$$P\cap Q = Q\cap P$$

$\therefore$ It is verified that intersection of sets is commutative.

3. **If A = {p, q, r, s}, B = {m, n, q, s, t} and C = {m, n, p, q, s}, then verify the associative property of union of sets.**

Sol. **Associative Property of union of sets**

$\boxed{A\cup(B\cup C) = (A\cup B)\cup C}$

$$B\cup C = \{m, n, q, s, t\} \cup \{m, n, p, q, s\} = \{m, n, p, q, s, t\}$$
$$A\cup(B\cup C) = \{p, q, r, s\} \cup \{m, n, p, q, s, t\} = \{m, n, p, q, r, s, t\} \quad \text{... (1)}$$
$$(A\cup B) = \{p, q, r, s\} \cup \{m, n, q, s, t\} = \{p, q, r, s, m, n, t\}$$
$$(A\cup B)\cup C = \{p, q, r, s, m, n, t\} \cup \{m, n, p, q, s\} = \{p, q, r, s, m, n, t\} \quad \text{... (2)}$$

From (1) & (2)

It is verified that $A\cup(B\cup C) = (A\cup B)\cup C$

4. **Verify the associative property of intersection of sets for A={−11, $\sqrt{2}$, $\sqrt{5}$, 7}, B = {$\sqrt{3}$, $\sqrt{5}$, 6, 13} and C = {$\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, 9}.**

Sol. **Associative Property of intersection of sets**

$\boxed{A\cap(B\cap C) = (A\cap B)\cap C}$

$$B\cap C = \{\sqrt{3},\sqrt{5},6,13\}\cap\{\sqrt{2},\sqrt{3},\sqrt{5},9\} = \{\sqrt{3},\sqrt{5}\}$$
$$A\cap(B\cap C) = \{-11,\sqrt{2},\sqrt{5},7\} \cap \{\sqrt{3},\sqrt{5}\} = \{\sqrt{5}\} \quad \text{... (1)}$$
$$A\cap B = \{-11,\sqrt{2},\sqrt{5},7\} \cap \{\sqrt{3},\sqrt{5},6,13\} = \{\sqrt{5}\}$$
$$(A\cap B)\cap C = \{\sqrt{5}\} \cap \{\sqrt{2},\sqrt{3},\sqrt{5},9\} = \{\sqrt{5}\} \quad \text{... (2)}$$

From (1) and (2), it is verified that $A\cap(B\cap C) = (A\cap B)\cap C$

5. **If A = {x : $x = 2^n$, $n \in \mathbb{W}$ and $n < 4$}, B = {x : $x = 2n$, $n \in \mathbb{N}$ and $n \leq 4$} and C = {0, 1, 2, 5, 6}, then verify the associative property of intersection of sets.**

Sol.

$$A = \{x : x = 2^n,\ n \in \mathbb{W},\ n < 4\}$$
$$\Rightarrow x = 2^0 = 1$$
$$x = 2^1 = 2$$
$$x = 2^2 = 4$$
$$x = 2^3 = 8$$
$$\therefore A = \{1, 2, 4, 8\}$$
$$B = \{x : x = 2n,\ n \in \mathbb{N} \text{ and } n \leq 4\}$$
$$\Rightarrow x = 2 \times 1 = 2$$
$$x = 2 \times 2 = 4$$
$$x = 2 \times 3 = 6$$
$$x = 2 \times 4 = 8$$
$$\therefore B = \{2, 4, 6, 8\}$$
$$C = \{0, 1, 2, 5, 6\}$$

Associative property of intersection of sets

$$A\cap (B \cap C) = (A\cap B) \cap C$$

$$B \cap C = \{2, 6\}$$

$$A \cap(B \cap C) = \{1, 2, 4, 8\} \cap \{2, 6\} = \{2\} \quad \dots (1)$$

$$A \cap B = \{1, 2, 4, 8\} \cap \{2, 4, 6, 8\} = \{2, 4, 8\}$$

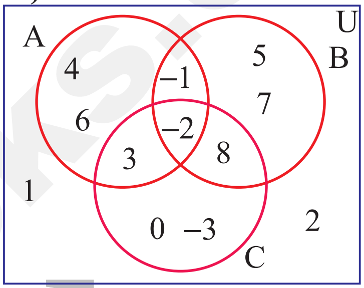
$$(A \cap B) \cap C = \{2, 4, 8\} \cap \{0, 1, 2, 5, 6\} = \{2\} \quad \dots (2)$$

From (1) and (2), It is verified that $A \cap(B \cap C) = (A \cap B) \cap C$

Exercise 1.5

1. Using the adjacent venn diagram, find the following sets :

(i) $A - B$ (ii) $B - C$ (iii) $A'\cup B'$ (vi) $A'\cap B'$
(v) $(B\cup C)'$ (vi) $A - (B\cup C)$ (vii) $A - (B\cap C)$



Sol. (i) $A - B = \{3, 4, 6\}$

(ii) $B - C = \{-1, 5, 7\}$

(iii) $A'\cup B'$

$A' = \{1, 2, 0, -3, 5, 7, 8\}$

$B' = \{-3, 0, 1, 2, 3, 4, 6\}$

$A'\cup B' = \{-3, 0, 1, 2, 3, 4, 5, 6, 7, 8\}$

(iv) $A'\cap B'$

$A'\cap B' = \{-3, 0, 1, 2\}$

(v) $(B \cup C)'$

$B \cup C = \{-3, -2, -1, 0, 3, 5, 7, 8\}$

$(B \cup C)' = U - (B \cup C)$

$= \{\cancel{-3}, \cancel{-2}, \cancel{-1}, 0, 1, 2, \cancel{3}, 4, \cancel{5}, 6, \cancel{7}, \cancel{8}\} - \{-3, -2, -1, 0, 3, 5, 7, 8\}$

$(B \cup C)' = \{1, 2, 4, 6\}$

(vi) $A - (B \cup C) = \{\cancel{-2}, \cancel{-1}, \cancel{3}, 4, 6\} - \{-3, -2, -1, 0, 3, 5, 7, 8\} = \{4, 6\}$

(vii) $A - (B\cap C)$

$B\cap C = \{-2, 8\}$

$A - (B\cap C) = \{\cancel{-2}, -1, 3, 4, 6\} - \{-2, 8\} = \{-1, 3, 4, 6\}$

2. If $K = \{a, b, d, e, f\}$, $L = \{b, c, d, g\}$ and $M \{a, b, c, d, h\}$ then find the following:
(i) $K\cup(L\cap M)$ (ii) $K\cap(L\cup M)$ (iii) $(K\cup L)\cap(K\cup M)$
(iv) $(K\cap L)\cup(K\cap M)$ and verify distributive laws.

Sol. $K = \{a, b, d, e, f\}$, $L = \{b, c, d, g\}$ and $M \{a, b, c, d, h\}$

(i) $K\cup(L\cap M)$

$L\cap M = \{b, c, d, g\} \cap\{a, b, c, d, h\} = \{b, c, d\}$

$K\cup(L\cap M) = \{a, b, d, e, f\}\cup \{b, c, d\} = \{a, b, c, d, e, f\}$

(ii) $K\cap(L\cup M)$

$L\cup M = \{a, b, c, d, g, h\}$

$K\cap(L\cup M) = \{a, b, d, e, f\} \cap\{a, b, c, d, g, h\} = \{a, b, d\}$

(iii) $(K\cup L)\cap(K\cup M)$

$K\cup L = \{a, b, c, d, e, f, g\}$

$K\cup M = \{a, b, c, d, e, f, h\}$

$(K\cup L) \cap(K\cup M) = \{a, b, c, d, e, f\}$

(iv) (K∩L)∪(K∩M)

$$(K\cap L) = \{b, d\}$$
$$(K\cap M) = \{a, b, d\}$$
$$(K\cap L)\cup(K\cap M) = \{b, d\}\cup\{a, b, d\} = \{a, b, d\}$$

Distributive laws

$$K\cup(L\cap M) = (K\cup L)\cap(K\cup M)$$
$$\{a, b, c, d, e, f\} = \{a, b, c, d, e, f, g\}\cap\{a, b, c, d, e, f, h\}$$
$$= \{a, b, c, d, e, f\}$$

Thus Verified.

$$K\cap(L\cup M) = (K\cap L)\cup(K\cap M)$$
$$\{a, b, d\} = \{a, b, c, d, e, f, g\}\cup\{a, b, c, d, e, f, h\}$$
$$= \{a, b, d\}$$

Thus Verified.

3. **If $A = \{x : x \in \mathbb{Z}, -2 < x \leq 4\}$, $B = \{x : x \in \mathbb{W}, x \leq 5\}$, $C = \{-4, -1, 0, 2, 3, 4\}$, then verify $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$.**

Sol.

$$A = \{x : x \in \mathbb{Z}, -2 < x \leq 4\} = \{-1, 0, 1, 2, 3, 4\}$$
$$B = \{x : x \in \mathbb{W}, x \leq 5\} = \{0, 1, 2, 3, 4, 5\}$$
$$C = \{-4, -1, 0, 2, 3, 4\}$$

$A\cup(B\cap C)$

$$B\cap C = \{0, 1, 2, 3, 4, 5\}\cap\{-4, -1, 0, 2, 3, 4\} = \{0, 2, 3, 4\}$$
$$A\cup(B\cap C) = \{-1, 0, 1, 2, 3, 4\}\cup\{0, 2, 3, 4\} = \{-1, 0, 1, 2, 3, 4\} \quad \text{... (1)}$$

$(A\cap B)\cup(A\cap C)$

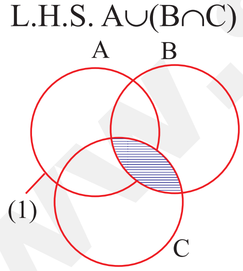
$$A\cap B = \{0, 1, 2, 3, 4\}$$
$$A\cap C = \{-1, 0, 2, 3, 4\}$$
$$(A\cap B)\cup(A\cap C) = \{0, 1, 2, 3, 4\}\cup\{-1, 0, 2, 3, 4\} = \{-1, 0, 1, 2, 3, 4\} \quad \text{... (2)}$$

From (1) and (2), it is verified that

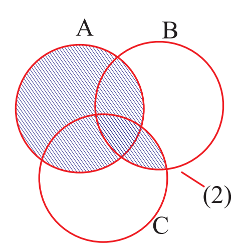
$$A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$$

4. **Verify $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$ using Venn diagrams.**

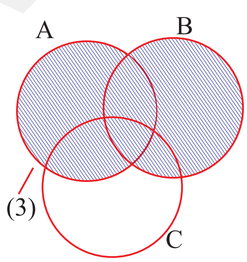
Sol. L.H.S. $A\cup(B\cap C)$



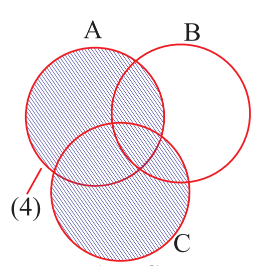
(B∩C)



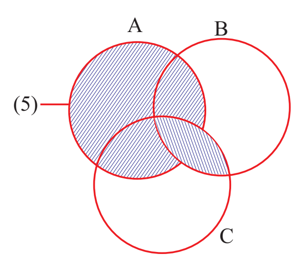
A∪ (B ∩ C)



A∪ B



A ∪ C



(A∪B) ∩(A∪C)

From (2) and (5), it is verified that $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

5. If A = {*b*, *c*, *e*, *g*, *h*}, B = {*a*, *c*, *d*, *g*, *i*} and C = {*a*, *d*, *e*, *g*, *h*}, then show that A − (B ∩ C) = (A − B) ∪ (A − C).

Sol.

$$A = \{b, c, e, g, h\}$$
$$B = \{a, c, d, g, i\}$$
$$C = \{a, d, e, g, h\}$$
$$B \cap C = \{a, d, g\}$$
$$A - (B \cap C) = \{b, c, e, g, h\} - \{a, d, g\} = \{b, c, e, h\} \quad \ldots (1)$$
$$A - B = \{b, c, e, g, h\} - \{a, c, d, g, i\} = \{b, e, h\}$$
$$A - C = \{b, c, e, g, h\} - \{a, d, e, g, h\} = \{b, c\}$$
$$(A - B) \cup (A - C) = \{b, c, e, h\} \quad \ldots (2)$$

From (1) and (2) it is verified that

$$A - (B \cap C) = (A - B) \cup (A - C)$$

6. If A = {x : x = 6n, $n \in \mathbb{W}$ and n < 6}, B = {x : x = 2n, $n \in \mathbb{N}$ and 2 < $n \leq$ 9} and C = {x : x = 3n, $n \in \mathbb{N}$ and 4 ≤ n < 10}, then show that A−(B∩C)= (A−B)∪(A−C)

Sol.

$$A = \{x : x = 6n, n \in \mathbb{W}, n < 6\}$$
$$x = 6n$$
$$n = \{0, 1, 2, 3, 4, 5\}$$
$$\Rightarrow \quad x = 6 \times 0 = 0$$
$$x = 6 \times 1 = 6$$
$$x = 6 \times 2 = 12$$
$$x = 6 \times 3 = 18$$
$$x = 6 \times 4 = 24$$
$$x = 6 \times 5 = 30$$
$$\therefore A = \{0, 6, 12, 18, 24, 30\}$$
$$B = \{x : x = 2n, n \in \mathbb{N}, 2 < n \leq 9\}$$
$$n = \{3, 4, 5, 6, 7, 8, 9\}$$
$$x = 2n$$
$$\Rightarrow \quad x = 2 \times 3 = 6$$
$$2 \times 4 = 8$$
$$2 \times 5 = 10$$
$$2 \times 6 = 12$$
$$2 \times 7 = 14$$
$$2 \times 8 = 16$$
$$2 \times 9 = 18$$
$$\therefore B = \{6, 8, 10, 12, 14, 16, 18\}$$
$$C = \{x : x = 3n, n \in \mathbb{N}, 4 \leq n < 10\}$$
$$\mathbb{N} = \{4, 5, 6, 7, 8, 9\}$$
$$\Rightarrow \quad x = 3 \times 4 = 12$$
$$x = 3 \times 5 = 15$$
$$x = 3 \times 6 = 18$$
$$x = 3 \times 7 = 21$$
$$x = 3 \times 8 = 24$$
$$x = 3 \times 9 = 27$$
$$x = 2 \times 9 = 18$$
$$\therefore C = \{12, 15, 18, 21, 24, 27\}$$

$$A-(B\cap C) = (A-B)\cup(A-C)$$
L.H.S R.H.S
$$B\cap C = \{12, 18\}$$
$$A-(B\cap C) = \{0, 6, 12, 18, 24, 30\} - \{12, 18\} = \{0, 6, 24, 30\} \quad \ldots (1)$$
$$(A-B) = \{0, 24, 30\}$$
$$(A-C) = \{0, 6, 30\}$$
$$(A-B)\cup(A-C) = \{0, 6, 24, 30\} \quad \ldots (2)$$
From (1) and (2), it is verified that
$$A-(B\cap C) = (A-B)\cup(A-C).$$

7. If A = {–2, 0, 1, 3, 5}, B = {–1, 0, 2, 5, 6} and C = {–1, 2, 5, 6, 7}, then show that A−(B∪C) = (A−B)∩(A−C).

Sol.
$$A = \{-2, 0, 1, 3, 5\}, B = \{-1, 0, 2, 5, 6\}$$
$$C = \{-1, 2, 5, 6, 7\}$$
$$B\cup C = \{-1, 0, 2, 5, 6, 7\}$$
$$A-(B\cup C) = \{-2, 1, 3\} \quad \ldots (1)$$
$$(A-B) = \{-2, 1, 3\}$$
$$(A-C) = \{-2, 0, 1, 3\}$$
$$(A-B)\cap(A-C) = \{-2, 1, 3\} \quad \ldots (2)$$
From (1) and (2), it is verified that
$$A-(B\cup C) = (A-B)\cap(A-C)$$

8. If $A=\left\{y : y=\frac{a+1}{2}, a\in\mathbb{W} \text{ and } a\le 5\right\}$, $B=\left\{y : y=\frac{2n-1}{2}, n\in\mathbb{W} \text{ and } n<5\right\}$ and $C=\left\{-1,-\frac{1}{2},1,\frac{3}{2},2\right\}$, then show that A−(B ∪ C) = (A−B) ∩ (A−C).

Sol.
$$A = \left\{y : y=\frac{a+1}{2}, a\in\mathbb{W}, a\le 5\right\}$$
$$a = \{0, 1, 2, 3, 4, 5\} \Rightarrow \quad y = \frac{a+1}{2} = \frac{1}{2}$$
$$y = \frac{1+1}{2} = \frac{\cancel{2}}{\cancel{2}} = 1$$
$$y = \frac{2+1}{2} = \frac{3}{2}$$
$$y = \frac{3+1}{2} = \frac{\overset{2}{\cancel{4}}}{\cancel{2}} = 2$$
$$y = \frac{4+1}{2} = \frac{5}{2}$$
$$y = \frac{5+1}{2} = \frac{\overset{3}{\cancel{6}}}{\cancel{2}} = 3 \quad \therefore A = \left\{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3\right\}$$
$$B = \left\{y : y=\frac{2n-1}{2}, n\in\mathbb{W} \text{ and } n<5\right\}$$
$$n = \{0, 1, 2, 3, 4\} \Rightarrow \quad y = \frac{2\times 0-1}{2} = \frac{-1}{2}$$

$$y = \frac{2\times1-1}{2} = \frac{1}{2}$$

$$y = \frac{2\times2-1}{2} = \frac{3}{2}$$

$$y = \frac{2\times3-1}{2} = \frac{5}{2}$$

$$y = \frac{2\times4-1}{2} = \frac{7}{2} \quad \therefore B = \left\{-\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}\right\}$$

$$C = \left\{-1, -\frac{1}{2}, 1, \frac{3}{2}, 2\right\}$$

$$B \cup C = \left\{-1, -\frac{1}{2}, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \frac{7}{2}\right\}$$

$$A-(B\cup C) = \{3\} \quad \ldots (1)$$

$$A - B = \{1, 2, 3\}$$

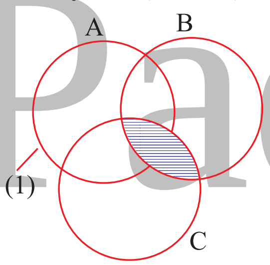
$$A - C = \left\{\frac{1}{2}, \frac{5}{2}, 3\right\}$$

$$(A-B)\cap(A-C) = \{3\} \quad \ldots (2)$$

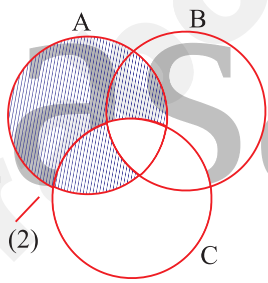
From (1) and (2), it is verified that $A-(B\cup C) = (A-B)\cap(A-C)$

9. Verify $A-(B\cap C) = (A-B)\cup(A-C)$ using Venn diagrams.

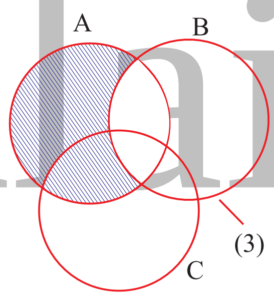
Sol.



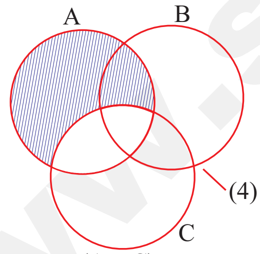
$B \cap C$



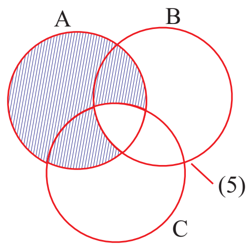
$A - (B \cap C)$



$(A - B)$



$(A - C)$



$(A - B) \cup (A - C)$

$\therefore A-(B\cap C) = (A-B)\cup(A-C)$ Hence it is proved.

10. If U = {4, 7, 8, 10, 11, 12, 15, 16}, A = {7, 8, 11, 12} and B = {4, 8, 12, 15}, then verify De Morgan's Laws for complementation.

Sol.

$$U = \{4, 7, 8, 10, 11, 12, 15, 16\}$$

$$A = \{7, 8, 11, 12\}, B = \{4, 8, 12, 15\}$$

De Morgan's Laws for complementation.

$$(A\cup B)' = A'\cap B'$$

$$A\cup B = \{4, 7, 8, 11, 12, 15\}$$

$$(A\cup B)' = \{4, 7, 8, 10, 11, 12, 15, 16\} - \{4, 7, 8, 11, 12, 15\}$$

$= \{10, 16\}$ ... (1)

$A' = \{4, 10, 15, 16\}$

$B' = \{7, 10, 11, 16\}$

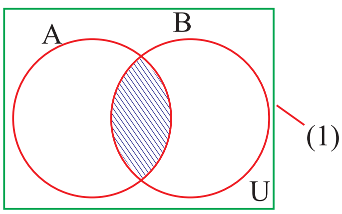
$A' \cap B' = \{10, 16\}$ ... (2)

From (1) and (2) it is verified that

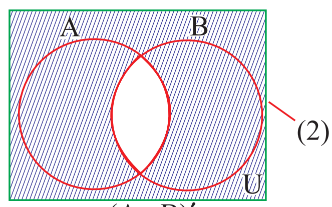
$(A \cup B)' = A' \cap B'$.

11. Verify $(A \cap B)' = A' \cup B'$ using Venn diagrams.

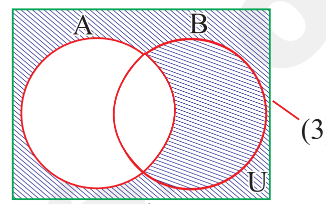
Sol. $(A \cap B)' = A' \cup B'$



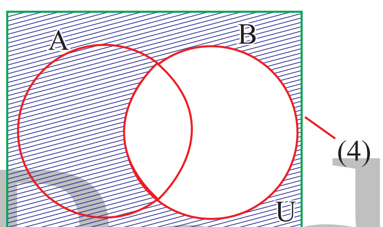
$(A \cap B)$



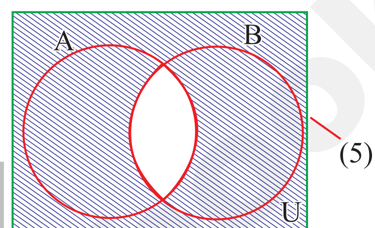
$(A \cap B)'$



A'



B'



$A' \cup B'$

$(2) = (5)$

$\therefore (A \cap B)' = A' \cup B'$

Exercise 1.6

1. (i) If $n(A) = 25$, $n(B) = 40$, $n(A \cup B) = 50$ and $n(B') = 25$, find $n(A \cap B)$ and $n(U)$.

(ii) If $n(A) = 300$, $n(A \cup B) = 500$, $n(A \cap B) = 50$ and $n(B') = 350$, find $n(B)$ and $n(U)$.

Sol. (i)

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$n(A \cap B) = 25 + 40 - 50 = 65 - 50 = 15$

$n(U) = n(B) + n(B') = 40 + 25 = 65$

(ii)

$n(U) = n(B) + n(B')$

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$n(B) = n(A \cup B) + n(A \cap B) - n(A) = 500 + 50 - 300 = 250$

$n(U) = 250 + 350 = 600$.

2. If $U = \{x : x \in \mathbb{N}, x \leq 10\}$, $A = \{2, 3, 4, 8, 10\}$ and $B = \{1, 2, 5, 8, 10\}$, then verify that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Sol.

$n(A) = 5, n(B) = 5$

$A \cup B = \{1, 2, 3, 4, 5, 8, 10\}, A \cap B = \{2, 8, 10\}$

$n(A \cup B) = 7, n(A \cap B) = 3$

L.H.S $= n(A \cup B) = 7$

R.H.S $= n(A) + n(B) - n(A \cap B) = 5 + 5 - 3 = 7$

$\therefore$ L.H.S = R.H.S proved.

3. Verify $n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$ for the following sets.

(i) A = {a, c, e, f, h}, B = {c, d, e, f} and C = {a, b, c, f}

(ii) A = {1, 3, 5} B = {2, 3, 5, 6} and C = {1, 5, 6, 7}.

Sol. $n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$

(i) $A = \{a, c, e, f, h\}, B = \{c, d, e, f\}, C = \{a, b, c, f\}$

$n(A) = 5, n(B) = 4, n(C) = 4$

$n(A\cap B) = 3$

$n(B\cap C) = 2$

$n(A\cap C) = 3$

$n(A\cap B\cap C) = 2$

$A\cap B = \{c, e, f\}$

$B\cap C = \{c, f\}$

$A\cap C = \{a, c, f\}$

$A\cap B\cap C = \{c, f\}$

$A\cup B\cup C = \{a, c, d, e, f, b, h\}$

$\therefore n(A\cup B\cup C) = 7$ ... (1)

$n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$

$= 5 + 4 + 4 - 3 - 2 - 3 + 2 = 15 - 8 = 7$ ... (2)

(1) = (2)

$\Rightarrow n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$

Hence it is verified.

(ii) $A = \{1, 3, 5\}, B = \{2, 3, 5, 6\}, C = \{1, 5, 6, 7\}$

$n(A) = 3, n(B) = 4, n(C) = 4$

$n(A\cap B) = 2$

$n(B\cap C) = 2$

$n(C\cap A) = 2$

$n(A\cap B\cap C) = 1$

$n(A\cup B\cup C) = 6$

$n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$

$6 = 3 + 4 + 4 - 2 - 2 - 2 + 1 = 12 - 6 = 6$

Hence it is verified.

4. In a class, all students take part in either music or drama or both. 25 students take part in music, 30 students take part in drama and 8 students take part in both music and drama. Find

(i) The number of students who take part in only music.

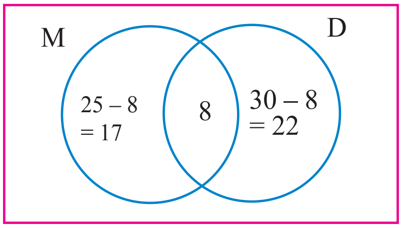
(ii) The number of students who take part in only drama.

(iii) The total number of students in the class.

Sol. Let the number of students take part in music is M.

Let the number of students take part in drama is D.

By using venn diagram



(i) The number of students take part in only music is 17.

(ii) The number of students take part in only drama is 22.

(iii) The total number of students in the class is 17 + 8 + 22 = 47.

5. In a party of 45 people, each one likes tea or coffee or both. 35 people like tea and 20 people like coffee. Find the number of people who

(i) like both tea and coffee. **(ii) do not like tea.**

(iii) do not like coffee.

Sol. Let the people who like tea be T.

Let the people who like coffee be C

By using formula : $\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$

(i) $n(T \cap C) = n(T) + n(C) - n(T \cup C) = 35 + 20 - 45 = 55 - 45 = 10$

The number of people who like both coffee and tea = 10.

(ii) The number of people who do not like Tea

$$n(T') = n(U) - n(T) = 45 - 35 = 10$$

(iii) The number of people who do not like coffee

$$n(C') = n(U) - n(C) = 45 - 20 = 25.$$

6. In an examination 50% of the students passed in Mathematics and 70% of students passed in Science while 10% students failed in both subjects. 300 students passed in atleast one subjects. Find the total number of students who appeared in the examination, if they took examination in only two subjects.

Sol. Let the students who appeared in the examination be 100%.

Let the percentage of students who failed in mathematics be M.

Let the percentage of students who failed in science be S.

$$\begin{aligned} \text{Failed in Maths} &= 100\,\% - \text{Pass}\% = 100\% - 50\% = 50\% \\ \text{Failed in Science}\% &= 100\% - 70\% = 30\% \\ \text{Failed in both}\% &= 10\% \\ n(M \cup S) &= n(M) + n(S) - n(M \cap S) \\ &= 50\% + 30\% - 10\% = 70\% \end{aligned}$$

% of students failed in atleast one subject = 70%

∴ The % of students who have passed in atleast one subject = 100% – 70% = 30%

$$30\% = 300$$

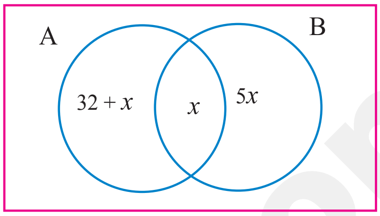
$$\therefore\ 100\% = \frac{100 \times \cancel{300}^{10}}{\cancel{30}} = 1000$$

∴ The total number of students who appeared in the examination = 1000 students.

7. A and B are two sets such that $n(A - B) = 32 + x$, $n(B - A) = 5x$ and $n(A \cap B) = x$. Illustrate the information by means of a venn diagram. Given that $n(A) = n(B)$, calculate the value of x.

Sol.

$$\begin{aligned} n(A - B) &= 32 + x \\ n(B - A) &= 5x \\ n(A \cap B) &= x \\ n(A) &= n(B) \\ 32 + x + x &= 5x + x \\ 32 + 2x &= 6x \\ 4x &= 32 \\ x &= 8 \end{aligned}$$



8. Out of 500 car owners investigated, 400 owned car A and 200 owned car B, 50 owned both A and B cars. Is this data correct?

Sol.

$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ n(A \cup B) &= 500 \text{ (given)} \qquad \ldots (1) \\ n(A) &= 400 \\ n(B) &= 200 \\ n(A \cap B) &= 50 \\ \therefore n(A) + n(B) - n(A \cap B) &= 400 + 200 - 50 = 550 \qquad \ldots(2) \end{aligned}$$

$1 \neq 2$ $\quad\therefore$ This data is incorrect.

9. In a colony, 275 families buy Tamil newspaper, 150 families buy English newspaper, 45 families buy Hindi newspaper, 125 families buy Tamil and English newspapers, 17 families buy English and Hindi newspapers, 5 families buy Tamil and Hindi newspapers and 3 families buy all the three newspapers. If each family buy atleast one of these newspapers then find

(i) Number of families buy only one newspaper

(ii) Number of families buy atleast two newspapers

(iii) Total number of families in the colony.

Sol. (i)

Tamil Newspaper buyers $n(A) = 275$

English Newspaper buyers $n(B) = 150$

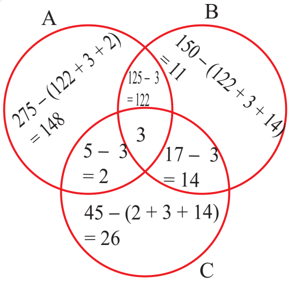
Hindi Newspaper buyers $n(C) = 45$

Tamil and English Newspaper buyers $n(A \cap B) = 125$

English and Hindi Newspaper buyers $n(B \cap C) = 17$

Hindi and Tamil Newspaper buyers $n(C \cap A) = 5$

All the three Newspaper buyers $n(A \cap B \cap C) = 3$



(i) Number of families buy only one newspaper
$= 148 + 11 + 26 = 185$

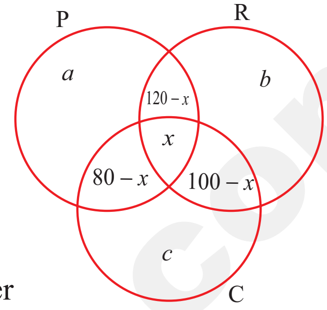
(ii) Number of families buy atleast two news papers
$= 122 + 14 + 2 + 3 = 141$

(iii) Total number of families in the colony
$= 148 + 11 + 26 + 122 + 14 + 2 + 3 = 326$

10. A survey of 1000 farmers found that 600 grew paddy, 350 grew ragi, 280 grew corn, 120 grew paddy and ragi, 100 grew ragi and corn, 80 grew paddy and corn. If each farmer grew atleast any one of the above three, then find the number of farmers who grew all the three.

Sol.

$$
\begin{aligned}
a &= 600 - (120 - x + x + 80 - x) \\
&= 600 - (200 - x) \\
&= 600 - 200 + x = 400 + x \\
b &= 350 - (120 - x + x + 100 - x) \\
&= 350 - (220 - x) \\
&= 350 - 230 + x = 130 + x \\
c &= 280 - (80 - x + x + 100 - x) \\
&= 2800 - (180 - x) = 280 - 180 + x = 100 + x
\end{aligned}
$$

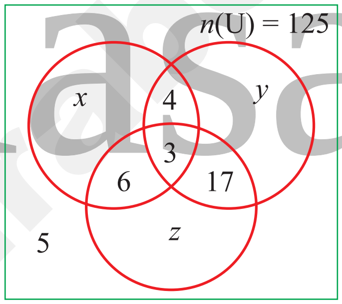


Each farmer grew atleast one of the above three, the number of farmers who grew all the three is x.

$$
\begin{aligned}
&= a + b + c + 120 - x + 100 - x + 80 - x + x = 1000 \\
& 400 + x + 130 + x + 100 + x + 120 - x + 100 - x + 80 - x + x = 1000 \\
\therefore 930 + x &= 1000 \\
x &= 1000 - 930 = 70
\end{aligned}
$$

$\therefore$ 70 farmers grew all the three crops

11. In the adjacent diagram, if n(U) = 125, y is two times of x and z is 10 more than x, then find the value of x, y and z.

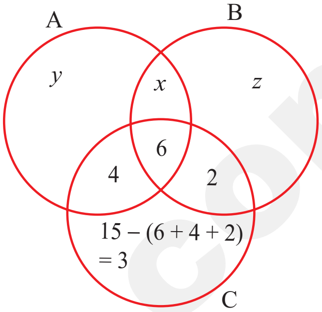


Sol.

$$
\begin{aligned}
n(U) &= 125 \\
y &= 2x \\
z &= x + 10 \\
\therefore x + y + z + 4 + 17 + 6 + 3 + 5 &= 125 \\
x + 2x + x + 10 + 35 &= 125 \\
4x + 45 &= 125 \\
4x &= 125 - 45 \\
4x &= 80 \\
x &= 20 \\
\therefore y &= 2x = 2 \times 20 = 40 \\
z &= x + 10 = 20 + 10 = 30 \\
\text{Hence } x &= 20;\ y = 40;\ z = 30
\end{aligned}
$$

12. Each student in a class of 35 plays atleast one game among chess, carrom and table tennis. 22 play chess, 21 play carrom, 15 play table tennis, 10 play chess and table tennis, 8 play carrom and table tennis and 6 play all the three games. Find the number of students who play (i) chess and carrom but not table tennis (ii) only chess (iii) only carrom (Hint: Use Venn diagram)

Sol.



A – Chess
B – Carrom
C – Table Tennis

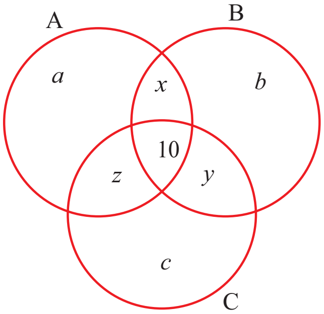
$$n(A) = 22$$
$$n(B) = 21$$
$$n(C) = 15$$
$$n(A \cap C) = 10$$
$$n(B \cap C) = 8$$
$$n(A \cap B \cap C) = 6$$

(i)
$$y = 22 - (x + 6 + 4) = 22 - (x + 10)$$
$$= 22 - x - 10 = 12 - x$$
$$z = 21 - (x + 6 + 2) = 21 - (8 + x)$$
$$= 21 - 8 - x = 13 - x$$
$$y + z + 3 + x + 2 + 4 + 6 = 35$$
$$12 - x + 13 - \cancel{x} + 15 + \cancel{x} = 35$$
$$40 - x = 35$$
$$x = 40 - 35 = 5$$

(i) Number of students who pay only chess and Carrom but not table tennis = 5
(ii) Number of students who play only chess = $12 - x = 12 - 5 = 7$
(iii) Number of students who play only carrom = $13 - x = 13 - 5 = 8$

13. In a class of 50 students, each one come to school by bus or by bicycle or on foot. 25 by bus, 20 by bicycle, 30 on foot and 10 students by all the three. Now how many students come to school exactly by two modes of transport?

Sol.



A – by bus
B – by bicycle
C – on foot

$$n(A) = 25$$
$$n(B) = 20$$
$$n(C) = 30$$
$$n(A \cap B \cap C) = 10$$
$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$
$$50 = 25 + 20 + 30 - (10 + x) - (10 + y) - (10 + z) + 10$$
$$50 = 75 - \cancel{10} - x - 10 - y - 10 - z + \cancel{10} = 75 - 20 - (x + y + z)$$
$$= 55 - (x + y + z)$$
$$x + y + z = 55 - 50 = 5$$

$\therefore$ The number of students who come to school exactly by two modes of transport = 5.

Exercise 1.7

Multiple Choice Questions :

1. Which of the following is correct?

(1) $\{7\} \in \{1,2,3,4,5,6,7,8,9,10\}$ (2) $7 \in \{1,2,3,4,5,6,7,8,9,10\}$
(3) $7 \notin \{1,2,3,4,5,6,7,8,9,10\}$ (4) $\{7\} \not\subseteq \{1,2,3,4,5,6,7,8,9,10\}$

[Ans. (2) $7 \in \{1,2,3,4,5,6,7,8,9,10\}$]

2. The set $P = \{x \mid x \in \mathbb{Z}, -1 < x < 1\}$ is a

(1) Singleton set (2) Power set (3) Null set (4) Subset

Hint : $P = \{0\}$ **[Ans. (1) Singleton set]**

3. If $U = \{x \mid x \in \mathbb{N}, x < 10\}$ and $A = \{x \mid x \in \mathbb{N}, 2 \le x < 6\}$ then $(A')'$ is

(1) $\{1,6,7,8,9\}$ (b) $\{1,2,3,4\}$ (c) $\{2,3,4,5\}$ (d) $\{\ \}$

Hint : $(A') = A = \{2, 3, 4, 5\}$ **[Ans. (3) $\{2,3,4,5\}$]**

4. If $B \subseteq A$ then $n(A \cap B)$ is

(1) $n(A - B)$ (2) $n(B)$ (3) $n(B - A)$ (4) $n(A)$

Hint : $B \subseteq A \Rightarrow A \cap B = B$ **[Ans. (2) $n(B)$]**

5. If A= $\{x,y,z\}$ then the number of non-empty subsets of A is

(1) 8 (2) 5 (3) 6 (4) 7

Hint : Number of non-empty subsets = 2 – 1 = 8 – 1 = 7 **[Ans. (4) 7]**

6. Which of the following is correct ?

(1) $\varnothing \subseteq \{a,b\}$ (2) $\varnothing \in \{a, b\}$ (3) $\{a\} \in \{a, b\}$ (4) $a \subseteq \{a, b\}$

Hint : Empty set is an improper subset **[Ans. (1) $\varnothing \subseteq \{a,b\}$]**

7. If $A \cup B = A \cap B$, then

(1) $A \neq B$ (2) $A = B$ (4) $A \subset B$ (4) $B \subset A$

[Ans. (2) A = B]

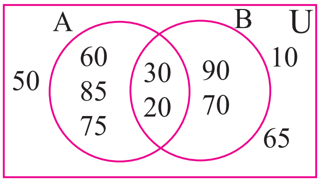
8. If $B - A$ is B, then $A \cap B$ is

(1) A (2) B (3) U (4) $\varnothing$

Hint : $B - A = B \Rightarrow$ A and B are disjoint sets. **[Ans. (4) $\varnothing$]**

9. From the adjacent diagram $n[P(A \Delta B)]$ is

(1) 8 (2) 16
(3) 32 (4) 64



Hint : $A \Delta B = \{60, 85, 75, 90, 70\}$

$\Rightarrow n(A \Delta B) = 5$

$\Rightarrow n(P(A \Delta B)) = 2^5 = 32$ **[Ans. (3) 32]**

10. If $n(A) = 10$ and $n(B) = 15$ then the minimum and maximum number of elements in $A \cap B$ is

(1) (10,15) (2) (15,10) (3) (10,0) (4) (0,10)

[Ans. (4) (0,10)]

11. Let $A = \{\varnothing\}$ and $B = P(A)$ then $A \cap B$ is

(1) $\{\varnothing, \{\varnothing\}\}$ (2) $\{\varnothing\}$ (3) $\varnothing$ (4) $\{0\}$

Hint : $P(A) = \{\varnothing\{\varnothing\}\}$ **[Ans. (2) $\{\varnothing\}$]**

12. In a class of 50 boys, 35 boys play carom and 20 boys play chess then the number of boys play both games is

(1) 5 (2) 30 (3) 15 (4) 10

Hint : $n(A\cup B) = n(A) + n(B) - n(A\cap B) \Rightarrow 50 = 35 + 20 - n(A\cap B) \Rightarrow n(A\cap B) = 5$

[Ans. (1) 5]

13. If $U = \{x : x \in \mathbb{N}$ and $x < 10\}$, $A = \{1, 2, 3, 5, 8\}$ and $B = \{2, 5, 6, 7, 9\}$, then $n[(A\cup B)']$ is

(1) 1 (2) 2 (3) 4 (4) 8

Hint :

$$\begin{aligned} U &= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \\ A &= \{1, 2, 3, 5, 8\} \\ B &= \{2, 5, 6, 7, 9\} \\ A \cup B &= \{1, 2, 3, 5, 6, 7, 8, 9\} \\ (A \cup B)' &= \{4\}, \\ n(A \cup B)' &= 1 \end{aligned}$$

[Ans. (1) 1]

14. For any three sets P, Q and R, $P-(Q\cap R)$ is

(1) $P-(Q\cup R)$ (2) $(P\cap Q)-R$

(3) $(P-Q)\cup(P-R)$ (4) $(P-Q)\cap(P-R)$

Hint : $P-(Q\cap R) = (P-Q)\cup(P-R)$ **[Ans. (3) $(P-Q)\cup(P-R)$]**

15. Which of the following is true?

(1) $A-B = A\cap B$ (2) $A-B = B-A$

(3) $(A\cup B)' = A'\cup B'$ (4) $(A\cap B)' = A'\cup B'$

Hint :

(1) $(A-B) = A\cap B$ ×

(2) $A-B = B-A$ ×

(3) $(A\cup B)' = A'\cup B'$ ×

(4) $(A\cap B)' = A'\cup B'$ ✓ **[Ans. (4) $(A\cap B)' = A'\cup B'$]**

16. If $n(A\cup B\cup C) = 100$, $n(A) = 4x$, $n(B) = 6x$, $n(C) = 5x$, $n(A\cap B) = 20$, $n(B\cap C) = 15$, $n(A\cap C) = 25$ and $n(A\cap B\cap C) = 10$, then the value of x is

(1) 10 (2) 15 (3) 25 (4) 30

Hint :

$$\begin{aligned} n(A\cup B\cup C) &= n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(C\cap A) + n(A\cap B\cap C) \\ 100 &= 4x + 6x + 5x - 20 - 15 - 25 + 10 \\ 100 &= 15x - 60 + 10 \\ 100 &= 15x - 50 \\ \therefore 15x &= 100 + 50 = 150 \\ x &= 10 \end{aligned}$$

[Ans. (1) 10]

17. For any three sets A, B and C, $(A-B)\cap(B-C)$ is equal to

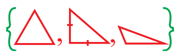
(1) A only (2) B only (3) C only (4) ϕ

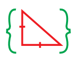
Hint : $(A-B)\cap(B-C)$ is equal to ϕ **[Ans. (4) ϕ]**

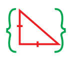
18. If J = Set of three sided shapes, K = Set of shapes with two equal sides and L = Set of shapes with right angle, then J ∩K ∩ L is

(1) Set of isoceles triangles (2) Set of equilateral triangles

(3) Set of isoceles right triangles (4) Set of right angled triangles

Hint : J = 

K = 

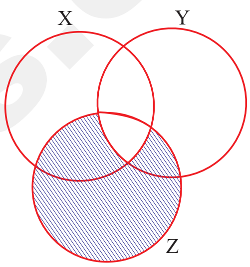
L = 

[Ans. (3) Set of isoceles right triangles]

19. The shaded region in the Venn diagram is

(1) $Z-(X\cup Y)$ (2) $(X\cup Y)\cap Z$ (3) $Z-(X\cap Y)$ (4) $Z\cup(X\cap Y)$

Hint : $Z-(X\cap Y)$

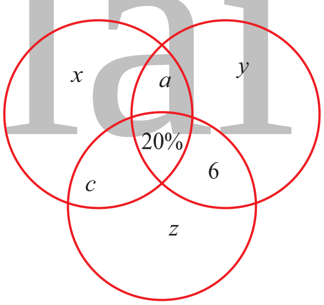


[Ans. (3) $Z-(X\cap Y)$]

20. In a city, 40% people like only one fruit, 35% people like only two fruits, 20% people like all the three fruits. How many percentage of people do not like any one of the above three fruits?

(1) 5 (2) 8

(3) 10 (4) 15



Hint :

$$40+35+20+x = 100\%$$
$$95\%+x = 100\%$$
$$x = 5\%$$

[Ans. (1) 5]

Text Book Activities

Activity - 1

1. Discuss and give as many examples of collections from your daily life situations, which are sets and which are not sets.

Sol. **Which are sets**

(i) Collection of pen (ii) Collection of dolls

(iii) Collection of books (iv) Collection of red flower etc.

Which are not sets

(i) Collection of good students in a class.

(ii) Collection of beautiful flowers in a garden etc.

Activity - 2

Write the following sets in respective forms.

Sol.

S. No	Descriptive Form	Set Builder Form	Roster Form
1	The set of all natural numbers less than 10	**A = { x : x is a natural number less than 10}**	**A = {1, 2, 3, 4, 5, 6, 7, 8, 9}**
2	**The set of all positive integers which are multiples of 3**	{x : x is a multiple of 3, $x \in$ N}	**{3, 6, 9, 12,18...}**
3	**The set of all natural even numbers. Less than 12**	**ℕ = {x : x is a natural even number, $x < 12$}**	{2,4,6,8,10}
4	The set of all days in a week.	**X = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}**	**X = {x : x is a day in a week}**
5	**The set of all Integers**	**A = {x : x is an on Integer}**	{…–3,–2,–1,0,1,2,3…}

Activity - 4

Fill in the blanks with appropriate cardinal numbers.

Sol.

S. No	n(A)	n(B)	n(A∪B)	n(A∩B)	n(A–B)	n(B–A)
1	30	45	65	**10**	**20**	**35**
2	20	**45**	55	10	**10**	**35**
3	50	62	**87**	25	**25**	**37**
4	30	43	70	**3**	**27**	**40**

Additional Questions and Answers

EXERCISE 1.1

1. Let A = {0, 1, 2, 3, 4, 5}. Insert the appropriate symbol ∈ or ∉ in the blank spaces.
(i) 0 ____ A (ii) 6 ____ A (iii) 3 ____ A (iv) 4 ____ A
(v) 7____ A

Sol. (i) 0 ∈ A (ii) 6 ∉ A (iii) 3 ∈ A (iv) 4 ∈ A (v) 7 ∉ A

2. Write the following in Set-Builder form.
(i) The set of all positive even numbers.
(ii) The set of all whole numbers less than 20.
(iii) The set of all positive integers which are multiple of 3.
(iv) The set of all odd natural numbers less than 15.
(v) The set of all letters in the word 'computer'.

Sol. (i) A = {x : x is a positive even number}
(ii) B = {x : x is a whole number and $x < 20$}
(iii) C = {x : x is a positive integer and multiple of 3}
(iv) D = {x : x is an odd natural number and $x < 15$}
(v) E = {x : x is a letter in the word "Computer"}

3. **Write the following sets in Roster form.**

(i) $A = \{x : x \in \mathbb{N},\ 2 < x < 10\}$

(ii) $B = \{x : x \in \mathbb{Z}, -\frac{1}{2} < x < \frac{11}{2}\}$

(iii) $C = \{x : x$ **is a prime number and a divisor of 6** $\}$

(iv) $x = \{x : x = 2^n, n \in \mathbb{N}$ **and** $n \leq 5\}$

(v) $M = \{x : x = 2y - 1, y \leq 5, y \in \mathbb{W}\}$

Sol. (i) $A = \{3, 4, 5, 6, 7, 8, 9\}$

(ii) $B = \{0, 1, 2, 3, 4, 5\}$

(iii) $C = \{2, 3\}$

(iv) Given, $x = 2^n$, $n \in \mathbb{N}$ and $n \leq 5$.

Here $n = 1, 2, 3, 4, 5$

$n = 1 \Rightarrow 2^1 = 2$

$n = 2 \Rightarrow 2^2 = 4$

$n = 3 \Rightarrow 2^3 = 8$

$n = 4 \Rightarrow 2^4 = 16$

$n = 5 \Rightarrow 2^5 = 32$

$X = \{2, 4, 8, 16, 32\}$

(v) Given, $x = 2y - 1$, $y \leq 5$ and $y \in \mathbb{W}$

Here $y = 0, 1, 2, 3, 4, 5$

$y = 0 \Rightarrow x = 2(0) - 1 = -1$

$y = 1 \Rightarrow x = 2(1) - 1 = 2 - 1 = 1$

$y = 2 \Rightarrow x = 2(2) - 1 = 4 - 1 = 3$

$y = 3 \Rightarrow x = 2(3) - 1 = 6 - 1 = 5$

$y = 4 \Rightarrow x = 2(4) - 1 = 8 - 1 = 7$

$y = 5 \Rightarrow x = 2(5) - 1 = 10 - 1 = 9$

$M = \{-1, 1, 3, 5, 7, 9\}$

EXERCISE 1.2

1. **Find the number of subsets and number of proper subsets of a set** $X = \{a, b, c, x, y, z\}$.

Sol. Given $X = \{a, b, c, x, y, z\}$.

Then, $n(X) = 6$

The number of subsets $= n[P(X)] = 2^6 = 64$

The number of proper subsets $= n[P(X)] - 1 = 2^6 - 1 = 64 - 1 = 63$

2. **Find the cardinal number of the following sets.**

(i) $A = \{x : x$ **is a prime factor of 12**$\}$.

(ii) $B = \{x : x \in \mathbb{W}, x \leq 5\}$.

(iii) $X = \{x : x$ **is an even prime number**$\}$

Sol. (i) Factors of 12 are 1, 2, 3, 4, 6, 12. So, the prime factors of 12 are 2,3.
We write the set A in roster form as $A = \{2,3\}$ and hence $n(A) = 2$.

(ii) In Tabular form $B = \{0, 1, 2, 3, 4, 5\}$
The set B has six elements and hence $n(B) = 6$

(iii) $X = \{2\}$ [2 is the only even prime number]
$\therefore n(X) = 1$

3. State whether the following sets are finite or infinite.

(i) A = {x : x is a multiple of 5, $x \in$ N}.

(ii) B = {0, 1, 2, 3, 4, ... 75}.

(iii) The set of all positive integers greater than 50.

Sol. (i) A = {5, 10, 15, 20, ...} $\therefore$ A is an infinite set

(ii) Finite

(iii) Let X be the set of all positive integers greater than 50

Then X = {51, 52, 53, ...}

$\therefore$ X is an infinite set.

4. Which of the following sets are equal?

(i) A = {1, 2, 3, 4}, B = {4, 3, 2, 1}

(ii) A = {4, 8, 12, 16}, B = {8, 4, 16, 18}

(iii) X = {2, 4, 6, 8}

Y = {x : x is a positive even integer and $0 < x < 10$}

Sol. (i) Since A and B contain exactly the same elements, A and B are equal sets.

(ii) A and B has different elements.

$\therefore$ A and B are not equal sets.

(iii) X = {2, 4, 6, 8}, Y = {2, 4, 6, 8}

$\therefore$ X and Y are equal sets.

5. Write $\subseteq$ or $\not\subseteq$ in each blank to make a true statement.

(i) {4, 5, 6, 7} ________ {4, 5, 6, 7, 8}

(ii) {a, b, c} ________ {b, e, f, g}

Sol. (i) {4, 5, 6, 7} $\subseteq$ {4, 5, 6, 7, 8}

(ii) {a, b, c} $\not\subseteq$ {b, e, f, g}

6. Write down the power set of A = {3, {4, 5}}.

Sol. The subsets of A are

$\varnothing$, {3}, {4, 5}, {3,{4, 5}}

P(A) = {, $\varnothing$, {3}, {4, 5}, {3{4, 5}}

EXERCISE 1.3

1. Find the union of the following sets.

(i) A = {1, 2, 3, 5, 6} and B = {4, 5, 6, 7, 8} **(ii) X = {3, 4, 5} and Y = $\varnothing$**

Sol. (i) A $\cup$ B = {1, 2, 3, 4, 5 , 6, 7, 8} (ii) X $\cup$ Y = {3, 4, 5}

2. Find A $\cap$ B if (i) A = {10, 11, 12, 13}, B = {12, 13, 14, 15}, (ii) A = {5, 9, 11}, B = $\varnothing$.

Sol. (i) A $\cap$ B = {12, 13} (ii) A $\cap$ B = $\varnothing$

3. Given the sets A = {4, 5, 6, 7} and B = {1, 3, 8, 9}, find A $\cap$ B.

Sol. A $\cap$ B = $\varnothing$

4. If A = { –2, –1, 0, 3, 4}, B = {–1, 3, 5}, find (i) A – B, (ii) B – A.

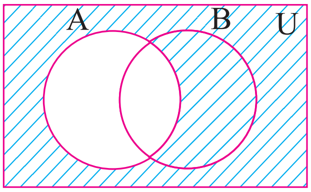
Sol. (i) A – B = {–2, 0, 4} (ii) B – A = {5}

5. If A = {2, 3, 5, 7, 11} and B = {5, 7, 9, 11, 13}, find A Δ B.

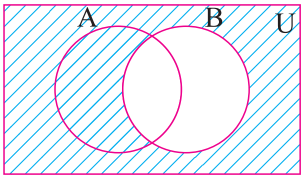
Sol. A Δ B = {2, 3, 9, 13}

6. Draw a venn diagram similar to one at the side and shade the regions representing the following sets (i) A′, (ii) B′, (iii) A′ ∪ B′, (iv) (A ∪ B)′, (v) A′ ∩ B′

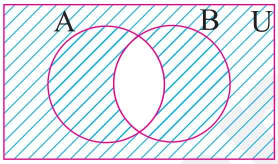
Sol. (i) A'



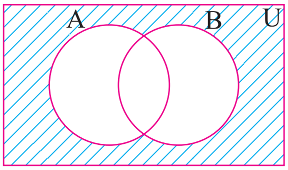
(ii) B'



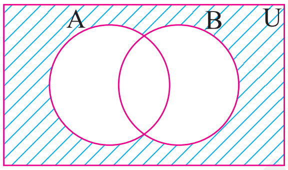
(iii) $A' \cup B'$



(iv) $(A \cup B)'$



(v) $A' \cap B'$



7. State which of the following sets are disjoint.

(i) $A = \{2, 4, 6, 8\}$, $B = \{x : x \text{ is an even number} < 10, x \in N\}$

(ii) $X = \{1, 3, 5, 7, 9\}$, $Y = \{0, 2, 4, 6, 8, 10\}$

(iii) $R = \{a, b, c, d, e\}$, $S = \{d, e, b, c, a\}$

Sol. (i) $A = \{2, 4, 6, 8\}$, $B = \{2, 4, 6, 8\}$

$A \cap B = \{2, 4, 6, 8\} \neq \varnothing$ $\quad \therefore$ A and B are not disjoint sets.

(ii) $X \cap Y = \{\ \} = \varnothing$, X and Y are disjoint sets.

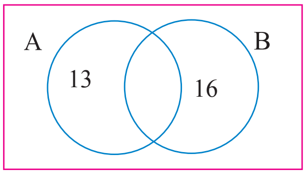
(iii) $R \cap S = \{a, b, c, d, e\} \neq \varnothing$ $\quad \therefore$ R and S are not disjoint sets.

8. If $A = \{a, b, c, d, e\}$ and $B = \{a, e, i, o, u\}$ find AB.

Sol. $A \cap B = \{a,b,c,d,e\} \cap \{a,e,i,o,u\} = \{a, e\}$

EXERCISE 1.4

1. If A and B are two sets containing 13 and 16 elements respectively, then find the minimum and maximum number of elements in A∪B?



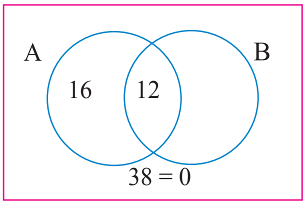
Sol.

$n(A) = 13$; $n(B) = 16$

Minimum $n(A \cup B) = 16$

Maximum $n(A \cup B) = 13 + 16 = 29$

2. If $n(U) = 38$, $n(A) = 16$, $n(A \cap B) = 12$, $n(B') = 20$, find $n(A \cup B)$.



Sol.

$n(U) = 38$

$n(A) = 16$

$n(A \cap B) = 12$

$n(B') = 20$

$n(A \cup B) = ?$

$n(B) = n(U) - n(B')$

$n(B) = 38 - 20$

$n(B) = 18$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B) = 16 - 12 + 18$

$n(A \cup B) = 4 + 18 = 22.$

Hint : $n(B) = n(U) - n(B')$
$= 38 - 20 = 18$

3. **Let A = {*b, d, e, g, h*} and B = {*a, e, c, h*} verify that $n(A - B) = n(A) - n(A \cap B)$**

Sol.

$$A = \{b, d, e, g, h\}, B = \{a, e, c, h\}$$
$$A - B = \{b, d, g\}$$
$$n(A-B) = 3 \quad \ldots (1)$$
$$A \cap B = \{e, h\}$$
$$n(A \cap B) = 2, n(A) = 5$$
$$n(A) - n(A \cap B) = 5 - 2 = 3 \quad \ldots (2)$$

Form (1) and (2) we get $n(A-B) = n(A) - n(A \cap B)$

4. **If A = {2, 5, 6, 7} and B = {3, 5, 7, 8}, then verify the commutative property of**

(i) union of sets (ii) intersection of sets

Sol. Given, A = {2, 5, 6, 7} and B = {3, 5, 7, 8}

(i) $A \cup B = \{2, 3, 5, 6, 7, 8\}$ ... (1)

$B \cup A = \{2, 3, 5, 6, 7, 8\}$ ... (2)

From (1) and (2) we have $A \cup B = B \cup A$

It is verified that union of sets is commutative.

(ii) $A \cap B = \{5, 7\}$ ... (3)

$B \cap A = \{5,7\}$ ... (4)

From (3) and (4) we get, $A \cap B = B \cap A$

It is verified that intersection of sets is commutative.

5. **If A = {*b, c, d, e*} and B = {*b, c, e,* g} and C = {*a, c, e*}, then verify $A \cup (B \cup C) = (A \cup B) \cup C$.**

Sol. Given, A = {*b, c, d, e*} and B = {*b, c, e,* g} and C = {*a, c, e*}

Now $B \cup C = \{a, b, c, e, g\}$

$A \cup (B \cup C) = \{a, b, c, d, e, g\}$ ... (1)

Then, $A \cup B = \{b, c, d, e, g\}$

$(A \cup B) \cup C = \{a, b, c, d, e, g\}$ ... (2)

From (1) and (2) it is verified that

$$A \cup (B \cup C) = (A \cup B) \cup C$$

EXERCISE 1.5

1. **If A = {1, 3, 5, 7, 9}, B = {*x* ; *x* is a composite number and *x* < 12} and C = {*x* : *x* ∈ N and 6 < *x* < 10} then verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.**

Sol. Given, A = {1, 3, 5, 7, 9} and B = {4, 6, 8, 9, 10}and C = {6, 7, 8, 9}

$B \cap C = \{4, \underline{6}, \underline{8}, \underline{9}, 10\} \cap \{\underline{6}, 7, \underline{8}, \underline{9}\} = \{6, 8, 9\}$

$A \cup (B \cap C) = \{1, 3, 5, 6, 7, 8, 9\}$ ... (1)

Then $(A \cup B) = \{1, 3, 5, 7, 9\} \cup \{4, 6, 8, 9, 10\} = \{1, 3, 4, 5, 6, 7, 8, 9, 10\}$

$(A \cup C) = \{1, 3, 5, 7, 9\} \cup \{6, 7, 8, 9\} = \{1, 3, 5, 6, 7, 8, 9\}$

$(A \cup B) \cap (A \cup C) = \{\underline{1}, \underline{3}, 4, \underline{5}, \underline{6}, \underline{7}, \underline{8}, \underline{9}, 10\} \cap \{\underline{1}, \underline{3}, \underline{5}, \underline{6}, \underline{7}, \underline{8}, \underline{9}\}$

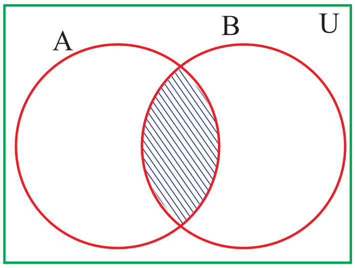
$= \{1, 3, 5, 6, 7, 8, 9\}$ ... (2)

From (1) and (2), it is verified that

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

2. If A, B and C are overlapping sets, draw venn diagram for : A ∩ B

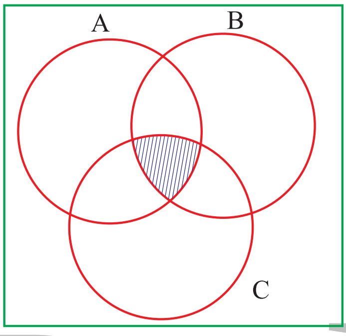
Sol.



(A∩B)

3. Draw Venn diagram for A ∩ B ∩ C

Sol.



A ∩ B ∩ C

4. If P = {x : $x \in \mathbb{N}$ and $1 < x < 11$}, Q = {x: $x = 2n$, $n \in \mathbb{N}$ and $n < 6$} and R = {4, 6, 8, 9, 10, 12}, then verify P – (Q ∩ R) = (P – Q) ∪ (P – R)

Sol. The roster form of sets P, Q and R are P = {2, 3, 4, 5, 6, 7, 8, 9, 10}, Q = {2,4, 6, 8, 10} and R = {4, 6, 8, 9, 10, 12}

First, we find Q ∩ R = {4, 6, 8, 10}
Then, P – (Q ∩ R) = {2, 3, 5, 7, 9} ... (1)
Next, P – Q = {3, 5, 7, 9}
and P – R = {2, 3, 5, 7}
and so, (P – Q) ∪ (P – Q) = {2, 3, 5, 7, 9} ... (2)

Hence from (1) and (2), it verified that P – (Q ∩ R) = (P – Q) ∪ (P – R)

Finding the elements of set Q

			Given, x	=	$2n$
n = 1	→		x	=	2 (1) = 2
n = 2	→		x	=	2 (2) = 4
n = 3	→		x	=	2 (3) = 6
n = 4	→		x	=	2 (4) = 8
n = 5	→		x	=	2 (5) = 10

Therefore, x takes values such as 2, 4, 6, 8, 10

5. If U = {x : $x \in \mathbb{Z}$, $-3 \le x \le 9$}, A = {x : $x = 2P + 1$, $P \in \mathbb{Z}$, $-2 \le P \le 3$}, B = {x : $x = q + 1$, $q \in \mathbb{Z}$, $0 \le q \le 3$}, verify De Morgan's laws for complementation.

Sol. Given, U = {–3, –2, –1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
A = {–3, –1, 1, 3, 5, 7} and B = {1, 2, 3, 4}
Law (i) (A∪B)′ = A′∩B′
Now, A∪B = {–3, –1, 1, 2, 3, 4, 5, 7}
(A∪B)′ = {–2, 0, 6, 8, 9} ... (1)

Then, $A' = \{-2, 0, 2, 4, 6, 8, 9\}$ and
$B' = \{-3, -2, -1, 0, 5, 6, 7, 8, 9\}$
$A' \cap B' = \{-2, 0, 6, 8, 9\}$ ... (2)

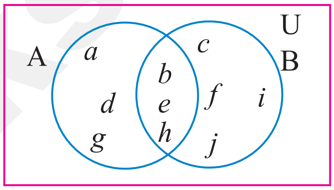
From (1) and (2) it is verified that
$(A \cup B)' = A' \cap B'$

Law (ii) $(A \cap B)' = A' \cup B'$
Now, $A \cap B = \{1, 3\}$
$(A \cap B)' = \{-3, -2, -1, 0, 2, 4, 5, 6, 7, 8, 9\}$ ... (3)
Then, $A' \cup B' = \{-3, -2, -1, 0, 2, 4,5, 6, 7, 8, 9\}$ ... (4)

From (3) and (4) it is verified that
$(A \cap B)' = A' \cup B'$

EXERCISE 1.6

1. From the given venn diagram. Find (i) A, (ii) B, (iii) A∪B, (iv) A∩B also verify that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$



Sol. (i) $A = \{a, b, d, e, g, h\}$
(ii) $B = \{b, c, e, f, h, i, j\}$
(iii) $A \cup B = \{a, b, c, d, e, f, g, h, i, j\}$
(iii) $A \cap B = \{b, e, h\}$

So, $n(A) = 6, n(B) = 7, n(A \cup B) = 10, n(A \cap B) = 3$
Now, $n(A) + n(B) - n(A \cap B) = 6 + 7 - 3 = 10$
Hence, $n(A) + n(B) - n(A \cap B) = n(A \cup B)$

2. If $n(A) = 12$, $n(B) = 17$ and $n(A \cup B) = 21$, find $n(A \cap B)$.

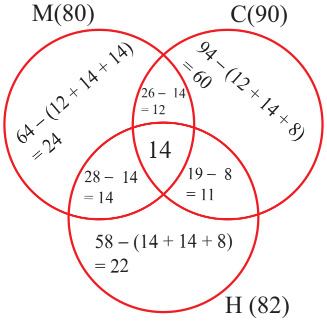
Sol. Given that $n(A) = 12, n(B) = 17$ and $n(A \cup B) = 21$
By using the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$n(A \cap B) = 12 + 17 - 21 = 8$

3. In a school, 80 students like Maths, 90 students like Science, 82 students like History, 21 like both Maths and Science, 19 like both Science and History 20 like both Maths and History and 8 liked all the three subjects. If each student like atleast one subject, then find (i) the number of students in the school (ii) the number of students who like only one subject.

Sol. Let M, S and H represent sets of students who like Maths, Science and History respectively.

Then, $n(M) = 80$, $n(S) = 90$, $n(H) = 82$, $n(M \cap S) = 21$, $n(S \cap H) = 19$, $n(M \cap H) = 20$, $n(M \cap S \cap H) = 8$

Let us represents the given data in a venn diagram.



(i) The number of student in the school
$= 52 + 59 + 55 + 12 + 11 + 8 + 8 = 205$

(ii) The number of students who like only one subject
$= 52 + 59 + 55 = 166$

4. State the formula to find $n(A \cup B \cup C)$.

Sol. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - (B \cap C) - n(A \cap C)$
Sol. $+ (A \cap B \cap B)$

5. **Verify $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - (B \cap C) - n(A \cap C) + (A \cap B \cap C)$ for the following sets A = {1, 3, 5, 6, 8}, B = {3, 4, 5, 6} and C = {1, 2, 3, 6}**

Sol. $(A \cup B \cup C) = \{1, 2, 3, 4, 5, 6, 8\}$

$\therefore n(A \cup B \cup C) = 7$

Also, $n(A) = 5$, $n(B) = 4$, $n(C) = 4$,

Further, $A \cap B = \{3, 5, 6\} \Rightarrow n(A \cap B) = 3$

$B \cap C = \{3, 6\} \Rightarrow n(B \cap C) = 2$

$A \cap C = \{3, 5, 6\} \Rightarrow n(A \cap C) = 3$

Also, $A \cap B \cap C = \{3, 6\} \Rightarrow n(A \cap B \cap C) = 2$

Now, $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$7 = 5 + 4 + 4 - 3 - 2 - 3 + 2$

$7 = 13 - 8 + 2$

$7 = 5 + 2$

$7 = 7$ Thus verified

EXERCISE 1.7

MULTIPLE CHOICE QUESTIONS :

1. **If A = {5, {5, 6}, 7} which of the following is correct?**
(1) {5, 6} ∈ A (2) {5} ∈ A (3) {7} ∈ A (4) {6} ∈ A
Hint : {5, 6} is an element of A. **[Ans. (1) {5, 6} ∈ A]**

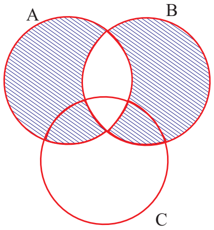
2. **If X = {*a*, {*b*, *c*}, *d*}, which of the following is a subset of X?**
(1) {*a*, *b*} (2) {*b*, *c*} (3) {*c*, *d*} (4) {*a*, *d*}
Hint : *b* is not an element of X. Similarly *c*. **[Ans. (4) {*a*, *d*}]**

3. **If a finite set A has *m* elements, then the number of non-empty proper subset of A is**
(1) 2^m (2) $2^m - 1$ (3) 2^{m-1} (4) $2(2^{m-1} - 1)$
Hint : $P(A) = 2^m$ $\therefore$ Proper non empty subset $= 2m - 2 = 2(2^{m-1} - 1)$
[Ans. (4) $2(2^{m-1} - 1)$]

4. **For any three A, B and C, A – (B ∪ C) is**
(1) $(A - B) \cup (A - C)$ (2) $(A - B) \cap (A \cup C)$
(3) $(A - B) \cup C$ (4) $A \cup (B - C)$
[Ans. (2) $(A - B) \cap (A - C)$]

5. **Which of the following is true?**
(1) $(A \cup B) = B \cup A$ (2) $(A \cup B)' = A' - B'$
(3) $(A \cap B)' = A' \cap B'$ (4) $A - (B \cap C) = (A - B) \cap (A - C)$
[Ans. (1) $(A \cup B) = B \cup A$]

6. **The shaded region in the venn diagram is**
(1) $A \cup B$ (2) $A \cap B$
(3) $(A \cap B)'$ (4) $(A - B) \cup (B - A)$
Hint :



[Ans. (4) $A - B \cup B - A$]

UNIT TEST

Time : 45 Minutes **CHAPTER - 1** **Marks: 25**

Section - A

(i) Answer all the questions.

(ii) Choose the correct Answer. $5 \times 1 = 5$

1. The set $P = \{x \mid x \in \mathbb{Z}, -1 < x < 1\}$ is a
   (1) Singleton set (2) Power set
   (3) Null set (4) Subset
2. Which of the following is correct ?
   (1) $\{7\} \in \{1,2,3,4,5,6,7,8,9,10\}$
   (2) $7 \in \{1,2,3,4,5,6,7,8,9,10\}$
   (3) $7 \notin \{1,2,3,4,5,6,7,8,9,10\}$
   (4) $\{7\} \notin \{1,2,3,4,5,6,7,8,9,10\}$
3. Which of the following is a correct statement?
   (1) $\varnothing \not\subseteq \{a, b\}$ (2) $\varnothing \in \{a, b\}$
   (3) $\{a\} \in \{a, b\}$ (4) $a \subseteq \{a, b\}$
4. If $U = \{x : x \in \mathbb{N}$ and $x < 10\}$, $A = \{1, 2, 3, 5, 8\}$ and $B = \{2, 5, 6, 7, 9\}$, then $n[(A \cup B)']$ is
   (1) 1 (2) 2
   (3) 4 (4) 8
5. For any three sets A, B and C, $(A - B) \cap (B - C)$ is equal to
   (1) A only (2) B only
   (3) C only (4) ϕ

Section - B

(i) Answer only five of the following

(ii) However Question number 12 is compulsory. $5 \times 2 = 10$

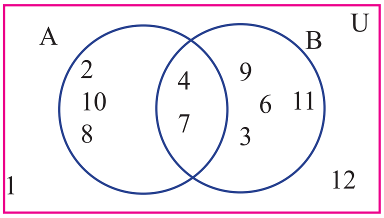
6. Represent the following sets in set builder form.
   (i) B = The set of all Cricket players in India who scored double centuries in One Day Internationals.
   (ii) $C = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4} ....\right\}$.
7. Which of the following sets are equivalent or unequal or equal sets?
   (i) A = The set of vowels in the English alphabets.
   B = The set of all letters in the word "VOWEL"
   (ii) $C = \{2, 3, 4, 5\}$
   $D = \{x : x \in \mathbb{W}, 1 < x < 5\}$
8. Write down the power set of the following sets.
   (i) $A = \{a, b\}$ (ii) $B = \{1, 2, 3\}$
9. If $U = \{x : x \notin \mathbb{N}, x \leq 10\}$, $A = \{2,3,4,8,10\}$ and $B = \{1,2,5,8,10\}$, then verify that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
10. Write the following sets in Roster form.
    (i) $C = \{x : x$ is a prime number and a divisor of 6 $\}$
    (ii) $x = \{x : x = 2n, n \in \mathbb{N}$ and $n \leq 5\}$
11. Which of the following sets are equivalent?
    (i) $A = \{2, 4, 6, 8, 10\}$,
    $B = \{1, 3, 5, 7, 9\}$
    (ii) $X = \{x : x \in \mathbb{N}, 1 < x < 6\}$,
    $Y = \{x : x$ is a vowel in the English Alphabet$\}$
12. If $A = \{-2, -1, 0, 3, 4\}$, $B = \{-1, 3, 5\}$, find (i) $A - B$, (ii) $B - A$.

Section - C

(i) Answer only two Questions of the following.

(ii) However Question number 16 is compulsory. $2 \times 5 = 10$

13. Using the given venn diagram, write the elements of
    (i) A (ii) B (iii) $A \cup B$ (iv) $A \cap B$
    (v) $A - B$ (vi) $B - A$ (vii) A' (viii) B'
    (ix) U



14. Let A and B be two overlapping sets and the universal set U. Draw appropriate Venn diagram for each of the following,

(i) $A \cup B$ (ii) $A \cap B$
(iii) $(A \cap B)'$ (iv) $(B - A)'$
(v) $A' \cup B'$ (vi) $A' \cap B'$
(vii) What do you observe from the diagram (iii) and (v)?

15. In an examination 50% of the students passed in mathematics and 70% of students passed in science while 10% students failed in both subjects. 300 students passed in atleast one subjects. Find the total number of students who appeared in the examination, if they took examination in only two subjects.

16. If $n(A) = 25$, $n(B) = 40$, $n(A \cup B) = 50$ and $n(B') = 25$, find $n(A \cap B)$ and $n(U)$.

ANSWERS

SECTION - A

1. (1) Singleton
2. (2) $7 \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
3. (1) $\varnothing \not\subseteq \{a, b\}$
4. (1) 1
5. (4) ϕ

SECTION - B

6. $\{x : x$ is an Indian player who scored double (i) in one day international$\}$
   (ii) $\{x : x = \frac{n}{n+1}, n \in \mathbb{N}\}$
7. (i) Equivalent sets
   (ii) Unequal sets
8. (i) $\{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1,3\}, \{1, 2, 3\}\}$
9. Thus verified
10. (i) $\{2, 3\}$;(ii) $\{2, 4, 8, 16, 32\}$
11. (i) Equivalent sets
    (ii) Not equivalent set
12. (i) $\{-2, 0, 4\}$ (ii) $\{5\}$

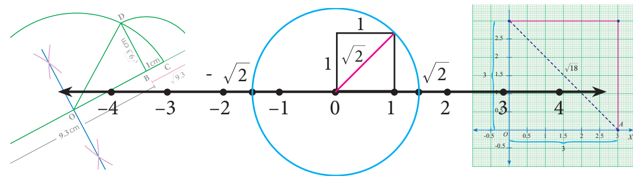
SECTION - C

13. (i) $\{2, 4, 7, 8, 10\}$
    (ii) $\{3, 4, 6, 7, 9, 11\}$
    (iii) $\{2, 3, 4, 6, 7, 8, 9, 10, 11\}$
    (iv) $\{4, 7\}$
    (v) $\{2, 8, 10\}$
    (vi) $\{3, 6, 9, 14\}$
    (vii) $\{1, 3, 6, 9, 11, 12\}$
    (viii) $\{1, 2, 8, 10, 12\}$
    (ix) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

14. ***Refer Sura's Guide Exercise No.1.3, Q. No.7***

15. 1000

16. 15; 65



2 REAL NUMBERS

Introduction

2.1 Number, numbers, everywhere! you have learnt baout many types of numbers so far. Now is the time to extend the ideas further.

2.2. Rational Numbers :

A rational number is a fraction indicating the quotient of two integers, excluding division by zero.

Exercise 2.1

1. Which arrow best shows the position of $\frac{11}{3}$ on the number line?

Sol. $\frac{11}{3} = 3.666..... = 3.7$ (nearly)

$\therefore$ D arrow best shows the position of $\frac{11}{3}$ on the number line.

2. Find any three rational numbers between $\frac{-7}{11}$ and $\frac{2}{11}$.

Sol. Three rational numbers between $\frac{-7}{11}$ and $\frac{2}{11}$ are $\frac{-6}{11}, \frac{-5}{11}, \frac{-4}{11}, ..., \frac{-1}{11}$

3. Find any five rational numbers between

(i) $\frac{1}{4}$ and $\frac{1}{5}$ (ii) 0.1 and 0.11 (iii) –1 and –2

Sol. (i) $a = \frac{1}{4}, b = \frac{1}{5}$

Let q_1, q_2, q_3, q_4 and q_5 be five rational numbers, $q_1 = \frac{1}{2}(a+b)$

$$= \frac{1}{2}\left(\frac{1}{4}+\frac{1}{5}\right) = \frac{1}{2}\left(\frac{5+4}{20}\right) = \frac{1}{2}\left(\frac{9}{20}\right) = \frac{9}{40}$$

$$q_2 = \frac{1}{2}(a+q_1) = \frac{1}{4} \text{ and } \frac{9}{40} = \frac{1}{2}\left(\frac{1}{4}+\frac{9}{40}\right) = \frac{1}{2}\left(\frac{10+9}{40}\right) = \frac{1}{2}\left(\frac{19}{40}\right) = \frac{19}{80}$$

$$q_3 = \frac{1}{2}(a+q_2) = \frac{1}{2}\left(\frac{1}{4}+\frac{19}{80}\right) = \frac{1}{2}\left(\frac{20+19}{80}\right) = \frac{1}{2}\left(\frac{39}{80}\right) = \frac{39}{160}$$

$$q_4 = \frac{1}{2}(a+q_3) = \frac{1}{4} \text{ and } \frac{39}{160} = \frac{1}{2}\left(\frac{1}{4}+\frac{39}{160}\right) = \frac{1}{2}\left(\frac{40+39}{160}\right) = \frac{1}{2}\left(\frac{79}{160}\right) = \frac{79}{320}$$

$$q_5 = \frac{1}{2}(a+q_4) = \frac{1}{4} \text{ and } \frac{79}{320} = \frac{1}{2}\left(\frac{1}{4}+\frac{79}{320}\right) = \frac{1}{2}\left(\frac{80+79}{320}\right) = \frac{1}{2}\left(\frac{159}{320}\right) = \frac{159}{640}$$

Hence five rational numbers between $\frac{1}{4}$ and $\frac{1}{5}$ are $\frac{9}{40}, \frac{19}{80}, \frac{39}{160}, \frac{79}{320}, \frac{159}{640}$

(ii) The rational numbers between 0.1 and 0.11 are 0.101, 0.102, 0.103, ...0.109.

(iii) – 1 and – 2

Let q_1, q_2, q_3, q_4 and q_5 be five rational numbers.

$$q_1 = \frac{1}{2}(\mathrm{a}+\mathrm{b}) = \frac{1}{2}((-1)+(-2)) = \frac{1}{2}(-3) = \frac{-3}{2}$$

$$q_2 = \frac{1}{2}(a+q_1) = \frac{1}{2}\left(-1+\frac{-3}{2}\right) = \frac{1}{2}\left(\frac{-2+(-3)}{2}\right) = \frac{1}{2}\left(\frac{-5}{2}\right) = \frac{-5}{4}$$

$$q_3 = \frac{1}{2}(a+q_2) = -1 \text{ and } \frac{-5}{4}$$

$$= \frac{1}{2}\left(-1-\frac{5}{4}\right) = \frac{1}{2}\left(\frac{-4+(-5)}{4}\right) = \frac{1}{2}\left(\frac{-9}{4}\right) = \frac{-9}{8}$$

$$q_4 = \frac{1}{2}(a+q_3) = -1 \text{ and } \frac{-9}{8}$$

$$= \frac{1}{2}\left(-1+\frac{-9}{8}\right) = \frac{1}{2}\left(\frac{(-8)+(-9)}{8}\right) = \frac{1}{2}\left(\frac{-17}{8}\right) = \frac{-17}{16}$$

$$q_5 = \frac{1}{2}(a+q_4) = -1 \text{ and } \frac{-17}{16}$$

$$= \frac{1}{2}\left(-1+\frac{-17}{16}\right) = \frac{1}{2}\left(\frac{(-16)+(-17)}{16}\right) = \frac{1}{2}\left(\frac{-33}{16}\right) = \frac{-33}{32}$$

The five rational numbers between –1 and –2 are $\frac{-3}{2}, \frac{-5}{4}, \frac{-9}{8}, \frac{-17}{16}, \frac{-33}{32}$.

2.3 Irrational numbers :

A number having a non-terminating and non-recurring decimal expansion is called an irrational number. (i.e) it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$

For example, $\sqrt{2}, \sqrt{3}$, e, π ......

(i.e) The square root of every positive but not a perfect square number is an irrational number.

Exercise 2.2

1. Express the following rational numbers into decimal and state the kind of decimal expansion.

(i) $\frac{2}{7}$ **(ii)** $-5\frac{3}{11}$ **(iii)** $\frac{22}{3}$ **(iv)** $\frac{327}{200}$

Sol. **(i)** $\frac{2}{7}$

$$\begin{array}{r} 0.285714 \\ 7\overline{)20} \\ 14 \\ \hline 60 \\ 56 \\ \hline 40 \\ 35 \\ \hline 50 \\ 49 \\ \hline 10 \\ 7 \\ \hline 30 \\ 28 \\ \hline 2 \\ \vdots \end{array}$$

$\frac{2}{7} = 0.\overline{285714}$,

Non-terminating and recurring

(ii) $-5\frac{3}{11} = \frac{-58}{11}$

$$\begin{array}{r} 5.27 \\ 11\overline{)58} \\ 55 \\ \hline 30 \\ 22 \\ \hline 80 \\ 77 \\ \hline 3 \\ \vdots \end{array}$$

$-5\frac{3}{11} = -5.\overline{27}$

Non-terminating and recurring.

(iii) $\frac{22}{3}$

$$\begin{array}{r} 7.3 \\ 3\overline{)22} \\ 21 \\ \hline 10 \\ 9 \\ \hline 1 \\ \vdots \end{array}$$

$\frac{22}{3} = 7.\overline{3}$,

Non-terminating and recurring.

(iv) $\frac{327}{200}$

$$\frac{327}{200} = 1.635 \text{, Terminating.}$$

$$\begin{array}{r|l} 200 & 327 \;(1.635 \\ & 200 \\ \hline & 1270 \\ & 1200 \\ \hline & 700 \\ & 600 \\ \hline & 1000 \\ & 1000 \\ \hline & 0 \end{array}$$

2. Express $\frac{1}{13}$ in decimal form. Find the length of the period of decimals.

Sol. $\frac{1}{13}$

$$\begin{array}{r|l} 13 & 100 \;(0.076923 \\ & 91 \\ \hline & 90 \\ & 78 \\ \hline & 120 \\ & 117 \\ \hline & 30 \\ & 26 \\ \hline & 40 \\ & 39 \\ \hline & 1 \\ & \vdots \end{array}$$

$\frac{1}{13} = 0.\overline{076923}$ has the length of the period of decimals = 6.

3. Express the rational number $\frac{1}{33}$ in recurring decimal form by using the recurring decimal expansion of $\frac{1}{11}$. Hence write $\frac{71}{33}$ in recurring decimal form.

Sol. The recurring decimal expansion of $\frac{1}{11} = 0.09090909.... = 0.\overline{09}$

$\therefore \frac{1}{33} = 0.03030303.... = 0.\overline{03}$ Also, $\frac{71}{33} = 2\frac{5}{33} = 2 + \frac{5}{33}$

$$= 2 + \left(5 \times \frac{1}{33}\right)$$
$$= 2 + (5 \times 0.\overline{03})$$
$$= 2 + (5 \times 0.030303\cdots)$$
$$= 2 + 0.151515\cdots\cdots$$
$$= 2.151515\cdots\cdots = 2.\overline{15}$$

4. Express the following decimal expression into rational numbers.

(i)	$\mathbf{0.\overline{24}}$	**(ii)**	$\mathbf{2.\overline{327}}$	**(iii)**	**–5.132**
(iv)	$\mathbf{3.1\overline{7}}$	**(v)**	$\mathbf{17.\overline{215}}$	**(vi)**	$\mathbf{–21.213\overline{7}}$

Sol. **(i)** $\mathbf{0.\overline{24}}$

$$\text{Let } x = 0.\overline{24} = 0.24242424\cdots\cdots \qquad ...(1)$$

(Here period of decimal is 2, multiply equation (1) by 100)

$$100x = 24.242424........ \qquad ....(2)$$

(2) – (1)

$$100x - x = 24.242424.... - 0.242424....$$
$$99x = 24$$
$$x = \frac{24}{99}$$

(ii) $\mathbf{2.\overline{327}}$

$$\text{Let } x = 2.327327327........ \qquad ...(1)$$

(Here period of decimal is 3, multiply equation (1) by 1000)

$$1000x = 2327.327........ \qquad ...(2)$$

(2) – (1)

$$1000x - x = 2327.327327\cdots - 2.327327\cdots$$
$$999x = 2325$$
$$x = \frac{2325}{999}$$

(iii) **–5.132**

$$x = -5.132 = \frac{-5132}{1000} = \frac{-1283}{250}$$

(iv) $\mathbf{3.1\overline{7}}$

$$\text{Let } x = 3.1777........ \qquad ...(1)$$

(Here the repeating decimal digit is 7, which is the second digit after the decimal point, multiply equation (1) by 10)

$$10x = 31.7777........ \qquad ...(2)$$

(Now period of decimal is 1, multiply equation (2) by 10)

$$100x = 317.7777........ \qquad ...(3)$$

(3) – (2)

$$100x - 10x = 317.777.... - 31.777....$$
$$90x = 286$$
$$x = \frac{286}{90} = \frac{143}{45}$$

(v) $\mathbf{17.\overline{215}}$

$$\text{Let } x = 17.215215........ \qquad ...(1)$$
$$1000x = 17215.215215........ \qquad ....(2)$$

(2) – (1)

$$1000x - x = 17215.215215... - 17.215...$$
$$999x = 17198$$
$$x = \frac{17198}{999}$$

(vi) $\mathbf{-21.213\overline{7}}$

$$\text{Let } x = -21.2137777........ \qquad ...(1)$$
$$10x = -212.137777........ \qquad ...(2)$$
$$100x = -2121.37777........ \qquad ...(3)$$
$$1000x = -21213.77777........ \qquad ...(4)$$
$$10000x = 212137.77777 \qquad ...(5)$$

(Now period of decimal is 1, multiply equation (4) it by 10)

(5) – (4)

$$10000x - 1000x = (-212137.7777...) - (-21213.7777...)$$
$$9000x = -190924$$
$$x = -\frac{190924}{9000}$$

5. Without actual division, find which of the following rational numbers have terminating decimal expansion.

(i) $\frac{7}{128}$ **(ii)** $\frac{21}{15}$ **(iii)** $4\frac{9}{35}$ **(v)** $\frac{219}{2200}$

Sol. **(i)** $\frac{7}{128}$

$$\begin{array}{r|l} 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

So $\frac{7}{128} = \frac{7}{2^7 5^0}$

This of the form 4m, $n \in \mathbb{W}$

So $\frac{7}{128}$ has a terminating decimal expansion.

(ii) $\frac{21}{15}$

$$\begin{array}{r|l} 5 & 15 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 3 & 21 \\ \hline & 7 \end{array}$$

$$\therefore \frac{21}{15} = \frac{\cancel{3} \times 7}{5 \times \cancel{3}} = \frac{7}{5} = \frac{7}{2^0 5^1}$$

So $\frac{21}{15}$ has a terminating decimal expansion.

(iii) $4\frac{9}{35} = \frac{149}{35}$

$$\begin{array}{r|l} 5 & 35 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$\frac{49}{35} = \frac{149}{5^1 7^1}$ $\therefore$ This is **not** of the form $\frac{P}{2^m 5^n}$

So, $4\frac{9}{35}$ has a non-terminating recurring decimal expansion.

(v) $\mathbf{\dfrac{219}{2200}}$

2	2200
2	1100
2	550
5	275
5	55
	11

$$\frac{219}{2200} = \frac{219}{2^3 5^2 11^1}$$

This is not of the form $\dfrac{P}{2^m 5^n}$

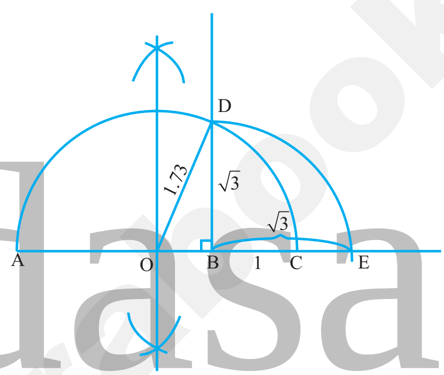
So, $\dfrac{219}{2200}$ has a non-terminating recurring decimal expansion.

Exercise 2.3

1. Represent the following irrational numbers on the number line.

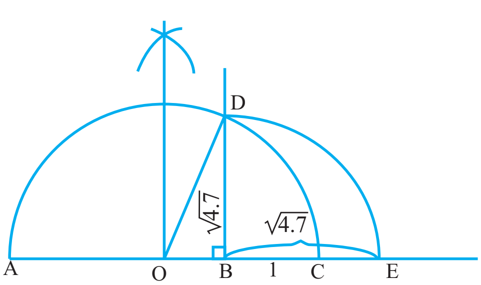
(i) $\sqrt{3}$ **(ii)** $\sqrt{4.7}$ **(iii)** $\sqrt{6.5}$

Sol. **(i)** $\sqrt{3}$



(i) Draw a line and mark a point A on it.
(ii) Mark a point B such that AB = 3 cm.
(iii) Mark a point C on this line such that BC = 1 unit.
(iv) Find the midpoint of AC by drawing perpendicular bisector of AC and let it be O.
(v) With O as center and OC = OA as radius, draw a semicircle.
(vi) Draw a line BD, which is perpendicular to AB at B.
(vii) Now BD = $\sqrt{3}$, which can be marked in the number line as the value of BE = BD = $\sqrt{3}$

(ii) $\sqrt{4.7}$



(i) Draw a line and mark a point A on it.
(ii) Mark a point B such that AB = 4.7 cm.

(iii) Mark a point C on this line such that BC = 1 unit.

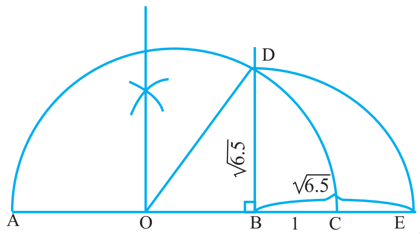
(iv) Find the midpoint of AC by drawing perpendicular bisector of AC and let it be O.

(v) With O as center and OC = OA as radius, draw a semicircle.

(vi) Draw a line BD, which is perpendicular to AB at B.

(vii) Now BD = $\sqrt{4.7}$, which can be marked in the number line as the value of BE = BD = $\sqrt{4.7}$

(iii) $\sqrt{\mathbf{6.5}}$



(i) Draw a line and mark a point A on it.

(ii) Mark a point B such that AB = 6.5 cm.

(iii) Mark a point C on this line such that BC = 1 unit.

(iv) Find the midpoint of AC by drawing perpendicular bisector of AC and let it be O.

(v) With O as center and OC = OA as radius, draw a semicircle.

(vi) Draw a line BD, which is perpendicular to AB at B.

(vii) Now BD = $\sqrt{6.5}$, which can be marked in the number line as the value of BE = BD = $\sqrt{6.5}$

2. Find any two irrational numbers between

(i) 0.3010011000111…. and 0.3020020002….

(ii) $\mathbf{\frac{6}{7}}$ **and** $\mathbf{\frac{12}{13}}$ **(iii)** $\sqrt{\mathbf{2}}$ **and** $\sqrt{\mathbf{3}}$

Sol. (i) 0.3010011000111…. and 0.3020020002….

Two irrational numbers 0.301202200222......, 0.301303300333........

(ii) $\mathbf{\frac{6}{7}}$ **and** $\mathbf{\frac{12}{13}}$

$$\frac{6}{7} = 0.857142 \ldots$$

$$\frac{12}{13} = 0.923076$$

Two irrational numbers between 0.8616611666111......, 0.8717711777111......

(iii) $\sqrt{2}$ **and** $\sqrt{3}$

	1. 4142......
1	2,00,00,00,00,00
	1
24	100
	96
281	400
	281
2824	11900
	11296
28282	60400
	56564
	383600
	⋮

	1.732.....
1	3,00,00,00
	1
27	200
	189
343	1100
	1029
3462	7100
	6924
28282	176
	⋮

$\sqrt{2}$ = 1.414......., $\sqrt{3}$ = 1.732........

∴ Two irrational numbers between 1.5155...., 1.6166.......

3. Find any two rational numbers between 2.2360679….. and 2.236505500….

Sol. Any two rational numbers are 2.2362, 2.2363

2.4 Real Numbers

The real numbers consist of all the rational numbers and all the irrational numbers.

Real numbers can be thought of as points on an infinitely long number line called the real line, where the points corresponding to integers are equally spaced.

Exercise 2.4

1. Represent the following numbers on the number line.

(i) 5.348 (ii) $6.\overline{4}$ upto 3 decimal places. (iii) $4.\overline{73}$ upto 4 decimal places.

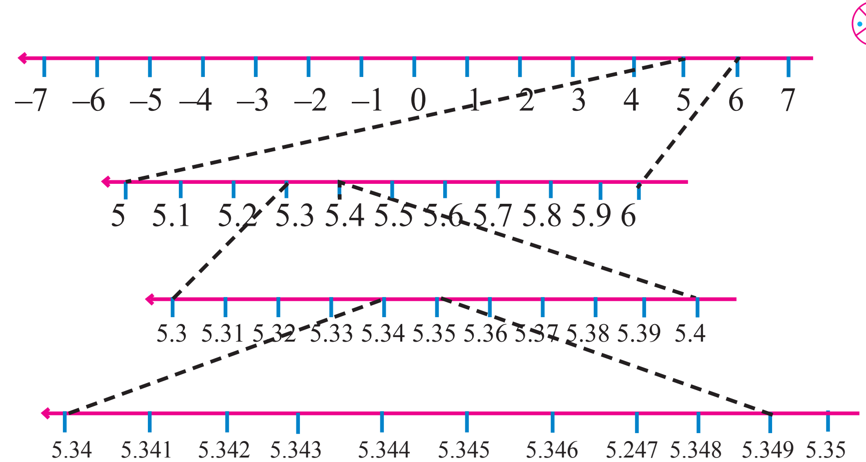
Sol. **(i) 5.348**

Step : 1

Step : 2

Step : 3

Step : 4



Step 1 : First we note that 5.348 lies between 5 and 6.

Step 2 : Divide the portion between 5 and 6 into 10 equal parts and use a magnifying glass to visualise that 5.348 lies between 5.3 and 5.4.

Step 3 : Divide the portion between 5.3 and 5.4 into 10 equal parts and use a magnifying glass to visualise that 5.348 lies between 5.34 and 5.35.

Step 4 : Divide the portion between 5.34 and 5.35 into 10 equal parts and use a magnifying glass to visualise that 5.348 lies between 5.348 and 5.349.

Step 5 : Divide the portion between 5.348 and 5.349 into 10 equal parts and use a magnifying glass to visualise that 5.348 lies between 5.349 and 5.348.

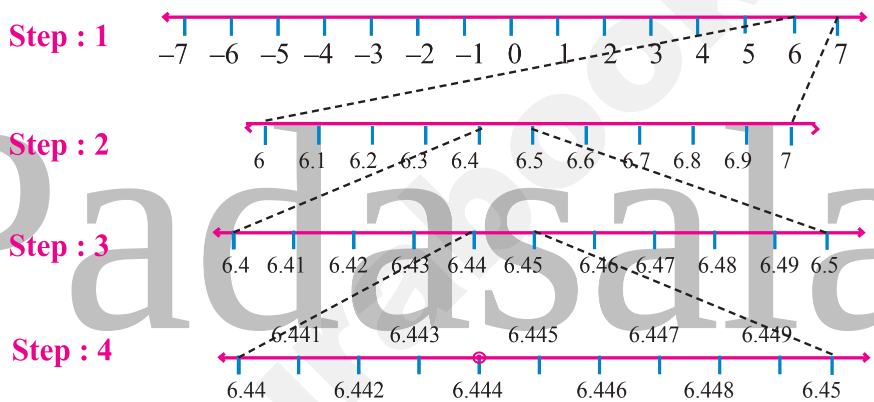
We note that 5.349 is visualised closed to 5.349 than to 5.348.

(ii) $6.\overline{4}$ upto 3 decimal places.

$6.\overline{4}$ = 6.444...........

= 6.444 (up to 3 decimal places).

The number lies between 6 and 7.



Step 1 : First we note that 6.444 lies between 6 and 7.

Step 2 : Divide the portion between 6 and 7 into 10 equal parts and use a magnifying glass to visualise that 6.4 lies between 6.4 and 6.5.

Step 3 : Divide the portion between 6.4 and 6.5 into 10 equal parts and use a magnifying glass to visualise that 6.4 lies between 6.44 and 6.45.

Step 4 : Divide the portion between 6.44 and 6.45 into 10 equal parts and use a magnifying glass to visualise that 6.444 lies between 6.445 and 6.444.

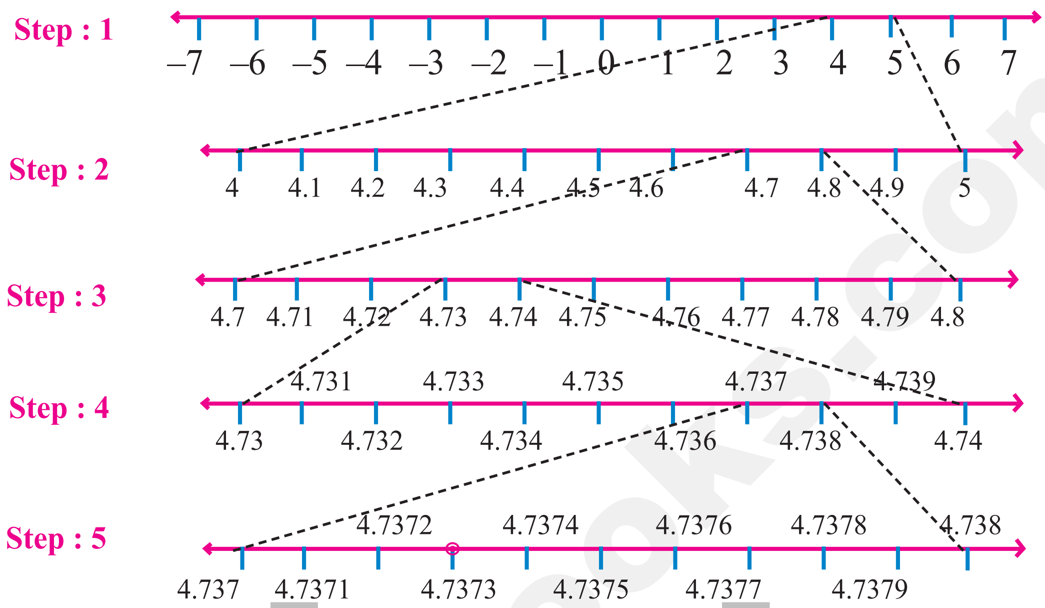
We note that $6.\overline{4}$ is visualised closed to 6.444 than to 6.445.

(iii) $4.\overline{73}$ upto 4 decimal places

$4.\overline{73}$ = 4.73737373...........

= 4.7373 (correct 4 decimal places).

The number lies between 4 and 5



Step 1 : First we note that $4.7\overline{3}$ lies between 4 and 5.

Step 2 : Divide the portion between 4 and 5 into 10 equal parts and use a magnifying glass to visualise that $4.7\overline{3}$ lies between 4.7 and 4.8.

Step 3 : Divide the portion between 4.7 and 4.8 into 10 equal parts and use a magnifying glass to visualise that $4.7\overline{3}$ lies between 4.73 and 4.74.

Step 4 : Divide the portion between 4.73 and 4.74 into 10 equal parts and use a magnifying glass to visualise that $4.7\overline{3}$ lies between 4.733 and 4.734.

Step 5 : Divide the portion between 4.733 and 4.734 into 10 equal parts and use a magnifying glass to visualise that $4.7\overline{3}$ lies between 4.7332 and 4.7334.

We note that $4.7\overline{3}$ is visualised closed to 4.7332 than to 4.7334.

Exercise 2.5

1. Write the following in the form of 5^n:

(i) 625 **(ii) $\frac{1}{5}$** **(iii) $\sqrt{5}$** **(iv) $\sqrt{125}$**

Sol. (i) $625 = 5^4$

$$\begin{array}{r|r} 5 & 625 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

(ii) $\frac{1}{5} = 5^{-1}$

(iii) $\sqrt{5} = 5^{\frac{1}{2}}$

(iv) $\sqrt{125} = \sqrt{5^3} = (5^3)^{\frac{1}{2}} = 5^{\frac{3}{2}}$

$$\begin{array}{r|r} 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

2. Write the following in the form of 4^n:

(i) 16 **(ii) 8** **(iii) 32**

Sol. (i) $16 = 4^2$

(ii) $8 = 4^1 \times 4^{\frac{1}{2}} = 4^{1+\frac{1}{2}} = 4^{\frac{3}{2}}$

(iii) $32 = 4 \times 4 \times 4^{\frac{1}{2}} = 4^{2+\frac{1}{2}} = 4^{\frac{5}{2}}$

$$\begin{array}{r|r} 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

3. Find the value of

(i) $(49)^{\frac{1}{2}}$ **(ii) $(243)^{\frac{2}{5}}$** **(iii) $9^{\frac{-3}{2}}$** **(iv) $\left(\frac{64}{125}\right)^{\frac{-2}{3}}$**

$$\begin{array}{r|r} 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

Sol. (i) $(49)^{\frac{1}{2}} = (7 \times 7)^{\frac{1}{2}} = 7$

(ii) $(243)^{\frac{2}{5}} = (3^5)^{\frac{2}{5}} = 3^{\cancel{5} \times \frac{2}{\cancel{5}}} = 3^2 = 9$

(iii) $9^{\frac{-3}{2}} = (3^2)^{\frac{-3}{2}} = 3^{\cancel{2} \times \frac{-3}{\cancel{2}}} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

(iv) $\left(\frac{64}{125}\right)^{\frac{-2}{3}} = \left(\frac{125}{64}\right)^{\frac{2}{3}} = \left(\left(\frac{5}{4}\right)^3\right)^{\frac{2}{3}} = \left(\frac{5}{4}\right)^{\cancel{3} \times \frac{2}{\cancel{3}}} = \left(\frac{5}{4}\right)^2 = \frac{25}{16}$

4. Use a fractional index to write:

(i) $\sqrt{5}$ **(ii)** $\sqrt[2]{7}$ **(iii)** $\left(\sqrt[3]{49}\right)^5$ **(iv)** $\left(\dfrac{1}{\sqrt[3]{100}}\right)^7$

Sol. (i) $\sqrt{5} = 5^{\frac{1}{2}}$ (ii) $\sqrt[2]{7} = 7^{\frac{1}{2}}$ (iii) $\left(\sqrt[3]{49}\right)^5 = 49^{\frac{5}{3}}$

(iv) $$\left(\frac{1}{\sqrt[3]{100}}\right)^7 = \left(\frac{1}{100^{\frac{1}{3}}}\right)^7 = \left(\frac{1^{\frac{1}{3}}}{100^{\frac{1}{3}}}\right)^7 = \left(\frac{1}{100}\right)^{\frac{7}{3}}$$

5. Find the 5th root of

(i) 32 **(ii)** 243 **(iii)** 100000 **(iv)** $\dfrac{1024}{3125}$

Sol. (i) $$\sqrt[5]{32} = 32^{\frac{1}{5}} = (2^5)^{\frac{1}{5}} = 2^{\cancel{5} \times \frac{1}{\cancel{5}}} = 2$$

(ii) $$\sqrt[5]{243} = 243^{\frac{1}{5}} = (3^5)^{\frac{1}{5}} = 3^{\cancel{5} \times \frac{1}{\cancel{5}}} = 3$$

(iii) $$\sqrt[5]{100000} = (100000)^{\frac{1}{5}} = (10^{\cancel{5}})^{\frac{1}{\cancel{5}}} = 10$$

(iv) $$\sqrt[5]{\frac{1024}{3125}} = \left(\frac{1024}{3125}\right)^{\frac{1}{5}} = \left(\left(\frac{4}{5}\right)^{\cancel{5}}\right)^{\frac{1}{\cancel{5}}} = \frac{4}{5}$$

Exercise 2.6

1. Simplify the following using addition and subtraction properties of surds:

(i) $5\sqrt{3} + 18\sqrt{3} - 2\sqrt{3}$ **(ii)** $4\sqrt[3]{5} + 2\sqrt[3]{5} - 3\sqrt[3]{5}$

(iii) $3\sqrt{75} + 5\sqrt{48} - \sqrt{243}$ **(iv)** $5\sqrt[3]{40} + 2\sqrt[3]{625} - 3\sqrt[3]{320}$

Sol. **(i)** $5\sqrt{3} + 18\sqrt{3} - 2\sqrt{3} = (5 + 18 - 2)\sqrt{3} = 21\sqrt{3}$

(ii) $4\sqrt[3]{5} + 2\sqrt[3]{5} - 3\sqrt[3]{5} = (4 + 2 - 3)\sqrt[3]{5} = 3\sqrt[3]{5}$

(iii) $3\sqrt{75} + 5\sqrt{48} - \sqrt{243}$

$$= 3\sqrt{5\times5\times3} + 5\sqrt{3\times2\times2\times2\times2} - \sqrt{3\times3\times3\times3\times3}$$
$$= 3 \times 5\sqrt{3} + 5 \times 2 \times 2\sqrt{3} - 3 \times 3\sqrt{3}$$
$$= 15\sqrt{3} + 20\sqrt{3} - 9\sqrt{3}$$
$$= (15 + 20 - 9)\sqrt{3} = 26\sqrt{3}$$

$$\begin{array}{r|r} 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|r} 5 & 75 \\ \hline 5 & 15 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|r} 3 & 48 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

(iv) $5\sqrt[3]{40}+2\sqrt[3]{625}-3\sqrt[3]{320}$

$= 5\sqrt[3]{2^3\times5}+2\sqrt[3]{5^3\times5}-3\sqrt[3]{2^3\times2^3\times5}$

$= 5\times2\times\sqrt[3]{5}+2\times5\sqrt[3]{5}-3\times2\times2\sqrt[3]{5}$

$= 10\sqrt[3]{5}+10\sqrt[3]{5}-12\sqrt[3]{5}$

$= (10+10-12)\sqrt[3]{5} = 8\sqrt[3]{5}$

2	40
2	20
2	10
5	5
	1

5	625
5	105
5	25
5	5
	1

2	320
2	160
2	80
2	40
2	20
2	10
5	5
	1

2. Simplify the following using multiplication and division properties of surds :

(i) $\sqrt{3}\times\sqrt{5}\times\sqrt{2}$ **(ii)** $\sqrt{35}\div\sqrt{7}$

(iii) $\sqrt[3]{27}\times\sqrt[3]{8}\times\sqrt[3]{125}$ **(iv)** $(7\sqrt{a}-5\sqrt{b})(7\sqrt{a}+5\sqrt{b})$

(v) $\left[\sqrt{\frac{225}{729}}-\sqrt{\frac{25}{144}}\right]\div\sqrt{\frac{16}{81}}$

Sol. (i) $\sqrt{3}\times\sqrt{5}\times\sqrt{2} = \sqrt{3\times5\times2}=\sqrt{30}$

(ii) $\sqrt{35}\div\sqrt{7} = \sqrt{\frac{35}{7}}=\sqrt{5}$

(iii) $\sqrt[3]{27}\times\sqrt[3]{8}\times\sqrt[3]{125}=\sqrt[3]{27\times8\times125} = \sqrt[3]{3^3\times2^3\times5^3} = 3\times2\times5 = 30$

(iv) $(7\sqrt{a}-5\sqrt{b})(7\sqrt{a}+5\sqrt{b}) = (7\sqrt{a})^2-(5\sqrt{b})^2 = 49a-25b$

(v) $\left[\sqrt{\frac{225}{729}}-\sqrt{\frac{25}{144}}\right]\div\sqrt{\frac{16}{81}}$

$= \left[\sqrt{\frac{15^2}{27^2}}-\sqrt{\frac{5^2}{12^2}}\right]\times\sqrt{\frac{9^2}{4^2}}$

$= \left(\frac{15}{27}-\frac{5}{12}\right)\times\frac{9}{4} = \left(\frac{5}{9}-\frac{5}{12}\right)\times\frac{9}{4}$

$= \left(\frac{20-15}{36}\right)\times\frac{9}{4} = \frac{5}{\cancel{36}_{4}}\times\frac{\cancel{9}}{4}=\frac{5}{6}$

3	729
3	243
3	81
3	27
3	9
3	3
	1

3. If $\sqrt{2}$ =1.414, $\sqrt{3}$ = 1.732, $\sqrt{5}$ = 2.236, $\sqrt{10}$ = 3.162, then find the values of the following correct to 3 places of decimals.

(i) $\sqrt{40}-\sqrt{20}$ **(ii)** $\sqrt{300}+\sqrt{90}-\sqrt{8}$

Sol. (i) $\sqrt{40}-\sqrt{20} = \sqrt{4\times10}-\sqrt{2\times10} = \sqrt{(4-2)10} = \sqrt{2\times10} = \sqrt{2}\times\sqrt{10}$

$= 1.414\times3.162 = 4.471068 = 4.471$

(ii) $\sqrt{300}+\sqrt{90}-\sqrt{8} = \sqrt{3\times100}+\sqrt{9\times10}+\sqrt{4\times2} = 10\sqrt{3}+3\sqrt{10}+2\sqrt{2}$

$= 10\times1.732+3\times3.162+2\times1.414$

$= 17.32+9.486+2.828 = 29.634$

4. Arrange surds in descending order :

(i) $\sqrt[3]{5}, \sqrt[9]{4}, \sqrt[6]{3}$ **(ii)** $\sqrt[2]{\sqrt[3]{5}}, \sqrt[3]{\sqrt[4]{7}}, \sqrt{\sqrt{3}}$

Sol. (i) $\sqrt[3]{5}, \sqrt[9]{4}, \sqrt[6]{3}$

$5^{\frac{1}{3}}$ $\therefore$ The order of the surds $\sqrt[3]{5}, \sqrt[9]{4}, \sqrt[6]{3}$ are 3, 9, 6.

$4^{\frac{1}{9}}$

$3^{\frac{1}{6}}$ l.c.m of 3, 9, 6 is 18

$$\therefore \frac{1}{3} = \frac{1\times 6}{3\times 6} = \frac{6}{18}$$

$$\frac{1}{9} = \frac{1\times 2}{9\times 2} = \frac{2}{18}$$

$$\frac{1}{6} = \frac{1\times 3}{6\times 3} = \frac{3}{18}$$

$$\left(5^{\frac{1}{3}}\right) = 5^{\frac{6}{18}} = (5^6)^{\frac{1}{18}} = (15625)^{\frac{1}{18}}$$

$$\left(4^{\frac{1}{9}}\right) = 4^{\frac{2}{18}} = (4^2)^{\frac{1}{18}} = 16^{\frac{1}{18}}$$

$$\left(3^{\frac{1}{6}}\right) = 3^{\frac{3}{18}} = (3^3)^{\frac{1}{18}} = 27^{\frac{1}{18}}$$

$\therefore$ The descending order of $\sqrt[3]{5}, \sqrt[9]{4}, \sqrt[6]{3}$ is $(15625)^{\frac{1}{18}} > (27)^{\frac{1}{18}} > 16^{\frac{1}{18}}$ i.e. $\sqrt[3]{5} > \sqrt[6]{3} > \sqrt[9]{4}$

(ii) $\sqrt[2]{\sqrt[3]{5}}, \sqrt[3]{\sqrt[4]{7}}, \sqrt{\sqrt{3}}$

The order of the surds $\sqrt[2]{\sqrt[3]{5}}, \sqrt[3]{\sqrt[4]{7}}, \sqrt[2]{\sqrt[2]{3}}$ are 6, 12, 4

l. c. m of 6, 12, 4 is 12

$$\sqrt[2]{\sqrt[3]{5}} = 5^{\frac{1}{6}} = 5^{\frac{1\times 2}{6\times 2}} = 5^{\frac{2}{12}} = (5^2)^{\frac{1}{12}} = 25^{\frac{1}{12}}$$

$$\sqrt[3]{\sqrt[4]{7}} = 7^{\frac{1}{12}}$$

$$\sqrt{\sqrt{3}} = 3^{\frac{1}{4}} = 3^{\frac{1\times 3}{4\times 3}} = 3^{\frac{3}{12}} = (3^3)^{\frac{1}{12}} = 27^{\frac{1}{12}}$$

$\therefore$ The ascending order of the surds

$\sqrt[2]{\sqrt[3]{5}}, \sqrt[3]{\sqrt[4]{7}}, \sqrt{\sqrt{3}}$ is $7^{\frac{1}{12}} < 25^{\frac{1}{2}} < 27^{\frac{1}{2}}$, that is $\sqrt[3]{\sqrt[4]{7}} < \sqrt[2]{\sqrt[3]{5}} < \sqrt{\sqrt{3}}$

5. Can you get a pure surd when you find

(i) the sum of two surds **(ii) the difference of two surds**

(iii) the product of two surds **(iv) the quotient of two surds**

Justify each answer with an example.

Sol. (i) Yes $4\sqrt[3]{21} + \left(-3\sqrt[3]{21}\right) = \sqrt[3]{21}$

(ii) Yes $\sqrt[4]{25}-6\sqrt[4]{25} = \sqrt[4]{25}(7-6)=\sqrt[4]{25}$

(iii) Yes $\sqrt[3]{5}\times\sqrt[3]{4} = \sqrt[3]{20}$, a pure surd

(iv) Yes $= \dfrac{\sqrt{2\times5}}{\sqrt{2}} = \dfrac{\cancel{\sqrt{2}}\times\sqrt{5}}{\cancel{\sqrt{2}}} = \sqrt{5}$

6. Can you get a rational number when you compute
(i) the sum of two surds (ii) the difference of two surds
(iii) the product of two surds (iv) the quotient of two surds
Justify each answer with an example.

Sol. (i) Yes $(5-\sqrt{3})+(5+\sqrt{3})$ = 10, a rational number

(ii) Yes $(5+\sqrt[3]{7})-(-6+\sqrt[3]{7})$ = 11, a rational number

(iii) Yes $(5+\sqrt{3})(5-\sqrt{3})$ = $25-3=22$, a rational number

(iv) Yes $\dfrac{5\sqrt{3}}{\sqrt{3}}$ = 5, a rational number

Exercise 2.7

1. Rationalise the denominator

(i) $\dfrac{1}{\sqrt{50}}$ **(ii)** $\dfrac{5}{3\sqrt{5}}$ **(iii)** $\dfrac{\sqrt{75}}{\sqrt{18}}$ **(iv)** $\dfrac{3\sqrt{5}}{\sqrt{6}}$

Sol. (i) $\dfrac{1}{\sqrt{50}} = \dfrac{1}{\sqrt{50}}\times\dfrac{\sqrt{50}}{\sqrt{50}} = \dfrac{\sqrt{50}}{50} = \dfrac{\sqrt{5\times5\times2}}{5\times5\times2} = \dfrac{\cancel{5}\sqrt{2}}{\cancel{5}\times5\times2} = \dfrac{\sqrt{2}}{10}$

(ii) $\dfrac{5}{3\sqrt{5}} = \dfrac{5}{3\sqrt{5}}\times\dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{\cancel{5}\sqrt{5}}{3\times\cancel{5}} = \dfrac{\sqrt{5}}{3}$

(iii) $\dfrac{\sqrt{75}}{\sqrt{18}} = \dfrac{\sqrt{3\times5\times5}}{\sqrt{3\times2\times3}} = \dfrac{5\sqrt{3}\times\sqrt{2}}{3\sqrt{2}\times\sqrt{2}} = \dfrac{5\sqrt{6}}{3\times2} = \dfrac{5\sqrt{6}}{6}$

(iv) $\dfrac{3\sqrt{5}}{\sqrt{6}} = \dfrac{3\sqrt{5}\times\sqrt{6}}{\sqrt{6}\times\sqrt{6}} = \dfrac{\cancel{3}\sqrt{30}}{\underset{2}{\cancel{6}}} = \dfrac{\sqrt{30}}{2}$

2. Rationalise the denominator and simplify

(i) $\dfrac{\sqrt{48}+\sqrt{32}}{\sqrt{27}-\sqrt{18}}$ **(ii)** $\dfrac{5\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ **(iii)** $\dfrac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$ **(iv)** $\dfrac{\sqrt{5}}{\sqrt{6}+2}-\dfrac{\sqrt{5}}{\sqrt{6}-2}$

Sol. (i) $\dfrac{\sqrt{48}+\sqrt{32}}{\sqrt{27}-\sqrt{18}} = \dfrac{\sqrt{48}+\sqrt{32}}{(\sqrt{27}-\sqrt{18})}\times\dfrac{\sqrt{27}+\sqrt{18}}{\sqrt{27}+\sqrt{18}}$

[Multiply the numerator and denominator by the rationatising factor $(\sqrt{27}+\sqrt{18})$.]

$= \dfrac{\sqrt{48\times27}+\sqrt{32}\times\sqrt{27}+\sqrt{48\times18}+\sqrt{32\times18}}{\sqrt{27^2}-\sqrt{18^2}}$

$$= \frac{\sqrt{3\times16\times3\times9}+\sqrt{2\times16\times3\times9}+\sqrt{3\times16\times2\times9}+\sqrt{2\times16\times2\times9}}{729-324}$$

$$= \frac{4\times3\times3+4\times3\sqrt{6}+4\times3\sqrt{6}+4\times2\times3}{405}$$

$$= \frac{36+12\sqrt{6}+12\sqrt{6}+24}{405} = \frac{60+24\sqrt{6}}{405}$$

(ii) $$\frac{5\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{(5\sqrt{3}+\sqrt{2})\times(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})\times(\sqrt{3}-\sqrt{2})}$$

$$= \frac{5\sqrt{3}\times\sqrt{3}+\sqrt{2}\times\sqrt{3}-5\sqrt{3}\times\sqrt{2}-\sqrt{2}^2}{\sqrt{3}^2-\sqrt{2}^2}$$

$$= \frac{5\times3+\sqrt{6}-5\sqrt{6}-2}{3-2} = \frac{13-4\sqrt{6}}{1} = 13-4\sqrt{6}$$

(iii) $$\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}} = \frac{(2\sqrt{6}-\sqrt{5})\times3\sqrt{5}+2\sqrt{6}}{(3\sqrt{5}-2\sqrt{6})\times3\sqrt{5}+2\sqrt{6}}$$

$$= \frac{2\sqrt{6}\times3\sqrt{5}-3\sqrt{5}\sqrt{5}-(2\sqrt{6})^2-2\sqrt{5}\times\sqrt{6}}{(3\sqrt{5})^2-(2\sqrt{6})^2}$$

$$= \frac{6\sqrt{30}-3\times5-4\times6-2\sqrt{30}}{9\times5-4\times6} = \frac{4\sqrt{30}-39}{45-24} = \frac{4\sqrt{30}-39}{21}$$

(iv) $$\frac{\sqrt{5}}{\sqrt{6}+2}-\frac{\sqrt{5}}{\sqrt{6}-2} = \frac{\sqrt{5}(\sqrt{6}-2)-\sqrt{5}(\sqrt{6}+2)}{(\sqrt{6}+2)(\sqrt{6}-2)}$$

$$= \frac{\cancel{\sqrt{30}}-2\sqrt{5}-\cancel{\sqrt{30}}-2\sqrt{5}}{\sqrt{6}^2-2^2} = \frac{-4\sqrt{5}}{6-4} = \frac{-\overset{2}{\cancel{4}}\sqrt{5}}{\cancel{2}} = -2\sqrt{5}$$

3. Find the value of *a* and *b* if $\frac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7}+b$.

Sol. $\frac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7}+b$

$$\text{L.H.S} = \frac{\sqrt{7}-2\times\sqrt{7}-2}{\sqrt{7}+2\times\sqrt{7}-2} = \frac{(\sqrt{7}-2)^2}{\sqrt{7}^2-2^2} = \frac{\sqrt{7}^2-2\sqrt{7}\times2+2^2}{7-4}$$

$$= \frac{7-4\sqrt{7}+4}{3} = \frac{11-4\sqrt{7}}{3} = \frac{11}{3}-\frac{4\sqrt{7}}{3}$$

$$\frac{-4\sqrt{7}}{3}+\frac{11}{3} = a\sqrt{7}+b$$

$$\therefore a\cancel{\sqrt{7}} = \frac{-4\cancel{\sqrt{7}}}{3} \Rightarrow a = \frac{-4}{3}$$

$$b = \frac{11}{3}$$

4. If $x = \sqrt{5}+2$, then find the value of $x^2 + \frac{1}{x^2}$.

Sol.
$$\text{If } x = \sqrt{5}+2$$
$$\frac{1}{x} = \frac{1}{\sqrt{5}+2} = \frac{1}{\sqrt{5}+2}\times\frac{\sqrt{5}-2}{\sqrt{5}-2} = \frac{\sqrt{5}-2}{\sqrt{5}^2-2^2} = \frac{\sqrt{5}-2}{5-4} = \frac{\sqrt{5}-2}{1}$$
$$x+\frac{1}{x} = \sqrt{5}+\cancel{2}+\sqrt{5}-\cancel{2} = 2\sqrt{5} \Rightarrow \left(x+\frac{1}{x}\right)^2 = (2\sqrt{5})^2 = 4\times 5 = 20$$

5. Given $\sqrt{2} = 1.414$, find the value of $\frac{8-5\sqrt{2}}{3-2\sqrt{2}}$ (to 3 places of decimals).

Sol.
$$\frac{8-5\sqrt{2}}{3-2\sqrt{2}} = \frac{(8-5\sqrt{2})\times(3+2\sqrt{2})}{(3-2\sqrt{2})\times(3+2\sqrt{2})} = \frac{24-15\sqrt{2}+16\sqrt{2}-10\times 2}{3^2-(2\sqrt{2})^2}$$
$$= \frac{24+\sqrt{2}-20}{9-4\times 2} = \frac{4+\sqrt{2}}{1} = 4+1.414 = 5.414$$

Exercise 2.8

1. Represent the following numbers in the scientific notation:

(i) 569430000000 **(ii) 2000.57**

(iii) 0.0000006000 **(iv) 0.0009000002**

Sol. (i) $569430000000 = 5.6943 \times 10^{11}$

(ii) $2000.57 = 2.00057 \times 10^3$

(iii) $0.0000006\cancel{000} = 6.0 \times 10^{-7}$

(iv) $0.0009000002 = 9.000002 \times 10^{-4}$

2. Write the following numbers in decimal form:

(i) 3.459×10^6 **(ii) 5.678×10^4**

(iii) 1.00005×10^{-5} **(iv) 2.530009×10^{-7}**

Sol. (i) $3.459 \times 10^6 = 3459000$

(ii) $5.678 \times 10^4 = 56780$

(iii) $1.00005 \times 10^{-5} = 0.0000100005$

(iv) $2.530009 \times 10^{-7} = 0.0000002530009$

3. Represent the following numbers in scientific notation:

(i) $(300000)^2 \times (20000)^4$ **(ii) $(0.000001)^{11} \div (0.005)^3$**

(iii) $\{(0.00003)^6 \times (0.00005)^4\} \div \{(0.009)^3 \times (0.05)^2\}$

Sol. (i) $(300000)^2 \times (20000)^4 = (3.0 \times 10^5)^2 \times (2.0 \times 10^4)^4$

$= 3^2 \times 10^{10} \times 2^4 \times 10^{16} = 9 \times 16 \times 10^{10+16}$

$= 144 \times 10^{26} = 1.44 \times 10^{28}$

(ii) $(0.000001)^{11} \div (0.005)^3$

$$\frac{(0.000001)^{11}}{(0.005)^3} = \frac{(1.0\times10^{-6})^{11}}{(5.0\times10^{-3})^3} = \frac{(1.0)^{11}\times10^{-66}}{(5.0)^3\times10^{-9}} = 1 \times 10^{-66+9} \times 5^3 = 125 \times 10^{-57}$$

$$= 1.25 \times 10^2 \times 10^{-57} = 1.25 \times 10^{(-57+2)} = 1.25 \times 10^{-55}$$

(iii) $\{(0.00003)^6 \times (0.00005)^4\} \div \{(0.009)^3 \times (0.05)^2\}$

$$= \frac{(3.0\times10^{-5})^6\times(5.0\times10^{-5})^4}{(9.0\times10^{-3})^3\times(5.0\times10^{-2})^2}$$

$$= \frac{3^6\times10^{-30}\times5^4\times10^{-20}}{9^3\times10^{-9}\times5^2\times10^{-4}} = \frac{3^6\times5^4\times10^{-30-20}}{(3^2)^3\times10^{-9-4}\times5^2} = \frac{\cancel{3^6}\times5^4\times10^{-50}}{\cancel{3^6}\times5^2\times10^{-13}}$$

$$= 5^{4-2}\times10^{-50+13}$$

$$= 5^2\times10^{-37} = 25\times10^{-37} = 2.5\times10^1\times10^{-37} = 2.5\times10^{-36}$$

4. Represent the following information in scientific notation:

(i) The world population is nearly 7000,000,000.

(ii) One light year means the distance 9460528400000000 km.

(iii) Mass of an electron is 0.000 000 000 000 000 000 000 000 000 00091093822 kg.

Sol. (i) The world population is nearly 7000,000,000 = 7.0×10^9

(ii) One light year means the distance 9460528400000000 km = 9.4605284×10^{15} km

(iii) Mass of an electron is
0.000 000 000 000 000 000 000 000 000 00091093822 kg. = $9.1093822 \times 10^{-31}$ kg

5. Simplfy :

(i) $(2.75 \times 10^7) + (1.23 \times 10^8)$ **(ii) $(1.598 \times 10^{17}) - (4.58 \times 10^{15})$**

(iii) $(1.02 \times 10^{10}) \times (1.20 \times 10^{-3})$ **(iv) $(8.41 \times 10^4) \div (4.3 \times 10^5)$**

Sol. (i)

$2.75 \times 10^7 = 27500000$

$1.23 \times 10^8 = 123000000$

$(2.75 \times 10^7) + (1.23 \times 10^8) =$

```
   27500000 +
  123000000
  ---------
  150500000   = 1.505 × 10^8
```

(ii) $(1.598 \times 10^{17}) - (4.58 \times 10^{15})$

$1.598 \times 10^{17} = 159800000000000000$

$4.58 \times 10^{15} = 4580000000000000$

$(1.598 \times 10^{17}) - (4.58 \times 10^{15})$

```
=       159800000000000000
    (–)   4580000000000000
    ----------------------
        155220000000000000  = 1.5522 × 10^17
```

(iii) $(1.02 \times 10^{10}) \times (1.20 \times 10^{-3}) = 1.02 \times 10^{10} \times 1.20 \times 10^{-3}$

$= 1.02 \times 1.2\cancel{0} \times 10^{10-3} = 1.224 \times 10^7$

(iv) $(8.41 \times 10^4) \div (4.3 \times 10^5) = \dfrac{8.41\times10^4}{4.3\times10^5} = \dfrac{841\times10^{-2}\times10^4}{43\times10^{-1}\times10^5}$

$$= \frac{841\times10^{-2+4}}{43\times10^{-1+5}} = \frac{841}{43}\times\frac{10^2}{10^4} = 19.5581395 \times 10^{2-4}$$

$= 1.95581395 \times 10^{1} \times 10^{-2}$

$= 1.95581395 \times 10^{-1} = 0.195581395$

Exercise 2.9

Multiple Choice Questions :

1. If n is a natural number then $\sqrt{n}$ is

(1) always a natural number (2) always an irrational number.

(3) always a rational number (4) may be rational or irrational

[Ans: (4) may be rational or irrational]

2. Which of the following is not true?

(1) Every rational number is a real number.

(2) Every integer is a rational number.

(3) Every real number is an irrational number.

(4) Every natural number is a whole number.

Hint : Real numbers contain rationals and irrationals.

[Ans: (3) Every real number is an irrational number]

3. Which one of the following, regarding sum of two irrational numbers, is true?

(1) always an irrational number (2) may be a rational or irrational number.

(3) always a rational number (4) always an integer.

[Ans: (2) may be a rational or irrational number]

4. Which one of the following has a terminating decimal expansion?

(1) $\frac{5}{64}$ (2) $\frac{8}{9}$ (3) $\frac{14}{15}$ (4) $\frac{1}{12}$

Hint : $\frac{5}{64} = \frac{5}{2^6}$ **[Ans: (1) $\frac{5}{64}$]**

5. Which one of the following is an irrational number?

(1) $\sqrt{25}$ (2) $\sqrt{\frac{9}{4}}$ (3) $\frac{7}{11}$ (4) π

Hint : π represents a irrational number **[Ans: (4) π]**

6. An irrational number between 2 and 2.5 is

(1) $\sqrt{11}$ (2) $\sqrt{5}$ (3) $\sqrt{2.5}$ (4) $\sqrt{8}$

Hint : $2^2 = 4$ and $2.5^2 = 6.25$ **[Ans: (2) $\sqrt{5}$]**

7. The smallest rational number by which $\frac{1}{3}$ should be multiplied so that its decimal expansion terminates after one place of decimal is

(1) $\frac{1}{10}$ (2) $\frac{3}{10}$ (3) 3 (4) 30

Hint : $\frac{3}{10}$ is the small number. **[Ans: (3) $\frac{3}{10}$]**

8. **If $\frac{1}{7} = 0.\overline{142857}$ then the value of $\frac{5}{7}$ is**

(1) $0.\overline{142857}$ (2) $0.\overline{714285}$ (3) $0.\overline{571428}$ (4) 0.714285

Hint : $5 \times \frac{1}{7} = 5 \times \overline{0.142857} = \overline{0.714285}$ **[Ans: (2) $0.\overline{714285}$]**

9. **Find the odd one out of the following.**

(1) $\sqrt{32} \times \sqrt{2}$ (2) $\frac{\sqrt{27}}{\sqrt{3}}$ (3) $\sqrt{72} \times \sqrt{8}$ (4) $\frac{\sqrt{54}}{\sqrt{18}}$

Hint : $\sqrt{72} \times \sqrt{8} = \sqrt{9 \times 8} \times \sqrt{8} = 3 \times 8 = 24$ **[Ans: (4) $\frac{\sqrt{54}}{\sqrt{84}}$]**

10. **$0.\overline{34} + 0.3\overline{4} =$**

(1) $0.6\overline{87}$ (2) $0.\overline{68}$ (3) $0.6\overline{8}$ (4) $0.68\overline{7}$

Hint : $0.343434 \ldots + 0.344444 \ldots = 0.6\overline{87}$ **[Ans: (1) $0.6\overline{87}$]**

11. **Which of the following statement is false?**

(1) The square root of 25 is 5 or –5 (2) $\sqrt{25} = 5$

(3) $-\sqrt{25} = -5$ (4) $\sqrt{25} = \pm 5$ **[Ans. (4) $\sqrt{25} = \pm 5$]**

12. **Which one of the following is not a rational number?**

(1) $\sqrt{\frac{8}{18}}$ (2) $\frac{7}{3}$ (3) $\sqrt{0.01}$ (4) $\sqrt{13}$

Hint : (1) $\sqrt{\frac{8}{18}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$ is a rational number (2) $\frac{7}{3}$ is a rational number

(3) $\sqrt{0.01} = \sqrt{\frac{1}{100}} = \frac{1}{10}$ is a rational number (4) $\sqrt{13}$ is not a rational number

[Ans. (4) $\sqrt{13}$]

13. **$\sqrt{27} + \sqrt{12} =$**

(1) $\sqrt{39}$ (2) $5\sqrt{6}$ (3) $5\sqrt{3}$ (4) $3\sqrt{5}$

Hint : $\sqrt{27} + \sqrt{12} = \sqrt{9 \times 3} + \sqrt{4 \times 3} = 3\sqrt{3} + 2\sqrt{3} = 5\sqrt{3}$ **[Ans. (3) $5\sqrt{3}$]**

14. **If $\sqrt{80} = k\sqrt{5}$, then $k =$**

(1) 2 (2) 4 (3) 8 (4) 16

Hint : $\sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5} = k\sqrt{5} \Rightarrow k = 4$ **[Ans. (2) 4]**

15. **$4\sqrt{7} \times 2\sqrt{3} =$**

(1) $6\sqrt{10}$ (2) $8\sqrt{21}$ (3) $8\sqrt{10}$ (4) $6\sqrt{21}$

Hint : $4\sqrt{7} \times 2\sqrt{3} = 8 \times \sqrt{7 \times 3} = 8\sqrt{21}$ **[Ans. (2) $8\sqrt{21}$]**

16. **When written with a rational denominator, the expression $\frac{2\sqrt{3}}{3\sqrt{2}}$ can be simplified as**

(1) $\frac{\sqrt{2}}{3}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{\sqrt{6}}{3}$ (4) $\frac{2}{3}$

Hint : $\frac{2\sqrt{3}}{3\sqrt{2}} = \frac{2\sqrt{3}}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{6}}{3 \times 2} = \frac{\cancel{2}\sqrt{6}}{\cancel{6}3}$ **[Ans. (3) $\frac{\sqrt{6}}{3}$]**

17. **When $(2\sqrt{5}-\sqrt{2})^2$ is simplified, we get**

(1) $4\sqrt{5}+2\sqrt{2}$ (2) $22-4\sqrt{10}$ (3) $8-4\sqrt{10}$ (4) $2\sqrt{10}-2$

Hint : $(2\sqrt{5}-\sqrt{2})^2 = (2\sqrt{5})^2 - 2\times 2\sqrt{5}\times\sqrt{2} + \sqrt{2}^2$
$= 4\times 5 - 4\sqrt{10} + 2 = 22-4\sqrt{10}$ **[Ans. (2) $22-4\sqrt{10}$]**

18. $(0.000729)^{\frac{-3}{4}} \times (0.09)^{\frac{-3}{4}} =$ ________.

(1) $\frac{10^3}{3^3}$ (2) $\frac{10^5}{3^5}$ (3) $\frac{10^2}{3^2}$ (4) $\frac{10^6}{3^6}$

Hint : $(0.000729)^{\frac{-3}{4}} \times (0.09)^{\frac{-3}{4}}$

$$= (7.29\times10^{-4})^{\frac{-3}{4}} \times (9\times10^{-2})^{\frac{-3}{4}} = (7.29)^{-\frac{3}{4}} \times 10^{-\cancel{4}\times\frac{-3}{\cancel{4}}} \times 9^{\frac{-3}{4}} \times 10^{-\cancel{2}\times\frac{3}{\cancel{4}_2}}$$

$$= (7.29)^{\frac{-3}{4}} \times 10^{+3} \times 9^{\frac{-3}{4}} \times 10^{\frac{3}{2}} = (729\times10^{-2})^{\frac{-3}{4}} \times 10^{3+\frac{3}{2}} \times 9^{\frac{-3}{4}}$$

$$= (9^3\times10^2)^{\frac{-3}{4}} \times 10^{\frac{9}{2}} \times 9^{\frac{-3}{4}} = (9^3)^{\frac{-3}{4}} \times 10^{\cancel{-2}\times\frac{-3}{\cancel{4}_2}} \times 10^{\frac{9}{2}} \times 9^{\frac{-3}{4}}$$

$$= 9^{\frac{-9}{4}} \times 10^{\frac{3}{2}+\frac{9}{2}} \times 9^{\frac{-3}{4}} = 9^{\frac{-9}{4}-\frac{3}{4}} \times 10^{\frac{\cancel{12}^{6}}{\cancel{2}}}$$

$$= 9^{\frac{-12}{4}} \times 10^6 = 9^{-3}\times10^6 = \frac{10^6}{9^3} = \frac{10^6}{(3^2)^3} = \frac{10^6}{3^6}$$

[Ans. (4) $\frac{10^6}{(3^6)}$]

19. **If $\sqrt{9^x} = \sqrt[3]{9^2}$, then $x =$ ________**

(1) $\frac{2}{3}$ (2) $\frac{4}{3}$ (3) $\frac{1}{3}$ (4) $\frac{5}{3}$

Hint : $(9^x)^{\frac{1}{2}} = (9^2)^{\frac{1}{3}} \Rightarrow 9^{\frac{x}{2}} = 9^{\frac{2}{3}} \Rightarrow \frac{x}{2} = \frac{2}{3} \Rightarrow 3x = 4$

$x = \frac{4}{3}$ **[Ans. (2) $\frac{4}{3}$]**

20. **The length and breadth of a rectangular plot are 5×10^5 and 4×10^4 metres respectively. Its area is ______.**

(1) $9\times10^1\ m^2$ (2) $9\times10^9\ m^2$ (3) $2\times10^{10}\ m^2$ (4) $20\times10^{20}\ m^2$

Hint : $l = 5\times10^5$ metres; $b = 4\times10^4$ metres

$\therefore$ Area $= l\times b = 5\times10^5\times4\times10^4 = 20\times10^{5+4} = 20\times10^9 = 2.0\times10^1\times10^9 = 2\times10^{10}\ m^2$

[Ans. (3) $2\times10^{10}\ m^2$]

Additional Questions and Answers

EXERCISE 2.1

1. Find only two rational numbers between $\frac{1}{4}$ and $\frac{3}{4}$.

Sol. A rational number between $\frac{1}{4}$ and $\frac{3}{4} = \frac{1}{2}\left(\frac{1}{4}+\frac{3}{4}\right) = \frac{1}{2}(1) = \frac{1}{2}$

Another rational number between $\frac{1}{2}$ and $\frac{3}{4} = \frac{1}{2}\left(\frac{1}{2}+\frac{3}{4}\right) = \frac{1}{2}\left(\frac{2+3}{4}\right) = \frac{1}{2} \times \frac{5}{4} = \frac{5}{8}$

The rational numbers $\frac{1}{2}$ and $\frac{5}{8}$ lies between $\frac{1}{4}$ and $\frac{3}{4}$.

2. Is zero a rational numbers? Give reasons for your answer.

Sol. Yes, since $\frac{0}{2} = 0$, (i.e) it can be written in the form $\frac{p}{q}$ where $q \neq 0$

EXERCISE 2.2

1. Express the following decimal expansion is the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(i) 0.75 (ii) 0.625 (iii) 0.5625 (iv) 0.28

Sol. (i) $0.75 = \frac{\cancel{75}^{3}}{\cancel{100}} = \frac{3}{4}$ (ii) $0.625 = \frac{\cancel{625}^{45}}{\cancel{1000}} = \frac{5}{8}$

(iii) $0.5625 = \frac{5625}{10000} = \frac{45}{80} = \frac{9}{16}$ (iv) $0.28 = \frac{\cancel{28}}{\cancel{100}_{25}} = \frac{7}{25}$

2. Convert $\overline{0.9}$ to a rational number.

Sol. (i) Let $x = 0.\overline{9}$. Then $x = 0.99999....$

Multiplying by 10 on both sides, we get

$$10x = 9.99999..... = 9 + 0.9999..... = 9 + x$$
$$9x = 9$$
$$x = 1. \text{ That is, } 0.\overline{9} = 1 \ (\because 1 \text{ is rational number}).$$

EXERCISE 2.3

1. Classify the following number as rational or irrational.

(i) $\sqrt{11}$ (ii) $\sqrt{81}$ (iii) 0.0625 (iv) $0.8\overline{3}$

Sol. (i) $\sqrt{11}$ is an irrational number. (11 is not a perfect square number)

(ii) $\sqrt{81} = 9 = \frac{9}{1}$, a rational number.

(iii) 0.0625 is a terminating decimal

$\therefore$ 0.0625 is a rational number.

(iv) $0.8\overline{3} = 0.8333......$
The decimal expansion is non-terminating and recurring.
$\therefore$ $0.8\overline{3}$ is a rational number.

2. Find the decimal expansion of $\sqrt{3}$.

Sol.

	1.7320508…
1	3.00,00,00,00,00,…
	1
27	200
	189
343	1100
	1029
3462	7100
	6924
346405	1760000
	1732025
34641008	279750000
	277128064
	2621936

3. Find any 4 irrational numbers between $\frac{1}{4}$ and $\frac{1}{3}$.

Sol. $\frac{1}{4} = 0.25$ and $\frac{1}{3} = 0.3333….. = 0.\overline{3}$

In between 0.25 and $0.\overline{3}$, there are infinitely many irrational numbers .

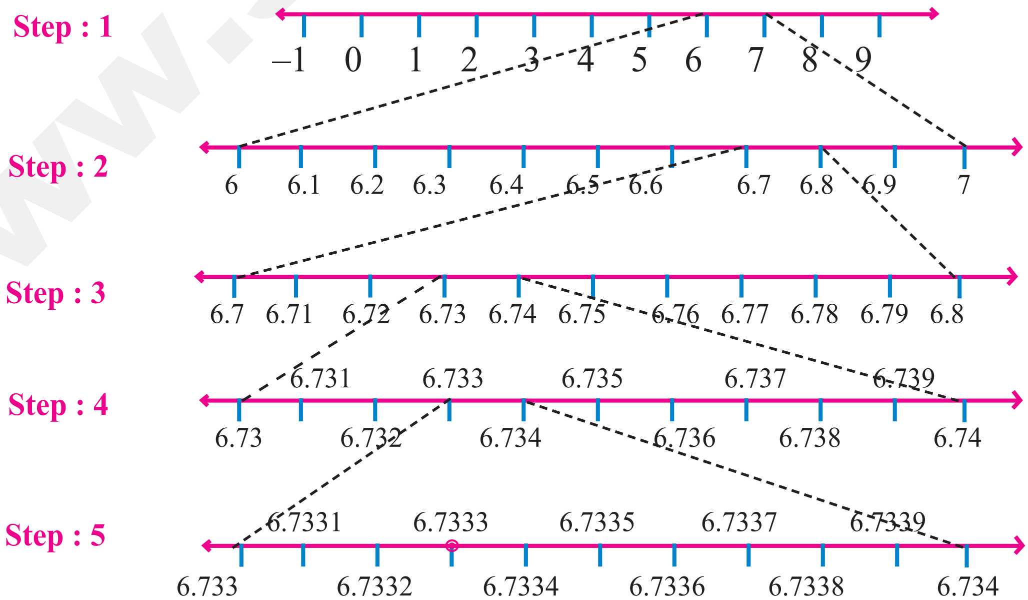
Four irrational numbers between 0.25 and $0.\overline{3}$ are

0.260100100010000l…..
0.270100100010000l…..
0.280100100010000l…..
0.310100100010000l…..

EXERCISE 2.4

1. Visualise $6.7\overline{3}$ on the number line, upto 4 decimal places.

Sol. We locate 6.73 on the number line, by the process of successive magnification.



Step 1 : First we note that $6.7\overline{3}$ lies between 6 and 7.

Step 2 : Divide the portion between 6 and 7 into 10 equal parts and use a magnifying glass to visualise that $6.7\overline{3}$ lies between 6.7 and 6.8.

Step 3 : Divide the portion between 6.7 and 6.8 into 10 equal parts and use a magnifying glass to visualise that $6.7\overline{3}$ lies between 6.73 and 6.74.

Step 4 : Divide the portion between 6.73 and 6.74 into 10 equal parts and use a magnifying glass to visualise that $6.7\overline{3}$ lies between 6.733 and 6.734.

Step 5 : Divide the portion between 6.733 and 6.734 into 10 equal parts and use a magnifying glass to visualise that $6.7\overline{3}$ lies between 6.7332 and 6.7334.

We note that $6.7\overline{3}$ is visualised closed to 6.7332 than to 6.7334.

2. Find whether x and y are rational or irrational in the following:

(i) $a = 2 + \sqrt{3}, b = 2 - \sqrt{3}; x = a + b, y = a - b$

(ii) $a = \sqrt{2} + 7, \ b = \sqrt{2} - 7; \ x = a + b, y = a - b$

Sol. (i) Given that $a = 2 + \sqrt{3}, b = 2 - \sqrt{3}$

$x = a + b = (2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$, a rational number.

$y = a - b = (2 + \sqrt{3}) - (2 - \sqrt{3}) = 2\sqrt{3}$, an irrational number.

(ii) Given that $a = \sqrt{2} + 7, b = \sqrt{2} - 7$

$x = a + b = (\sqrt{2} + 7) + (\sqrt{2} - 7) = 2\sqrt{2}$, an irrational number.

$y = a - b = (\sqrt{2} + 7) - (\sqrt{2} - 7) = 14$, a rational number.

EXERCISE 2.5

1. Evaluate : (i) 10^{-4} (ii) $\left(\frac{1}{9}\right)^{-3}$ (iii) $(0.01)^{-2}$

Sol. (i) $$10^{-4} = \frac{1}{10^4} = \frac{1}{10000} = 0.0001$$

(ii) $$\left(\frac{1}{9}\right)^{-3} = \frac{1}{\left(\frac{1}{9}\right)^3} = \frac{1}{\left(\frac{1}{729}\right)} 729$$

(iii) $$(0.01)^{-2} = \left(\frac{1}{100}\right)^2 = \frac{1}{\left(\frac{1}{100}\right)^{-2}} = \frac{1}{\frac{1}{10000}} = 10000$$

2. Evaluate : (i) 10^{-4} (ii) $\left(\frac{1}{9}\right)^{-3}$ (iii) $(0.01)^{-2}$

Sol. (i) $$10^{-4} = \frac{1}{10^4} = \frac{1}{10000} = 0.0001$$

(ii) $\left(\frac{1}{9}\right)^{-3} = \frac{1}{\left(\frac{1}{9}\right)^{3}} = \frac{1}{\left(\frac{1}{729}\right)}729$

(iii) $(0.01)^{-2} = \left(\frac{1}{100}\right)^{2} = \frac{1}{\left(\frac{1}{100}\right)^{-2}} = \frac{1}{\frac{1}{10000}} = 10000$

3. Find the value of $625^{\frac{3}{4}}$:

Sol. $625^{\frac{3}{4}} = \left(\sqrt[4]{625}\right)^{3} = \left(\sqrt[4]{5^4}\right)^{3} = 5^3 = 5 \times 5 \times 5 = 125$

4. Find the value of $729^{\frac{-5}{6}}$:

Sol. $729^{\frac{-5}{6}} = \frac{1}{729^{\frac{5}{6}}} = \frac{1}{\left(\sqrt[6]{729}\right)^{5}} = \frac{1}{\left(\sqrt[6]{3^6}\right)^{5}} = \frac{1}{3^5} = \frac{1}{243}$

5. Use a fractional index to write :

(i) $\left(5\sqrt{125}\right)^{7}$ **(ii)** $\sqrt[3]{7}$

Sol. (i) $\left(5\sqrt{125}\right)^{7} = 125^{\frac{7}{5}}$

(ii) $\sqrt[3]{7} = 7^{\frac{1}{3}}$

EXERCISE 2.6

1. Can you reduce the following numbers to surds of same order.

(i) $\sqrt{5}$ **(ii)** $\sqrt[3]{5}$ **(iii)** $\sqrt[4]{5}$

Sol. (i) $\sqrt{5} = 5^{\frac{1}{2}} = 5^{\frac{6}{12}} = \sqrt[12]{5^6} = \sqrt[12]{15625}$

(ii) $\sqrt[3]{5} = 5^{\frac{1}{3}} = 5^{\frac{4}{12}} = \sqrt[12]{5^4} = \sqrt[12]{625}$

(iii) $\sqrt[4]{5} = 5^{\frac{1}{4}} = 5^{\frac{3}{12}} = \sqrt[12]{5^3} = \sqrt[12]{125}$

Now the surds have same order

2. Express the surds in the simplest form

(i) $\sqrt{27}$ **(ii)** $\sqrt[3]{128}$

Sol. (i) $\sqrt{27} = \sqrt{3 \times 3 \times 3} = 3\sqrt{3}$

(ii) $\sqrt[3]{128} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} = 4\sqrt[3]{2}$

3. **Show that $\sqrt[3]{2} > \sqrt[5]{3}$.**

Sol. $\sqrt[3]{2} = \sqrt[15]{2^5} = \sqrt[15]{32} \Rightarrow \sqrt[5]{3} = \sqrt[15]{3^3} = \sqrt[15]{27}$

$\sqrt[15]{32} > \sqrt[15]{27}$

Therefore, $\sqrt[3]{2} > \sqrt[5]{3}$

4. **Express the following surds in its simplest form $\sqrt[4]{324}$.**

Sol. $\sqrt[4]{324} = \sqrt[4]{81\times 4} = \sqrt[4]{3^4\times 4} = \sqrt[4]{3^4}\times\sqrt[4]{4}$ $\left[\because \sqrt[n]{a^n}\times\sqrt[n]{b} = \sqrt[n]{ab}\right]$

$= 3\times\sqrt[4]{4}$ $\left[\because \sqrt[n]{a^n} = a\right]$

order = 4 ; radicand = 4; Coefficient = 3

5. **Simplify $\sqrt{63} - \sqrt{175} + \sqrt{28}$**

Sol. $\sqrt{63} - \sqrt{175} + \sqrt{28} = \sqrt{9\times 7} - \sqrt{25\times 7} + \sqrt{4\times 7} = 3\sqrt{7} - 5\sqrt{7} + 2\sqrt{7}$

$= (3\sqrt{7} + 2\sqrt{7}) - 5\sqrt{7} = 5\sqrt{7} - 5\sqrt{7} = 0$

6. **Arrange in ascending order : $\sqrt[3]{2}, \sqrt[2]{4}, \sqrt[4]{3}$**

Sol. The order of the surds $\sqrt[3]{2}, \sqrt[2]{4}$ and $\sqrt[4]{3}$ are 3, 2, 4

L.C.M. of 3, 2, 4 = 12.

$\sqrt[3]{2} = \left(2^{\frac{1}{3}}\right) = \left(2^{\frac{4}{12}}\right) = \sqrt[12]{2^4} = \sqrt[12]{16} \Rightarrow \sqrt[2]{4} = \left(4^{\frac{1}{2}}\right) = \left(4^{\frac{6}{12}}\right) = \sqrt[12]{4^6} = \sqrt[12]{4096}$

$\sqrt[4]{3} = \left(3^{\frac{1}{4}}\right) = \left(3^{\frac{3}{12}}\right) = \sqrt[12]{3^3} = \sqrt[12]{27}$

The ascending order of the surds $\sqrt[3]{2}, \sqrt[4]{3}, \sqrt[2]{4}$ is $\sqrt[12]{16} < \sqrt[12]{27} < \sqrt[12]{4096}$ that is $\sqrt[3]{2}, \sqrt[4]{3}, \sqrt[2]{4}$.

EXERCISE 2.7

1. **Subtract $6\sqrt{7}$ from $9\sqrt{7}$. Is the answer rational or irrational?**

Sol. $9\sqrt{7} - 6\sqrt{7} = (9-6)\sqrt{7} = 3\sqrt{7}$

The answer is irrational.

2. **Simplify : $\sqrt{44} + \sqrt{99} - \sqrt{275}$.**

Sol. $\sqrt{44} + \sqrt{99} - \sqrt{275} = \sqrt{4\times 11} + \sqrt{9\times 11} - \sqrt{11\times 25}$

$= (2\sqrt{11} + 3\sqrt{11}) - 5\sqrt{11} = 5\sqrt{11} - 5\sqrt{11} = 0$

3. **Compute and give the answer in the simplest form : $3\sqrt{162}\times 7\sqrt{50}\times 6\sqrt{98}$**

Sol. $3\sqrt{162}\times 7\sqrt{50}\times 6\sqrt{98} = (3\times 9\sqrt{2}\times 7\times 5\sqrt{2}\times 6\times 7\sqrt{2})$

$= 3\times 7\times 6\times 9\times 5\times 7\times\sqrt{2}\times\sqrt{2}\times\sqrt{2} = 79380\sqrt{2}$

EXERCISE 2.8

1. Write in scientific notation : $(60000000)^3$

Sol.
$$(60000000)^3 = (6.0 \times 10^7)^4 = (6.0)^4 \times (10^7)^4$$
$$= 1296 \times 10^{28} = 1.296 \times 10^3 \times 10^{28} = 1.296 \times 10^{31}$$

2. Write in scientific notation : $(0.00000004)^3$

Sol.
$$(0.00000004)^3 = (4.0 \times 10^{-8})^3 = (4.0)^3 \times (10^{-8})^3$$
$$= 64 \times 10^{-24} = 6.4 \times 10 \times 10^{-24} = 6.4 \times 10^{-23}$$

3. Write in scientific notation : $(500000)^5 \times (3000)^3$

Sol.
$$(500000)^5 \times (3000)^3 = (5.0 \times 10^5)^3 \times (3.0 \times 10^3)^3$$
$$= (5.0)^2 \times (10^5)^2 \times (3.0)^3 \times (10^3)^3$$
$$= 25 \times 10^{10} \times 27 \times 10^9 = 675 \times 10^{19}$$
$$= 675.0 \times 10^{19} = 6.75 \times 10^2 \times 10^{19} = 6.75 \times 10^{21}$$

4. Write in scientific notation : $(6000000)^3 \div (0.00003)^2$

Sol.
$$(6000000)^3 \div (0.00003)^2 = (6.0 \times 10^6)^3 \div (3.0 \times 10^{-5})^2$$
$$= (6.0 \times 10^6)^3 \div (3.0 \times 10^{-5})^2 = 216 \times 10^{18} \div 9 \times 10^{-10}$$
$$= \frac{216 \times 10^9}{9 \times 10^{-10}} = 24 \times 10^{18} \times 10^{10} = 24 \times 10^{28}$$
$$= 24.0 \times 10^{28} = 2.4 \times 10 \times 10^{28} = 2.4 \times 10^{29}$$

EXERCISE 2.9

MULTIPLE CHOICE QUESTIONS :

1. A number having non-terminating and recurring decimal expansion is

(1) an integer (2) a rational number
(3) an irrational number (4) a whole number

Hint : Irrational number have nonterminating and non recurring decimal expansion.

[Ans: (2) a rational number]

2. If a number has a non-terminating and non-recurring decimal expansion, then it is

(1) a rational number (2) a natural number
(3) an irrational number (4) an integer

Hint : Rational number gave terminating or recurring and non-terminating decimal expansion. **[Ans: (3) an irrational number]**

3. Decimal form of $\frac{-3}{4}$ is

(1) – 0.75 (2) – 0.50 (3) – 0.25 (4) – 0.125

Hint : $\frac{1}{4} = 0.25$; $\frac{1}{2} = 0.5$; $\frac{3}{4} = 0.75$ **[Ans: (1) – 0.75]**

4. Which one of the following has a terminating decimal expansion?

(1) $\frac{5}{32}$ (2) $\frac{7}{9}$ (3) $\frac{8}{15}$ (4) $\frac{1}{12}$

Hint : $32 = 2^5 \Rightarrow \frac{5}{32}$ has terminating decimal expansion **[Ans: (1) $\frac{5}{32}$]**

5. Which one of the following is an irrational number?

(1) π (2) $\sqrt{9}$ (3) $\frac{1}{4}$ (4) $\frac{1}{5}$ **[Ans: (1) π]**

6. Which one of the following are irrational numbers?

(i) $\sqrt{2+\sqrt{3}}$ (ii) $\sqrt{4+\sqrt{25}}$ (iii) $\sqrt[3]{5+\sqrt{7}}$ (iv) $\sqrt{8-\sqrt[3]{8}}$

(a) (ii), (iii) and (iv) (b) (i), (ii) and (iv)

(c) (i), (ii) and (iii) (d) (i), (iii) and (iv)

Hint : $\sqrt{4+\sqrt{25}} = \sqrt{9} = 3; \sqrt{8-\sqrt[3]{8}} = \sqrt{8-2} = \sqrt{6}$ **[Ans: (4) (i), (iii) and (iv)]**

7. Which of the following is not an irrational number?

(1) $\sqrt{2}$ (2) $\sqrt{5}$ (3) $\sqrt{3}$ (4) $\sqrt{25}$ **[Ans. (4) $\sqrt{25}$]**

8. In simple form, $\sqrt[3]{54}$ is?

(1) $3\sqrt[3]{2}$ (2) $\sqrt[3]{27}$ (3) $3\sqrt{2}$ (4) $\sqrt{3}$ **[Ans. (1) $3\sqrt[3]{2}$]**

9. $\sqrt[3]{192}+\sqrt[3]{24}$

(1) $3\sqrt[3]{6}$ (2) $6\sqrt[3]{3}$ (3) $\sqrt[3]{216}$ (4) $\sqrt[6]{216}$ **[Ans. (2) $6\sqrt[3]{3}$]**

10. $5\sqrt{21}\times 6\sqrt{10}$

(1) $30\sqrt{210}$ (2) 30 (3) $\sqrt{210}$ (4) $210\sqrt{30}$

[Ans. (1) $30\sqrt{210}$]

Text Book Activities

Activity - 1

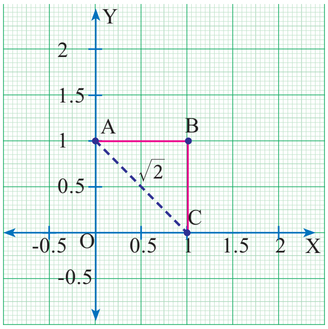
Is it interesting to see this pattern? $\sqrt{4\frac{4}{15}} = 4\sqrt{\frac{4}{15}}$ and $\sqrt{5\frac{5}{24}} = 5\sqrt{\frac{5}{24}}$ Verify it. Can you frame 4 such new surds?

Sol. (i) $\sqrt{6\frac{6}{35}} = 6\sqrt{\frac{6}{35}}$ (ii) $\sqrt{7\frac{7}{48}} = 7\sqrt{\frac{7}{48}}$ (iii) $\sqrt{8\frac{8}{63}} = 8\sqrt{\frac{8}{63}}$

(iv) $\sqrt{9\frac{9}{80}} = 9\sqrt{\frac{9}{80}}$ (v) $\sqrt{10\frac{10}{99}} = 10\sqrt{\frac{10}{99}}$

Activity - 2

Take a graph sheet and mark O, A, B, C as follows:



In the square OABC,

OA = AB = BC = OC = 1 unit

Consider right angled ΔOAC

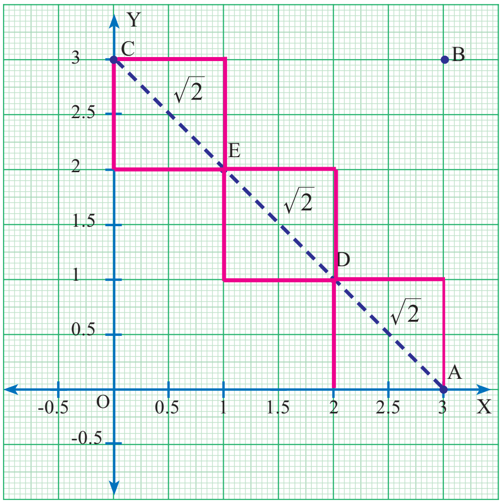
$$AC = \sqrt{1^2+1^2}$$

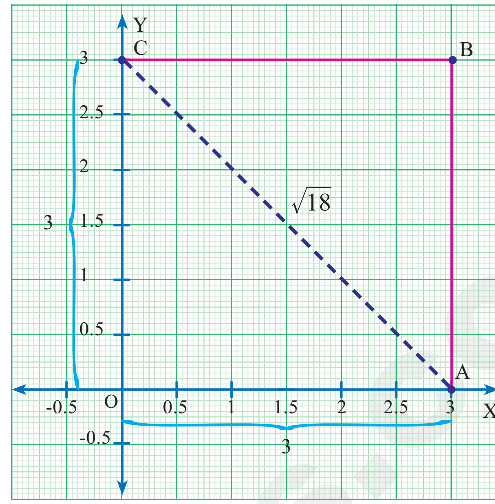
$= \sqrt{2}$ unit [By Pythagoras theorem]

The length of the diagonal (hypotenuse)

AC $= \sqrt{2}$ which is a surd.

Consider the following graphs:





Let us try to find the length of AC in two different ways :

$AC = AD + DE + EC$

(diagonals of units squares)

$= \sqrt{2} + \sqrt{2} + \sqrt{2}$

$AC = 3\sqrt{2}$ units

$AC = \sqrt{OA^2 + OC^2} = \sqrt{3^2 + 3^2}$

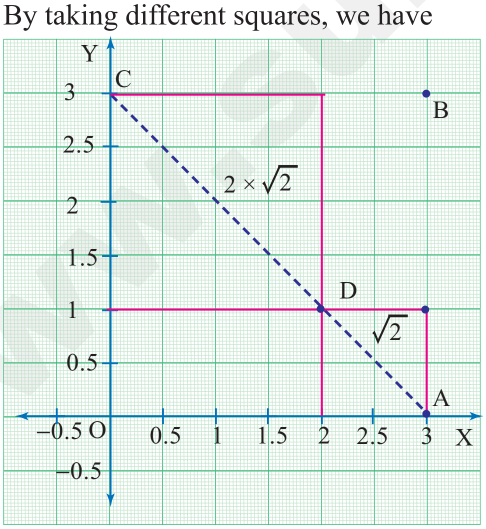
$= \sqrt{9+9}$

$AC = \sqrt{18}$ units

Are they equal? Discuss. Can you verify the same by taking different squares of different lengths?

Sol. $\sqrt{18} = \sqrt{3 \times 3 \times 2} = 3\sqrt{2}$ Hence they are equal.

By taking different squares, we have



$AC = CD + AD$

(diagonals of squares of lengths 2 and 1)

$= 2\sqrt{2} + \sqrt{2}$ Hence verified.

Activity - 3

The following list shows the mean distance of the planets of the solar system from the Sun. Complete the following table. Then arrange in order of magnitude starting with the distance of the planet closest to the Sun.

Sol.

Planet	Decimal form (in Km)	Scientific Notation (in Km)
Jupiter	778000000	7.78×10^8
Mercury	58000000	5.8×10^7
Mars	228000000	2.28×10^8
Uranus	2870000000	2.87×10^9
Venus	108000000	1.08×10^8
Neptune	4500000000	4.5×10^9
Earth	150000000	1.5×10^8
Saturn	143000000	1.43×10^8

Arranging the planets in order of distance from the sun.

Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune and Pluto.

UNIT TEST

Time : 45 Minutes CHAPTER - 2 Marks: 25

Section - A

(i) Answer all the questions.
(ii) Choose the correct Answer. 5 × 1 = 5

1. Which one of the following, regarding sum of two irrational numbers, is true?
   (1) always an irrational number
   (2) may be a rational or irrational number.
   (3) always a rational number
   (4) always an integer.

2. Which one of the following is an irrational number?
   (1) $\sqrt{25}$ (2) $\sqrt{\frac{9}{4}}$ (3) $\frac{7}{11}$ (4) π

3. If $\frac{1}{7} = 0.\overline{142857}$ then the value of $\frac{5}{7}$ is
   (1) $0.\overline{142857}$ (2) $0.\overline{714285}$
   (3) $0.\overline{571428}$ (4) 0.714285

4. When written with a rational denominator, the expression $\frac{2\sqrt{3}}{3\sqrt{2}}$ can be simplified as
   (1) $\frac{\sqrt{2}}{3}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{\sqrt{6}}{3}$ (4) $\frac{2}{3}$

5. If $\sqrt{9^x} = \sqrt[3]{9^2}$, then $x =$ ________
   (1) $\frac{2}{3}$ (2) $\frac{4}{3}$ (3) $\frac{1}{3}$ (4) $\frac{5}{3}$

Section - B

(i) Answer only five of the following
(ii) However Question number 12 is compulsory. 5 × 2 = 10

6. Simplify using addition and subtraction properties of surds $5\sqrt{3}+18\sqrt{3}-2\sqrt{3}$

7. Simplify using multiplication and division properties of surds $\sqrt{3}\times\sqrt{5}\times\sqrt{2}$

8. Find any two irrational number between $\frac{6}{7}$ and $\frac{12}{13}$.

9. Find the value of
   (i) $(49)^{\frac{1}{2}}$ (ii) $(243)^{\frac{2}{5}}$

10. Without actual division, $\frac{7}{128}$ find rational numbers have terminating decimal expansion.

11. Which arrow best shows the position of $\frac{11}{3}$ on the number line?

12. Convert $2.\overline{327}$ decimal expression into rational numbers.

Section - C

(i) Answer only two Questions of the following.
(ii) However Question number 16 is compulsory. 2 × 5 = 10

13. Find any five rational numbers between
    (i) $\frac{1}{4}$ and $\frac{1}{5}$ (ii) 0.1 and 0.11

14. Represent $\sqrt{3}$ irrational numbers on the number line.

15. Represent the following numbers on the number line.
    $4.\overline{73}$ upto 4 decimal places.

16. Arrange surds in descending order :
    (i) $\sqrt[3]{5}, \sqrt[9]{4}, \sqrt[6]{3}$
    (ii) $\sqrt[2]{\sqrt[3]{5}}, \sqrt[3]{\sqrt[4]{7}}, \sqrt{\sqrt{3}}$

ANSWERS

SECTION - A

1. (2) May be a rational or irrational number
2. (1) π
3. (2) $0.\overline{714285}$
4. (3) $\frac{\sqrt{6}}{3}$
5. (2) $\frac{4}{3}$

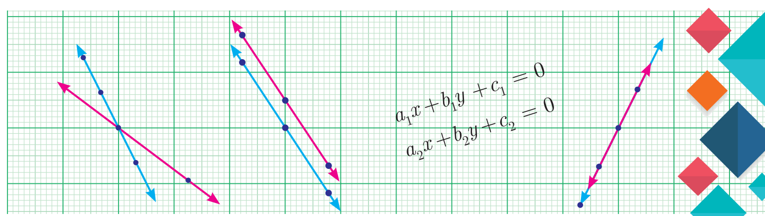
SECTION - B

6. $21\sqrt{3}$
7. $\frac{43}{50}$
8. 0.867142..., 0.8771242...
9. (i) 7; (ii) 9
10. $\frac{7}{2^7 5^0}$
11. D
12. $\frac{2325}{999}$

SECTION - C

13. (i) $\frac{9}{40}, \frac{19}{80}, \frac{39}{160}, \frac{79}{320}, \frac{159}{640}$

    (ii) 0.101, 0.102, 0.103, ..., 0.109
14. ***Refer Sura's Guide Ex. 2.3, Q.No.1 (i)***
15. ***Refer Sura's Guide Ex.2.4, Q.No.1 (iii)***
16. ***Refer Sura's Guide Ex. 2.6, Q.No.4 (i) & (ii)***



3 ALGEBRA

3.1 Introduction

Algebra, a branch of mathematics consists of proving the most obvious thing in the least obvious way. Algebra is a generalized form of Arithmetic. In Arithmetic we use numbers which have one single definite value while in algebra, we use letters or alphabets which may have any value, we assign to them. These letters are called literal numbers or variables.

This technique called algebra enable many difficult problems in the real world to be analysed and solved using the same methods like we do for any other branch of mathematics.

Constants	:	Any real number is a constant.
Variables	:	Letters used for representing unknown real numbers are called variables.
Co-efficients	:	The constant part of a term that is multiplied by the variable part of the term is called co-efficient.
Standard form of a polynomial	:	A polynomial $f(x)$ in the increasing or decreasing order of the power of x. This way of writing a polynomial is called the standard form of a polynomial.
Degree of the polynomial	:	In case of a polynomial in more than one variable, the sum of the powers of the variables in each term is considered and the highest sum so obtained is called the degree of the polynomial.
A very special polynomial	:	It is zero polynomial having all its Co-efficients to be zero.

3.2 Types of polynomials :

(i) Polynomial on the basis of number of terms

(1) Monomial – A polynomial having only one term.

(2) Binomial – A polynomial having two terms.

(3) Trinomial – A polynomial having three terms.

(ii) Polynomial based on degree.

(1) Constant – A polynomial of degree zero is called constant polynomial.

(2) Linear – A polynomial of degree one is called linear polynomial.

(3) Quadratic – A polynomial of degree two is called quadratic polynomial.

(4) Cubic – A polynomial of degree three is called cubic polynomial.

3.2.1 Arithmetic of Polynomials :

We can add polynomials, subtract one from another, multiply polynomials, divide one by another.

3.3.1 Addition of Polynomials :

Only like terms can be added. But addition of unlike terms gives a new polynomial.

3.3.2 Subtraction of Polynomials :

Only like terms can be subtracted. But subtraction of unlike terms gives a new polynomial.

3.3.3 Multiplication of Polynomials :

Any two polynomials can be multiplied and the product will also be a polynomial.

Exercise 3.1

1. Which of the following expressions are polynomials. If not give reason:

(i) $\frac{1}{x^2}+3x-4$ (ii) $x^2(x-1)$ (iii) $\frac{1}{x}(x+5)$ (iv) $\frac{1}{x^{-2}}+\frac{1}{x^{-1}}+7$

(v) $\sqrt{5}x^2+\sqrt{3}x+\sqrt{2}$ (vi) $m^2-\sqrt[3]{m}+7m-10$

Sol.

	Given Polynomial	Polynomial / Not	Reason
(i)	$\frac{1}{x^2}+3x-4$ $=x^{-2}+3x-4$	Not a polynomial	Negative integral power
(ii)	$x^2(x-1)$	Polynomial	Non-negative integral power
(iii)	$\frac{1}{x}(x+5)$ $=x^{-1}(x^1+5)=x^{-1+1}+5x^{-1}$ $=x^0+5x^{-1}$	Not a polynomial	One of the power of x is negative
(iv)	$\frac{1}{x^{-2}}+\frac{1}{x^{-1}}+7=x^2+x^1+7$	Polynomial	Non-negative integral power
(v)	$\sqrt{5}x^2+\sqrt{3}x+\sqrt{2}$	Polynomial	Non-negative integral power
(vi)	$m^2-\sqrt[3]{m}+7m-10$ $m^2-m^{\frac{3}{2}}+7m-10$	Not a polynomial	One of the power of m is a fraction $\frac{1}{3}$

2. **Write the coefficient of x^2 and x in each of the following polynomials.**

(i) $4+\frac{2}{5}x^2-3x$ (ii) $6-2x^2+3x^3-\sqrt{7}x$ (iii) πx^2-x+2

(iv) $\sqrt{3}x^2+\sqrt{2}x+0.5$ (v) $x^2-\frac{7}{2}x+8$

Sol.

	Polynomial	Co-efficient of x^2	Co-efficient of x
(i)	$4+\frac{2}{5}x^2-3x$	$\frac{2}{5}$	-3
(ii)	$6-2x^2+3x^3-\sqrt{7}x$	-2	$-\sqrt{7}$
(iii)	πx^2-x+2	π	-1
(iv)	$\sqrt{3}x^2+\sqrt{2}x+0.5$	$\sqrt{3}$	$\sqrt{2}$
(v)	$x^2-\frac{7}{2}x+8$	1	$-\frac{7}{2}$

3. **Find the degree of the following polynomials.**

(i) $1-\sqrt{2}y^2+y^7$ (ii) $\frac{x^3-x^4+6x^6}{x^2}$ (iii) $x^3(x^2+x)$

(iv) $3x^4+9x^2+27x^6$ (v) $2\sqrt{5}p^4-\frac{8p^3}{\sqrt{3}}+\frac{2p^2}{7}$

Sol.

	Polynomial	Degree
(i)	$1-\sqrt{2}y^2+y^7$	7
(ii)	$\frac{x^3-x^4+6x^6}{x^2}$ $=\frac{x^{\cancel{3}1}}{\cancel{x^2}}-\frac{x^{\cancel{4}2}}{\cancel{x^2}}+\frac{6x^{\cancel{6}4}}{\cancel{x^2}}$ $=x-x^2+6x^4$	4
(iii)	$x^3(x^2+x)=x^5+x^4$	5
(iv)	$3x^4+9x^2+27x^6$	6
(v)	$2\sqrt{5}p^4-\frac{8p^3}{\sqrt{3}}+\frac{2p^2}{7}$	4

4. **Rewrite the following polynomial in standard form.**

(i) $x-9+\sqrt{7}x^3+6x^2$ (ii) $\sqrt{2}x^2-\frac{7}{2}x^4+x-5x^3$ (iii) $7x^3-\frac{6}{5}x^2+4x-1$

(iv) $y^2+\sqrt{5}y^3-11-\frac{7}{3}y+9y^4$

Sol.

	Polynomial	Standard form
(i)	$x-9+\sqrt{7}x^3+6x^2$	$\sqrt{7}x^3+6x^2+x-9$
(ii)	$\sqrt{2}x^2-\frac{7}{2}x^4+x-5x^3$	$\frac{-7}{2}x^4-5x^3+\sqrt{2}x^2+x$
(iii)	$7x^3-\frac{6}{5}x^2+4x-1$	$7x^3-\frac{6}{5}x^2+4x-1$
(iv)	$y^2+\sqrt{5}y^3-11-\frac{7}{3}y+9y^4$	$9y^4+\sqrt{5}y^3+y^2-\frac{7}{3}y-11$

5. **Add the following polynomials and find the degree of the resultant polynomial.**

(i) $p(x) = 6x^2-7x+2$ $q(x) = 6x^3-7x+15$

(ii) $h(x) = 7x^3-6x+1$ $f(x) = 7x^2+17x-9$

(iii) $f(x) = 16x^4-5x^2+9$ $g(x) = -6x^3+7x-15$

Sol.

	Polynomials	Addition	Degree of the resultant polynomial
(i)	$p(x) = 6x^2-7x+2$ $q(x) = 6x^3-7x+15$	$6x^2-7x+2$ $6x^3+0x^2-7x+15$ $6x^3+6x^2-14x+17$	3
(ii)	$h(x) = 7x^3-6x+1$ $f(x) = 7x^2+17x-9$	$7x^3+0x^2-6x+1$ $7x^2+17x-9$ $7x^3+7x^2+11x-8$	3
(iii)	$f(x) = 16x^4-5x^2+9$ $g(x) = -6x^3+7x-15$	$16x^4+0x^3-5x^2+0x+9$ $-6x^3+0x^2+7x-15$ $16x^4-6x^3-5x^2+7x-6$	4

6. **Subtract the second polynomial from the first polynomial and find the degree of the resultant polynomial.**

(i) $p(x) = 7x^2 + 6x - 1$ $\quad q(x) = 6x - 9$

(ii) $f(y) = 6y^2 - 7y + 2$ $\quad g(y) = 7y + y^3$

(iii) $h(z) = z^5 - 6z^4 + z$ $\quad f(z) = 6z^2 + 10z - 7$

Sol.

	Polynomial	Subtraction	Degree of the resultant polynomial
(i)	$p(x) = 7x^2 + 6x - 1$ $q(x) = 6x - 9$	$7x^2 + 6x - 1$ $6x - 9$ $(-)\ (+)$ $7x^2 + 0x + 8$	2
(ii)	$f(y) = 6y^2 - 7y + 2$ $g(y) = 7y + y^3$	$6y^2 - 7y + 2$ $\underset{(-)}{y^3} \underset{(-)}{+ 0y^2} \underset{(-)}{+ 7y} \underset{(-)}{+ 0}$ $-y^3 + 6y^2 - 14y + 2$	3
(iii)	$h(z) = z^5 - 6z^4 + z$ $f(z) = 6z^2 + 10z - 7$	$h(z) - f(z) \Rightarrow z^5 - 6z^4 + 0z^3 + 0z^2 + z$ $+ 6z^2 + 10z - 7$ $(-)\ (-)\ (+)$ $z^5 - 6z^4 + 0z^3 - 6z^2 - 9z + 7$ The degree of resultant polynomial is 5.	5

7. **What should be added to $2x^3 + 6x^2 - 5x + 8$ to get $3x^3 - 2x^2 + 6x + 15$?**

Sol. $(2x^3 + 6x^2 - 5x + 8) + Q(x) = 3x^3 - 2x^2 + 6x + 15$

$\therefore Q(x) = (3x^3 - 2x^2 + 6x + 15) - (2x^3 + 6x^2 - 5x + 8)$

$$\begin{array}{r} 3x^3 - 2x^2 + 6x + 15 \\ \underset{(-)}{2x^3} \underset{(-)}{+ 6x^2} \underset{(+)}{- 5x} \underset{(-)}{+ 8} \\ \hline x^3 - 8x^2 + 11x + 7 \\ \hline \end{array}$$

The required polynomial is $x^3 - 8x^2 + 11x + 7$

8. **What must be subtracted from $2x^4 + 4x^2 - 3x + 7$ to get $3x^3 - x^2 + 2x + 1$?**

Sol.
$$\begin{aligned} (2x^4 + 4x^2 - 3x + 7) - Q(x) &= 3x^3 - x^2 + 2x + 1 \\ Q(x) &= (2x^4 + 4x^2 - 3x + 7) - 3x^3 - x^2 + 2x + 1 \\ \text{The required polynomial} &= 2x^4 + 4x^2 - 3x + 7 - 3x^3 + x^2 - 2x - 1 \\ &= 2x^4 - 3x^3 + 5x^2 - 5x + 6 \end{aligned}$$

9. Multiply the following polynomials and find the degree of the resultant polynomial:

(i) $p(x) = x^2 - 9$ $\quad q(x) = 6x^2 + 7x - 2$

(ii) $f(x) = 7x + 2$ $\quad g(x) = 15x - 9$

(iii) $h(x) = 6x^2 - 7x + 1$ $\quad f(x) = 5x - 7$

Sol. (i) $p(x) = x^2 - 9$ $\quad q(x) = 6x^2 + 7x - 2$

$p(x) \times q(x) = (x^2 - 9)\,(6x^2 + 7x - 2)$

		x^2	x	constant
		1	0	–9
		6	7	–2
		–2	0	18
	7	0	–63	
6	0	–54		
6	7	–56	–63	18

The required polynomial is $6x^4 + 7x^3 - 56x^2 - 63x + 18$, degree 4.

(ii) $f(x) = 7x + 2$ $\quad g(x) = 15x - 9$

$f(x) \times g(x) = (7x + 2)\,(15x - 9)$

	x	Constant
	7	2
	15	–9
	–63	–18
105	30	
105	–33	–18

The required polynomial is $105x^2 - 33x - 18$, degree 2.

(iii) $h(x) = 6x^2 - 7x + 1$ $\quad f(x) = 5x - 7$

$h(x) \times f(x) = (6x^2 - 7x + 1)\,(5x - 7)$

	x^2	x	1
	6	–7	1
		5	–7
	–42	49	–7
30	–35	5	
30	–77	54	–7

The polynomial is $30x^3 - 77x^2 + 54x - 7$, degree 3.

10. The cost of a chocolate is Rs. $(x + y)$ and Amir bought $(x + y)$ chocolates. Find the total amount paid by him in terms of x and y. If $x = 10$, $y = 5$ find the amount paid by him.

Sol. Amount paid = Number of chocolates × Cost of a chocolate

$= (x + y)(x + y) = (x + y)^2 = x^2 + 2xy + y^2$

If $x = 10$, $y = 5$

The total amount paid by him

$= 10^2 + 2 \times 10 \times 5 + 5^2 = 100 + 100 + 25 = $ Rs. 225

11. The length of a rectangle is $(3x+2)$ units and it's breadth is $(3x–2)$ units. Find its area in terms of x. What will be the area if $x = 20$ units.

Sol. Area of a rectangle = length × breadth

$= (3x + 2) \times (3x - 2) = (3x)^2 - 2^2 = [9x^2 - 4]$ Sq. units

If $x = 20$, Area $= 9 \times 20^2 - 4 = 9 \times 400 - 4$

$= 3600 - 4 = 3596$ Sq. units

12. $p(x)$ is a polynomial of degree 1 and $q(x)$ is a polynomial of degree 2. What kind of the polynomial $p(x) \times q(x)$ is ?

Sol. $p(x)$ is a polynomial of degree 1.

$q(x)$ is a polynomial of degree 2.

Then the $p(x) \times q(x)$ will be the polynomial of degree $(1 + 2) = 3$ (or) Cubic polynomial

3.2.2 Value and Zeros of a Polynomial

The number of zeros depend on the line intersecting "x" axis.

Zeros of a polynomial ≤ the degree of the polynomial

Value of a polynomial $p(x)$ at $x = a$ is $p(a)$ obtained on replacing x by a ($a \in R$)

Zeros of Polynomial :

When the value of $p(x)$ at $x = 1$ is zero, we can say that 1 is one of the zeros of $p(x)$.

Example $q(x) = x^2 - 3x + 2$.

Roots of a polynomial equation :

If 'a' is zero of polynomial $p(x)$ if $p(a) = 0$ and a is zero of the polynomial 'a' is the root of polynomial equation $p(x) = 0$.

(i) A zero of a polynomial can be any real number not necessarily zero.

(ii) A non-zero constant polynomial has no zero.

(iii) By convention, every real number is zero of the zero polynomial.

Exercise 3.2

1. Find the value of the polynomial $f(y) = 6y - 3y^2 + 3$ at (i) $y = 1$ (ii) $y = -1$ (iii) $y = 0$

Sol. (i) At $y = 1$,

$f(1) = 6(1) - 3(1)^2 + 3 = 6 - \cancel{3} + \cancel{3} = 6$

(ii) At $y = -1$,

$f(-1) = 6(-1) - 3(-1)^2 + 3 = -6 - 3(1) + 3 = -6 - \cancel{3} + \cancel{3} = -6$

(iii) At $y = 0$,

$f(0) = 6(0) - 3(0)^2 + 3 = 0 - 0 + 3 = 3$

2. If $p(x) = x^2 - 2\sqrt{2}x + 1$, find $p(2\sqrt{2})$

Sol.
$$p(2\sqrt{2}) = (2\sqrt{2})^2 - 2\sqrt{2}(2\sqrt{2}) + 1$$
$$= 4 \times 2 - 4 \times 2 + 1$$
$$= \cancel{8} - \cancel{8} + 1 = 1$$

3. Find the zeros of the polynomial in each of the following :

(i) $p(x) = x - 3$, (ii) $p(x) = 2x + 5$, (iii) $q(y) = 2y - 3$,
(iv) $f(z) = 8z$, (v) $p(x) = ax$ where $a \neq 0$, (vi) $h(x) = ax + b, a \neq 0, a, b \in R$

Sol. (i)
$$x = 3.$$
$$p(3) = 3 - 3 = 0$$

$\therefore$ The zero of the polynomial is $x = 3$.

(ii) $p(x) = 2x + 5$,

if $2x + 5 = 0$.
$$2x = -5$$
$$x = \frac{-5}{2}$$

$$\therefore p\left(\frac{-5}{2}\right) = \cancel{2}\left(\frac{-5}{\cancel{2}}\right) + 5$$
$$= -5 + 5 = 0$$

$\therefore$ $x = \frac{-5}{2}$ is the zero of the polynomial $p(x) = 2x + 5$.

(iii) $q(y) = 2y - 3$,

if $2y - 3 = 0$.
$$2y = 3$$
$$y = \frac{3}{2}$$

$$\therefore q\left(\frac{3}{2}\right) = \cancel{2}\left(\frac{3}{\cancel{2}}\right) - 3 = 0$$

$\therefore$ $y = \frac{3}{2}$ is the zero of the given polynomial.

(iv) $f(z) = 8z$,
$f(0) = 8(0) = 0$

if $8z = 0$.
$$z = \frac{0}{8} = 0$$

$\therefore$ $z = 0$ is the zero of the given polynomial.

(v) $p(x) = ax$ when $a \neq 0$
$p(0) = a(0) = 0$

$$ax = 0$$
$$x = \frac{0}{a}$$
$$x = 0$$

$\therefore$ $x = 0$ is the zero of the given polynomial.

(vi) $h(x) = ax + b, a \neq 0, a, b \in \mathbb{R}$

if $ax + b = 0$
$$ax = -b$$
$$x = \frac{-b}{a}$$

$$\therefore h\left(\frac{-b}{a}\right) = \cancel{a}\left(\frac{-b}{\cancel{a}}\right) + b$$
$$= -b + b = 0$$

$\therefore$ $x = \frac{-b}{a}$ is the zero of the given polynomial.

4. Find the roots of the polynomial equations.

(i) $5x - 6 = 0$ (ii) $x + 3 = 0$ (iii) $10x + 9 = 0$ (iv) $9x - 4 = 0$

Sol. (i)
$$5x - 6 = 0$$
$$5x = 6 \qquad \therefore x = \frac{6}{5}$$

(ii)
$$x + 3 = 0$$
$$\therefore x = -3$$

(iii)
$$10x + 9 = 0$$
$$10x = -9 \qquad \therefore x = \frac{-9}{10}$$

(iv)
$$9x - 4 = 0$$
$$9x = 4 \qquad \therefore x = \frac{4}{9}$$

5. Verify whether the following are zeros of the polynomial indicated against them, or not.

(i) $p(x) = 2x - 1, x = \frac{1}{2}$, (ii) $p(x) = x^3 - 1, x = 1$,

(iii) $p(x) = ax + b, \; x = \frac{-b}{a}$ (iv) $p(x) = (x+3)(x-4), x = 4, x = -3$

Sol. (i)
$$p(x) = 2x - 1, x = \frac{1}{2}$$
$$p\left(\frac{1}{2}\right) = \cancel{2}\left(\frac{1}{\cancel{2}}\right) - 1 = 1 - 1 = 0$$
$$\therefore x = \frac{1}{2} \text{ is the zero of the given polynomial.}$$

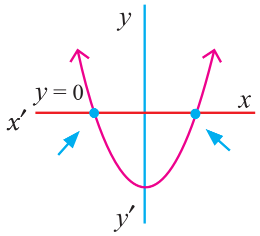
(ii)
$$p(x) = x^3 - 1, x = 1$$
$$p(1) = 1^3 - 1 = 1 - 1 = 0$$
$\therefore x = 1$ is the zero of the given polynomial.

(iii)
$$p(x) = ax + b, \; x = \frac{-b}{a}$$
$$p\left(\frac{-b}{a}\right) = \cancel{a}\left(\frac{-b}{\cancel{a}}\right) + b$$
$$= -b + b = 0$$
$\therefore x = \frac{-b}{a}$ is the zero of the given polynomial.

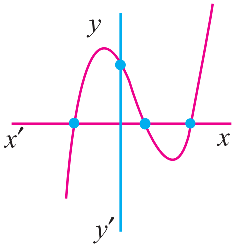
(iv)
$$p(x) = (x+3)(x-4), x = 4, x = -3$$
$$p(-3) = (-3+3)(-3-4) = 0(-7) = 0$$
$$p(4) = (4+3)(4-4) = 7(0) = 0$$
$\therefore \; x = -3, x = 4$ are the zeros of the given polynomial.

6. Find the number of zeros of the following polynomials represented by their graphs.

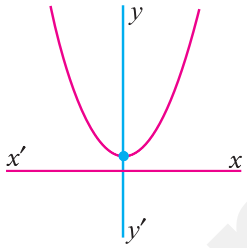
(i)



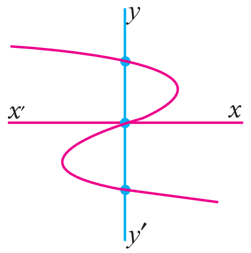
(ii)



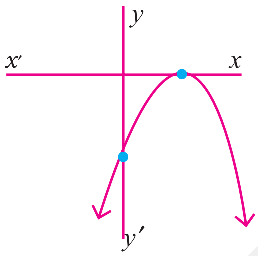
(iii)



(iv)



(v)



Sol. (i) The curve cuts the x-axis at two points. $\therefore$ The equation has 2 zeros.

(ii) Since the curve cuts the x-axis at 3 different points. The number of zeros of the given curve is three.

(iii) Since the curve doesn't cut the x axis. The number of zeros of the given curve is zero.

(iv) The curve cut the x-axis at one point. $\therefore$ The equation has one zero.

(v) The curve cut the x axis at one point. $\therefore$ The equation has one zero.

3.3 Remainder Theorem :

If a polynomial $p(x$ of degree greater than or equal to one is divided by a linear polynomial $(x - a)$ then the remainder is $p\ (a)$, where a is any real number.

Exercise 3.3

1. Check whether $p(x)$ is a multiple of g(x) or not.
$p(x) = x^3 - 5x^2 + 4x - 3$, g($x$) $= x - 2$

Sol. **$p(x) = x^3 - 5x^2 + 4x - 3$; $g(x) = x - 2$**

$$\begin{aligned} \text{Let } g(x) &= 0 \\ x - 2 &= 0 \\ x &= 2 \\ p(2) &= 2^3 - 5(2^2) + 4(2) - 3 \\ &= 8 - 5 \times 4 + 8 - 3 = 8 - 20 + 5 = -7 \neq 0 \end{aligned}$$

$\Rightarrow p(x)$ is not a multiple of $g(x)$

2. By remainder theorem, find the remainder when, $p(x)$ is divided by g(x) where,

(i) $p(x) = x^3 - 2x^2 - 4x - 1$; g(x) $= x + 1$

(ii) $p(x) = 4x^3 - 12x^2 + 14x - 3$; g($x$) $= 2x - 1$

(iii) $p(x) = x^3 - 3x^2 + 4x + 50$; g($x$) $= x - 3$

Sol. (i) $p(x) = x^3 - 2x^2 - 4x - 1$; $g(x) = x + 1$

$$\begin{aligned} p(-1) &= (-1)^3 - 2(-1)^2 - 4(-1) - 1 \\ &= -1 - 2 \times 1 + 4 - 1 \\ &= -4 + 4 = 0 \end{aligned}$$

$\therefore$ Remainder = 0.

Let $g(x) = x + 1$
$x + 1 = 0$
$x = -1$

(ii) $p(x) = 4x^3 - 12x^2 + 14x - 3$; $g(x) = 2x - 1$

$$\begin{aligned} p\left(\frac{1}{2}\right) &= 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 14\left(\frac{1}{2}\right) - 3 \\ &= 4 \times \frac{1}{8} - 12 \times \frac{1}{4} + 7 - 3 \\ &= \frac{1}{2} - 3 + 4 \\ &= \frac{1 - 6 + 8}{2} = \frac{3}{2} \end{aligned}$$

Let $g(x) = 2x - 1$
$2x - 1 = 0$
$2x = 1$
$\therefore x = \frac{1}{2}$

$\therefore$ Remainder = $\frac{3}{2}$.

(iii) $p(x) = x^3 - 3x^2 + 4x + 50$; $g(x) = x - 3$

$$\begin{aligned} p(3) &= 3^3 - 3(3^2) + 4(3) + 50 \\ &= 27 - 27 + 12 + 50 \\ &= 62 \end{aligned}$$

Let $g(x) = x - 3$
$x - 3 = 0$
$x = 3$

$\therefore$ Remainder = 62.

3. Find the remainder when $3x^3 - 4x^2 + 7x - 5$ is divided by $(x + 3)$

Sol. $(3x^3 - 4x^2 + 7x - 5) \div (x + 3)$

$$\begin{array}{r|l} & 3x^2 - 13x + 46 \\ \hline x+3 & 3x^3 - 4x^2 + 7x - 5 \\ & \underset{(-)}{3x^3} \underset{(-)}{+ 9x^2} \\ \hline & -13x^2 + 7x \\ & \underset{(+)}{-13x^2} \underset{(+)}{- 39x} \\ \hline & 46x - 5 \\ & \underset{(-)}{46x} \underset{(-)}{+ 138} \\ \hline & -143 \end{array}$$

Hint

(i) $\frac{3x^3}{x} = 3x^2$

(ii) $(x+3)3x^2 = 3x^3 + 9x^2$

(iii) $\frac{-13x^2}{x} = -13x$

(iv) $(x+3)(-13x) = -13x^2 - 39x$

(v) $\frac{46x}{x} = 46$

(vi) $(x+3)46 = 46x + 138$

$\therefore$ The remainder is -143.

4. What is the remainder when $x^{2018} + 2018$ is divided by $x - 1$.

Sol. $x^{2018} + 2018$ is divided by $x - 1$

$p(x) = x^{2018} + 2018$

Let $g(x) = x - 1 = 0$
$x = 1$

$p(1) = 1^{2018} + 2018$
$= 1 + 2018 = 2019$

5. For what value of k is the polynomial $p(x) = 2x^3 - kx^2 + 3x + 10$ exactly divisible by $(x - 2)$.

Sol. Let $g(x) = x - 2 = 0$
$x = 2$
Since $p(x)$ is exactly divisible by $(x - 2)$

$$p(2) = 2(2^3) - k(2^2) + 3(2) + 10$$
$$= 16 - 4k + 6 + 10$$
$$= 32 - 4k = 0$$
$$= \cancel{-}4k = \cancel{-}32$$
$$k = \frac{32}{4} = 8.$$

6. If two polynomials $2x^3 + ax^2 + 4x - 12$ and $x^3 + x^2 - 2x + a$ leave the same remainder when divided by $(x - 3)$, find the value of a. and also find the remainder.

Sol. Let $f(x) = 2x^3 + ax^2 + 4x - 12$ and $g(x) = x^3 + x^2 - 2x + a$

When $f(x)$ is divided by $x - 3$, the remainder is $f(3)$.

$$\text{Now } f(3) = 2(3)^3 + a(3)^2 + 4(3) - 12 = 54 + 9a + 12 - 12$$
$$f(3) = 9a + 54 \quad ...(1)$$

When $g(x)$ is divided by $x - 3$, the remainder is $g(3)$.

$$\text{Now } g(3) = 3^3 + 3^2 - 2(3) + a = 27 + 9 - 6 + a$$
$$g(3) = a + 30 \quad ...(2)$$

Since, the remainder's are same (1) = (2)

$$\text{Given that } f(3) = g(3)$$
$$\text{That is } 9a + 54 = a + 30$$
$$9a - a = 30 - 54 \Rightarrow 8a = -24 \quad \therefore a = -3$$

Substituting $a = -3$ in $f(3)$, we get

$$f(3) = 9(-3) + 54 = -27 + 54$$
$$f(3) = 27$$

$\therefore$ The remainder is 27.

7. Determine whether $(x - 1)$ is a factor of the following polynomials:

(i) $x^3 + 5x^2 - 10x + 4$ (ii) $x^4 + 5x^2 - 5x + 1$

Sol. (i) Let $P(x) = x^3 + 5x^2 - 10x + 4$

By factor theorem $(x - 1)$ is a factor of $P(x)$, if $P(1) = 0$

$$P(1) = 1^3 + 5(1^2) - 10(1) + 4 = 1 + 5 - 10 + 4$$
$$P(1) = 0$$

$\therefore (x - 1)$ is a factor of $x^3 + 5x^2 - 10x + 4$

(ii) Let $P(x) = x^4 + 5x^2 - 5x + 1$

By factor theorem, $(x - 1)$ is a factor of $P(x)$, if $P(1) = 0$

$$P(1) = 1^4 + 5(1^2) - 5(1) + 1 = 1 + 5 - 5 + 1 = 2 \neq 0$$

$\therefore (x - 1)$ is not a factor of $x^4 + 5x^2 - 5x + 1$

8. Using factor theorem, show that $(x - 5)$ is a factor of the polynomial $2x^3 - 5x^2 - 28x + 15$

Sol. Let $P(x) = 2x^3 - 5x^2 - 28x + 15$

By factor theorem, $(x - 5)$ is a factor of $P(x)$, if $P(5) = 0$

$$P(5) = 2(5)^3 - 5(5)^2 - 28(5) + 15$$
$$= 2 \times 125 - 5 \times 25 - 140 + 15$$
$$= 250 - 125 - 140 + 15 = 265 - 265 = 0$$

$\therefore (x - 5)$ is a factor of $2x^3 - 5x^2 - 28x + 15$

9. Determine the value of m, if $(x + 3)$ is a factor of $x^3 - 3x^2 - mx + 24$.

Sol.

$$\text{Let } P(x) = x^3 - 3x^2 - mx + 24$$

By using factor theorem,

$(x + 3)$ is a factor of $P(x)$, then $P(-3) = 0$

$$P(-3) = (-3)^3 - 3(-3)^2 - m(-3) + 24 = 0$$

$$\Rightarrow -27 - 3 \times 9 + 3m + 24 = 0 \quad \Rightarrow \quad 3m = 54 - 24$$

$$\Rightarrow \quad m = \frac{30}{3} = 10$$

10. If both $(x - 2)$ and $\left(x - \frac{1}{2}\right)$ are the factors of $ax^2 + 5x + b$, then show that $a = b$.

Sol.

$$\text{Let } P(x) = ax^2 + 5x + b$$

$$(x - 2) \text{ is a factor of } P(x), \text{ if } P(2) = 0$$

$$P(2) = a(2)^2 + 5(2) + b = 0$$

$$4a + 10 + b = 0$$

$$4a + b = -10 \quad \ldots (1)$$

$$\left(x - \frac{1}{2}\right) \text{ is a factor of } P(x), \text{ if } P\left(\frac{1}{2}\right) = 0$$

$$P\left(\frac{1}{2}\right) = a\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + b = 0$$

$$\frac{a}{4} + \frac{5}{2} + b = 0$$

$$\frac{a}{4} + b = \frac{-5}{2}$$

$$\frac{a + 4b}{4} = \frac{-5}{2}$$

$$2a + 8b = -20$$

$$a + 4b = -10 \quad \ldots (2)$$

From (1) and (2)

$$4a + b = -10 \quad \ldots (1)$$

$$a + 4b = -10 \quad \ldots (2)$$

$$(1) \text{ and } (2) \Rightarrow \quad 4a + b = a + 4b$$

$$3a = 3b$$

$$\therefore a = b. \quad \text{Hence it is proved.}$$

11. If $(x - 1)$ divides the polynomial $kx^3 - 2x^2 + 25x - 26$ without remainder, then find the value of k.

Sol.

$$\text{Let } P(x) = kx^3 - 2x^2 + 25x - 26$$

By factor theorem, $(x - 1)$ divides $P(x)$ without remainder, $P(1) = 0$

$$P(1) = k(1)^3 - 2(1)^2 + 25(1) - 26 = 0$$

$$k - 2 + 25 - 26 = 0$$

$$k - 3 = 0$$

$$k = 3$$

12. Check if $(x + 2)$ and $(x - 4)$ are the sides of a rectangle whose area is $x^2 - 2x - 8$ by using factor theorem.

Sol. Let $P(x) = x^2 - 2x^2 - 8$

By using factor theorem,$(x + 2)$ is a factor of $P(x)$, if $P(-2) = 0$

$P(-2) = (-2)^2 - 2(-2) - 8 = 4 + 4 - 8 = 0$

and also $(x - 4)$ is a factor of $P(x)$, if $P(4) = 0$

$P(4) = 4^2 - 2(4) - 8 = 16 - 8 - 8 = 0$

$\therefore$ $(x + 2)$, $(x - 4)$ are the sides of a rectangle whose area is $x^2 - 2x - 8$.

Exercise 3.4

1. Expand the following :

(i) $(2x + 3y + 4z)^2$ **(ii)** $(-p + 2q + 3r)^2$

(iii) $(2p + 3)(2p - 4)(2p - 5)$ **(iv)** $(3a + 1)(3a - 2)(3a + 4)$

Sol. (i) $(2x + 3y + 4z)^2$

$$\boxed{(a + b + c)^2 \equiv a^2 + b^2 + c^2 + 2ab + 2bc + 2ca}$$

$$\begin{aligned}\therefore (2x + 3y + 4z)^2 &= (2x)^2 + (3y)^2 + (4z)^2 + 2(2x)(3y) + 2(3y)(4z) + 2(4z)(2x) \\ &= 4x^2 + 9y^2 + 16z^2 + 12xy + 24yz + 16zx\end{aligned}$$

(ii) $(-p + 2q + 3r)^2$

$$\boxed{(a + b + c)^2 \equiv a^2 + b^2 + c^2 + 2ab + 2bc + 2ca}$$

$$\begin{aligned}(-p + 2q + 3r)^2 &= (-p)^2 + (2q)^2 + (3r)^2 + 2(-p)(2q) + 2(2q)(3r) + 2(-p)(3r) \\ &= p^2 + 4q^2 + 9r^2 - 4pq + 12qr - 6rp\end{aligned}$$

(iii) $(2p + 3)(2p - 4)(2p - 5)$

$$\boxed{(x + a)(x + b)(x + c) \equiv x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc}$$

$$\begin{aligned}(2p + 3)(2p - 4)(2p - 5) &= (2p)^3 + (3 - 4 - 5)(2p)^2 + [(3 \times -4)^2 + (-4 \times -5) + \\ &\quad (-5 \times 3)]\, 2p + 3 \times -4 \times -5 \\ &= 8p^3 + (-6)(4p^2) + [-12 + 20 + (-15)]\, 2p + 60 \\ &= 8p^3 - 24p^2 + (-7)2p + 60 \\ (2p + 3)(2p - 4)(2p - 5) &= 8p^3 - 24p^2 - 14p + 60\end{aligned}$$

(iv) $(3a + 1)(3a - 2)(3a + 4)$

$$\boxed{(x + a)(x + b)(x + c) \equiv x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc}$$

$$\begin{aligned}(3a + 1)(3a - 2)(3a + 4) &= (3a)^3 + (1 - 2 + 4)(3a)^2 + [1 \times (-2) + (-2 \times 4) + \\ &\quad + 4 \times 1](3a) + 1 \times -2 \times 4 \\ &= 27a^3 + 3(9a^2) + (-2 - 8 + 4)3a - 8 \\ &= 27a^3 + 27a^2 - 8a - 8\end{aligned}$$

2. Using algebraic identity, find the coefficients of x^2, x and constant term without actual expansion.

(i) $(x + 5)(x + 6)(x + 7)$ (ii) $(2x + 3)(2x - 5)(2x - 6)$

Sol. (i) $(x + 5)(x + 6)(x + 7)$

$$\boxed{(x + a)(x + b)(x + c) \equiv x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc}$$

$$\begin{aligned}
\text{Co-efficient of } x^2 &= a + b + c = 5 + 6 + 7 = 18 \\
\text{Co-efficient of } x^2 &= ab + bc + ca = (5 \times 6) + (6 \times 7) + (7 \times 5) \\
&= 30 + 42 + 35 = 107 \\
\text{Constant term} &= abc = 5 \times 6 \times 7
\end{aligned}$$

Co-efficient of constant term = 210

(ii) $(2x + 3)(2x - 5)(2x - 6)$

$$\begin{aligned}
\therefore \text{Co-efficient of } x^2 &= 4(a + b + c) = 4(3 + (-5) + (-6)) \\
&= 4 \times (-8) = -32 \\
\text{Co-efficient of } x &= 2(ab + bc + ca) \\
&= 2[3\times(-5) + (-5)(-6) + (-6)(3)] \\
&= 2[-15 + 30 - 18] = 2 \times (-3) = -6 \\
\text{Constant term} &= abc = 3 \times (-5) \times (-6) = 90
\end{aligned}$$

3. If $(x + a)(x + b)(x + c) = x^3 + 14x^2 + 59x + 70$, find the value of

(i) $a + b + c$ (ii) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

(iii) $a^2 + b^2 + c^2$ (iv) $\frac{a}{bc} + \frac{b}{ac} + \frac{c}{ab}$

Sol.

$$(x + a)(x + b)(x + c) = x^3 + 14x^2 + 59x + 70 \quad \dots (1)$$

$$\boxed{(x + a)(x + b)(x + c) \equiv x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc} \quad \dots (2)$$

(i) Comparing (1) & (2)

We get, $a + b + c = 14$

(ii) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{bc + ac + ab}{abc} = \frac{59}{70}$

(iii) $\boxed{(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca}$

$$\begin{aligned}
a^2 + b^2 + c^2 &= (a + b + c)^2 - 2(ab + bc + ca) \\
&= 14^2 - 2(59) = 196 - 118 = 78
\end{aligned}$$

(iv) $\frac{a}{bc} + \frac{b}{ac} + \frac{c}{ab} = \frac{a^2 + b^2 + c^2}{abc} = \frac{78}{70} = \frac{39}{35}$

4. Expand

(i) $(3a - 4b)^3$ (ii) $\left(x + \frac{1}{y}\right)^3$

Sol. (i) $(3a - 4b)^3$

We know that

$$\boxed{(x - y)^3 \equiv x^3 - 3x^2y + 3xy^2 - y^3}$$

$$\begin{aligned}
(3a - 4b)^3 &= (3a)^3 - 3(3a)^2(4b) + 3(3a)(4b)^2 - (4b)^3 \\
&= 27a^3 - 108a^2b + 144ab^2 - 64b^3
\end{aligned}$$

(ii) $\left(x+\frac{1}{y}\right)^3$

$$\boxed{(x+y)^3 \equiv x^3+3x^2y+3xy^2+y^3}$$

$$\left(x+\frac{1}{y}\right)^3 = x^3+\frac{3x^2}{y}+\frac{3x}{y^2}+\frac{1}{y^3}$$

5. Evaluate the following by using identities:

(i) 98^3 (ii) 1001^3

Sol. (i)

$$98^2 = (100-2)^3$$

$$\boxed{(a-b)^3 \equiv a^3-3a^2b+3ab^2-b^3}$$

$$98^3 = (100-2)^3 = 100^3 - 3\times100^2\times2+3\times100\times2^2-2^3$$

$$= 1000000-3\times10000\times2+300\times4-8$$

$$= 1000000–60000+1200– 8 =1001200-60008 = 941192$$

(ii)

$$1001^3 = (1000+1)^3$$

$$\boxed{(a+b)^3 \equiv a^3+3a^2b+3ab^2+b^3}$$

$$(1000+1)^3 = 1000^3+3(1000^2)\times1+3\times1000\times1^2+1^3$$

$$= 1000,000,000+3,000,000+3000+1=1,003,003,001$$

6. If $(x + y + z) = 9$ and $(xy + yz + zx) = 26$ then find the value of $x^2+ y^2 + z^2$.

Sol.

$$(x+y+z) = 9 \text{ and } (xy+yz+zx) = 26$$

$$x^2+y^2+z^2 = (x+y+z)^2 -2(xy+yz+zx)$$

$$= 9^2-2\times26 = 81-52 = 29$$

7. Find $27a^3 + 64b^3$, if $3a + 4b = 10$ and $ab = 2$.

Sol.

$$3a+4b = 10, ab = 2$$

$$(3a+4b)^3 = (3a)^3+3(3a)^2(4b)+3(3a)(4b)^2+(4b)^3$$

$$(27a^3+64b^3) = (3a+4b)^3-3(3a)(4b)(3a+4b)$$

$$\boxed{\because x^3+y^3 \equiv (x+y)^3-3xy-(x+y)}$$

$$= 10^3-36\,ab\,(10) = 1000-36\times2\times10$$

$$= 1000-720 = 280$$

8. Find $x^3 - y^3$, if $x - y = 5$ and $xy = 14$.

Sol.

$$x-y = 5, xy = 14$$

$$x^3-y^3 = (x-y)^3+3xy(x-y) = 5^3+3\times14\times5$$

$$= 125+210 = 335$$

9. If $a+\frac{1}{a}=6$, then find the value of $a^3+\frac{1}{a^3}$.

Sol.

$$\boxed{a^3+b^3 = (a+b)^3-3ab(a+b)}$$

$$a^3+\left(\frac{1}{a}\right)^3 = \left(a+\frac{1}{a}\right)^3-3\cancel{a}\times\frac{1}{\cancel{a}}\left(a+\frac{1}{a}\right)$$

$$a^3+\frac{1}{a^3} = 6^3-3\times6 = 216-18 = 198$$

10. **If $x^2 + \frac{1}{x^2} = 23$, then find the value of $x + \frac{1}{x}$ and $x^3 + \frac{1}{x^3}$.**

Sol.

$$\left(x+\frac{1}{x}\right)^2 = x^2 + 2\cancel{x} \times \frac{1}{\cancel{x}} + \frac{1}{x^2}$$

$$\left(x+\frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2} = \left(x^2+\frac{1}{x^2}\right) + 2$$

$$\left(x+\frac{1}{x}\right)^2 = 23 + 2 = 25$$

$$\left(x+\frac{1}{x}\right) = \sqrt{25} = 5$$

$$x^3 + \frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\cancel{x} \times \frac{1}{\cancel{x}}\left[x+\frac{1}{x}\right]$$

$$= 5^3 - 3\,(5) = 125 - 15 = 110$$

11. **If $\left(y-\frac{1}{y}\right)^3 = 27$, then find the value of $y^3 - \frac{1}{y^3}$.**

Sol.

$$\left(y-\frac{1}{y}\right)^3 = 27 \text{ (Given)}$$

$$y^3 - \frac{1}{y^3} = \left(y-\frac{1}{y}\right)^3 + 3y + \frac{1}{y}\left(y-\frac{1}{y}\right)$$

$$\boxed{\because x^3 - y^3 \equiv (x-y)^3 + 3xy\,(x-y)} = 27 + 3\left(y-\frac{1}{y}\right)$$

$$\left[\because \left(y-\frac{1}{y}\right)^3 = 27\,;\ y - \frac{1}{y} = \sqrt[3]{27} = 3\right]$$

$$= 27 + 3 \times 3 = 27 + 9 = 36$$

12. **Simplify : (i) $(2a + 3b + 4c)(4a^2 + 9b^2 + 16c^2 - 6ab - 12bc - 12bc - 8ca)$**

(ii) $(x - 2y + 3z)(x^2 + 4y^2 + 9z^2 + 2xy + 6yz - 3xz)$

Sol. **(i)** **$(2a + 3b + 4c)(4a^2 + 9b^2 + 16c^2 - 6ab - 12bc - 12bc - 8ca)$**

We know that

$$\boxed{(a+b+c)(a^2+b^2+c^2-ab-bc-ca) = a^3+b^3+c^3 \times 3abc}$$

$\therefore (2a + 3b + 4c)(4a^2 + 9b^2 + 16\,c^2 - 6\,ab - 12\,bc - 8\,ca)$

$$= (2a)^3 + (3b)^3 + (4c)^3 - 3 \times 2a \times 3b \times 4c$$

$$= 8a^3 + 27b^3 + 64c^3 - 72\,abc$$

(ii) **$(x - 2y + 3z)(x^2 + 4y^2 + 9z^2 + 2xy + 6yz - 3xz)$**

$$\boxed{(a+b+c)(a^2+b^2+c^2-ab-bc-ca) = a^3+b^3+c^3-3abc}$$

$\therefore (x - 2y + 3z)(x^2 + 4y^2 + 9z^2 + 2xy + 6yz - 3xz)$

$$= x^3 + (-2y)^3 + (3z)^3 - 3 \times x \times (-2y)\,(3z)$$

$$= x^3 - 8y^3 + 27z^3 + 18\,xyz$$

13. By using identity evaluate the following:

(i) $7^3 - 10^3 + 3^3$ **(ii)** $1 + \frac{1}{8} - \frac{27}{8}$

Sol. (i) $7^3 - 10^3 + 3^3$

$$\boxed{(a+b+c)(a^2+b^2+c^2-ab-bc-ca) = a^3+b^3+c^3-3abc}$$

$$\text{If } a+b+c=0, \text{ then } a^3+b^3+c^3 = 3\,abc$$

$$\therefore 7-10+3 = 0$$

$$\Rightarrow 7^3-10^3+3^3 = 3\times 7\times -10\times 3$$

$$= 9\times -70 = -630$$

(ii) $1 + \frac{1}{8} - \frac{27}{8}$

$$1+\frac{1}{8}-\frac{27}{8} = 1^3+\left(\frac{1}{2}\right)^3+\left(\frac{-3}{2}\right)^3$$

$$\boxed{\text{Here } 1+\frac{1}{2}-\frac{3}{2} = \frac{2+1-3}{2}=\frac{0}{2}=0}$$

$$\therefore 1+\frac{1}{8}-\frac{27}{8} = 1^3+\left(\frac{1}{2}\right)^3+\left(\frac{-3}{2}\right)^3 = 3\times 1\times\frac{1}{2}\times\frac{-3}{2} = \frac{-9}{4}$$

14. If $2x - 3y - 4z = 0$, then find $8x^3 - 27y^3 - 64z^3$.

Sol. If $2x-3y-4z = 0$ then $8x^3-27y^3-64z^3 = ?$

$$\boxed{\text{If } x+y+z = 0 \text{ then } x^3+y^3+z^3 = 3xyz}$$

$$8x^3-27y^3-64z^3 = (2x)^3+(-3y)^3+(-4z)^3$$

$$= 3\times 2x\times -3y\times -4z = 72\,xyz$$

Exercise 3.5

1. Factorise the following expressions:

(i) $2a^2 + 4a^2b + 8a^2c$ **(ii)** $ab - ac - mb + mc$

Sol. (i) $2a^2+4a^2b+8a^2c = 2a^2[1+2b+4c]$

(ii) $ab-ac-mb+mc = a(b-c)-m(b-c) = (b-c)(a-m)$

2. Factorise the following:

(i) $x^2 + 4x + 4$ **(ii)** $3a^2 - 24ab + 48b^2$

(iii) $x^5 - 16x$ **(iv)** $m^2 + \frac{1}{m^2} - 23$

(v) $6 - 216x^2$ **(vi)** $a^2 + \frac{1}{a^2} - 18$

Sol. (i) $x^2+4x+4 = (x+2)(x+2) = (x+2)^2$

$$\boxed{\because (a+b)^2 \equiv a^2+2ab+b^2}$$

(ii) $3a^2-24ab+48b^2 = 3[a^2-8ab+16b^2]$

$= 3[a-4b]^2$ $(\because (a-b)^2 = a^2-2ab+b^2)$

(iii) $x^5-16x = x[x^4-16] = x[(x^2)^2-4^2]$

$= x(x^2+4)(x^2-4) = x(x^2+4)(x+2)(x-2)$

(iv) $m^2 + \frac{1}{m^2} - 23 = \left(m + \frac{1}{m}\right)^2 - 2 - 23 = \left(m + \frac{1}{m}\right)^2 - 25$

$= \left(m + \frac{1}{m}\right)^2 - 5^2 = \left(m + \frac{1}{m} - 5\right)\left(m + \frac{1}{m} + 5\right)$

(v) $6 - 216x^2 = 6\,(1 - (6x)^2) = 6\,(1 + 6x)\,(1 - 6x)$

(vi) $a^2 + \frac{1}{a^2} - 18 = \left(a - \frac{1}{a}\right)^2 + 2 - 18 = \left(a - \frac{1}{a}\right)^2 - 16 = \left(a - \frac{1}{a} + 4\right)\left(a - \frac{1}{a} - 4\right)$

3. Factorise the following:

(i) $4x^2 + 9y^2 + 25z^2 + 12xy + 30yz + 20\,xz$

(ii) $25x^2 + 4y^2 + 9z^2 - 20xy + 12yz - 30xz$

Sol. (i) $4x^2 + 9y^2 + 25z^2 + 12\,xy + 30\,yz + 20\,xz$

$= (2x)^2 + (3y)^2 + (5z)^2 + 2\,(2x)\,(3y) + 2\,(3y) + (5z) + 2 \times 3y \times 5z$

$= (2x + 3y + 5z)^2$

$\because (a + b + 1)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(ii) $25x^2 + 4y^2 + 9z^2 - 20xy + 12\,yz - 30xz$

$= (5x)^2 + (-2y)^2 + (-3z)^2 + 2(5x)\,(-2y) + 2\,(-2y)\,(-3z) + 2\,(-3z)\,(5x)$

$= (5x - 2y - 3z)^2$

4. Factorise the following:

(i) $8x^3 + 125y^3$ **(ii) $27x^3 - 8y^3$** **(iii) $a^6 - 64$**

Sol. (i) $8x^3 + 125y^3 = (2x)^3 + (5y)^3$ $\boxed{\therefore a^3 - b^3 = (a + b)\,(a^2 - ab + b^2)}$

$= (2x + 5y)\,[(2x)^2 - (2x)(5y) + (5y)^2]$

$= (2x + 5y\,)^2\,(4x^2 - 10xy + 25y^2)$

(ii) $27x^3 - 8y^3 = (3x)^3 - (2y)^2$

$= (3x - 2y\,)\,((3x)^2 + 3x \times 2y + (2y)^2)$

$= (3x - 2y\,)\,(9x^2 + 6xy + 4xy + 4y^2)$

(iii) $a^6 - 64 = (a^2)^3 - 4^3$ $(a^3 - b^3 = (a - b)\,(a^2 + ab + b^2)$

$= (a^2 - 4)\,(a^4 + 4a^2 + 4^2)$

$= (a + 2)\,(a - 2)\,(a^2 + 4 - 2a)\,(a^2 - 4 + 2a)$

5. Factorise the following:

(i) $x^3 + 8y^3 + 6xy - 1$ **(ii) $l^3 - 8m^3 - 27n^3 - 18lmn$**

Sol. (i) $x^3 + 8y^3 + 6xy - 1 = x^3 + (2y)^3 + (-1)^3 - 3\,(x)\,(2y)\,(-1)$

$= (x + 2y - 1\,)\,(x^2 + 4y^2 + 1 - 2xy + 2y + x)$

(ii) $l^3 - 8m^3 - 27n^3 - 18lmn = l^3 + (-2m)^3 + (-3n)^3 - 3\,(l)\,(-2m)\,(-3n)$

$= (l - 2m - 3n)\,(l^2 + (-2\,m)^2 + (-3n)^2 - l \times -2m -$
$(-2m \times -3n) - (-3n \times l))$

$= (l - 2m - 3n)\,(l^2 + 4m^2 + 9n^2 + 2lm - 6mn + 3nl)$

3.5.2 Factorising the Quadratic Polynomial (Trinomial) of the type $ax^2 + bx + c, a \neq 0$

The linear factors of $ax^2 + bx + c$ will be in the form $(kx + m)$ and $(lx + n)$

Thus, $ax^2 + bx + c = (kx + m)(lx + n) = klx^2 + (lm + kn)x + mn$

Exercise 3.6

1. Factorise the following:

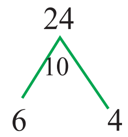
(i) $x^2 + 10x + 24$ (ii) $z^2 + 4z - 12$

(iii) $p^2 - 6p - 16$ (iv) $t^2 + 72 - 17t$

(v) $y^2 - 16y - 80$ (vi) $a^2 + 10a - 600$

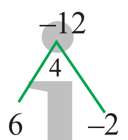
Sol. (i) $x^2 + 10x + 24$

$$\begin{aligned} x^2 + 10x + 24 &= x^2 + 6x + 4x + 24 \\ &= x(x+6) + 4(x+6) \\ &= (x+6)(x+4) \end{aligned}$$



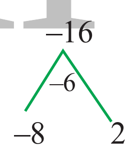
(ii) $z^2 + 4z - 12$

$$\begin{aligned} z^2 + 4z - 12 &= z^2 + 6z - 2z - 12 \\ &= z(z+6) - 2(z+6) \\ &= (z+6)(z-2) \end{aligned}$$



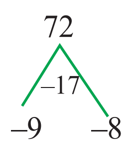
(iii) $p^2 - 6p - 16$

$$\begin{aligned} p^2 - 6p - 16 &= p^2 - 8p + 2p - 16 \\ &= p(p-8) + 2(p-8) \\ &= (p-8)(p+2) \end{aligned}$$



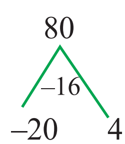
(iv) $t^2 + 72 - 17t$

$$\begin{aligned} t^2 + 72 - 17t &= t^2 - 17t + 72 \\ &= t^2 - 9t - 8t + 72 \\ &= t(t-9) - 8(t-9) \\ &= (t-9)(t-8) \end{aligned}$$



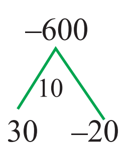
(v) $y^2 - 16y - 80$

$$\begin{aligned} y^2 - 16y - 80 &= y^2 - 20y + 4y - 80 \\ &= y(y-20) + 4(y-20) \\ &= (y-20)(y+4) \end{aligned}$$



(vi) $a^2 + 10a - 600$

$$\begin{aligned} a^2 + 10a - 600 &= a^2 + 30a - 20a - 600 \\ &= a(a+30) - 20(a+30) \\ &= (a+30)(a-20) \end{aligned}$$

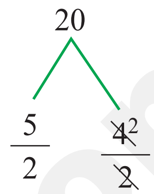


2. Factorise the following:

(i)	$2a^2 + 9a + 10$	**(ii)**	$5x^2 - 29xy - 42y^2$
(iii)	$9 - 18x + 18x^2$	**(iv)**	$6x^2 + 16xy + 8y^2$
(v)	$12x^2 + 36x^2 y + 27y^2 x^2$	**(vi)**	$(a + b)^2 + 9(a + b) + 18$

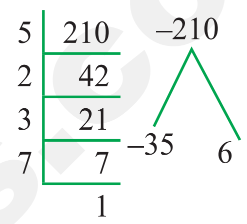
Sol. **(i)** $2a^2 + 9a + 10$

$$\begin{aligned} 2a^2 + 9a + 10 &= 2a^2 + 4a + 5a + 10 \\ &= 2a(a + 2) + 5(a + 2) \\ &= (a + 2)(2a + 5) \end{aligned}$$



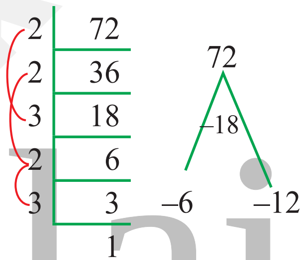
(ii) $5x^2 - 29xy - 42y^2$

$$\begin{aligned} &= 5x^2 - 35xy + 6xy - 42y^2 \\ &= 5x(x - 7) + 6y(x - 7) \\ &= (x - 7)(5x + 6y) \end{aligned}$$



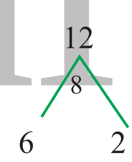
(iii) $9 - 18x + 8x^2$

$$\begin{aligned} &= 8x^2 - 18x + 9 \\ &= 8x^2 - 6x - 12x + 9 \\ &= 2x(4x - 3) - 3(4x - 3) \\ &= (4x - 3)(2x - 3) \end{aligned}$$



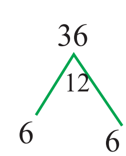
(iv) $6x^2 + 16xy + 8y^2$

$$\begin{aligned} &= 2(3x^2 + 8xy + 4y^2) \\ &= 2(3x^2 + 8xy + 4y^2) \\ &= 2(3x^2 + 6xy + 2xy + 4y^2) \\ &= 2(3x(x + 2y) + 2y(x + 2y)) \\ &= 2(x + 2y)(3x + 2y) \end{aligned}$$



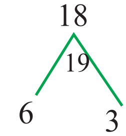
(v) $12x^2 + 36x^2 y + 27y^2 x^2$

$$\begin{aligned} &= 27y^2x^2 + 36x^2y + 12x^2 = 3x^2(9y^2 + 12y + 4) \\ &= 3x^2(9y^2 + 6y + 6y + 4) = 3x^2(3y(3y + 2) + 2(3y + 2)) \\ &= 3x^2(3y + 2)(3y + 2) = 3x^2(3y + 2)(3y + 2) \end{aligned}$$



(vi) $(a + b)^2 + 9(a + b) + 18$

$$\begin{aligned} &= (a + b)^2 + 6(a + b) + 3(a + b) + 18 \\ &= (a + b)((a + b) + 6) + 3((a + b) + 6) \\ &= ((a + b) + 6)((a + b) + 3) = (a + b + 6)(a + b + 3) \end{aligned}$$



3. Factorise the following:

(i)	$(p - q)^2 - 6(p - q) - 16$	**(ii)**	$m^2 + 2mn - 24n^2$
(iii)	$\sqrt{5}a^2 + 2a - 3\sqrt{5}$	**(iv)**	$a^4 - 3a^2 + 2$
(v)	$8m^3 - 2m^2n - 15mn^2$	**(vi)**	$\dfrac{1}{x^2} + \dfrac{1}{y^2} + \dfrac{2}{xy}$

Sol. **(i)** $(p-q)^2 - 6(p-q) - 16$

$= (p-q)^2 - 8(p-q) + 2(p-q) - 16$
$= (p-q)((p-q)-8) + 2((p-q)-8)$
$= (p-q-8)(p-q+2)$

−16
−6
−8 2

(ii) $m^2 + 2mn - 24n^2$

$= m^2 + 6mn - 4mn - 24n^2$
$= m(m+6n) - 4n(m+6n) = (m+6n)(m-4n)$

−24
2
6 −4

(iii) $\sqrt{5}a^2 + 2a - 3\sqrt{5}$

$= \sqrt{5}\,a^2 + 2a - 3\sqrt{5}$ $\qquad \sqrt{5} \times -3\sqrt{5}$
$= \sqrt{5}\,a^2 + 5a - 3a - 3\sqrt{5}$ $\qquad = -3 \times 5$
$= \sqrt{5}\,a(a+\sqrt{5}) - 3(a+\sqrt{5})$ $\qquad = -15$
$= (a+\sqrt{5})(\sqrt{5}\,a - 3)$

−15
2
+5 −3

(iv) $a^4 - 3a^2 + 2$

$= a^4 - 2a^2 - 1a^2 + 2 = a^2(a^2-2) - 1(a^2-2)$
$= (a^2-2)(a^2-1) = (a^2-2)(a+1)(a-1)$

2
−3
−2 −1

(v) $8m^3 - 2m^2n - 15mn^2$

$= m(8m^2 - 2mn - 15n^2)$
$= m(8m^2 - 12mn + 10mn - 15n^2)$
$= m(4m(2m-3n) + 5n(2m-3n))$
$= m(4m+5n)(2m-3n)$

−120
−2
−12 10

(vi) $\dfrac{1}{x^2} + \dfrac{1}{y^2} + \dfrac{2}{xy} = \left(\dfrac{1}{x}\right)^2 + 2\left(\dfrac{1}{x}\right)\left(\dfrac{1}{y}\right) + \left(\dfrac{1}{y}\right)^2 = \left(\dfrac{1}{x} + \dfrac{1}{y}\right)^2 \quad (\because (a+b)^2 = a^2 + 2ab + b^2)$

Exercise 3.7

1. Find the quotient and remainder of the following.

(i) $(4x^3 + 6x^2 - 23x + 18) \div (x+3)$ **(ii)** $(8y^3 - 16y^2 + 16y - 15) \div (2y-1)$
(iii) $(8x^3 - 1) \div (2x-1)$ **(iv)** $(-18z + 14z^2 + 24z^3 + 18) \div (3z+4)$

Sol. **(i)** $(4x^3 + 6x^2 - 23x + 18) \div (x+3)$

$$
\begin{array}{r|l}
 & 4x^2 - 6x - 5 \\
\hline
x+3 & 4x^3 + 6x^2 - 23x + 18 \\
 & \underset{(-)}{4x^3} \underset{(-)}{+12x^2} \\
\hline
 & -6x^2 - 23x \\
 & \underset{(+)}{-6x^2} \underset{(+)}{-18x} \\
\hline
 & -5x + 18 \\
 & \underset{(+)}{-5x} \underset{(+)}{-15} \\
\hline
 & 33
\end{array}
$$

Quotient $= 4x^2 - 6x - 5$
Remainder $= 33$

Hint
(i) $\dfrac{4x^3}{x} = 4x^2$
(ii) $(x+3)4x^2 = 4x^3 + 12x^2$
(iii) $\dfrac{-6x^2}{x} = -6x$
(iv) $-6x(x+3) = -6x^2 - 18$
(v) $\dfrac{-5x}{x} = -5$
(vi) $-5(x+3) = -5x - 15$

(ii) $(8y^3 - 16y^2 + 16y - 15) \div (2y - 1)$

$$
\begin{array}{r|l}
 & 4y^2 - 6y + 5 \\
\hline
2y-1 & 8y^3 - 16y^2 + 16y - 15 \\
 & \underset{(-)}{8y^3} \underset{(+)}{- 4y^2} \\
\hline
 & -12y^2 + 16y \\
 & \underset{(+)}{-12y^2} \underset{(-)}{+ 6y} \\
\hline
 & 10y - 15 \\
 & \underset{(-)}{10y} \underset{(+)}{- 5} \\
\hline
 & -10
\end{array}
$$

Hint

(i) $\frac{8y^3}{2y} = 4y^2$

(ii) $(2y-1) \times 4y^2 = 8y^3 - 4y^2$

(iii) $\frac{-12y^2}{2y} = -6y$

(iv) $(2y-1)(-6y) = -12y^2 + 6y$

(v) $\frac{10y}{2y} = 5$

(vi) $(2y-1) \times 5 = 10y - 5$

Quotient $= 4y^2 - 6y + 5$
Remainder $= -10$

(iii) $(8x^3 - 1) \div (2x - 1)$

$$
\begin{array}{r|l}
 & 4x^2 + 2x + 1 \\
\hline
2x-1 & 8x^3 + 0x^2 + 0x - 1 \\
 & \underset{(-)}{8x^3} \underset{(+)}{- 4x^2} \\
\hline
 & 4x^2 + 0x \\
 & \underset{(-)}{4x^2} \underset{(+)}{- 2x} \\
\hline
 & 2x - 1 \\
 & \underset{(-)}{2x} \underset{(+)}{- 1} \\
\hline
 & 0
\end{array}
$$

Hint

(i) $\frac{8x^3}{2x} = 4x^2$

(ii) $(2x-1)4x^2 = 8x^3 - 4x^2$

(iii) $\frac{4x^2}{2x} = 2x$

(iv) $(2x-1)(2x) = 4x^2 - 2x$

(v) $\frac{2x}{2x} = 1$

(vi) $(2x-1)(1) = 2x - 1$

Quotient $= 4x^2 + 2x + 1$
Remainder $= 0$

(iv) $(24z^3 + 14z^2 - 18z + 18) \div (3x + 4)$

$$
\begin{array}{r|l}
 & 8z^2 - 6z + 2 \\
\hline
3z+4 & 24z^3 + 14z^2 - 18z + 18 \\
 & \underset{(-)}{24z^3} \underset{(-)}{+ 32z^2} \\
\hline
 & -18z^2 - 18z \\
 & \underset{(+)}{-18z^2} \underset{(+)}{- 24z} \\
\hline
 & 6z + 18 \\
 & \underset{(-)}{6z} \underset{(-)}{+ 8} \\
\hline
 & 10
\end{array}
$$

Hint

(i) $\frac{24z^3}{3z} = 8z^2$

(ii) $(3z+4)8z^2 = 24z^3 + 32z^2$

(iii) $\frac{-18z^2}{3z} = -6z$

(iv) $(3z+4)(-6z) = -18z^2 - 24z$

(v) $\frac{6z}{3z} = 2$

(vi) $(3z+4)(2) = 6z + 8$

Quotient $= 8z^2 - 6z + 2$
Remainder $= 10$

2. The area of a rectangle is $x^2 + 7x + 12$. If its breadth is $(x+3)$, then find its length.

Sol. Area of a rectangle $= x^2 + 7x + 12$

Its breadth $= x + 3$

Area $=$ breadth $\times$ length

$x^2 + 7x + 12 = (x + 3) \times$ length

$\therefore$ Length $= (x^2 + 7x + 12) \div (x + 3)$

$$\begin{array}{r|l} & x+4 \\ \hline x+3 & x^2+7x+12 \\ & x^2+3x \\ & (-)\quad(-) \\ \hline & \quad 4x+12 \\ & \quad 4x+12 \\ & \quad (-)\quad(-) \\ \hline & \quad\quad 0 \end{array}$$

Hint

(i) $\frac{x^2}{x} = x$

(ii) $(x+3)x = x^2 + 3x$

(iii) $\frac{4x}{x} = 4$

(iv) $(x+3)4 = 4x + 12$

Quotient $= x + 4$; Remainder $= 0$; $\therefore$ Length $= x + 4$

3. The base of a parallelogram is $(5x+4)$. Find its height, if the area is $25x^2-16$.

Sol. The base of a parallelogram is $(5x + 4)$

Area $= 25x^2 - 16$

Area $= b \times h = 25x^2 - 16$

base $= 5x + 4$

height $= \frac{25x^2 - 16}{\text{base}} = \frac{25x^2 - 16}{5x + 4}$

$$\begin{array}{r|l} & 5x-4 \\ \hline 5x+4 & 25x^2+0x-16 \\ & 25x^2+20x \\ & (-)\quad(-) \\ \hline & \quad -20x-16 \\ & \quad -20x-16 \\ & \quad (+)\quad(+) \\ \hline & \quad\quad 0 \end{array}$$

Hint

(i) $\frac{25x^2}{5x} = 5x$

(ii) $5x(5x+4) = 25x^2 + 20x$

(iii) $\frac{-20x}{5x} = -4$

(iv) $(5x+4)(-4) = -20x - 16$

$\therefore$ height of the parallelogram $= 5x - 4$.

4. The sum of $(x+5)$ observations is (x^3+125). Find the mean of the observations.

Sol. The sum of $(x + 5)$ observations is $(x^3 + 125)$

$$\text{Mean} = \frac{\text{Sum of the observations}}{\text{No. of observations}} = \frac{\sum x}{n}$$

$$= \frac{x^3 + 125}{x + 5} = \frac{x^3 + 0x^2 + 0x + 125}{x + 5}$$

$$\begin{array}{r|l} & x^2 - 5x + 25 \\ \hline x+5 & x^3 + 0x^2 + 0x + 125 \\ & x^3 + 5x^2 \\ & {\scriptstyle(-)\quad(-)} \\ \hline & -5x^2 + 0x \\ & -5x^2 - 25x \\ & {\scriptstyle(+)\quad(+)} \\ \hline & 25x + 125 \\ & 25x + 125 \\ & {\scriptstyle(-)\quad(-)} \\ \hline & 0 \end{array}$$

Hint

(i) $\frac{x^3}{x} = x^2$

(ii) $(x+5)x^2 = x^3 + 5x^2$

(iii) $\frac{-5x^2}{x} = -5x$

(iv) $(x+5)(-5x) = -5x^2 - 25x$

(v) $\frac{25x}{x} = 25$

(vi) $(x+5)25 = 25x + 125$

Quotient $= x^2 - 5x + 25$

Remainder $= 0$

$\therefore$ Mean of the observations $= x^2 - 5x + 25$.

5. Find the quotient and remainder for the following using synthetic division:

(i) $(x^2 + x^2 - 7x - 3) \div (x - 3)$ **(ii)** $(x^3 + 2x^2 - x - 4) \div (x + 2)$

(iii) $(3x^3 - 2x^2 + 7x - 5) \div (x + 3)$ **(iv)** $(8x^4 - 2x^2 + 6x + 5) \div (4x + 1)$

Sol. **(i)** $(x^2 + x^2 - 7x - 3) \div (x - 3)$

Let $p(x) = x^3 + x^2 - 7x - 3$

$q(x) = x - 3$ To find the zero of $x - 3$

$p(x)$ in standard form ((i.e.) descending order)

$x^3 + x^2 - 7x - 3$

Co-efficients are 1 1 –7 – 3

3	1	1	–7	–3
	0	3	12	15
	1	4	5	12 (remainder)

Quotient is $x^2 + 4x + 5$

Remainder is 12

(ii) $(x^2 + 2x^2 - x - 4) \div (x + 2)$

$p(x) = x^3 + 2x^2 - x - 4$

Co-efficients are 1 2 –1 – 4

To find zero of $x + 2$, put $x + 2 = 0$; $x = -2$

–2	1	2	–1	–4
	0	–2	0	2
	1	0	–1	–2

$\therefore$ Quotient is $x^2 - 1$

Remainder is –2

(iii) $(3x^3 - 2x^2 + 7x - 5) \div (x + 3)$

To find zero of the divisor $(x + 3)$, put $x + 3 = 0$; $\therefore x = -3$

Dividend in Standard form $3x^3 - 2x^2 + 7x - 5$

Co-efficients are 3 –2 7 –5

Synthetic Division

$$\begin{array}{r|rrrr} -3 & 3 & -2 & 7 & -5 \\ & 0 & -9 & 33 & -120 \\ \hline & 3 & -11 & 40 & \boxed{-125} \end{array}$$

Quotient is $3x^2 - 11x + 40$

Remainder is –125

(iv) $(8x^4 - 2x^2 + 6x + 5) \div (4x + 1)$

To find zero of the divisor $4x + 1$, put $4x + 1 = 0$; $4x = -1$; $x = -\frac{1}{4}$.

Dividend in Standard form $8x^4 + 0x^3 - 2x^2 + 6x + 5$

Co-efficients are 8 0 –2 6 5

Synthetic Division

$$\begin{array}{r|rrrrr} -\frac{1}{4} & 8 & 0 & -2 & 6 & 5 \\ & 0 & -2 & \frac{1}{2} & +\frac{3}{8} & -\frac{51}{32} \\ \hline & 8 & -2 & -\frac{3}{2} & \frac{51}{8} & \boxed{\frac{109}{32}} \end{array}$$

$$8x^4 + 0x^3 - 2x^2 + 6x + 5 = \left(x+\frac{1}{4}\right)\left(8x^3 - 2x^2 - \frac{3x}{2} + \frac{51}{8}\right) + \frac{109}{32}$$

$$= \frac{(4x+1)}{\cancel{4}} \times \cancel{4}\left(2x^3 - \frac{x^2}{2} - \frac{3x}{8} + \frac{51}{32}\right) + \frac{109}{32}$$

$$= (4x+1)\left(2x^3 - \frac{x^2}{2} - \frac{3x}{8} + \frac{51}{32}\right) + \frac{109}{32}$$

$\therefore$ Quotient is $= 2x^3 - \frac{x^2}{2} - \frac{3x}{8} + \frac{51}{32}$

Remainder is $\frac{109}{32}$

6. If the quotient obtained on dividing $(8x^4 - 2x^2 + 6x - 7)$ by $(2x + 1)$ is $(4x^3 + px^2 - qx + 3)$, then find p, q and also the remainder.

Sol. Let $p(x) = 8x^4 - 2x^2 + 6x - 7$

Standard form $= 8x^4 + 0x^3 - 2x^2 + 6x - 7$

Co-efficients are 8 0 –2 6 –7

$$Q(x) = 2x + 1,$$

To find zero of $2x + 1$, put $2x + 1 = 0$; $2x = -1$; $x = -\frac{1}{2}$

Synthetic division

$$\begin{array}{c|ccccc} -\frac{1}{2} & 8 & 0 & -2 & 6 & -7 \\ & 0 & -4 & 2 & 0 & -3 \\ \hline & 8 & -4 & 0 & 6 & \boxed{-10} \end{array}$$

$$\text{Quotient} = \frac{1}{2}[8x^3 - 4x^2 + 6] = 4x^3 - 2x^2 + 3$$

Quotient $4x^3 - 2x^2 + 3$ is compared with the given quotient $4x^3 + px^2 - qx + 3$

Co-efficients of x^2 is $p = -2$

Co-efficients of x is $q = 0$

Remainder is – 10

$$\boxed{\begin{aligned} p &= -2 \\ q &= 0 \\ r &= -10 \end{aligned}}$$

7. **If the quotient obtained on dividing $3x^3 + 11x^2 + 34x + 106$ by $x - 3$ is $3x^2 + ax + b$, then find a, b and also the remainder.**

Sol. Let $p(x) = 3x^3 + 11x^2 + 34x + 106$

$p(x)$ in standard form

Co-efficients are 3 11 34 106

$q(x) = x - 3$, its zero $x = 3$

Synthetic division

$$\begin{array}{c|cccc} 3 & 3 & 11 & 34 & 106 \\ & 0 & 9 & 60 & 282 \\ \hline & 3 & 20 & 94 & \boxed{388} \end{array}$$

Quotient is $3x^2 + 20x + 94$, it is compared with the given quotient $3x^2 + ax + b$

Co-efficient of x is $a = 20$

Constant term is $b = 94$

Remainder $r = 388$

Exercise 3.8

1. **Factorise each of the following polynomials using synthetic division:**

(i) $x^3 - 3x^2 - 10x + 24$ (ii) $2x^3 - 3x^2 - 3x + 2$

(iii) $-7x + 3 + 4x^3$ (iv) $x^3 + x^2 - 14x - 24$

(v) $x^3 - 7x + 6$ (vi) $x^3 - 10x^2 - x + 10$

Sol. **(i)** $\mathbf{x^3 - 3x^2 - 10x + 24}$

$$\text{Let } p(x) = x^3 - 3x^2 - 10x + 24$$

Sum of all the co-efficients = $1 - 3 - 10 + 24 = 25 - 13 = 12 \neq 0$

Hence $(x - 1)$ is not a factor.

Sum of co-efficient of even powers with constant = $-3 + 24 = 21$

Sum of co-efficients of odd powers = $1 - 10 = -9$

$$21 \neq -9$$

Hence $(x + 1)$ is not a factor.

$$p(2) = 2^3 - 3(2^2) - 10 \times 2 + 24 = 8 - 12 - 20 + 24$$

$$= 32 - 32 = 0 \qquad \therefore (x - 2) \text{ is a factor.}$$

Now we use synthetic division to find other factor

$$\begin{array}{r|rrrr}
2 & 1 & -3 & -10 & 24 \\
 & 0 & 2 & -2 & -24 \\
\hline
-3 & 1 & -1 & -12 & \boxed{0} \\
 & 0 & -3 & 12 & \\
\hline
 & 1 & -4 & \boxed{0} &
\end{array}$$

Thus $(x - 2)(x + 3)(x - 4)$ are the factors.

$$\therefore x^3 - 3x^2 - 10x + 24 = (x - 2)(x + 3)(x - 4)$$

(ii) $\mathbf{2x^2 - 3x^2 - 3x + 2}$

$$\text{Let } p(x) = 2x^3 - 3x^2 - 3x + 2$$

Sum of all the co-efficients are

$$2 - 3 - 3 + 2 = 4 - 6 = -2 \neq 0$$

$\therefore (x - 1)$ is not a factor

Sum of co-efficients of even powers of x with constant = $-3 + 2 = -1$

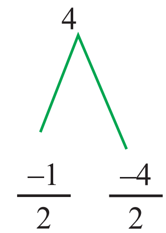
Sum of co-efficients of odd powers of $x = 2 - 3 = -1$

$$(-1) = (-1)$$

$\therefore (x + 1)$ is a factor

Let us find the other factors using synthetic division

$$\begin{array}{r|rrrr}
-1 & 2 & -3 & -3 & 2 \\
 & 0 & -2 & 5 & -2 \\
\hline
 & 2 & -5 & 2 & \boxed{0}
\end{array}$$



$$\text{Quotient is } 2x^2 - 5x + 2 = 2x - 4x - x + 2 = 2x(x - 2) - 1(x - 2)$$

$$= (x - 2)(2x - 1)$$

$$\therefore 2x^3 - 3x^2 - 3x + 2 = (x + 1)(x - 2)(2x - 1)$$

(iii) $-7x + 3 + 4x^3$

$$\text{Let } p(x) = 4x^3 + 0x^2 - 7x + 3$$
$$\text{Sum of the co-efficients are} = 4 + 0 - 7 + 3 = 7 - 7 = 0$$

$\therefore (x - 1)$ is a factor

Sum of co-efficients of even powers of x with constant $= 0 + 3 = 3$

Sum of co-efficients of odd powers of x with constant $= 4 - 7 = -3$

$3 \neq -3$

$\therefore (x + 1)$ is not a factor

Using synthetic division, let us find the other factors.

1	4	0	–7	3
	0	4	4	–3
	4	4	–3	0

–12

$\frac{\cancel{6}^3}{\cancel{4}_2}$ $\frac{\cancel{-2}^1}{\cancel{4}_2}$

Quotient is $4x^2 + 4x - 3$

$$= 4x^2 + 6x - 2x - 3$$
$$= 2x(2x + 3) - 1(2x + 3)$$
$$= (2x + 3)(2x - 1)$$

–12

6 –2

$\therefore$ The factors are $(x - 1)$, $(2x + 3)$ and $(2x - 1)$

$$\therefore -7x + 3 + 4x^3 = (x + 1)(2x + 3)(2x - 1)$$

(iv) $x^3 + x^2 - 14x - 24$

$$\text{Let } p(x) = x^3 + x^2 - 14x - 24$$
$$\text{Sum of the co-efficients are} = 1 + 1 - 14 - 24 = -36 \neq 0$$

$\therefore (x - 1)$ is not a factor

Sum of co-efficients of even powers of x with constant $= 1 - 24 = -23$

Sum of co-efficients of odd powers of $x = 1 - 14 = -3$

$-23 \neq -13$

$\therefore (x + 1)$ is also not a factor

$$p(2) = 2^3 + 2^2 - 14(2) - 24 = 8 + 4 - 28 - 24$$
$$= 12 - 52 \neq 0, (x - 2) \text{ is a not a factor}$$
$$p(-2) = (-2)^3 + (-2)^2 - 14(-2) - 24$$
$$= -8 + 4 + 28 - 24 = 32 - 32 = 0$$

$\therefore (x + 2)$ is a factor

To find the other factors let us use synthetic division.

$x^3 + x^2 - 14x - 24$

–2	1	1	–14	–24
	0	–2	2	24
–3	1	–1	–12	0
	0	–3	12	
	1	–4	0	

–12

–4 3

$\therefore$ The factors are $(x + 2), (x + 3), (x - 4)$

$$\therefore x^3 + x^2 - 14x - 24 = (x + 2)(x + 3)(x - 4)$$

(v) $x^3 - 7x + 6$

$$\begin{aligned} \text{Let } p(x) &= x^3 + 0x^2 - 7x + 6 \\ \text{Sum of the co-efficients are} &= 1 + 0 - 7 + 6 = 7 - 7 = 0 \end{aligned}$$

$\therefore (x - 1)$ is a factor

Sum of co-efficients of even powers of x with constant $= 0 + 6 = 6$

Sum of coefficient of odd powers of $x = 1 - 7 = -7$

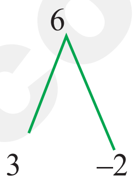
$$6 \neq -7$$

$\therefore (x + 1)$ is not a factor

To find the other factors, let us use synthetic division.

$x^3 + x^2 - 14x - 24$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -7 & 6 \\ & 0 & 1 & 1 & -6 \\ \hline 2 & 1 & 1 & -6 & \boxed{0} \\ & 0 & 2 & 6 & \\ \hline & 1 & 3 & 0 & \end{array}$$



$\therefore$ The factors are $(x - 1), (x - 2), (x + 3)$

$$\therefore x^3 + 0x^2 - 7x + 6 = (x - 1)(x - 2)(x + 3)$$

(vi) $x^3 - 10x^2 - x + 10$

$$\begin{aligned} \text{Let } p(x) &= x^3 - 10x^2 - x + 10 \\ \text{Sum of the co-efficients} &= 1 - 0 - 1 + 10 \\ &= 11 - 11 = 0 \end{aligned}$$

$\therefore (x - 1)$ is a factor

Sum of co-efficients of even powers of x with constant $= -10 + 10 = 0$

Sum of co-efficients of odd powers of $= 1 - 1 = 0$

$\therefore (x + 1)$ is a factor

Synthetic division

$$\begin{array}{r|rrrr} -1 & 1 & -10 & -1 & 10 \\ & 0 & 1 & -9 & -10 \\ \hline -1 & 1 & -9 & -10 & \boxed{0} \\ & 0 & -1 & 10 & \\ \hline & 1 & -10 & \boxed{0} & \end{array}$$

$$\therefore x^3 + 10x^2 - x + 10 = (x - 1)(x + 1)(x - 10)$$

Exercise 3.9

1. Find the GCD for the following:

(i)	p^5, p^{11}, p^9	**(ii)**	$4x^3, y^3, z^3$
(iii)	$9a^2b^2c^3, 15a^3b^2c^4$	**(iv)**	$64x^8, 240x^6$
(v)	$ab^2c^3, a^2b^3c, a^3bc^2$	**(vi)**	$35x^5y^3z^4, 49x^2yz^3, 14xy^2z^2$
(vii)	$25ab^3c, 100a^2bc, 125ab$	**(viii)**	$3abc, 5xyz, 7pqr$

Sol. (i) p^5, p^{11}, p^9

$$\text{G.C.D. of } p^5, p^{11}, p^9 = p^5$$

$$\boxed{\begin{aligned} p^5 &= \underline{P^5} \\ p^9 &= \underline{P^5} \times p^4 \\ p^{11} &= \underline{P^5} \times p^6 \end{aligned}}$$

(ii) $4x^3, y^3, z^3$

$$\begin{aligned} \text{G.C.D of } 4x^3 &= \underline{1} \times 4x^3 \\ y^3 &= \underline{1} \times y^3 \\ z^3 &= \underline{1} \times z^3 \\ \therefore \text{G.C.D} &= 1 \end{aligned}$$

(iii) $9a^2b^2c^3$, $15a^3b^2c^4$

$$9a^2b^2c^3 = \underline{3} \times 3a^2b^2c^3$$
$$15a^3b^2c^4 = \underline{3} \times 5a^3b^2c^4$$
$$\text{G.C.D. of } 9a^2b^2c^3, 15a^3b^2c^4 = 3 \times a^2b^2c^3 = 3a^2b^2c^3$$

(iv) $64x^8$, $240x^6$

$$64x^8 = \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{x^6} \times x^2$$
$$240x^6 = \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times 3 \times 5\,\underline{x^6}$$
$$\text{G.C.D. of } 64x^8, 240x^6 \text{ is} = 2 \times 2 \times 2 \times 2\,x^6 = 16x^6$$

2	240
2	120
2	60
2	30
3	15
5	5
	1

(v) ab^2c^3, a^2b^3c, a^3bc^2

$$ab^2c^3 = \underline{a} \times \underline{b} \times b \times \underline{c} \times c \times c$$
$$a^2b^3c = \underline{a} \times a \times \underline{b} \times \underline{b} \times b \times \underline{c}$$
$$a^3bc^2 = \underline{a} \times \underline{a} \times a \times \underline{b} \times \underline{c} \times c$$
$$\text{G.C.D. of } ab^2c^3, a^2b^3c, a^3bc^2 \text{ is} = a \times b \times c = abc$$

(vi) $35x^5y^3z^4$, $49x^2yz^3$, $14xy^2z^2$

$$35x^5y^3z^4 = 5 \times \underline{7}\,x^5y^3z^4$$
$$49x^2yz^3 = \underline{7} \times 7x^2y\,z^3$$
$$14xy^2z^2 = 2 \times \underline{7}\,x\,y^2\,z^2$$
$$\text{G.C.D is} = 7x\,y\,z^2$$

(vii) $25ab^3c$, $100a^2bc$, $125ab$

$$25ab^3c = \underline{5} \times \underline{5}\,ab^3c$$
$$100a^2bc = 2 \times \underline{5} \times 2 \times \underline{5}\,a^2\,b\,c$$
$$125\,ab = \underline{5} \times \underline{5} \times 5\,ab$$
$$\text{G.C.D is} = 5 \times 5\,ab = 25\,ab$$

(viii) $3abc$, $5xyz$, $7pqr$

$$3\,abc = \underline{1} \times \underline{3}\,a\,bc$$
$$5\,xyz = \underline{1} \times 5\,x\,y\,z$$
$$7\,p\,q\,r = \underline{1} \times 7\,p\,q\,r$$
$$\text{G.C.D is} = 1$$

2. Find the GCD of the following :

(i) $(2x + 5), (5x + 2)$ (ii) $a^{m+1}, a^{m+2}, a^{m+3}$

(iii) $2a^2 + a, 4a^2 - 1$ (iv) $3a^2, 5b^3, 7c^4$

(v) $x^4 - 1, x^2 - 1$ (vi) $a^3 - 9ax^2, (a - 3x)^2$

Sol. (i) $(2x + 5), (5x + 2)$

$$(2x + 5) = \underline{1} \times (2x + 5)$$
$$(5x + 2) = \underline{1} \times (5x + 2)$$
$$\therefore \text{G.C.D} = 1$$

(ii) $a^{m+1}, a^{m+2}, a^{m+3}$

$$a^{m+1} = \underline{a^m} \times \underline{a^1}$$
$$a^{m+2} = \underline{a^m} \times \underline{a^1} \times \underline{a^1}$$
$$a^{m+3} = \underline{a^m} \times \underline{a^1} \times a^1 \times a^1$$
$$\text{G.C.D} = a^m \times a^1 = a^{m+1}$$

(iii) $2a^2 + a, 4a^2 - 1$

$$2a^2 + a = a(2a+1)$$
$$4a^2 - 1 = (2a)^2 - 1^2 = (2a+1)(2a-1)$$
$$\text{G.C.D} = (2a+1)$$

(iv) $3a^2, 5b^3, 7c^4$

$$3a^2 = \underline{1} \times 3a \times a$$
$$5b^3 = \underline{1} \times 5\,b \times b \times b$$
$$7c^4 = \underline{1} \times 7\,c \times c \times c \times c$$
$$\therefore \text{G.C.D} = 1$$

(v) $x^4 - 1, \ x^2 - 1$

$$x^4 - 1 = (x^2)^2 - 1^2 = (x^2+1)(x^2-1)$$
$$= (x^2+1)(x^2-1^2) = (x^2+1)\,\underline{(x+1)}\,\underline{(x-1)}$$
$$x^2 - 1 = x^2 - 1^2 = (x+1)(x-1)$$
$$\text{G.C.D} = (x+1)(x-1) = x^2 - 1$$

(vi) $a^3 - 9ax^2, (a-3x)^2$

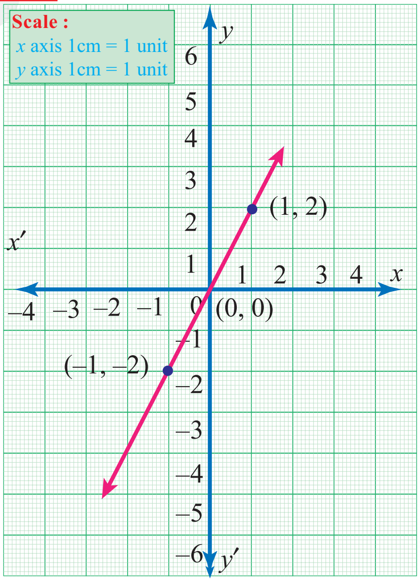
$$a^3 - 9ax^2 = a(a^2 - (3x)^2) = a(a+3x)\,\underline{(a-3x)}$$
$$(a-3x)^2 = \underline{(a-3x)}\,(a-3x)$$
$$\text{G.C.D} = (a-3x)$$

Exercise 3.10

1. Draw the graph for the following
(i) $y = 2x$ (ii) $y = 4x - 1$ (iii) $y = \left(\frac{3}{2}\right)x + 3$
(iv) $3x + 2y = 14$

Sol. (i)
Put $x = -1$, $y = 2 \times -1 = -2$
When $x = 0$, $y = 2 \times 0 = 0$
When $x = 1$, $y = 2 \times 1 = 2$

x	-1	0	1
y	-2	0	2



The points (x, y) to be plotted: $(-1, -2), (0, 0), (1, 2)$

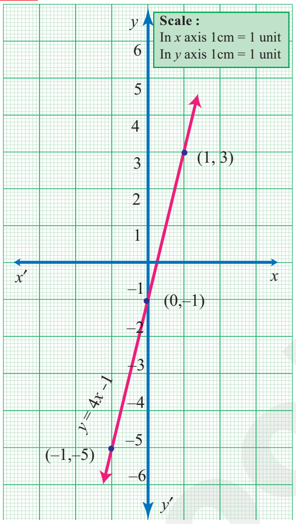
(ii) When $x = -1 \Rightarrow y = 4\,(-1) - 1$

$$y = -4 - 1 = -5$$

$$x = 0 \Rightarrow y = 4 \times 0 - 1 = -1$$

$$x = 1 \Rightarrow y = 4 \times 1 - 1 = 3$$

x	-1	0	1
y	-5	-1	3



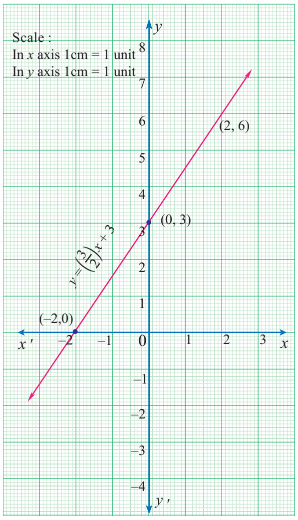
The points (x, y) to be plotted:
$(-1, -5), (0, -1), (1, 3)$

(iii) $x = -2 \Rightarrow \quad y = \left(\dfrac{3}{2}\right)(-2) + 3 = -3 + 3 = 0$

$$x = 0 \Rightarrow y = \left(\frac{3}{2}\right)(0) + 3 = 0 + 3 = 3$$

$$x = 2 \Rightarrow y = \left(\frac{3}{2}\right)(2) + 3 = 3 + 3 = 6$$

x	-2	0	2
y	0	3	6



The points to be plotted: $(-2, 0), (0, 3), (2, 6)$

(iv) $2y = -3x + 14$

$$y = \frac{-3x+14}{2}$$

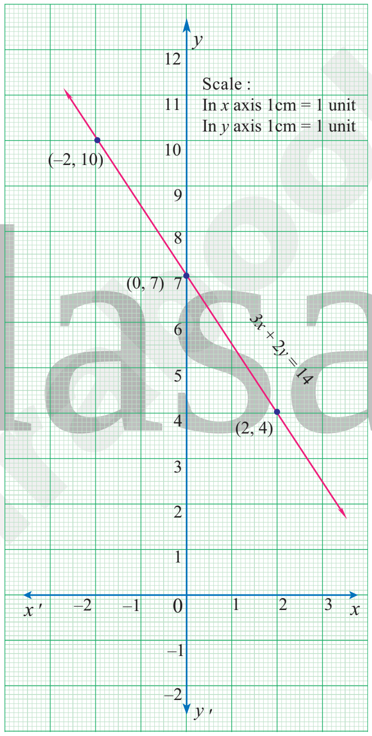
$$\Rightarrow \quad y = \left(\frac{-3}{2}\right)x + 7$$

When $x = -2$, $y = \left(\frac{-3}{2}\right)(-2) + 7 = 3 + 7 = 10$

When $x = 0$, $y = \left(\frac{-3}{2}\right)(0) + 7 = 7$

When $x = 2$, $y = -3 + 7 = 4$

x	–2	0	2
y	10	7	4



The points to be plotted: (–2, 10), (0, 7), (2, 4)

2. Solve graphically

(i) $x + y = 7; x - y = 3$ **(ii)** $3x + 2y = 4; 9x + 6y - 12 = 0$

(iii) $\frac{x}{2} + \frac{y}{4} = 1; \frac{x}{2} + \frac{y}{4} = 2$ **(vi)** $x - y = 0; y + 3 = 0$

(v) $y = 2x + 1; y + 3x - 6 = 0$ **(vi)** $x = -3, y = 3$

Sol. (i) We can find x and y intercepts and thus of the two points on the lines (1), (2)

$x + y = 7$ ... (1) , $x - y = 3$ ... (2)

To draw the graph of (1)

Put $x = 0$ in (1)

$0 + y = 7 \Rightarrow y = 7$

Thus A (0, 7) is a point on the line

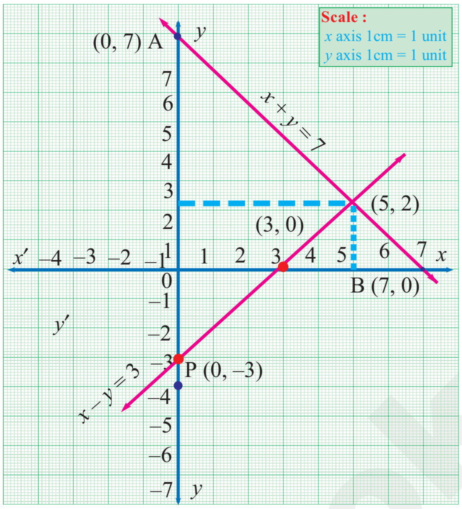
Put $y = 0$ in (1)

$x + 0 = 7 \Rightarrow x = 7$

Thus B (7, 0) is another point on the line

Plot A and B. Join them to produce the line (1).

To draw the graph of (2), we can adopt the same procedure.



When $x = 0$, (2) $\Rightarrow x - y = 3$

$0 - y = 3 \Rightarrow y = -3$

P (0, –3) is a point on the line.

Put $y = 0$ in (2) ; $x - 0 = 3$

$x = 3$

$\therefore$ Q (3, 0) is another point on the line (2)

Plot P, Q

The point of intersection (5, 2) of lines (1), (2) is a solution

(ii) $3x + 2y = 4$ .... (1)

$9x + 6y = 12$............ (2)

To draw the graph of (1)

Put $x = 0$ in (1) $\Rightarrow 3(0) + 2y = 4$

$2y = 4$

$y = 2$

$\therefore$ A (0, 2) is a point on the line (1)

Put $y = 0$ in (1) $\Rightarrow 3x + 2(0) = 4$

$3x = 4$

$x = \frac{4}{3} = 1.3$

$\therefore$ B (1.3, 0) is another point on the line (1)

Plot the points A, B. Join them to produce the line (1)

To draw the graph of (2)

Put $x = 0$ in (2) $\Rightarrow 9(0) + 6y = 12$

$6y = 12$

$y = 2$

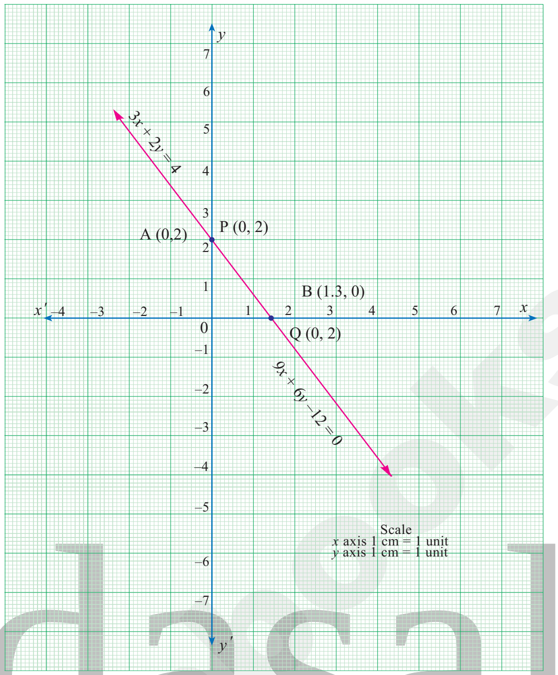
$\therefore$ P (0, 2) is a point on the line (2)

Put $y = 0$ in (2) $\Rightarrow 9x + 6(0) = 12$

$$9x = 12$$

$$x = \frac{\cancel{12}^{4}}{\cancel{9}_{3}} = \frac{4}{3}$$

$$x = 1.3$$



Q (1.3,0) is another point on the line (2)
Plot P, Q. Join them to produce the line (2).
The point of intersection of the lines (1), (2) is a solution.
[A, B],[P, Q] represent the same line.
∴ All the points on one line are also on the other.
This means we have an infinite number of solutions.

(iii) $\frac{x}{2} + \frac{y}{4} = 1$..........................(1)

$\frac{x}{2} + \frac{y}{4} = 2$..........................(2)

$(1) \Rightarrow \frac{x}{2} + \frac{y}{4} = 1 \Rightarrow \frac{2x+y}{4} = 1$

$\Rightarrow 2x + y = 4$

$y = -2x + 4$

$\therefore \quad (1) \Rightarrow y = -2x + 4$

$(2) \Rightarrow \frac{x}{2} + \frac{y}{4} = 2$

$\frac{2x+y}{4} = 2 \Rightarrow 2x + y = 8$

$(2) \Rightarrow y = -2x + 8$

∴ Comparing (1), (2) we can conclude their slopes are equal

∴ The lines are parallel and will not meet at any point and hence no solution exists.

Let us draw the graphs of (1) & (2)

(1) $\Rightarrow 2x + y = 4$, Put $x = 0$ in (1) $\Rightarrow y = 4$

$\therefore$ A (0, 4) is a point on (1)

Put $y = 0$ in (1) $\Rightarrow 2x = 4$

$x = 2$

$\therefore$ B (2, 0) is another point on (1)

Plot A, B ; Join them to produce the line (1)

(2) $\Rightarrow 2x + y = 8$, Put $x = 0$ in (2)

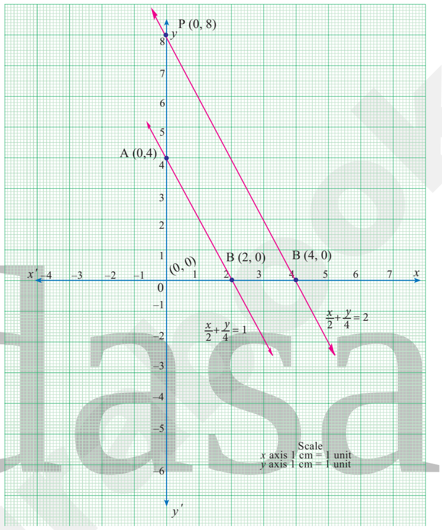
$2(0) + y = 8$

$y = 8$, P (0,8) is a point

Put $y = 0$ in (2) $\Rightarrow 2x + 0 = 8$

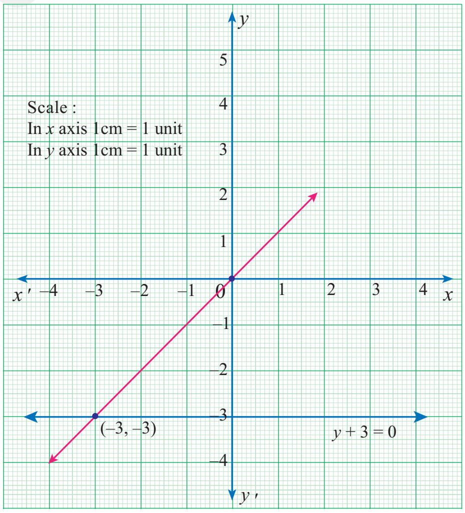
$2x = 8$

$x = 4$, Q (4, 0) is another point on the line (2)



Plot P, Q : Join them to produce the line (2)

(iv) $x - y = 0$.............. (1) $\Rightarrow -y = -x \Rightarrow y = x$



Put $x = 0$ in (1), $0 - y = 0$

$-y = 0 \Rightarrow y = 0$

A (0, 0) is a point on the line (1)

Put $y = 0$ in (1) $\Rightarrow x - 0 = 0 \Rightarrow x = 0$

B (–3, –3) is also the same point as A

$(2) \Rightarrow y + 3 = 0$

$y = -3$

$\therefore$ from (1) $y = -3 = x$

$\therefore$ B (–3, –3) is the solution

(v) $y = 2x + 1$ ...(1)

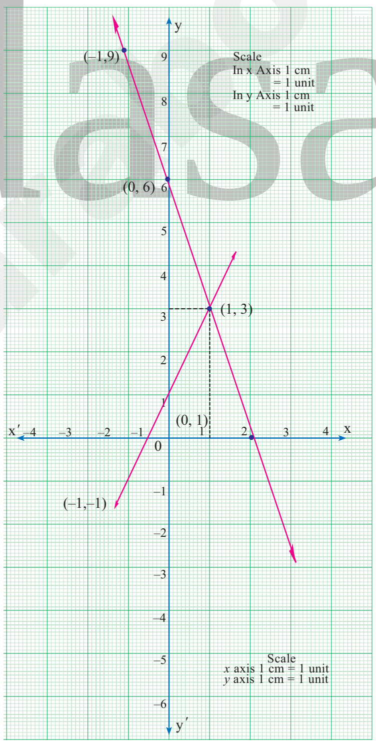
x	–1	0	1
$2x$	–2	0	2
1	1	1	1
$y = 2x + 1$	–1	1	3

$y = -3x + 6$ ...(2)

x	–1	0	1
$-3x$	3	0	–3
6	6	6	6
$y = -3x + 6$	9	6	3

The points to be plotted
(–1,–1), (0, 1), (1, 3)

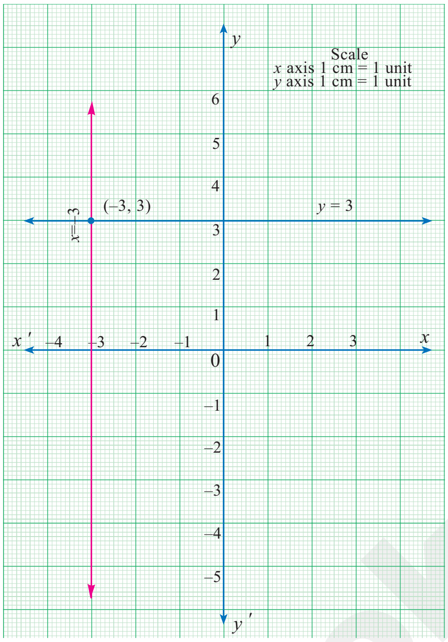
The points to be plotted
(–1, 9), (0, 6), (1, 3)



The points of intersection (1, 3) of the lines (1) and (2) is a solution. The solution is the point that is common to both the lines.

$\therefore$ The solution is as $x = 1, y = 3$.

(viii) The point of intersection (–3, 3) is a solution.



$x + y = 7$ ... (1) , $x - y = 3$ ... (2)

To draw the graph of (1)

Put $x = 0$ in (1)

$0 + y = 7 \Rightarrow y = 7$

Thus A (0, 7) is a point on the line

Put $y = 0$ in (1)

$x + 0 = 7 \Rightarrow x = 7$

Thus B (7, 0) is another point on the line

Plot A and B. Join them to produce the line (1).

To draw the graph of (2), we can adopt the same procedure.

When $x = 0$, ...(2) $\Rightarrow x - y = 3$

$0 - y = 3 \Rightarrow y = -3$

P (0, –3) is a point on the line.

Put $y = 0$ in (2) ; $x - 0 = 3$

$x = 3$

$\therefore$ Q (3, 0) is another point on the line (2) Plot P, Q

3. **Two cars are 100 miles apart. If they drive towards each other they will meet in 1 hour. If they drive in the same direction they will meet in 2 hours. Find their speed by using graphical method.**

Sol. Let x, y be the speed of the two cars. If the two cars travel towards each other they will meet in 1 hr. The distance between them $d = 100$; $\frac{d}{s} = t$

i.e., $\frac{100}{x+y} = 1 \Rightarrow x + y = 100$ ...(1)

If the two cars travel in the same direction they will meet in 2 hrs.

i.e., $x - y = \frac{\cancel{100}^{50}}{\cancel{2}} \Rightarrow x - y = 50$ ...(2)

(1) $\Rightarrow x + y = 100$

$y = -x + 100$

x	100	0
$-x$	-100	0
100	100	100
$y = -x + 100$	0	100

The points to be plotted (0, 100), (100, 0)

(2) $\Rightarrow x - y = 50 \Rightarrow -y = -x + 50 \Rightarrow y = x - 50$

x	100	0	1
-50	-50	-50	0
$y = x - 50$	50	-50	0

The points to be plotted (–100, –50), (0, –50),

$x + y = 100$ ...(1)

Put $x = 0$ in (1), then $0 + y = 100 \Rightarrow y = 100$

A (0, 100) is a point on (1)

Put $y = 0$ in (1), then $x + 0 = 100 \Rightarrow x = 100$

B (100, 0) is another point on (1)

Plot A & B. Join them to produce the line (1)

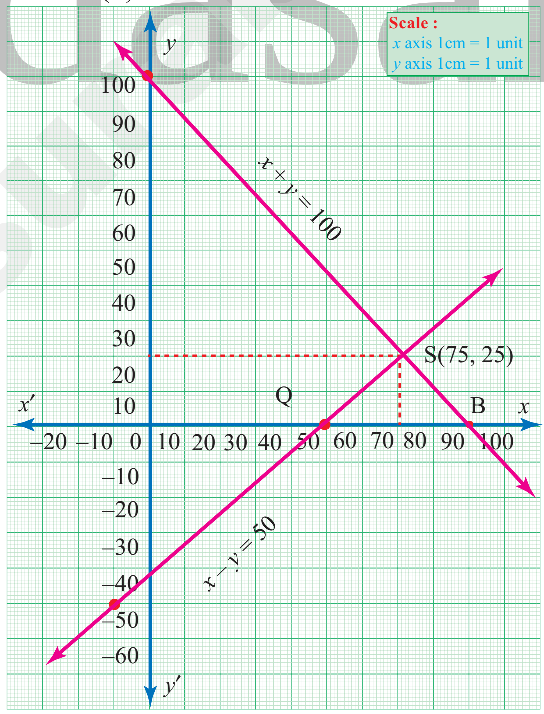
Similarly by $x - y = 50$ ...(2)

Put $x = 0$ in (2), then $0 - y = 50 \Rightarrow y = -50$

P (0, –50) is a point on (2)

Put $y = 0$ in (2), then $x - 0 = 50 \Rightarrow x = 50$

Q (50, 0) is another point on (2)



Plot P & Q. Join them to produce the line (2)

The point of intersection (75,25) of the two lines (1) & (2) is the solution.

∴ The solution i.e., the speed of the two cars x and y is given by $x = 75$ km and $y = 25$ km

Exercise 3.11

1. Solve, using the method of substitution

(i) $2x - 3y = 7$; $5x + y = 9$ (ii) $1.5x + 0.1y = 6.2$; $3x - 0.4y = 11.2$

(iii) 10% of x + 20% of $y = 24$; $3x - y = 20$

(iv) $\sqrt{2}x - \sqrt{3}y = 1$; $\sqrt{3}x - \sqrt{8}y = 0$

Sol. (i)

$$2x - 3y = 7 \quad ...(1)$$
$$5x + y = 9 \quad ...(2)$$

Step (1)

From the equation (2)

$$5x + y = 9$$
$$y = -5x + 9 \quad ...(3)$$

Step (2)

substitute (3) in (1)

$$2x - 3(-5x+9) = 7$$
$$2x + 15x - 27 = 7$$
$$17x = 7 + 27$$
$$17x = 34$$
$$x = \frac{34}{17} = 2; \; x = 2$$

Step (3)

substitute $x = 2$ in (3)

$$y = -5(2) + 9 = -10 + 9 = -1$$

Solution: $x = 2; y = -1$

(ii)

$$1.5x + 0.1y = 6.2 \quad ...(1)$$
$$3x - 0.4y = 11.2 \quad ...(2)$$

Multiply (1)×10 ⇒ $15x + y = 62$ ...(1)

(2) × 10 ⇒ $30x - 4y = 112$ ...(4)

Step (1)

From equation (3)

$$15x + y = 62$$
$$y = -15x + 62 \quad ....(5)$$

Step (2)

substitute (5) in (4)

$$30x - 4(-15x + 62) = 112$$
$$30x + 60x - 248 = 112$$
$$90x = 112 + 248$$
$$90x = 360$$
$$x = \frac{\cancel{360}^{4}}{\cancel{90}}$$
$$x = 4$$

Step (3)

substitute $x = 4$ in (5)

$$y = -15(4) + 62$$
$$= -60 + 62$$
$$y = 2$$

Solution: $x = 4; y = 2$

(iii) $\Rightarrow \frac{\cancel{10}}{\cancel{100}} x + \frac{\cancel{20}}{\cancel{100}_5} \frac{y}{5} = 24$

$$\frac{x}{10} + \frac{y}{5} = 24$$

$$\frac{x+2y}{10} = 24$$

$$x + 2y = 240 \quad ....(1)$$
$$3x - y = 20 \quad ....(2)$$

Step (1)

From equation (2)

$$3x - y = 20$$
$$-y = 20 - 3x$$
$$y = 3x - 20 - (3)$$

Step (2)

substitute (3) in (1)

$$x + 2(3x - 20) = 240$$
$$x + 6x - 40 = 240$$
$$7x = 240 + 40$$
$$x = \frac{280}{7}$$
$$x = 40$$

Step (3)

substitute $x = 40$ in (3)

$$y = 3(40) - 20$$
$$= 120 - 20 = 100$$

Solution : $x = 40$ and $y = 100$

(iv) $\sqrt{2}x - \sqrt{3}y = 1 \quad ....(1)$

$\sqrt{3}x - \sqrt{8}y = 0 \quad ....(2)$

Step (1)

From the equation (2)

$$\sqrt{3}x - \sqrt{8}y = 0$$
$$\cancel{-}\sqrt{8}y = \cancel{-}\sqrt{3}x$$
$$y = \frac{\sqrt{3}}{\sqrt{8}} x \quad ....(3)$$

Step (2)

substitute (3) in (1)

$$\sqrt{2}x - \sqrt{3}y = 1$$

$$\sqrt{2}x - \sqrt{3}\left(\frac{\sqrt{3}}{\sqrt{8}}\right)x = 1$$

$$\frac{\sqrt{2}x}{1} - \frac{3}{\sqrt{8}}x = 1$$

$$\frac{\sqrt{16}x - 3x}{\sqrt{8}} = \frac{4x-3x}{\sqrt{8}} = 1 \quad x = \sqrt{8}$$

Step (3)

substitute $x = \sqrt{8}$ in (1)

$$\sqrt{2}\left(\sqrt{8}\right) - \sqrt{3}y = 1$$

$$\sqrt{16} - \sqrt{3}y = 1$$

$$4 - \sqrt{3}y = 1$$

$$-\sqrt{3}y = 1 - 4$$

$$\cancel{-}\sqrt{3}y = \cancel{-}3 \quad y = \sqrt{3}$$

Solution: $x = \sqrt{8}$ and $y = \sqrt{3}$

2. **Raman's age is three times the sum of the ages of his two sons. After 5 years his age will be twice the sum of the ages of his two sons. Find the age of Raman.**

Sol. Let Raman's age = x

Let the sum of his two sons age = y

now $x = 3y \quad \Rightarrow x - 3y = 0$ ...(1)

After 5 years,

$$x + 5 = 2(y + 10)$$
$$x + 5 = 2y + 20$$
$$x - 2y = 20 - 5$$
$$x - 2y = 15 \quad ...(2)$$

Step (1)

From equation (1) $x = 3y$

Step (2)

Substitute $x = 3y$ in (2)

$$3y - 2y = 15$$
$$y = 15$$

Step (3)

Substitute $y = 15$ in (1)

$$x = 3y = 3 \times 15$$
$$x = 45$$

$\therefore$ **Raman's age is 45 years.**

3. The middle digit of a number between 100 and 1000 is zero and the sum of the other digit is 13. If the digits are reversed, the number so formed exceeds the original number by 495. Find the number.

Sol. Let the number be $x0y$

$$x + y = 13 \quad \ldots (1)$$

If the digits are reversed the number so formed is y0x

$$x0y = 100x + 10\times0 + 1\times y$$
$$y0x = 100y + 10\times0 + 1\times x$$
$$100y + x - (100x + y) = 495$$
$$100y + x - 100x - y = 495$$
$$-99x + 99y = 495 \quad \ldots(2)$$
$$(1) \times -99 \Rightarrow -99x - 99y = -1287$$
$$(2) \Rightarrow -99x + 99y = 495$$
$$\cancel{-}198x = \cancel{-}792$$
$$x = \frac{792}{198} = 4$$

Substitute $x = 4$ in (1)

$$4 + y = 13 = 13 - 4 = 9$$

The number is 409.

Exercise 3.12

1. Solve by the method of elimination

(i) $2x - y = 3; 3x + y = 7$ **(ii)** $x - y = 5; 3x + 2y = 25$

(iii) $\frac{x}{10} + \frac{y}{5} = 14; \frac{x}{8} + \frac{y}{6} = 15$ **(iv)** $3(2x + y) = 7xy; 3(x + 3y) = 11xy$

(v) $\frac{4}{x} + 5y = 7; \frac{3}{x} + 4y = 5$ **(vi)** $13x + 11y = 70; 11x + 13y = 74$

Sol. (i) $2x - y = 3 \quad \ldots (1)$

$3x + y = 7 \quad \ldots (2)$

$(1) + (2) \Rightarrow$

$$2x - y = 3$$
$$3x + y = 7$$
$$5x = 10$$
$$x = \frac{10}{5} = 2$$

Substitute $x = 2$ in (1)

$$2(2) - y = 3$$
$$4 - y = 3$$
$$-y = 3 - 4$$
$$-\cancel{y} = -\cancel{1}$$

$\therefore$ **Solution: $x = 2; y = 1$**

Verification:

Substitute $x = 2, y = 1$ in (2)

$3(2) + 1 = 7 = \text{RHS}$

$\therefore$ **Verified.**

(ii) $x - y = 5$ ... (1)

$$3x + 2y = 25 \quad \text{... (2)}$$

$$(1) \times 3 \Rightarrow \quad 3x - 3y = 15 \quad \text{... (3)}$$

$$(-) \quad (-) \quad (-)$$

$$(2) \Rightarrow \quad 3x + 2y = 25$$

$$(3) - (2) \quad -5y = -10$$

$$y = 2$$

Substitute $y = 2$ in (1)

$$x - 2 = 5$$

$$x = 5 + 2$$

$$x = 7$$

∴ **Solution: $x = 7, \quad y = 2$**

Verification:

Substitute $x = 7, y = 2$ in (2)

$3(7) + 2(2) = 21 + 4 = 25 = \text{RHS}$

∴ **Verified.**

(iii) $\frac{x}{10} + \frac{y}{5} = 14 \Rightarrow \frac{x+2y}{10} = 14$

$$x + 2y = 140 \quad \text{... (1)}$$

$$\frac{x}{8} + \frac{y}{6} = 15 \Rightarrow \frac{3x+4y}{24} = 15$$

$$3x + 4y = 360 \quad \text{... (2)}$$

$$(-) \quad (-) \quad (-)$$

$$(1) \times 3 \Rightarrow \quad 3x + 6y = 420 \quad \text{... (3)}$$

$$\cancel{-}2y = \cancel{-}60$$

$$y = 30$$

Substitute $y = 30$ in (1)

$$x + 2(30) = 140$$

$$x + 60 = 140$$

$$x = 140 - 60$$

$$x = 80$$

∴ **Solution: $x = 80; y = 30$**

Verification:

Substitute $x = 80, y = 30$ in (2)

$3(80) + 4(30) = 240 + 120 = 360 = \text{RHS}$

∴ **Verified.**

(iv) $3(2x + y) = 7xy \Rightarrow 6x + 3y = 7xy$ ... (1)

$3(x + 3y) = 11xy \Rightarrow 3x + 9y = 11xy$ ... (2)

$$(1) \div xy \Rightarrow \quad \frac{6\cancel{x}}{\cancel{x}y} + \frac{3\cancel{y}}{x\cancel{y}} = \frac{7\cancel{xy}}{\cancel{xy}}$$

$$\frac{6}{y}+\frac{3}{x} = 7 \qquad \text{...(3)}$$

$$(2) \div xy \Rightarrow \quad \frac{3\cancel{x}}{\cancel{x}y}+\frac{9\cancel{y}}{x\cancel{y}} = \frac{11\cancel{xy}}{\cancel{xy}}$$

$$\frac{3}{y}+\frac{9}{x} = 11 \qquad \text{... (4)}$$

In (3), (4) Put $\frac{1}{x} = a, \frac{1}{y} = b$

$$(3) \Rightarrow \quad 6b + 3a = 7 \qquad \text{... (5)}$$
$$(4) \Rightarrow \quad 3b + 9a = 11 \qquad \text{... (6)}$$
$$(6) \times 2 \Rightarrow \quad 6b + 18a = 22$$
$$(-) \quad (-) \quad (-)$$
$$(5) \Rightarrow \quad 6b + 3a = 7$$
$$(3) - (2) \quad 15a = 15$$
$$a = \frac{15}{15} = 1$$

Substitute $a = 1$ in (5)

$$6b + 3(1) = 7$$
$$6b + 3 = 7$$
$$6b = 7 - 3$$
$$b = \frac{4}{6} = \frac{2}{3}$$
$$\therefore \quad a = \frac{1}{x} = 1 \Rightarrow x = 1$$
$$b = \frac{1}{y} = \frac{2}{3} \Rightarrow y = \frac{3}{2}$$

$\therefore$ **Solution: $x = 1; y = \frac{3}{2}$**

(v) Put $\frac{1}{x} = a$

$$\frac{4}{x} + 5y = 7 \Rightarrow 4a + 5y = 7 \qquad \text{... (1)}$$
$$\frac{3}{x} + 4y = 5 \Rightarrow 3a + 4y = 5 \qquad \text{... (2)}$$
$$(1) \times 3 \Rightarrow \quad 12a + 15y = 21$$
$$(-) \quad (-) \quad (-)$$
$$(2) \times 4 \Rightarrow \quad 12a + 16y = 20$$
$$-1y = 1$$
$$y = -1$$

Substitute $y = -1$ in (1)

$$4a + 5(-1) = 7$$
$$4a - 5 = 7$$
$$4a = 7 + 5$$
$$a = \frac{12}{4} = 3$$

$$a = \frac{1}{x} = 3 \Rightarrow x = \frac{1}{3}$$

$$\therefore y = -1$$

$\therefore$ **Solution:** $x = \frac{1}{3}; y = -1$

(vi)
$$13x + 11y = 70 \quad \text{... (1)}$$
$$11x + 13y = 74 \quad \text{... (2)}$$

(1) ×11 ⇒ $143x + 121y = 770$ ... (3)

$\quad (-) \quad (-) \quad (-)$

(2) × 13 ⇒ $143x + 169y = 962$ ... (4)

(3) – (4) $\quad -48y = -192$

$$y = \frac{192}{48} = 4$$

Substitute $y = 4$ in (1)

$$13x + 11(4) = 70$$
$$13x + 44 = 70$$
$$13x = 70 - 44 = 26$$
$$x = \frac{26}{13} = 2$$

$\therefore$ **Solution:** $x = 2; y = 4$

2. The monthly income of A and B are in the ratio 3:4 and their monthly expenditures are in the ratio 5:7. If each saves ₹ 5,000 per month, find the monthly income of each.

Sol. Let the monthly income of A and B be $3x$ and $4x$ respectively.

Let the monthly expenditure of A and B be $5y$ and $7y$ respectively.

$\therefore$
$$3x - 5y = 5000 \quad \text{... (1)}$$
$$4x - 7y = 5000 \quad \text{...(2)}$$

(1) × 4 ⇒ $12x - 20y = 20000$

$\quad (-) \quad (+) \quad (-)$

(2) × 37 ⇒ $12x - 21y = 15000$

$$1y = 5000$$

Substitute $y = 5000$ in (1)

$$3x - 5(5000) = 5000$$
$$3x - 25000 = 5000$$
$$3x = 5000 + 25000$$
$$3x = 30000$$
$$x = 10000$$

$\therefore$ Monthly income of A is $3x = 3 \times 10000 =$ ₹ 30000

Monthly income of B is $4x = 4 \times 10000 =$ ₹ 40000

3. Five years ago, a man was seven times as old as his son, while five year hence, the man will be four times as old as his son. Find their present age.

Sol.

$$\begin{aligned} \text{Let the man's present age } &= x \\ \text{Five years ago his age is } &= x - 5 \\ \text{Let his son's age be } &= y \\ \text{5 years ago his son's age } &= y - 5 \end{aligned}$$

$$\begin{aligned} \therefore \quad x - 5 &= 7(y - 5) \\ x - 5 &= 7y - 35 \\ x - 7y &= -35 + 5 \\ x - 7y &= -30 \quad \text{...(1)} \end{aligned}$$

After 5 years, man's age will be $= x + 5$

His son's age will be $= y + 5$

$$\begin{aligned} \therefore \quad x + 5 &= 4(y + 5) \\ x + 5 &= 4y + 20 \\ x - 4y &= 20 - 5 \\ \Rightarrow x - 4y &= 15 \quad \text{... (2)} \\ (1) \Rightarrow x - 7y &= -30 \\ (-) \quad (+) \quad & \quad (-) \\ (2) \Rightarrow x - 4y &= 15 \\ \hline \cancel{-}3y &= \cancel{-}45 \\ y &= 15 \end{aligned}$$

Substitute $y = 15$ in (1)

$$\begin{aligned} x - 7(15) &= -30 \\ x - 105 &= -30 \\ x &= -30 + 105 \\ x &= 75 \end{aligned}$$

$\therefore$ Man's Age = 75, His son's Age = 15

Exercise 3.13

1. Solve by cross-multiplication method

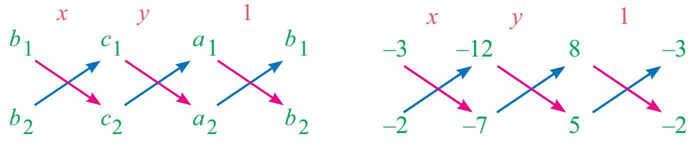
(i) $8x - 3y = 12$; $5x = 2y + 7$ (ii) $6x + 7y - 11 = 0$; $5x + 2y = 13$

(iii) $\frac{2}{x} + \frac{3}{y} = 5$; $\frac{3}{x} - \frac{1}{y} + 9 = 0$

Sol. (i)

$$\begin{aligned} 8x - 3y &= 12 \quad \text{... (1)} \\ 5x - 2y &= 7 \quad \text{... (2)} \\ 8x - 3y - 12 &= 0 \\ 5x - 2y - 7 &= 0 \end{aligned}$$

For cross multiplication method, we write the co-efficients as



$$\frac{x}{(-3)(-7)-(-2)(-12)} = \frac{y}{(-12)(5)-(-7)(8)} = \frac{1}{(8)(-2)-(5)(-3)}$$

$$\frac{x}{21-24} = \frac{y}{-60+56} = \frac{1}{-16+15}$$

$$\frac{x}{-3} = \frac{y}{-4} = \frac{1}{-1}$$

$$\therefore \frac{x}{-3} = \frac{1}{-1} \qquad \frac{y}{-4} = \frac{1}{-1}$$

$$-x = -3 \qquad -y = -4$$

$\therefore$ Solutions: $x = 3; y = 4$

(ii) $6x + 7y - 11 = 0$

$5x + 2y - 13 = 0$

For cross multiplication method, we write the co-efficients as

$$\begin{array}{cccc} & x & y & 1 \\ 7 & -11 & 6 & 7 \\ 2 & -13 & 5 & 2 \end{array}$$

$$\frac{x}{-91-(-22)} = \frac{y}{-55-(-68)} = \frac{1}{12-35}$$

$$\frac{x}{-91+22} = \frac{y}{-55+78} = \frac{1}{-23}$$

$$\frac{x}{-69} = \frac{y}{23} = \frac{1}{-23}$$

$$\therefore \frac{x}{-69} = \frac{1}{-23} \qquad \frac{y}{23} = \frac{1}{-23}$$

$$x = \frac{\cancel{-}\cancel{69}^{3}}{\cancel{-}\cancel{23}} \qquad y = \frac{\cancel{23}^{1}}{-\cancel{23}}$$

$$x = 3 \qquad y = -1$$

$\therefore$ Solutions: $x = 3; y = -1$

(iii) $\frac{2}{x} + \frac{3}{y} - 5 = 0$ ...(1)

$\frac{3}{x} - \frac{1}{y} + 9 = 0$ ... (2)

In (1), (2) Put $\frac{1}{x} = a, \frac{1}{y} = b$

(1) $\Rightarrow$ $2a + 3b - 5 = 0$

(2) $\Rightarrow$ $3a - b + 9 = 0$

For cross multiplication method, we write the co-efficients as

$$\frac{a}{(3)(9)-(-1)(-5)} = \frac{b}{(-5)(3)-(9)(2)} = \frac{1}{(2)(-1)-(3)(3)}$$

$$\frac{a}{27-5} = \frac{b}{-15-18} = \frac{1}{-2-9}$$

$$\frac{a}{22} = \frac{b}{-33} = \frac{1}{-11}$$

$$\therefore \frac{a}{22} = \frac{1}{-11} \qquad \frac{b}{-33} = \frac{1}{-11}$$

$$a = \frac{\cancel{22}^{2}}{-\cancel{11}} \qquad b = \frac{\cancel{-33}^{3}}{\cancel{-11}}$$

$$a = -2 \qquad b = 3$$

$$a = \frac{1}{x} = -2 \qquad b = \frac{1}{y} = 3$$

$$\therefore x = -\frac{1}{2} \qquad y = \frac{1}{3}$$

Solutions: $x = -\frac{1}{2}; y = \frac{1}{3}$

2. Akshaya has 2 rupee coins and 5 rupee coins in her purse. If in all she has 80 coins totalling ₹ 220, how many coins of each kind does she have.

Sol. Let the number of 2 rupee coins be x

Let the number of 5 rupee coins be y

$$x + y = 80$$
$$2x + 5y = 220$$
$$x + y - 80 = 0$$
$$2x + 5y - 220 = 0$$

For cross multiplication method, we write the co-efficients as

$$\begin{array}{cccccc} & x & & y & & 1 \\ 1 & & -80 & & 1 & & 1 \\ 5 & & -220 & & 2 & & 5 \end{array}$$

$$\frac{x}{-220+400} = \frac{y}{-160+220} = \frac{1}{5-2}$$

$$\frac{x}{180} = \frac{y}{60} = \frac{1}{3}$$

$$\therefore \frac{x}{180} = \frac{1}{3} \qquad \frac{y}{60} = \frac{1}{3}$$

$$x = \frac{\cancel{180}^{60}}{\cancel{3}} \qquad y = \frac{\cancel{60}^{20}}{\cancel{3}}$$

$$x = 60 \qquad y = 20$$

$\therefore$ No. of 2 rupee coins = 60 No. of 5 rupee coins = 20

3. It takes 24 hours to fill a swimming pool using two pipes. If the pipe of larger diameter is used for 8 hours and the pipe of the smaller diameter is used for 18 hours. Only half of the pool is filled. How long would each pipe take to fill the swimming pool.

Sol. Let the time taken by the larger pipe be x hours and Set the time taken by the smaller pipe be y hours.

$$\therefore \quad \frac{1}{x} + \frac{1}{y} = \frac{1}{24}$$

In 1 hour the larger pipe can fill it = $\frac{1}{x}$

In 1 hour the smaller pipe can fill it = $\frac{1}{y}$

$$\therefore \frac{8}{x}+\frac{18}{y} = \frac{1}{2}$$

Put $\frac{1}{x} = a$, $\frac{1}{y} = b$

$$a+b = \frac{1}{24}$$
$$24a + 24 = 1$$
$$24a + 24b - 1 = 0 \quad \text{---------} (1)$$
$$8a + 18b = \frac{1}{2}$$
$$16a + 36b = 1$$
$$16a + 36b - 1 = 0 \quad \text{----------} (2)$$

For cross multiplication method, we write the co-efficients as

$$\frac{a}{-24+36} = \frac{b}{-16+24} = \frac{1}{864-384}$$
$$\frac{a}{12} = \frac{b}{8} = \frac{1}{480}$$
$$\therefore \frac{a}{12} = \frac{1}{480}, \quad \frac{b}{8} = \frac{1}{480}$$
$$a = \frac{\cancel{12}}{\cancel{480}_{40}} = \frac{1}{40} \qquad b = \frac{\cancel{8}}{\cancel{480}_{60}} = \frac{1}{60}$$
$$\therefore x = 40 \qquad y = 60$$

$\therefore$ To fill the full tank the larger pipe takes 40hrs

To fill the full tank the smaller pipe takes 60hrs

Exercise 3.14

Solve by any one of the methods

1. **The sum of a two digit number and the number formed by interchanging the digits is 110. If 10 is subtracted from the first number, the new number is 4 more than 5 times the sums of the digits of the first number. Find the first number.**

Sol. Let the two digit number be $x\ y$

Its place value = $10x + y$

After interchanging the digits the number will be $y\ x$

Its place value = $10y + x$

Their sum = $10x + y + 10y + x = 110$

$$11x + 11y = 110$$
$$x + y = 10 \quad \ldots (1)$$

If 10 is subtracted from the first number, the new number is $10x + y - 10$

The sums of the digits of the first number is $x + y$.

$$\text{Its 4 more than 5 times is} = 5(x+y)+4$$
$$\therefore 10x + y - 10 = 5x + 5y + 4$$
$$10x + y - 5x - 5y = 4 + 10$$
$$5x - 4y = 14 \quad \dots (2)$$

$(1) \times 5 \Rightarrow$ $\quad 5x + 5y = 50$

$\quad (-) \quad (+) \quad (-)$

$(2) \Rightarrow$ $\quad 5x - 4y = 14$

$$9y = 36$$
$$y = \frac{\cancel{36}^{4}}{\cancel{9}} = 4$$

Substitute $y = 4$ in (1)

$$x + 4 = 10$$
$$x = 10 - 4$$
$$x = 6$$

$\therefore$ The first number is 64

2. The sum of the numerator and denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes $\frac{1}{2}$. Find the fraction.

Sol. Let the numerator be x

Denominator be y

$$x + y = 12 \quad \dots (1)$$
$$\frac{x}{y+3} = \frac{1}{2}$$
$$2x = y + 3$$
$$2x - y = 3 \quad \dots (2)$$

$(1)\times 2 \Rightarrow$ $\quad 2x + 2y = 24$

$\quad (-) \quad (+) \quad (-)$

$(2) \Rightarrow$ $\quad 2x - y = 3 \quad \dots(2)$

$$3y = 21$$
$$y = 7$$

Substitute $y = 7$ in (1)

$$x + 7 = 12$$
$$x = 12 - 7$$
$$x = 5$$

$\therefore$ The fraction is $\frac{x}{y} = \frac{5}{7}$

3. ABCD is a cyclic quadrilateral such that $\angle A = (4y + 20)°$, $\angle B = (3y - 5)°$, $\angle C = (4x)°$ and $\angle D = (7x + 5)°$. Find the four angles.

Sol. In a cyclic quadrilateral the sum of the four angles is 360° and the sum of the opposite angles is 180°.

$$\angle A + \angle C = 180°$$
$$4y + 20 + 4x = 180$$
$$4x + 4y = 180 - 20$$
$$x + y = \frac{\cancel{160}^{40}}{\cancel{4}}$$
$$x + y = 40 \quad \text{... (1)}$$
$$\angle B + \angle D = 180°$$
$$3y - 5 + 7x + 5 = 180$$
$$7x + 3y = 180 \quad \text{... (2)}$$
$$(1)\times 7 \Rightarrow \quad 7x + 7y = 280$$
$$(-) \quad (-) \quad (-)$$
$$(2) \Rightarrow \quad 7x + 3y = 180 \quad \text{...(2)}$$
$$4y = 100$$
$$y = 25°$$

Substitute $y = 25°$ in (1)

$$x + 25 = 40$$
$$x = 40 - 25$$
$$x = 15°$$
$$\angle A = (4y + 20)° = (4\times25 + 20) = 100 + 20 = 120°$$
$$\angle B = (3y - 5)° = (3\times25° - 5) = 75° - 5° = 70°$$
$$\angle C = (4x)° = 4\times 15° = 60°$$
$$\angle D = (7x + 5)° = (7\times15 + 5) = 105° + 5° = 110°$$

$\therefore$ $\angle A = 120°, \angle B = 70°, \angle C = 60°, \angle D = 110°$

4. On selling a T.V. at 5% gain and a fridge at 10% gain, a shopkeeper gains ₹2000. But if he sells the T.V. at 10% gain and the fridge at 5% loss, he gains ₹ 1500 on the transaction. Find the actual price of the T.V. and the fridge.

Sol. Let the actual price of a T.V. = x

Let the actual price of a Fridge = y

$$\frac{5}{100}x + \frac{10}{100}y = 2000$$
$$\frac{5}{100}(x + 2y) = 2000$$
$$x + 2y = 2000 \times \frac{100}{5}$$
$$x + 2y = 40000 \quad \text{... (1)}$$
$$\frac{10}{100}x - \frac{5}{100}y = 1500$$

$$\frac{5}{100}(2x-y) = 1500$$
$$2x - y = 1500 \times \frac{100}{5}$$
$$2x - y = 30000 \quad \dots (2)$$
$$(1)\times 2 \Rightarrow \quad 2x + 4y = 80000$$
$$\quad (-) \quad (+) \quad (-)$$
$$(2) \Rightarrow \quad 2x - y = 30000 \quad \dots (2)$$
$$5y = 50000$$
$$y = 10000$$

Substitute $y = 10000$ in (1)

$$x + 2(10000) = 40000$$
$$x + 20000 = 40000$$
$$x = 40000 - 20000$$
$$x = 20000$$

$\therefore$ **Actual price of T.V. = ₹ 20000**

Actual price of Fridge = ₹ 10000

5. **Two numbers are in the ratio 5 : 6. If 8 is subtracted from each of the numbers, the ratio becomes 4 : 5. Find the numbers.**

Sol. Let the two numbers be x, y

$$\frac{x}{y} = \frac{5}{6}$$
$$\Rightarrow \quad 6x = 5y$$
$$6x - 5y = 0 \quad \dots (1)$$
$$\frac{x-8}{y-8} = \frac{4}{5}$$
$$\Rightarrow \quad 5(x-8) = 4(y-8)$$
$$5x - 40 = 4y - 32$$
$$5x - 4y = 40 - 32$$
$$5x - 4y = 8 \quad \dots (2)$$
$$(1)\times 5 \Rightarrow \quad 30x - 25y = 0$$
$$\quad (-) \quad (+) \quad (-)$$
$$(2)\times 6 \Rightarrow \quad 30x - 24y = 48 \quad \dots (2)$$
$$-y = -48$$
$$y = 48$$

Substitute $y = 48$ in (1)

$$6x - 5(48) = 0$$
$$6x - 240 = 0$$
$$6x = 240$$

$$x = \frac{240}{6} = 40$$

$$\frac{x}{y} = \frac{40}{48} = \frac{5}{6}$$

$\therefore$ **The numbers are in the ratio 5 : 6**

6. **4 Indians and 4 Chinese can do a piece of work in 3 days. While 2 Indians and 5 Chinese can finish it in 4 days. How long would it take for 1 Indian to do it? How long would it take for 1 Chinese to do it?**

Sol. Let for one Indian the rate of working be $\frac{1}{x}$

Let for one Chinese the rate of working be $\frac{1}{y}$

$$\therefore \frac{4}{x}+\frac{4}{y} = \frac{1}{3}$$

$$\frac{2}{x}+\frac{5}{y} = \frac{1}{4}$$

Put $\frac{1}{x} = a, \frac{1}{y} = b$

$$4a + 4b = \frac{1}{3}$$

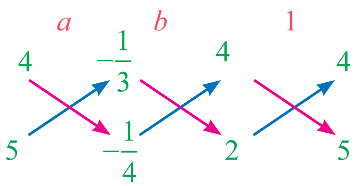
$$4a + 4b - \frac{1}{3} = 0 \quad \text{... (1)}$$

$$2a + 5b = \frac{1}{4}$$

$$2a + 5b - \frac{1}{4} = 0 \quad \text{... (2)}$$

For cross multiplication method, we write the co-efficients as

$$\frac{a}{-1+\frac{5}{3}} = \frac{b}{-\frac{2}{3}+1} = \frac{1}{20-8}$$



$$\frac{3a}{2} = 3b = \frac{1}{12}$$

$$\therefore \frac{3a}{2} = \frac{1}{12} \qquad 3b = \frac{1}{12}$$

$$a = \frac{1}{12} \times \frac{2}{3} = \frac{1}{18} \qquad b = \frac{1}{36}$$

$\therefore$ 1 Chinese can do the same piece of work $= \frac{1}{y} = b = \frac{1}{36}$ = **36 days**

1 Indian can do the piece of work $= \frac{1}{x} = a = \frac{1}{18}$ = **18 days**

Exercise 3.15

Multiple Choice Questions :

1. If $x^3 + 6x^2 + kx + 6$ is exactly divisible by $(x + 2)$, then $k = ?$

(1) –6 (2) –7 (3) –8 (4) 11

Hint :

$$P(-2) = (-2)^3 + 6(-2)^2 + k(-2) + 6 = 0$$
$$\Rightarrow -8 + 24 - 2k + 6 = 0$$
$$\Rightarrow 22 = 2k$$
$$\Rightarrow k = 11$$

[Ans: (4) 11]

2. The root of the polynomial equation $2x + 3 = 0$ is

(1) $\frac{1}{3}$ (2) $-\frac{1}{3}$ (3) $-\frac{3}{2}$ (4) $-\frac{2}{3}$

[Ans: (3) $-\frac{3}{2}$]

3. The type of the polynomial $4 - 3x^3$ is

(1) constant polynomial (2) linear polynomial
(3) quadratic polynomial (4) cubic polynomial.

Hint : Polynomial of degree 3 is called cubic. **[Ans: (4) cubic polynomial]**

4. If $x^{51} + 51$ is divided by $x + 1$, then the remainder is

(1) 0 (2) 1 (3) 49 (4) 50

Hint : $P(-1) = (-1)^{51} + 51 = -1 + 51 = 50$ **[Ans: (4) 50]**

5. The zero of the polynomial $2x + 5$ is

(1) $\frac{5}{2}$ (2) $-\frac{5}{2}$ (3) $\frac{2}{5}$ (4) $-\frac{2}{5}$

[Ans:(2) $-\frac{5}{2}$]

6. The sum of the polynomials $p(x) = x^3 - x^2 - 2$, $q(x) = x^2 - 3x + 1$

(1) $x^3 - 3x - 1$ (2) $x^3 + 2x^2 - 1$ (3) $x^3 - 2x^2 - 3x$ (4) $x^3 - 2x^2 + 3x - 1$

Hint : **[Ans: (1) $x^3 - 3x - 1$]**

$$\begin{array}{r} x^3 - \cancel{x^2} + 0x - 2 \\ \cancel{x^2} - 3x + 1 \\ \hline x^3 - 3x - 1 \end{array} +$$

7. Degree of the polynomial $(y^3 - 2)(y^3 + 1)$ is

(1) 9 (2) 2 (3) 3 (4) 6

Hint : $(y^3 - 2)(y^3 + 1) = (y^3 - 2)(y^3 - 2) \times 1 = y^6 - 2y^3 + y^3 - 2 = y^6 - y^3 - 2$

[Ans: (4) 6]

8. Let the polynomials be

(A) $-13q^5 + 4q^2 + 12q$ (B) $(x^2 + 4)(x^2 + 9)$

(C) $4q^8 - q^6 + q^2$ (D) $-\frac{5}{7}y^{12} + y^3 + y^5$

Then ascending order of their degree is

(1) A,B,D,C (2) A,B,C,D (3) B,C,D,A (4) B,A,C,D

Hint : Degree of (A), (B) (C) & (D) are respectively be 5, 4, 8, 12

[Ans: (4) B,A,C,D]

9. If $p(a) = 0$ then $(x - a)$ is a __________ of $p(x)$

(1) divisor (2) quotient (3) remainder (4) factor

[Ans. (4) factor]

10. Zeros of $(2 - 3x)$ is __________

(1) 3 (2) 2 (3) $\frac{2}{3}$ (4) $\frac{3}{2}$

Hint :
$$2 - 3x = 0$$
$$-3x = -2$$
$$x = \frac{2}{3}$$

[Ans. (3) $\frac{2}{3}$]

11. Which of the following has $x - 1$ as a factor?

(1) $2x - 1$ (2) $3x - 3$ (3) $4x - 3$ (4) $3x - 4$

Hint :
$$p(x) = 3x - 3$$
$$p(1) = 3(1) - 3 = 0$$

$\therefore$ $(x - 1)$ is a factor of $p(x)$ **[Ans. (2) $3x - 3$]**

12. If $x - 3$ is a factor of $p(x)$, then the remainder is

(1) 3 (2) -3 (3) $p(3)$ (4) $p(-3)$

[Ans. (3) $p(3)$]

13. $(x + y)(x^2 - xy + y^2)$ is equal to

(1) $(x + y)^3$ (2) $(x - y)^3$ (3) $x^3 + y^3$ (4) $x^3 - y^3$

[Ans. (3) $x^3 + y^3$]

14. $(a + b - c)^2$ is equal to __________

(1) $(a - b + c)^2$ (2) $(-a - b + c)^2$ (3) $(a + b + c)^2$ (4) $(a - b - c)^2$

Hint : $(a + b - c)^2 = [-(-a - b + c)]^2 = (-a - b + c)^2$

[Ans. (2) $(-a - b + c)^2$]

15. In an expression $ax^2 + bx + c$ the sum and product of the factors respectively,

(1) a, bc (2) b, ac (3) ac, b (4) bc, a

[Ans. (2) b, ac]

16. If $(x + 5)$ and $(x - 3)$ are the factors of $ax^2 + bx + c$, then values of a, b and c are

(1) 1,2,3 (2) 1,2,15 (3) 1,2, -15 (4) 1, -2,15

Hint :
$$p(-5) = a(-5^2) + b(-5) + c = 25a - 5b + c = 0 \quad \ldots (1)$$
$$p(3) = a(3^2) + bc + 3 + c = 9 + 3b + c = 0 \quad \ldots (2)$$
$$25a - 5b = 9a + 3b$$
$$25a - 9a = 3b + 5b$$
$$16a = 8b$$
$$\frac{a}{b} = \frac{8}{16} = \frac{1}{2}$$

Substitute $a = 1, b = 2$ in (1)
$$25(1) - 5(2) = -c$$
$$25 - 10 = 15 = -c$$
$$c = -15$$

[Ans. (3) 1, 2, -15]

17. **Cubic polynomial may have maximum of ___________ linear factors.**

(1) 1 (2) 2 (3) 3 (4) 4

[Ans. (3) 3]

18. **Degree of the constant polynomial is __________**

(1) 3 (2) 2 (3) 1 (4) 0

[Ans. (4) 0]

19. **Find the value of m from the equation $2x+3y = m$. If its one solution is $x = 2$ and $y = -2$.**

(1) 2 (2) -2 (3) 10 (4) 0

Hint : $2x + 3y = m,\quad x = 2, y = -2$

$$m = 2(2) + 3(-2)$$
$$= 4 - 6 = -2$$

[Ans. (2) –2]

20. **Which of the following is a linear equation?**

(1) $x + \frac{1}{x} = 2$ (2) $x(x-1) = 2$ (3) $3x + 5 = \frac{2}{3}$ (4) $x^3 - x = 5$

Hint : $x + \frac{1}{x} = 2 \Rightarrow x^2 - 2x + 1 = 0$; $x(x-1) = 2 \Rightarrow x^2 - x - 2 = 0$ **[Ans. (3) $3x + 5 = \frac{2}{3}$]**

21. **Which of the following is a solution of the equation $2x - y = 6$?**

(1) (2,4) (2) (4,2) (3) (3, −1) (4) (0,6)

Hint : $2x - y = 6$

$$2(4) - 2 = 8 - 2 = 6 = \text{RHS}$$

[Ans. (2) (4,2)]

22. **If (2,3) is a solution of linear equation $2x + 3y = k$ then, the value of k is**

(1) 12 (2) 6 (3) 0 (4) 13

Hint : $2x + 3y = k$

$$2(2) + 3(3) = 4 + 9 = 13$$

[Ans. (4) 13]

23. **Which condition does not satisfy the linear equation $ax + by + c = 0$**

(1) $a \neq 0, b = 0$ (2) $a = 0, b \neq 0$ (3) $a = 0, b = 0, c \neq 0$ (4) $a \neq 0, b \neq 0$

Hint : $a = 0, b = 0, c \neq 0 \Rightarrow (0)x + (0)y + c = 0$ False **[Ans. (3) $a = 0, b = 0, c \neq 0$]**

24. **Which of the following is not a linear equation in two variable**

(1) $ax + by + c = 0$ (2) $0x + 0y + c = 0$

(3) $0x + by + c = 0$ (4) $ax + 0y + c = 0$ **[Ans. (2) $0x + 0y + c = 0$]**

Hint : *a and b* both can not be zero

25. **The value of k for which the pair of linear equations $4x + 6y - 1 = 0$ and $2x + ky - 7 = 0$ represents parallel lines is**

(1) $k = 3$ (2) $k = 2$ (3) $k = 4$ (4) $k = -3$

Hint : $4x + 6y = 1$

$$6y = -4x + 1$$
$$y = \frac{-4}{6}x + \frac{1}{6} \quad \ldots(1)$$

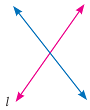
$$2x + ky - 7 = 0$$
$$ky = -2x + 7$$
$$y = \frac{-2}{k}x + \frac{7}{k} \quad ...(2)$$

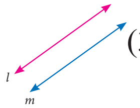
Since the lines (1) and (2) parallel

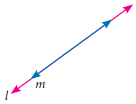
$$m_1 = m_2$$
$$\frac{-4}{6} = \frac{-2}{k}$$
$$k = \cancel{-2} \times \frac{\cancel{-6}^{\,3}}{\cancel{4}_{\cancel{2}}} = 3$$

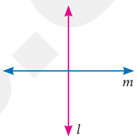
[Ans. (1) $k = 3$]

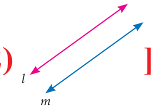
26. A pair of linear equations has no solution then the graphical representation is

(1) 

(2) 

(3) 

(4) 

Hint : Parallel lines have no solution **[Ans. (2)** 

]

27. If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ where $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ then the given pair of linear equation has __________ solution(s)

(1) no solution (2) two solutions (3) unique (4) infinite

Hint : $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$; unique solution **[Ans. (3) unique]**

28. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ where $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ then the given pair of linear equation has __________ solution(s).

(1) no solution (2) two solutions (3) infinite (4) unique

Hint : $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$: parallel **[Ans. (1) no solution]**

29. GCD of any two prime numbers is __________

(1) -1 (2) 0 (3) 1 (4) 2

[Ans. (3) 1]

30. The GCD of $x^4 - y^4$ and $x^2 - y^2$ is

(1) $x^4 - y^4$ (2) $x^2 - y^2$ (3) $(x + y)^2$ (4) $(x + y)^4$

Hint :
$$x^4 - y^4 = (x^2)^2 - (y^2)^2 = (x^2 + y^2)(x^2 - y^2)$$
$$x^2 - y^2 = x^2 - y^2$$
$$\text{G.C.D. is} = x^2 - y^2$$

[Ans. (2) $x^2 - y^2$]

Additional Questions and Answers

EXERCISE 3.1

1. Classify the following polynomials based on number of terms.

(i) $x^3 - x^2$	(ii) $5x$	(iii) $4x^4 + 2x^3 + 1$	(iv) $4x^3$
(v) $x + 2$	(vi) $3x^2$	(vii) $y^4 + 1$	(viii) $y^{20} + y^{18} + y^2$
(ix) 6	(x) $2u^3 + u^2 + 3$	(xi) $u^{23} - u^4$	(xii) y

Sol. $5x$, $3x^2$, $4x^3$, y and 6 are monomials because they have only one term.
$x^3 - x^2$, $x + 2$, $y^4 + 1$ and $u^{23} - u^4$ are binomials as they contain only two terms.
$4x^4 + 2x^3 + 1$, $y^{20} + y^{18} + y^2$ and $2u^3 + u^2 + 3$ are trinomials as they contain only three terms.

2. Classify the following polynomials based on their degree.

(i) $p(x) = 3$	(ii) $p(y) = \frac{5}{2}y^2 + 1$	(iii) $p(x) = 2x^3 - x^2 + 4x + 1$
(iv) $p(x) = 3x^2$	(v) $p(x) = x + 3$	(vi) $p(x) = -7$
(vii) $p(x) = x^3 + 1$	(viii) $p(x) = 5x^2 - 3x + 2$	(ix) $p(x) = 4x$
(x) $p(x) = \frac{3}{2}$	(xi) $p(x) = \sqrt{3}\,x + 1$	(xii) $p(y) = y^3 + 3y$

Sol. $p(x) = 3$, $p(x) = -7$, $p(x) = \frac{3}{2}$ are constant polynomials

$p(x) = x + 3$, $p(x) = 4x$, $p(x) = \sqrt{3}\,x + 1$ are linear polynomials, since the highest degree of the variable x is one.

$p(x) = 5x^2 - 3x + 2$, $p(y) = \frac{5}{2}y^2 + 1$, $p(x) = 3x^2$ are quadratic polynomials, since the highest degree of the variable is two.

$p(x) = 2x^3 - x^2 + 4x + 1$, $p(x) = x^3 + 1$, $p(y) = y^3 + 3y$ are cubic polynomials, since the highest degree of the variable is three.

3. Find the product of given polynomials $p(x) = 3x^3+2x-x^2+ 8$ and $q(x) = 7x + 2$.

Sol. $(7x+2)(3x^3 + 2x - x^2 + 8) = 7x(3x^3 + 2x - x^2 + 8) + 2x$
$(3x^3 + 2x - x^2+8) = 21x^4 + 14x^2-7x^3+ 56x+ 6x^3+ 4x - 2x^2+ 16 = 21x^4 - x^3 + 12x^2 + 60x + 16$

4. Let $P(x) = 4x^2 - 3x + 2x^3 +5$ and $q(x) = x^2+2x + 4$ find $p(x) - q(x)$.

Sol.

Given Polynomial	Standard form
$p(x) = 4x^2 - 3x + 2x^3 + 5$	$2x^2 + 4x - 3x + 5$
$q(x) = x^2 + 2x + 4$	$x^2 + 2x + 4$
$p(x) - q(x) = 2x^3 + 3x^2 - 5x + 1$	

EXERCISE 3.2

1. If $p(x) = 5x^3 - 3x^2 + 7x - 9$, find (i) $p(-1)$ (ii) $p(2)$.

Sol. Given that $p(x) = 5x^3 - 3x^2 + 7x - 9$

(i) $p(-1) = 5(-1)^3 - 3(-1)^2 + 7(-1) - 9 = -5 - 3 - 7 - 9$

$\therefore p(-1) = -24$

(ii) $p(2) = 5(2)^3 - 3(2)^2 + 7(2) - 9 = 40 - 12 + 14 - 9$

$\therefore p(2) = 33$

2. Find the zeros of the following polynomials.

(i) $p(x) = 2x - 3$ (ii) $p(x) = x - 2$

Sol. (i) Given that $p(x) = 2x - 3 = 2\left(x - \frac{3}{2}\right)$.

we have $p\left(\frac{3}{2}\right) = 2\left(\frac{3}{2} - \frac{3}{2}\right) = 2\,(0) = 0$

Hence $\frac{3}{2}$ is the zero of $p(x)$.

(ii) Given that $p(x) = x - 2$. Now,

$p(2) = 2 - 2 = 0$

Hence 2 is the zero of $p(x)$.

EXERCISE 3.3

1. Find the remainder using remainder theorem, when

(i) $4x^3 - 5x^2 + 6x - 2$ is divided by $x - 1$. (ii) $x^3 - 7x^2 - x + 6$ is divided by $x + 2$.

Sol. (i) Let $p(x) = 4x^3 - 5x^2 + 6x - 2$. The zero of $(x - 1)$ is 1.

When $p(x)$ is divided by $(x - 1)$ the remainder is $p(1)$. Now,

$p(1) = 4(1)^3 - 5(1)^2 + 6(1) - 2 = 4 - 5 + 6 - 2 = 3$

$\therefore$ The remainder is 3.

(ii) Let $p(x) = x^3 - 7x^2 - x + 6$. The zero of $x + 2$ is -2.

When $p(x)$ is divided by $x + 2$, the remainder is $p(-2)$. Now,

$p(-2) = (-2)^3 - 7(-2)^2 - (-2) + 6 = -8 - 7\,(4) + 2 + 6$

$= -8 - 28 + 2 + 6 = -28.$

$\therefore$ The remainder is -28.

2. Find the value of a if $2x^3 - 6x^2 + 5ax - 9$ leaves the remainder 13 when it is divided by $x - 2$.

Sol. Let $p(x) = 2x^3 - 6x^2 + 5ax - 9$

When $p(x)$ is divided by $(x - 2)$ the remainder is $p(2)$.

Given that $p(2) = 13$

$\Rightarrow 2(2)^3 - 6(2)^2 + 5a(2) - 9 = 13$

$\Rightarrow 2(8) - 6(4) + 10a - 9 = 13$

$\Rightarrow 16 - 24 + 10a - 9 = 13$

$$\Rightarrow \quad 10a - 17 = 13$$
$$\Rightarrow \quad 10a = 30$$
$$\Rightarrow \quad \therefore\ a = 3$$

3. If the polynomials $f(x) = ax^3 + 4x^2 + 3x - 4$ and $g(x) = x^3 - 4x + a$ leave the same remainder when divided by $x - 3$. Find the value of a. Also find the remainder.

Sol. Let $f(x) = ax^3 + 4x^2 + 3x - 4$ and $g(x) = x^3 - 4x + a$, When $f(x)$ is divided by $(x - 3)$, the remainder is $f(3)$.

$$\text{Now } f(3) = a(3)^3 4(3)^2 + 3(3) - 4 = 27a + 36 + 9 - 4$$
$$f(3) = 27a + 41 \quad \text{...(1)}$$

When g(x) is divided by $(x - 3)$, the remainder is g(3).

$$\text{Now } g(3) = 3^3 - 4(3) + a = 27 - 12 + a = 15 + a \quad \text{...(2)}$$

Since the remainders are same, (1) = (2)

Given that, $f(3) = g(3)$

That is $27a + 41 = 15 + a$

$$27a - a = 15 - 41$$
$$26a = -26$$
$$a = \frac{-26}{26} = -1$$

Substituting $a = -1$, in $f(3)$, we get

$$f(3) = 27(-1) + 41 = -27 + 41$$
$$f(3) = 14$$

$\therefore$ The remainder is 14.

4. Show that $x + 4$ is a factor of $x^3 + 6x^2 - 7x - 60$.

Sol. Let $p(x) = x^3 + 6x^2 - 7x - 60$

By factor theorem $(x + 4)$ is a factor of $p(x)$, if $p(-4) = 0$

$$p(-4) = (-4)^3 + 6(-4)^2 - 7(-4) - 60 = -64 + 96 + 28 - 60 = 0$$

Therefore, $(x + 4)$ is a factor of $x^3 + 6x^2 + - 7x - 60$

5. In $(5x + 4)$ a factor of $5x^3 + 14x^2 - 32x - 32$.

Sol. Let $p(x) = 5x^3 + 14x^2 - 32x - 32$

By factor theorem, $5x + 4$ is a factor, if $p\left(\frac{-4}{5}\right) = 0$

$$p\left(\frac{-4}{5}\right) = 5\left(\frac{-4}{5}\right)^3 + 14\left(\frac{-4}{5}\right)^2 - \left(32\left(\frac{-4}{5}\right) - 32\right)$$
$$= 5\left(\frac{-64}{125}\right) + 14\left(\frac{16}{25}\right) + 32\left(\frac{4}{5}\right) - 32$$
$$= \frac{-64}{25} + \frac{224}{25} + \frac{128}{5} - 32 = \frac{-65}{25} + \frac{224}{25} + \frac{640}{25} - \frac{800}{25}$$
$$= \frac{-65 + 224 + 640 - 800}{25} = 0$$
$$p\left(\frac{-4}{5}\right) = 0$$

Therefore, $5x + 4$ is a factor of $5x^3 + 14x^2 - 32x - 32$

6. Find the value of k, if $(x-3)$ is a factor of polynomial $x^3 - 9x^2 + 26x + k$.

Sol. Let $p(x) = x^3 - 9x^2 + 26x + k$

By factor theorem, $(x-3)$ is a factor of $p(x)$, if $p(3) = 0$

$$p(3) = 0$$
$$3^3 - 9(3)^2 + 26(3) + k = 0$$
$$27 - 81 + 78 + k = 0$$
$$k = -24$$

To find the zero of $x-3$:
Put $x - 3 = 0$
we get $x = 3$

7. Show that $(x-3)$ is a factor of $x^3 + 9x^2 - x - 105$.

Sol. Let $p(x) = x^3 + 9x^2 - x - 105$

By factor theorem, $x-3$ is a factor of $p(x)$, if $p(3) = 0$

$$p(3) = 3^3 + 9(3)^3 - 3 - 105$$
$$= 27 + 81 - 3 - 105$$
$$= 108 - 108$$
$$p(3) = 0$$

To find the zero of $x-3$:
Put $x - 3 = 0$
we get $x = 3$

Therefore, $x-3$ is a factor of $x^3 + 9x^2 - x - 105$.

8. Show that $(x+2)$ is a factor of $x^3 - 4x^2 - 2x + 20$.

Sol. Let $p(x) = x^3 - 4x^2 - 2x + 20$

By factor theorem,
$(x+2)$ is factor of $p(x)$, if $p(-2) = 0$

$$p(-2) = (-2)^3 - 4(-2)^2 - 2(-2) + 20 = -8 - 4(4) + 4 + 20$$
$$p(-2) = 0$$

Therefore, $(x+2)$ is a factor of $x^3 - 4x^2 - 2x + 20$

EXERCISE 3.4

1. Expand the following using identities :

(i) $(7x + 2y)^2$ (ii) $(4m - 3m)^2$ (iii) $(4a + 3b)(4a - 3b)$

(iv) $(k + 2)(k - 3)$

Sol. (i) $(7x + 2y)^2 = (7x)^2 + 2(7x)(2y) + (2y)^2 = 49x^2 + 28xy + 4y^2$

(ii) $(4m - 3m)^2 = (4m)^2 - 2(4m)(3m) + (3m)^2 = 16m^2 - 24\,mn + 9n^2$

(iii) $(4a + 3b)(4a - 3b)$ [We have $(a + b)(a - b) = a^2 - b^2$]

Put $[a = 4a,\ b = 3b]$

$(4a + 3b)(4a - 3b) = (4a)^2 - (3b)^2 = 16a^2 - 9b^2$

(iv) $(k + 2)(k - 3)$ [We have $(x + a)(x - b) = x^2 + (a - b)x - ab$]

Put $[x = k,\ a = 2,\ b = 3]$

$(k + 2)(k - 3) = k^2 + (2 - 3)x - 2 \times 3 = k^2 - x - 6$

2. Expand : $(a + b - c)^2$

Sol. Replacing 'c' by '$-c$' in the expansion of

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$
$$(a + b + (-c))^2 = a^2 + b^2 + (-c)^2 + 2ab + 2b(-c) + 2(-c)a$$
$$= a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$$

3. Expand : $(x + 2y + 3z)^2$

Sol. We know that,

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

Substituting, $a = x,\ b = 2y$ and $c = 3z$

$$(x+2y+3z)^2 = x^2 + (2y)^2 + (3z)^2 + 2\,(x)\,(2y) + 2\,(2y)\,(3z) + 2(3z)\,(x)$$
$$= x^2 + 4y^2 + 9z^2 + 4\,xy + 12y^2 + 6zx$$

4. Find the area of square whose side length is $m + n - q$.

Sol. Area of square $=$ side $\times$ side

$$= (m+n-q)^2$$

We know that,

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$
$$[m+n+(-q)]^2 = m^2 + n^2 + (-q)^2 + 2\,mn + 2n\,(-q) + 2\,(-q)m$$
$$= m^2 + n^2 + q^2 + 2\,mn - 2nq - 2\,qm$$

Therefore, Area of square $= [m^2 + n^2 + q^2 + 2\,mn - 2nq - 2\,qm]$ sq. units.

EXERCISE 3.5

1. Factorise the following

(i) $25m^{-2} - 16n^2$ (ii) $x^4 - 9x^2$

Sol. (i) $25m^2 - 16n^2 = (5m)^2 - (4n)^2$ $[\because a^2 - b^2 = (a-b)\,(a+b)]$

$$= (5m - 4n)\,(5m + 4n)$$

(ii) $x^4 - 9x^2 = x^2\,(x^2 - 9) = x^2\,(x^2 - 3^2) = x^2\,(x-3)\,(x+3)$

2. Factorise the following.

(i) $64m^3 + 27n^3$ (ii) $729x^3 - 343y^3$

Sol. (i) $64m^3 + 27n^3 = (4m)^3 + (3n)^3 = (4m + 3n)\,((4m)^2 - (4m)\,(3n) + (3n)^2)$

$[\because a^3 + b^3 = (a+b)\,(a^2 - ab + b^2)]$

$$= (4m + 3n)\,(16m^2 - 12mn + 9n^2)$$

(ii) $729x^3 - 343y^3 = (9x)^3 - (7y)^3 = (9x - 7y)\,((9x)^2 + (9x)\,(7y) + (7y)^2)$

$$= (9x - 7y)\,(81x^2 + 63xy + 49y^2)$$

$[\because a^3 - b^3 = (a-b)\,(a^2 + ab + b^2)]$

3. Factorise $81x^2 + 90x + 25$

Sol. $81x^2 + 90x + 25 = (9x)^2 + 2\,(9x)\,(5) + 5^2$

$= (9x + 5)^2$ $[\because a^2 + 2ab + b^2 = (a+b)^2]$

4. Find $a^3 + b^3$ if $a + b = 6,\ ab = 5$.

Sol. Given, $a + b = 6,\ ab = 5$

$$a^3 + b^3 = (a+b)^3 - 3ab(a+b) = (6)^3 - 3(5)(6) = 126$$

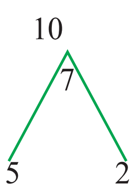
5. Factorise $(a-b)^2 + 7\,(a-b) + 10$

Sol. Let $a - b = p$, we get $p^2 + 7p + 10$,

$$p^2 + 7p + 10 = p^2 + 5p + 2p + 10$$
$$= p\,(p+5) + 2\,(p+5) = (p+5)\,(p+2)$$

Put $p = a - b$ we get,

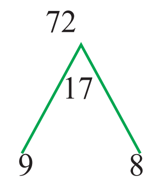
$$(a-b)^2 + 7\,(a-b) + 10 = (a - b + 5)\,(a - b + 2)$$



6. Factorise $6x^2 + 17x + 12$

Sol.
$$\begin{aligned} 6x^2 + 17x + 12 &= 6x^2 + 9x + 8x + 12 \\ &= 3x\,(2x+3) + 4\,(2x+3) \\ &= (2x+3)\,(3x+4) \end{aligned}$$

$$\begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

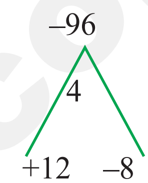


EXERCISE 3.6

1. Factorise $x^2 + 4x - 96$

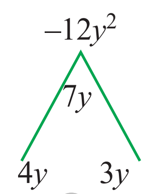
Sol.
$$\begin{aligned} x^2 + 4x - 96 &= x^2 + 12x - 8x - 96 \\ &= x\,(x+12) - 8\,(x+12) \\ &= (x+12)\,(x-8) \end{aligned}$$

$$\begin{array}{r|l} 2 & 96 \\ \hline 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$



2. Factorise $x^2 + 7xy - 12y^2$

Sol.
$$\begin{aligned} x^2 + 7xy + 12y^2 &= x^2 + 4yx + 3yx + 12y^2 \\ &= x\,(x+4y) + 3y\,(x+4y) \\ &= (x+4y)\,(x+3y) \end{aligned}$$



3. Find the quotient and remainder when $5x^3 - 9x^2 + 10x + 2$ is divided by $x + 2$ using synthetic division.

Sol.
$$\begin{aligned} p\,(x) &= 5x^3 - 9x^2 + 10x + 2 \\ d\,(x) &= x + 2 \\ \text{Standard form of } p\,(x) &= 5x^2 - 9x^2 + 10x + 2 \text{ and} \\ d\,(x) &= x + 2 \end{aligned}$$

To find the zero of $x + 2$:
Put $x + 2 = 0$
we get $x = -2$

$$\begin{array}{r|rrrr} -2 & 5 & -9 & 10 & 2 \\ & 0 & -10 & 38 & -96 \\ \hline & 5 & -19 & 48 & \boxed{-94} \end{array} \text{ (remainder)}$$

$$5x^3 - 9x^2 + 10x + 2 = (x+2)\,(5x^2 - 19x + 48) - 94$$

Hence the quotient is $5x^2 - 19x + 48$ and remainder is -94.

4. Find the quotient and remainder when $-7 + 3x - 2x^2 + 5x^3$ is divided by $-1 + 4x$ using synthetic division.

Sol.
$$\begin{aligned} p\,(x) &= -7 + 3x - 2x^2 + 5x^3 \\ d\,(x) &= -1 + 4x \\ \text{Standard form of } p\,(x) &= 5x^3 - 2x^2 + 3x - 7 \\ d\,(x) &= 4x - 1 \end{aligned}$$

To find the zero of $-1 + 4x$:
Put $-1 + 4x = 0$
we get $4x = 1$; $x = \dfrac{1}{4}$

$$\begin{array}{c|cccc}
\frac{1}{4} & 5 & -2 & 3 & -7 \\
 & 0 & \frac{5}{4} & \frac{-3}{16} & \frac{45}{64} \\
\hline
 & 5 & \frac{-3}{4} & \frac{45}{16} & \frac{-403}{64}
\end{array}$$

$$5x^3 - 2x^2 + 3x - 7 = \left(x - \frac{1}{4}\right)\left(5x^2 - \frac{3}{4}x + \frac{45}{16}\right) - \frac{403}{64}$$

$$= \left(\frac{4x-1}{4}\right)4\left(\frac{5}{4}x^2 - \frac{3}{16}x + \frac{45}{64}\right) - \frac{403}{64} = (4x-1)\left(\frac{5}{4}x^2 - \frac{3}{16}x + \frac{45}{64}\right) - \frac{403}{64}$$

Hence the quotient is $\frac{5}{4}x^2 - \frac{3}{16}x + \frac{45}{64}$ and remainder is $\frac{-403}{64}$

5. If the quotient on dividing $5x^4 + 4x^3 + 2x + 1$ by $x + 3$ is $5x^3 + 9x^2 + bx - 97$ then find the values of a, b and also remainder.

Sol. $p(x) = 5x^4 + 4x^3 + 2x + 1$

Standard form of $p(x) = 5x^4 + 4x^3 + 0x^2 + 2x + 1$

To find the zero of $x + 3$;
Put $x + 3 = 0$
we get $x = -3$.

$$\begin{array}{c|ccccc}
-3 & 5 & 4 & 0 & 2 & 1 \\
 & 0 & -15 & 33 & -99 & 291 \\
\hline
 & 5 & -11 & 33 & -97 & 292
\end{array}$$

Quotient $5x^3 + 11x^2 + 33x - 97$ is compared with given quotient $5x^3 + ax^2 + bx - 97$
co-efficient of x^2 is $-11 = a$ and co-efficient of x is $33 = b$.
Therefore $a = -11$, $b = 33$ and remainder $= 292$.

EXERCISE 3.7

1. Find the quotient and the remainder when $10 - 4x + 3x^2$ is divided by $x - 2$.

Sol. Let us first write the terms of each polynomial in descending order (or ascending order). Thus the given problem becomes $(3x^2 - 4x + 10) \div (x - 2)$

$$\begin{array}{r|l}
 & 3x + 2 \\
\hline
x - 2 & \cancel{3x^2} - 4x + 10 \\
 & \cancel{3x^2} - 6x \\
 & (-) \quad (+) \\
\hline
 & \cancel{2x} + 10 \\
 & \cancel{2x} - 4 \\
 & (-) \quad (+) \\
\hline
 & 14
\end{array}$$

Hint

(i) $\frac{3x^2}{x} = 3x$

(ii) $3x(x-2) = 3x^2 - 6x$

(iii) $\frac{2x}{x} = 2$

(iv) $2(x-2) = 2x - 4$

$\therefore$ Quotient $= 3x + 2$ and Remainder $= 14$, i.e $3x^2 - 4x + 10 = (x - 2)(3x + 2) + 14$ and is in the form

Dividend = (Divisor × Quotient) + Remainder

2. If $8x^3 - 14x^2 - 19x - 8$ is divided by $4x + 3$ then find the quotient and the remainder.

Sol.

$$\begin{array}{r|l}
 & \quad 2x^2 - 5x - 1 \\ \hline
4x+3 & \cancel{8x^3} - 14x^2 - 19x - 8 \\
 & \cancel{8x^3} + 6x^2 \\
 & (-) \quad (-) \\ \hline
 & \quad \cancel{-20x^2} - 19x \\
 & \quad \cancel{-20x^2} - 15x \\
 & \quad (+) \quad (+) \\ \hline
 & \qquad \cancel{-4x} - 8 \\
 & \qquad \cancel{-4x} - 3 \\
 & \qquad (+) \quad (+) \\ \hline
 & \qquad\quad -5
\end{array}$$

Hint

(i)	$\dfrac{8x^3}{4x}$	$= 2x^2$
(ii)	$2x^2(4x+3)$	$= 8x^3 + 6x^2$
(iii)	$\dfrac{-20x^2}{4x}$	$= -5x$
(iv)	$-5x(4x+3)$	$= -20x^2 - 15x$
(v)	$\dfrac{-4x}{4x}$	$= -1$
(vi)	$-1(4x+3)$	$= -4x - 3$

$\therefore$ Quotient $= 2x^2 - 5x - 1$, Remainder $= -5$

EXERCISE 3.8

1. Factorise $2x^3 - x^2 - 12x - 9$ into linear factors.

Sol.
Let $p(x) = 2x^3 - x^2 - 12x - 9$

Sum of the co-efficients $= 2 - 1 - 12 - 9 = -20 \neq 0$

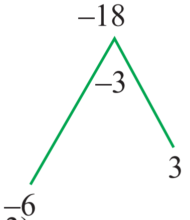
Hence $x - 1$ is not a factor

Sum of co-efficients of even powers with constant $= -1 - 9 = -10$

Sum of co-efficients of odd powers $= 2 - 12 = -10$

Hence $x + 1$ is a factor of x.

Now we use synthetic division to find the other factors.



$$\begin{array}{r|rrrr}
-1 & 2 & -1 & -12 & -9 \\
 & 0 & -2 & 3 & 9 \\ \hline
 & 2 & -3 & -9 & \boxed{0}
\end{array}$$

Then $p(x) = (x + 1)(2x^2 - 3x - 9)$

Now $2x^2 - 3x - 9 = 2x^2 - 6x + 3x - 9 = 2x(x - 3) + 3(x - 3)$

$= (x - 3)(2x + 3)$

Hence $2x^3 - x^2 - 12x - 9 = (x + 1)(x - 3)(2x + 3)$

2. Factorize $x^3 + 3x^2 - 13x - 15$.

Sol. Let $p(x) = x^3 + 3x^2 - 13x - 15$

Sum of all the co-efficients $= 1 + 3 - 13 - 15 = -24 \neq 0$

Hence $x - 1$ is not a factor

Sum of co-efficients of even powers with constant $= 3 - 15 = -12$

Sum of coefficients of odd powers $= 1 - 13 = -12$

Hence $x + 1$ is a factor of $p(x)$

Now use synthetic division to find the other factors.

–1	1	3	–13	–15
	0	–1	–2	15
	1	2	–15	0

$$\text{Hence } p(x) = (x+1)(x^2 + 2x - 15)$$
$$\text{Now } x^3 + 3x^2 - 13x - 15 = (x+1)(x^2 + 2x - 15)$$
$$\text{Now } x^2 + 2x - 15 = x^2 + 5x - 3x - 15 = x(x+5) - 3(x+5) = (x+5)(x-3)$$
$$\text{Hence } x^3 + 3x^2 - 13x - 15 = (x+1)(x+5)(x-3)$$

3. Factorize $x^3 + 13x^2 + 32x + 20$ into linerar factors.

Sol. Let, $p(x) = {}^3 + 13x^2 + 32x + 20$

Sum of all the coefficients $= 1 + 13 + 32 + 20 = 66 \neq 0$

Hence, $(x - 1)$ is not a factor.

Sum of coefficients of even powers with constant $= 13 + 20 = 33$

Sum of coefficients of odd powers $= 1 + 32 = 33$

Hence, $(x + 1)$ is a factor of $p(x)$

Now we use synthetic division to find the other factors

Method I

–1	1	13	32	20	
	0	–1	–12	–20	
–2	1	12	20	0	(remainder)
	0	–2	–20		
	1	10	0		(remainder)

$p(x) = (x+1)(x+2)(x+10)$

Hence,

$$x^3 + 13x^2 + 32x + 20 = (x+1)(x+2)(x+10)$$

Method II

–1	1	13	32	20	
	0	–1	–12	–20	
	1	12	20	0	(remainder)

Then $p(x) = (x+1)(x^3 + 12x^2 + 20)$

Now

$$x^2 + 12x^2 + 20 = x^2 + 10x + 2x + 20$$
$$= x(x+10) + 2(x+10)$$
$$= (x+2)(x+10)$$

Hence, $x^3 + 13x^2 + 32x + 20$
$$= (x+1)(x+2)(x+10)$$

EXERCISE 3.9

1. Find GCD of $25x^3y^2z$, $45x^2y^4z^3b$

Sol.
$$25x^3y^2z = 5 \times 5x^3y^2z = \underline{5} \times 5 \times \underline{x^2} \times x \times \underline{y^2} \times z$$
$$45x^3y^4z^3 = 5 \times 3 \times 3 \times x^2y^4z^3 = \underline{5} \times 9 \times \underline{x^2} \times \underline{y^2} \times y^2 \times \underline{z} \times z^2$$
$$\text{Therefore GCD} = 5x^2y^2z$$

2. Find the GCD of $(y^3 - 1)$ and $(y - 1)$.

Sol.

$$y^3 - 1 = (y-1)(y^2 + y + 1)$$
$$y - 1 = y - 1$$
$$\text{Therefore, GCD} = y - 1$$

3. Find the GCD of $3x^2 - 48$ and $x^2 - 7x + 12$.

Sol.

$$3x^2 - 48 = 3(x^2 - 16) = 3(x^2 - 4^2) = 3(x+4)(x-4)$$
$$x^2 - 7x + 12 = x^2 - 3x - 4x + 12 = x(x-3) - 4(x-3) = (x-3)(x-4)$$
$$\text{Therefore, GCD} = x - 4$$

4. Find the GCD of a^x, a^{x+y}, a^{x+y+z}.

Sol.

$$a^x = \underline{a^x}$$
$$a^{x+y} = \underline{a^x}.\, a^y$$
$$a^{x+y+z} = \underline{a^x}.\, a^y.a^z \qquad \therefore \text{GCD} = a^x$$

EXERCISE 3.10

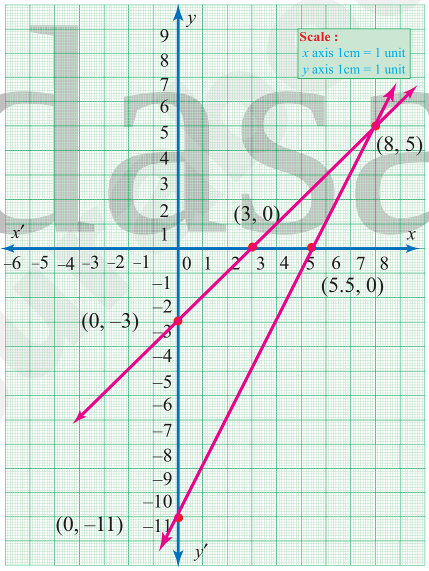
5. Solve graphically. $x - y = 3$; $2x - y = 11$

Sol. $x - y = 3$

x	0	3
y	–3	0

$2x - y = 11$

x	0	5.5
y	–11	0



EXERCISE 3.11

1. Solve using the method of substitution. $5x - y = 5$, $3x + y = 11$

Sol.

$$5x - y = 5 \qquad \ldots(1)$$
$$3x + y = 11 \qquad \ldots(2)$$

(1) ⇒ $y = 5x - 5$

(2) ⇒ $3x + 5x - 5 = 11$

$$8x = 11 + 5$$
$$8x = 16$$
$$x = \frac{16}{8} = 2$$

(1) ⇒ $5(2) - y = 5 \Rightarrow 10 - 5 = y \Rightarrow y = 5$ $\qquad \therefore$ Solution is $x = 2, y = 5$

EXERCISE 3.12

2. Solve by the method of elimination. $2x + y = 10$; $5x - y = 11$

Sol.

$$\begin{aligned} 2x + y &= 10 && \dots (1) \\ 5x - y &= 11 && \dots (2) \end{aligned}$$

$$(1) + (2) \Rightarrow \begin{array}{r} 2x + y = 10 \\ 5x - y = 11 \\ \hline 7x = 21 \end{array}$$

$$x = \frac{21}{7} = 3$$

$$\begin{aligned} (1) \Rightarrow \quad 2(3) + y &= 10 \\ 6 + y &= 10 \\ y &= 10 - 6 = 4 \\ \therefore \text{Solution is } x &= 3, y = 4 \end{aligned}$$

EXERCISE 3.13

3. Solve $5x - 2y = 10$; $3x + y = 17$ by the method of cross multiplication.

Sol.

$$5x - 2y - 10 = 0$$

$$3x + y - 17 = 0$$

$$\begin{array}{cccccc} & x & & y & & 1 \\ -2 & & -10 & & 5 & & -2 \\ 1 & & -17 & & 3 & & 1 \end{array}$$

$$\frac{x}{34 + 10} = \frac{y}{-30 + 85} = \frac{1}{5 + 6}$$

$$\frac{x}{44} = \frac{y}{55} = \frac{1}{11}$$

$$\frac{x}{44} = \frac{1}{11} \Rightarrow x = \frac{44}{11} = 4$$

$$\frac{y}{55} = \frac{1}{11} \Rightarrow y = \frac{55}{11} = 5 \qquad \therefore \text{Solution is } x = 4, y = 5$$

EXERCISE 3.14

1. The age of Arjun is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

Sol. Let the age of the father be x

Let the sum of the age of his sons be y

At present $x = 2y \Rightarrow x - 2y = 0$ ... (1)

After 20 years $x + 20 = y + 40$

$x - y = 40 - 20$

$$x - y = 20 \quad \ldots (2)$$

$$\begin{aligned}
(1) \Rightarrow \quad & \cancel{x} - 2y = 0 \\
& (-) \quad (+) \quad (-) \\
(2) \Rightarrow \quad & \cancel{x} - y = 20 \\
\hline
& -y = -20 \\
& y = 20
\end{aligned}$$

Substitute $y = 20$ in (1)

$$x - 2(20) = 0$$

$$x - 40 = 0$$

$$x = 40$$

EXERCISE 3.15

Multiple Choice Questions :

1. The polynomial $3x - 2$ is a

(1) linear polynomial (2) quadratic polynomial
(3) cubic polynomial (4) constant polynomial

Hint : A polynomial is linear if its degree is 1 **[Ans: (1) linear polynomial]**

2. The polynomial $4x^2 + 2x - 2$ is a

(1) linear polynomial (2) quadratic polynomial
(3) cubic polynomial (4) constant polynomial

Hint : A polynomial is quadratic of its highest power of x is 2

[Ans: (2) quadratic polynomial]

3. The zero of the polynomial $2x - 5$ is

(1) $\frac{5}{2}$ (2) $-\frac{5}{2}$ (3) $\frac{2}{5}$ (4) $-\frac{2}{5}$

Hint : Zero is given by $2x - 5 = 0 \Rightarrow x = \frac{5}{2}$ **[Ans: (1) $\frac{5}{2}$]**

4. The root of the polynomial equation $3x - 1 = 0$ is

(1) $x = -\frac{1}{3}$ (2) $x = \frac{1}{3}$ (3) $x = 1$ (4) $x = 3$

Hint : $x = \frac{1}{3}$ gives $3\left(\frac{1}{3}\right) - 1 = 1 - 1 = 0$ **[Ans: (2) $x = \frac{1}{3}$]**

5. Zero of $(7 + 4x)$ is__________

(1) $\frac{4}{7}$ (2) $\frac{-7}{4}$ (3) 7 (4) 4

$\left[\textbf{Ans. (2)}\ \frac{-7}{4}\right]$

6. Which of the following has as a factor?

(1) $x^2 + 2x$ (2) $(x - 1)^2$ (3) $(x + 1)^2$ (4) $(x^2 - 2^2)$

[Ans. (1) $x^2 + 2x$]

7. If $x - 2$ is a factor of $q(x)$, then the remainder is ________

(1) $q(-2)$ (2) $x - 2$ (3) 0 (4) -2 **[Ans. (3) 0]**

8. $(a-b)(a^2+ab+b^2) =$ __________

(1) $a^3+b^3+c^3-3abc$ (2) a^2-b^2

(3) a^3+b^3 (4) a^3-b^3 **[Ans. (4) a^3-b^3]**

9. **The polynomial whose factors are $(x+2)(x+3)$ is** __________

(1) x^2+5x+6 (2) x^2-4 (3) x^2-9 (4) x^2+6x+5

[Ans. (1) x^2+5x+6]

10. $(-a-b-c)^2$ **is equal to** __________

(1) $(a-b+c)^2$ (2) $(a+b-c)^2$ (3) $(-a+b+c)^2$ (4) $(a+b+c)^2$

[Ans. (4) $(a+b+c)^2$]

Text Book Activities

Activity - 1

Write the Variable, Coefficient and Constant in the given algebraic expression,

Sol.

Expression	$x+7$	$3y-2$	$5x^2$	$2xy+11$	$-\frac{1}{2}p+7$	$-8+3a$
Variable	x	y	x	x, y	p	a
Coefficient	1	3	5	2	$-\frac{1}{2}$	3
Constant	7	-2	0	11	7	-8

Activity - 2

Write the following polynomials in standard form.

Sol.

S. No.	Polynomial	Standard Form
1	$5m^4-3m+7m^2+8$	$5m^4+7m^2-3m+8$
2	$\frac{2}{3}y+8y^3-12+\sqrt{5}y^2$	$8y^3+\sqrt{5}y^2+\frac{2}{3}y-12$
3	$12p^2-8p^5-10p^4-7$	$-8p^5-10p^4+12p^2-7$

Activity - 3

1. **Find the value of k for the given system of linear equations satisfying the condition below:**

(i) $2x+ky=1;\ 3x-5y=7$ has a unique solution

(ii) $kx+3y=3;\ 12x+ky=6$ has no solution

(iii) $(k-3)x+3y=k;\ kx+ky=12$ has infinite number of solution

Sol. (i) $\frac{3}{2} \neq \frac{-5}{k} \quad \Rightarrow k \neq \frac{-10}{3}$

(ii)
$$\begin{aligned} \frac{k}{12} = \frac{3}{k} &\neq \frac{3}{6} \\ k^2 &= 36 \\ k &= \pm 6 \\ \frac{3}{k} &\neq \frac{3}{6} \Rightarrow k \neq 6 \\ k &= -6 \end{aligned}$$

(iii)
$$\frac{3}{5} = \frac{-(a+1)}{1-2a} = \frac{2b-1}{3b}$$

$$\begin{aligned} \frac{3}{5} &= \frac{-(a+1)}{1-2a} & \qquad \frac{2b-1}{3b} &= \frac{3}{5} \\ 3-6a &= -5a-5 & 10b-5 &= 9b \\ a &= 8 & b &= 5 \end{aligned}$$

2. **Find the value of a and b for which the given system of linear equation has infinite number of solutions $3x - (a+1)y = 2b - 1$, $5x + (1-2a)y = 3b$**

Sol.
$$\begin{aligned} \frac{3}{5} &= \frac{-(a+1)}{1-2a} = \frac{2b-1}{3b} \\ 3-6a &= -5a-5 \\ ab &= 10b-5 \\ a &= 8 \\ b &= 5 \end{aligned}$$

Activity - 4

For the given linear equations, find another linear equation satisfying each of the given condition

Sol.

Given linear equation	Another linear equation		
	Unique solution	Infinitely many solutions	No solution
$2x + 3y = 7$	$3x + 4y = 8$	$4x + 6y = 14$	$6x + 9y = 15$
$3x - 4y = 5$	$4x + 3y = 7$	$6x - 8y = 10$	$9x - 12y = 10$
$y - 4x = 2$	$3y + 5x = 4$	$2y - 8x = 4$	$5y - 20x = 4$
$5y - 2x = 8$	$7y + 3x = 16$	$10y - 4x = 16$	$15y - 6x = 32$

UNIT TEST

Time : 45 Minutes | CHAPTER - 3 | Marks: 25

Section - A

(i) **Answer all the questions.**
(ii) **Choose the correct Answer.** **5 × 1 = 5**

1. **Let the polynomials be**

(1) $-13q^5 + 4q^2 + 12q$ (2) $(x^2 + 4)(x^2 + 9)$

(3) $4q^8 - q^6 + q^2$ (4) $-\frac{5}{7}y^{12} + y^3 + y^5$

Then ascending order of their degree is

(1) A,B,D,C (2) A,B,C,D (3) B,C,D,A (4) B,A,C,D

2. **If $x^{51} + 51$ is divided by $x + 1$, then the remainder is**
(1) 0 (2) 1 (3) 49 (4) 50

3. **The sum of the polynomials $p(x) = x^3 - x^2 - 2$, $q(x) = x^2 - 3x + 1$**

(1) $x^3 - 3x - 1$ (2) $x^3 + 2x^2 - 1$ (3) $x^3 - 2x^2 - 3x$ (4) $x^3 - 2x^2 + 3x - 1$

4. **Degree of the constant polynomial is __________**

(1) 3 (2) 2 (3) 1 (4) 0

5. **The value of k for which the pair of linear equations $4x + 6y - 1 = 0$ and $2x + ky - 7 = 0$ represents parallel lines is**

(1) $k = 3$ (2) $k = 2$ (3) $k = 4$ (4) $k = -3$

Section - B

(i) **Answer only five of the following**
(ii) **However Q.No. 12 is compulsory.**

5 × 2 = 10

6. The cost of a chocolate is Rs. $(x + y)$ and Amir bought $(x + y)$ chocolates. Find the total amount paid by him in terms of x and y. If $x = 10$, $y = 5$ find the amount paid by him.
7. Find the value of the polynomial $f(y) = 6y - 3y^2 + 3$ at (i) $y = 1$ (ii) $y = -1$ (iii) $y = 0$
8. Find the quotient and remainder of the $(8x^3 - 1) \div (2x - 1)$
9. What is the remainder when $x^{2018} + 2018$ is divided by $x - 1$
10. Check whether $p(x)$ is a multiple of $g(x)$ or not $p(x) = x^3 - 5x^2 + 4x - 3$, $g(x) = x - 2$
11. Expand the following : $(-p + 2q + 3r)^2$
12. Find the GCD for the following: p^5, p^{11}, p^9

Section - C

(i) Answer only two Questions of the following.

(ii) However Q.No. 16 is compulsory. **$2 \times 5 = 10$**

13. The sum of $(x+5)$ observations is (x^3+125). Find the mean of the observations.

14. Verify whether the following are zeros of the polynomial indicated against them, or not.

 (i) $p(x) = 2x - 1, x = \frac{1}{2}$, (ii) $p(x) = x^3 - 1, x = 1$, (iii) $p(x) = ax + b, x = \frac{-b}{a}$

 (iv) $p(x) = (x + 3)(x - 4), x = 4, x = -3$

15. Add the following polynomials and find the degree of the resultant polynomial.

 (i) $p(x) = 6x^2 - 7x + 2$

 $q(x) = 6x^3 - 7x + 15$

 (ii) $h(x) = 7x^3 - 6x + 1$

 $f(x) = 7x^2 + 17x - 9$

 (iii) $f(x) = 16x^4 - 5x^2 + 9$

 $g(x) = -6x^3 + 7x - 15$

16. Determine whether $(\boldsymbol{x} - 1)$ is a factor of the following polynomials:

 (i) $x^3 + 5x^2 - 10x + 4$

 (ii) $x^4 + 5x^2 - 5x + 1$

ANSWERS

SECTION - A

1. (3) B, C, D, A
2. (4) 50
3. (1) $x^3 - 3x - 1$
4. (4) 0
5. (1) $k = 3$

SECTION - B

6. ₹ 225
7. (i) 6 (ii) – 6 (iii) 3
8. $4x^2 + 2x + 1; 0$
9. 2019
10. Not a multiple

11. $p^2 + 4q^2 + 9r^2 - 4pq + 12\,qr - 6rp$

12. 1

SECTION - C

13. $x^2 - 5x + 25$

14. (i) $\therefore x = \frac{1}{2}$ is the zero of the given polynomial.

(ii) $\therefore x = 1$ is the zero of the given polynomial.

(iii) $\therefore\ x = \frac{-b}{a}$ is the zero of the given polynomial.

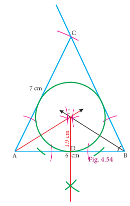
(iv) $\therefore x = -3, x = 4$ are the zeros of the given polynomial.

15. (i) $6x^3 + 6x^2 - 14x + 17; 3$

(ii) $7x^3 + 7x^2 + 11x - 8; 3$

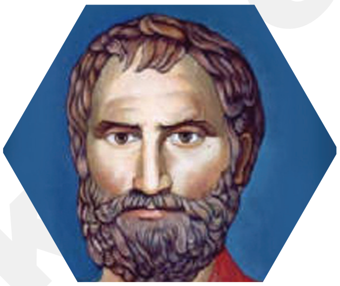
(iii) $16x^4 - 6x^3 - 5x^2 + 7x - 6; 4$

16. Factor; Not factor


4 GEOMETRY

4.1 Introduction



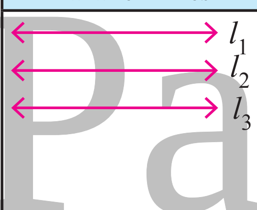
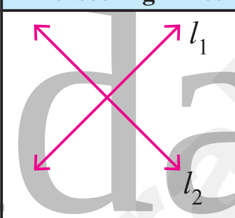
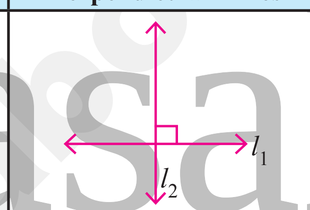
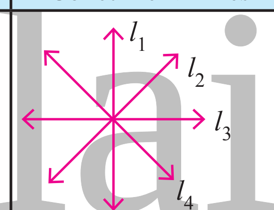
We describe different shapes by their properties.

Parallel lines : Two or more lines lying in the same plane that never meet.

Intersecting lines : Two lines which meet at a common point.

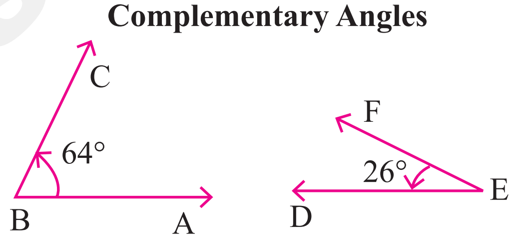
Perpendicular lines : Two lines which intersect each other at right angle.

Concurrent lines : Three or more lines passing through the same point.

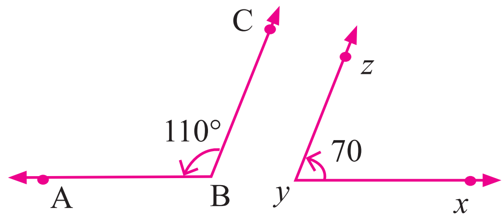
Parallel Lines	Intersecting Lines	Perpendicular Lines	Concurrent Lines
			
$l_1 \parallel l_2 \parallel l_3$		$l_1 \perp l_2$	

Types of angles :

Complementary Angles



Supplementary Angles



UNIT TEST

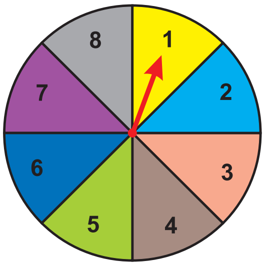
Time : 1 hr. **CHAPTER - 9** **Marks: 40**

Part - A

I. Answer the following questions **10 × 2 = 20**

1. You are walking along a street. If you just choose a stranger crossing you, what is the probability that his next birthday will fall on a Sunday?
2. What is the probability of drawing a King or a Queen or a Jack from a deck of cards?
3. What is the probability of throwing an even number with a single standard dice of six faces?
4. There are 24 balls in a pot. If 3 of them are Red, 5 of them are Blue and the remaining are Green then, what is the probability of picking out (i) a Blue ball, (ii) a Red ball and (iii) a Green ball?
5. When two coins are tossed, what is the probability that two heads are obtained?
6. Two dice are rolled, find the probability that the sum is
   i) equal to 1 (ii) equal to 4 (iii) less than 13
7. A manufacturer tested 7000 LED lights at random and found that 25 of them were defective. If a LED light is selected at random, what is the probability that the selected LED light is a defective one.
8. What is the probability that the spinner will not land on a multiple of 3?

9. Frame two problems in calculating probability, based on the spinner shown here.



10. A company manufactures 10000 Laptops in 6 months. In that 25 of them are found to be defective. When you choose one Laptop from the manufactured, what is the probability that selected Laptop is a good one.

Part - B

II. Answer the following questions **4 × 5 = 20**

11. In a survey of 400 youngsters aged 16-20 years, it was found that 191 have their voter ID card. If a youngster is selected at random, find the probability that the youngster does not have their voter ID card.

12. The probability of guessing the correct answer to a certain question is $\frac{x}{3}$. If the probability of not guessing the correct answer is $\frac{x}{5}$, then find the value of x

13. If a probability of a player winning a particular tennis match is 0.72. What is the probability of the player loosing the match?

14. 1500 families were surveyed and following data was recorded about their maids at home

Type of maids	Only part time	Only full time	Both
Number of families	860	370	250

A family is selected at random. Find the probability that the family selected has

(i) Both types of maids (ii) Part time maids (iii) No maids

ANSWERS

Part - A

1. $\frac{1}{7}$
2. $\frac{3}{13}$
3. $\frac{1}{2}$
4. (i) $\frac{5}{24}$ (ii) $\frac{1}{8}$ (iii) $\frac{2}{3}$
5. $\frac{1}{4}$
6. (i) 0 (ii) $\frac{1}{12}$ (iii) 1
7. $\frac{1}{280}$
8. $\frac{3}{4}$
9. (i) What is the probability that the spinner will land on an even number?
(ii) What is the probability that the spinner will not land on a prime number.
10. 0.9975

Part - B

11. $\frac{209}{400}$
12. $\frac{15}{8}$
13. 0.28
14. (i) $\frac{1}{6}$ (ii) $\frac{43}{75}$ (iii) $\frac{1}{75}$

Though the Trimester Pattern is changed from this year, we have given this Question Paper for Practice Purpose only

9TH STD

I TERM SUMMATIVE ASSESSMENT 2018-19

MATHEMATICS

Reg. No.

Time : 2.00 hr Marks : 60

SECTION - I

I. Choose the best answer **10 × 1 = 10**

1. If B–A is B, then is A ∩ B is
(1) A (2) B (3) U (4) φ

2. If n(A) = 10 and n(B) = 15 then the minimum and maximum number of elements in A ∩ B is
(1) (10,15) (2) (15, 10) (3) (10,0) (4) (0,10)

3. Which one of the following has a terminating decimal expansion?
(1) $\frac{5}{64}$ (2) $\frac{8}{9}$ (3) $\frac{14}{15}$ (4) $\frac{1}{12}$

4. $0.\overline{34}+0.3\overline{4} =$

(1) $0.6\overline{87}$ (2) $0.\overline{68}$ (3) $0.6\overline{8}$ (4) $0.68\overline{7}$

5. The root of the polynomial equation $2x + 3 = 0$ is
(1) $\frac{1}{3}$ (2) $\frac{-1}{3}$ (3) $\frac{-3}{2}$ (4) $\frac{-2}{3}$

6. Degree of the polynomial $(y^3 - 2)(y^3 + 1)$ is
(1) 9 (2) 2 (3) 3 (4) 6

7. The exterior angle of a triangle is equal to the sum of two
(1) Exterior angles (2) Interior opposite angles
(3) Alternate angles (4) Interior angles

8. The interior angles made by the side in a parallelogram is 90° then the parallelogram is a
(1) rhombus (2) rectangle (3) trapezium (4) kite

9. Point (–3, 5) lie in the _______ quadrant
(1) I (2) II (3) III (4) IV

10. If $(x + 2, 4) = (5, y - 2)$ then the co-ordinates (x, y) are _______
(1) (7,12) (2) (6,3) (3) (3,6) (4) (2,1)

19. $\frac{1}{2}$
20. $\frac{1}{4}$
21. $\frac{209}{400}$
22. No. of 2 rupee coins = 60; No. of 5 rupee coins = 20
23. (0, –1)
24. $\frac{5}{3}$
25. 7.2 sq.m

Part - C

26. $x = 2$
27. $x = 4; y = 2$
28. (1, –1)
29. (1, 1)
30. $\sin\theta = \frac{4}{5}$; $\cos\theta = \frac{3}{5}$; $\tan\theta = \frac{4}{3}$

    $\operatorname{cosec}\theta = \frac{5}{4}$; $\sec\theta = \frac{5}{3}$; $\cot\theta = \frac{3}{4}$
31. (i) 85° 57′ (ii) 47° 33′

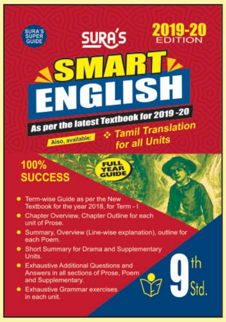
    (iii) 4° 7′ (iv) 87° 45′

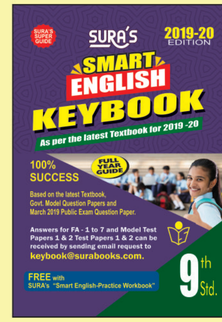
    (v) 82° 30′
32. ₹ 26400
33. 138 cm^2
34. 354 m^2
35. (i) $\frac{5}{24}$ (ii) $\frac{1}{8}$ (iii) $\frac{2}{3}$

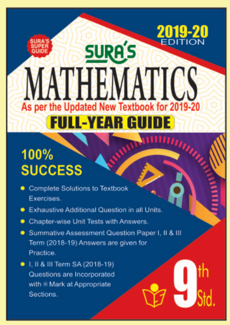
Part - D

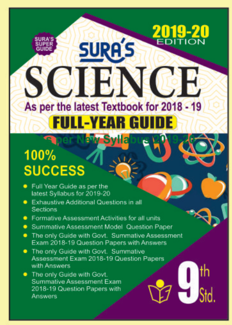
36. (–3, 3)
37. $\left(\frac{19}{2}, \frac{13}{2}\right)$

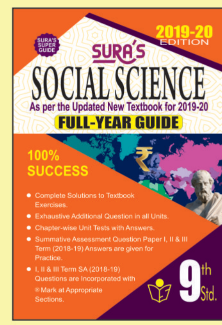
Our other Guides for 9th Std.

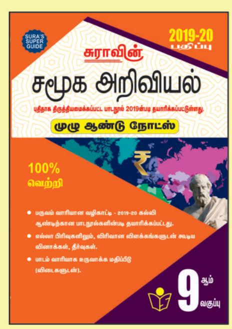


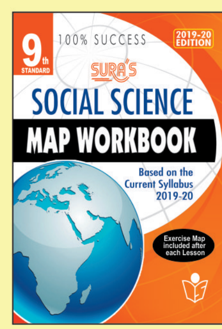












Sura Publications
1620, 'J' Block,
16th Main Road, Anna Nagar,
Chennai - 600 040. INDIA
Phones: 044-26162173, 26161099
Mobile: 81242 01000 / 81243 01000
e-mail: enquiry@surabooks.com
website: www.surabooks.com

Category:
School Guides

SG 56

ISBN 978-81-8449-559-1

9 788184 495591

Price: ₹299.00